

Part C: Instrumental Variables

C2: IV with Heterogeneous Effects

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Questions when effects are heterogeneous

- In Angrist (1990), does the draft lottery IV identify a causal parameter?
- Is it the ATE or the affect for some subpopulation?
- Is this subpopulation interesting and policy relevant?
- How can we describe it theoretically?
- What can we say about it in the data?

C2 Outline

1 Basic LATE result

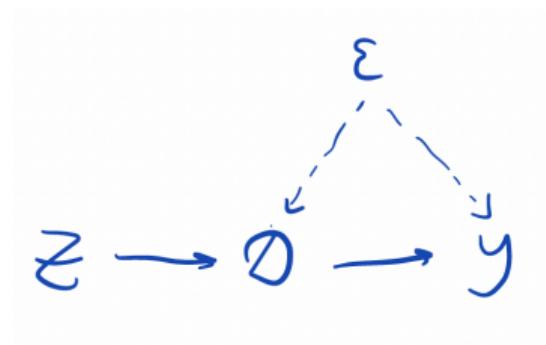
2 Characterizing compliers

3 Extensions

Readings: MHE Chapters 4.4, 4.5; IW Lecture 5

Recap of general IV assumptions

Consider $J = K = 1$ and a causal first-stage with potential outcomes $D(z)$:



- **Exclusion:** in writing $Y_i(d)$ rather than $Y_i(d, z)$
- **Independence:** $Z_i \perp\!\!\!\perp Y_i(d)$ for all d and $Z_i \perp\!\!\!\perp D_i(z)$ for all z
- **Relevance:** $D_i(z) \neq D_i(z')$ for some z, z' with positive probability

The four groups

- Consider binary D and Z , e.g. Vietnam draft lottery
- $D_i(0)$ and $D_i(1)$ are binary \implies (up to) four latent groups:
 - ▶ $D_i(0) = D_i(1) = 0$: **never-takers**
 - ▶ $D_i(0) = 0 < D_i(1) = 1$: **compliers**
 - ▶ $D_i(0) = 1 > D_i(1) = 0$: **defiers**
 - ▶ $D_i(0) = D_i(1) = 1$: **always-takers**
- First-stage: $\pi = \mathbb{E}[D_i(1) - D_i(0)] = Pr(\text{Complier}_i) - Pr(\text{Defier}_i)$

Understanding the groups: True/False

- “A defier is someone who gets the encouragement ($Z = 1$) but doesn’t take the treatment ($D = 0$)”
- “Someone who took the treatment but had the lowest possible outcome is a defier”

Reduced-form

- Reduced-form (a.k.a. **intent-to-treat** effect, **ITT**):

$$\begin{aligned}\rho &= \mathbb{E}[Y_i | Z_i = 1] - \mathbb{E}[Y_i | Z_i = 0] \\ &= Pr(NT_i) \cdot \mathbb{E}[Y_i(0) - Y_i(0) | NT_i] \\ &\quad + Pr(Complier_i) \cdot \mathbb{E}[Y_i(1) - Y_i(0) | Complier_i] \\ &\quad + Pr(Defier_i) \cdot \mathbb{E}[Y_i(0) - Y_i(1) | Defier_i] \\ &\quad + Pr(AT_i) \cdot \mathbb{E}[Y_i(1) - Y_i(1) | AT_i] \\ &= Pr(Complier_i) \mathbb{E}[\tau_i | Complier_i] - Pr(Defier_i) \mathbb{E}[\tau_i | Defier_i]\end{aligned}$$

Ratio

$$\tau_{IV} = \frac{Pr(Complier_i) \mathbb{E}[\tau_i | Complier_i] - Pr(Defier_i) \mathbb{E}[\tau_i | Defier_i]}{Pr(Complier_i) - Pr(Defier_i)}$$

doesn't have an intuitive interpretation, except constant effects or...

Monotonicity assumption

- Assume **monotonicity**: the instrument affects the treatment weakly in the same direction for everyone
 - ▶ Either no defiers (when $\pi > 0$) or no compliers (when $\pi < 0$)
 - ▶ Without loss, consider positive first-stage
- **LATE theorem** (Imbens and Angrist (1994)): Under exclusion, independence, and monotonicity, IV identifies the (internally valid) average causal effect among compliers:

$$\tau_{IV} = \mathbb{E} [\tau_i | \text{Complier}_i] \equiv \text{Local average treatment effect}$$

Is monotonicity plausible?

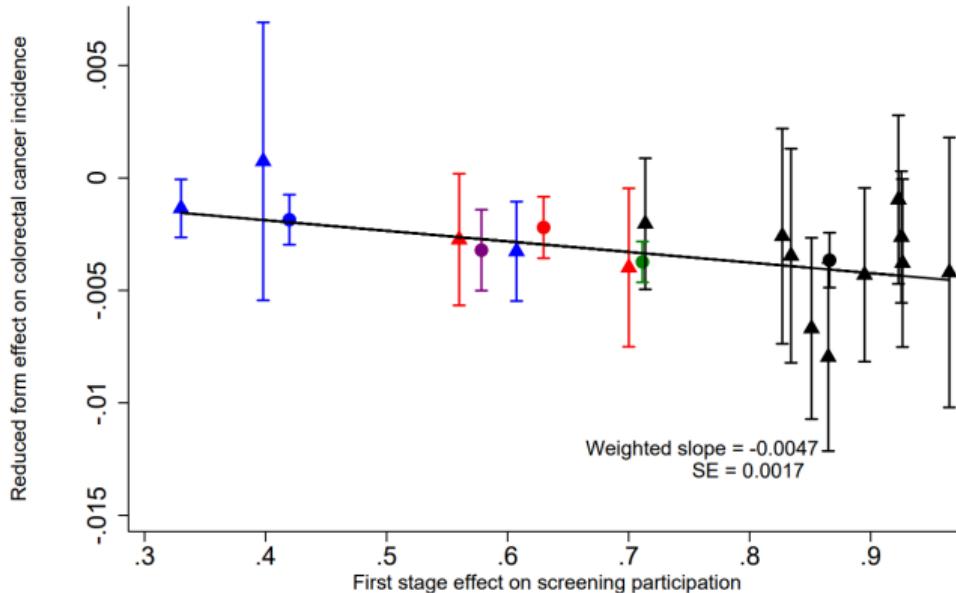
- In the new pill example?
 - ▶ Does the treated group = complier population?
 - ▶ How does LATE relate to the ATT?
- In Angrist (1990) Vietnam draft study?
- In Angrist and Evans (1998) with same-sex IV?

Is LATE more useful than ITT?

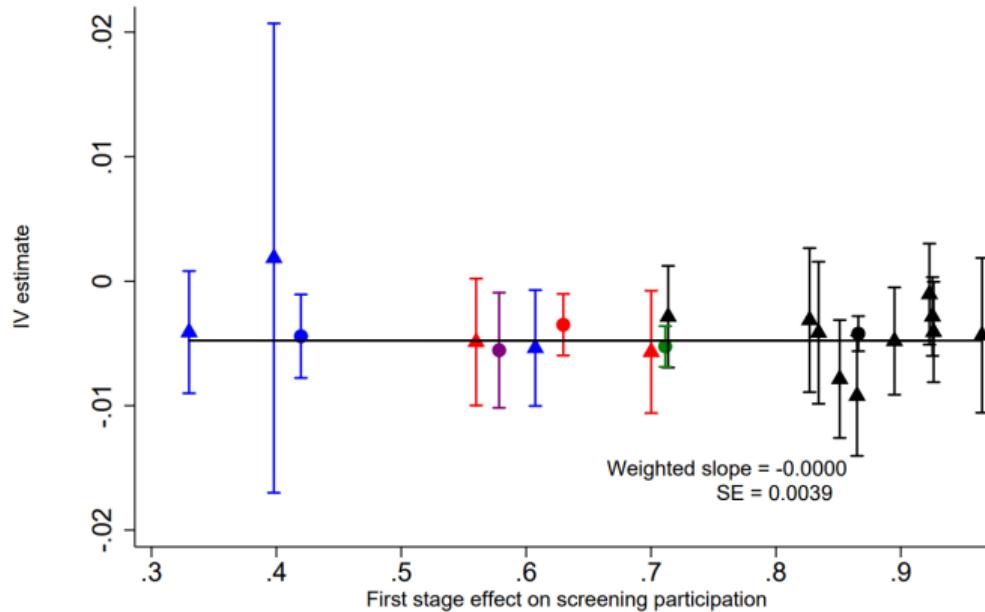
- Reduced-form (ITT) is always causal if independence holds
 - ▶ Always holds in RCTs, e.g. in encouragement designs
 - ▶ While LATE further requires exclusion and monotonicity
 - ▶ So why ever do IV?
- IV answers more interesting questions than ITT
 - ▶ Wage effects of serving in the army vs. of low lottery numbers
 - ▶ More interesting in part because hopefully more externally valid with respect to alternative implementations...

Example: Angrist and Hull (2023)

- Does screening for prostate & colorectal cancer improve health outcomes?
 - ▶ A dozen of RCTs where screening is offered for free find very different ITTs



IV estimates of the effect of screening



Open question: but is this IV estimate policy relevant?

External validity: Is LATE useful?

- Criticism: “*The LATE may, or may not, be a parameter of interest... and in general, there is no reason to suppose that it will be*” (Deaton (2010))
 - ▶ E.g. better to focus on ATE: report partially identified set (“Manski bounds”) or use a structural model for point identification
- Imbens (2010) “Better LATE than nothing”:
 - ▶ You should first report what the data directly gives, i.e. LATE. Then do whatever
 - ▶ E.g. “*I would prefer to keep [structural] assumptions separate and report both the LATE, with its high degree of internal but possibly limited external validity, and possibly add a set of estimates for the overall average effect with the corresponding additional assumptions*”

External validity: Is LATE useful? (2)

- Angrist and Pischke (2010):
 - ▶ “A good structural model might tell us something about economic mechanisms as well as causal effects. But if the information about mechanisms is to be worth anything, the structural estimates should line up with those derived under weaker assumptions”
- LATE sometimes has economic and policy relevance (e.g., Imbens (2010)), Kline and Walters (2016))
- And we can learn a lot about the complier subpopulation...

Outline

1 Basic LATE result

2 Characterizing compliers

3 Extensions

Counting the three groups

- Assume monotonicity (no defiers). Which groups occupy each cell?

	$D_i = 0$ (didn't serve)	$D_i = 1$ (served)
$Z_i = 0$ (high lottery #)	NT + C	AT
$Z_i = 1$ (low lottery #)	NT	AT + C

- Counting the groups:
 - $n \equiv Pr(NT_i) = Pr(D_i = 0 | Z_i = 1)$
 - $a = Pr(AT_i) = Pr(D_i = 1 | Z_i = 0)$
 - $\pi = Pr(C_i) = Pr(D_i = 1 | Z_i = 1) - Pr(D_i = 1 | Z_i = 0) = \text{first stage}$
- E.g. in Angrist (1990), for white men born in 1950, the first-stage = 0.16
 - By Bayes rule, compliers are 32% of veterans and 10% of non-veterans

Characterizing compliers

- Do compliers have unusual...
 - ▶ potential outcomes: $Y_i(1)$ (relative to AT) and $Y_i(0)$ (relative to NT)?
 - ▶ predetermined characteristics X_i (relative to NT and AT)?
- Start with $\mathbb{E}[Y_i(1) | AT_i]$ and $\mathbb{E}[Y_i(1) | C_i]$:

	$D_i = 0$ (didn't serve)	$D_i = 1$ (served)
$Z_i = 0$ (high lottery #)	NT + C	AT
$Z_i = 1$ (low lottery #)	NT	AT + C

Thus, $\mathbb{E}[Y_i | Z_i = 0, D_i = 1] = \mathbb{E}[Y_i(1) | AT_i]$ and...

Characterizing compliers (2)

$$\mathbb{E}[Y_i | Z_i = 1, D_i = 1] = \mathbb{E}[Y_i(1) | AT_i] \frac{a}{a+\pi} + \mathbb{E}[Y_i(1) | C_i] \frac{\pi}{a+\pi} \implies$$

$$\mathbb{E}[Y_i(1) | C_i] = \frac{\mathbb{E}[Y_i | Z_i = 1, D_i = 1] \cdot (a + \pi) - \mathbb{E}[Y_i | Z_i = 0, D_i = 1] \cdot a}{\pi}$$

Given $a + \pi = Pr(D_i = 1 | Z_i = 1)$ and $a = Pr(D_i = 1 | Z_i = 0)$,

$$= \frac{\mathbb{E}[Y_i D_i | Z_i = 1] - \mathbb{E}[Y_i D_i | Z_i = 0]}{\mathbb{E}[D_i | Z_i = 1] - \mathbb{E}[D_i | Z_i = 0]}$$

Implement by IV: $Y_i D_i$ on D_i , IVed with Z_i

- *Intuition:* think potential outcomes of $Y_i D_i$ and the causal effect on this outcome

Characterizing compliers (3)

- Similarly, for $\mathbb{E}[Y_i(0) | C_i]$: run $Y_i \cdot (1 - D_i)$ on $1 - D_i$, instrumented with Z_i
- Can get entire distributions $Y_i(0)$ and $Y_i(1)$ for compliers (Abadie (2002))
 - ▶ See Kitagawa (2015) for a test of IV validity based on this
- For predetermined characteristics X_i (that are $\perp\!\!\!\perp Z$), have multiple options:
 - ▶ $X_i D_i$ on D_i , instrumented with Z_i
 - ▶ $X_i(1 - D_i)$ on $1 - D_i$, instrumented with Z_i
 - ▶ A combination of the two (see Angrist, Hull, and Walters (2023))
 - ▶ Note: this trick works with all design-based OLS and IV specifications, even with non-binary D and Z (Hull, 2025)

Characterizing compliers: Example

- Angrist, Pathak, and Walters (2013) and Angrist, Hull, and Walters (2023):
 D_i = studying in a charter school, Z_i = getting an offer in a lottery, Y_i = test scores

Table 3: Characteristics of Lottery Compliers at Massachusetts Urban Charter Schools

	Compliers				
	Untreated (1)	Treated (2)	Pooled (3)	Always-takers (4)	Never-takers (5)
Female	0.506 (0.023)	0.510 (0.021)	0.508 (0.016)	0.539 (0.024)	0.463 (0.017)
Black	0.401 (0.022)	0.380 (0.021)	0.390 (0.016)	0.623 (0.023)	0.490 (0.017)
Hispanic	0.250 (0.02)	0.300 (0.018)	0.275 (0.013)	0.183 (0.019)	0.228 (0.014)
Asian	0.022 (0.007)	0.024 (0.005)	0.023 (0.004)	0.004 (0.003)	0.024 (0.005)
White	0.229 (0.018)	0.216 (0.016)	0.223 (0.012)	0.154 (0.016)	0.215 (0.014)
Special education	0.190 (0.018)	0.181 (0.016)	0.186 (0.012)	0.158 (0.018)	0.177 (0.013)
English language learner	0.143 (0.015)	0.148 (0.013)	0.145 (0.010)	0.054 (0.011)	0.088 (0.010)
Subsidized lunch	0.689 (0.021)	0.705 (0.019)	0.697 (0.014)	0.698 (0.022)	0.666 (0.016)
Baseline math score	-0.274 (0.047)	-0.312 (0.041)	-0.293 (0.032)	-0.394 (0.045)	-0.301 (0.036)
Baseline English score	-0.352 (0.050)	-0.349 (0.043)	-0.350 (0.033)	-0.362 (0.046)	-0.299 (0.038)
Share of sample		0.546	0.197	0.257	

Extrapolation via reweighting

- If compliers have different X_i than overall population or the treated group, we can reweight the IV estimator to make them look similar
 - ▶ If effect heterogeneity is fully explained by X_i , i.e. homogeneous effects given X_i , this yields ATE and ATT
- Aronow and Carnegie (2013): compliers have a higher female share \Leftrightarrow females have a higher complier share
 - ▶ Estimate $Pr(\text{Complier}_i \mid X_i)$ via first stage heterogeneity
 - ▶ Use $1/\hat{Pr}(\text{Complier}_i \mid X_i)$ as weights in IV
- Angrist and Fernandez-Val (2010): if two IVs give different answers (e.g., twin births vs. same sex), is it because compliers have different covariates?
 - ▶ Develop an overidentification-type test

Outline

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3 Extensions

Extensions

- Non-binary treatments and instruments with linear heterogeneity
- Single multi-valued or continuous instrument
- Single multi-valued or continuous treatment
- Multiple instruments
- Multiple treatments
- Including necessary covariates
- Non-causal first stage

Linear heterogeneity

Consider arbitrary D_i and Z_i but assume linear heterogeneity of causal effects and first-stage coeffs:

- First-stage: $D_i(z) = D_i(0) + \pi_i z$ (without loss if Z_i is binary)
⇒ $D_i = \pi_i Z_i + u_i$ for $u_i = D_i(0)$
- Structural equation: $Y_i(d) = Y_i(0) + \tau_i d$ (without loss if D_i is binary)
⇒ $Y_i = \tau_i D_i + \varepsilon_i$ for $\varepsilon_i = Y_i(0)$
- Reduced-form: $Y_i = \tau_i \pi_i Z_i + e_i$ for $e_i = \tau_i u_i + \varepsilon_i$
- By independence, $Z_i \perp\!\!\!\perp (\tau_i, \varepsilon_i, \pi_i, u_i)$
- *Claim:* $\tau_{IV} = \frac{\mathbb{E}[\tau_i \pi_i]}{\mathbb{E}[\pi_i]}$, i.e. a π_i -weighted average of τ_i if monotonicity holds (π_i have the same sign for all i)

Proof

We know first-stage identifies $\mathbb{E} [\pi_i]$; reduced-form identifies $\mathbb{E} [\tau_i \pi_i]$ since

$$Y_i = \tau_i D_i + \varepsilon_i = \tau_i (\pi_i Z_i + u_i) + \varepsilon_i = \tau_i \pi_i Z_i + e_i$$

In more detail, let $\mathbb{E} [Z_i] = \mu$. Consider the numerator of $\tau_{IV} = \text{Cov} [Y_i, Z_i] / \text{Cov} [D_i, Z_i]$:

$$\begin{aligned}\text{Cov} [Y_i, Z_i] &= \mathbb{E} [Y_i (Z_i - \mu)] \\ &= \mathbb{E} [(\tau_i \pi_i Z_i + e_i) (Z_i - \mu)] \\ &= \mathbb{E} [\tau_i \pi_i] \text{Var} [Z_i].\end{aligned}$$

Similarly, $\text{Cov} [D_i, Z_i] = \mathbb{E} [\pi_i] \text{Var} [Z_i]$ and thus

$$\tau_{IV} = \frac{\mathbb{E} [\tau_i \pi_i]}{\mathbb{E} [\pi_i]}.$$

Single multi-valued instrument

- Suppose D_i is binary but Z_i takes values $z_0 \leq \dots \leq z_p$
 - ▶ E.g. $D_i = \text{attend college}$, $Z_i = \text{number of colleges nearby}$
- Monotonicity: $D_i(z)$ never switches from 1 to 0 as $z \uparrow$
- **Option 1:** select the sample of $Z_i \in \{z_{p-1}, z_p\}$ and compare groups $Z_i = z_p$ vs. $Z_i = z_{p-1}$
 - ▶ Yields $\tau_p = \mathbb{E}[Y_i(1) - Y_i(0) \mid D_i(z_p) > D_i(z_{p-1})]$
 - ▶ Non-overlapping complier groups for each p

Single multi-valued instrument (2)

- **Option 2:** Use linear IV with a single instrument Z_i

- ▶ Yields a weighted average of τ_p :

$$\frac{\text{Cov}[Y_i, Z_i]}{\text{Cov}[D_i, Z_i]} = \sum_{p=1}^P \omega_p \tau_p, \quad \omega_p = \frac{\Pr(D_i(z_p) > D_i(z_{p-1})) \cdot \text{Cov}[\mathbf{1}[Z_i \geq z_p], Z_i]}{\sum_{\ell=1}^P \Pr(D_i(z_\ell) > D_i(z_{\ell-1})) \cdot \text{Cov}[\mathbf{1}[Z_i \geq z_\ell], Z_i]}$$

- ▶ Weights can be computed; larger for big complier groups, around the median of Z_i
 - ▶ *Exercise:* prove using result from part A1 of the course

Single continuous instrument

- Result on linear IV weights extends naturally to continuous Z_i
 - ▶ E.g. Z_i = distance to nearest college (Card, 1993)
- Can compare $Z_i = z + \epsilon$ to $Z_i = z - \epsilon$ (for small ϵ) to get **marginal treatment effects (MTE)**
- If variation in Z_i is super rich, such that $Pr(D_i = 1 | Z_i)$ takes all values $\in [0, 1]$, weighted averages of MTEs identify the ATE, ATT, and other estimands
 - ▶ *Intuition:* there are no always-takers or never-takers
 - ▶ See e.g. Heckman and Vytlacil (2005) and Mogstad and Torgovitsky (2024)

Single multi-valued treatment (Angrist and Imbens (1995))

- Conversely, suppose Z_i is binary but D_i takes values $d_0 \leq \dots \leq d_Q$
 - ▶ E.g. Angrist and Krueger (1991): D_i = years of schooling, Z_i = born in 1st quarter
- Monotonicity: $D_i(1) \geq D_i(0)$ for all i (compliers: $D_i(1) > D_i(0)$)
- We can measure how Z_i shifts the CDF of D_i :

$$Pr(D_i \geq d_q | Z_i = 1) - Pr(D_i \geq d_q | Z_i = 0) = Pr(D_i(1) \geq d_q > D_i(0))$$

- ▶ Larger for values d_q where the IV pushes more people from below to above
- ▶ Complier groups $D_i(1) \geq d_q > D_i(0)$ can overlap across different q

Single multi-valued treatment (2)

- IV (Wald) estimand:

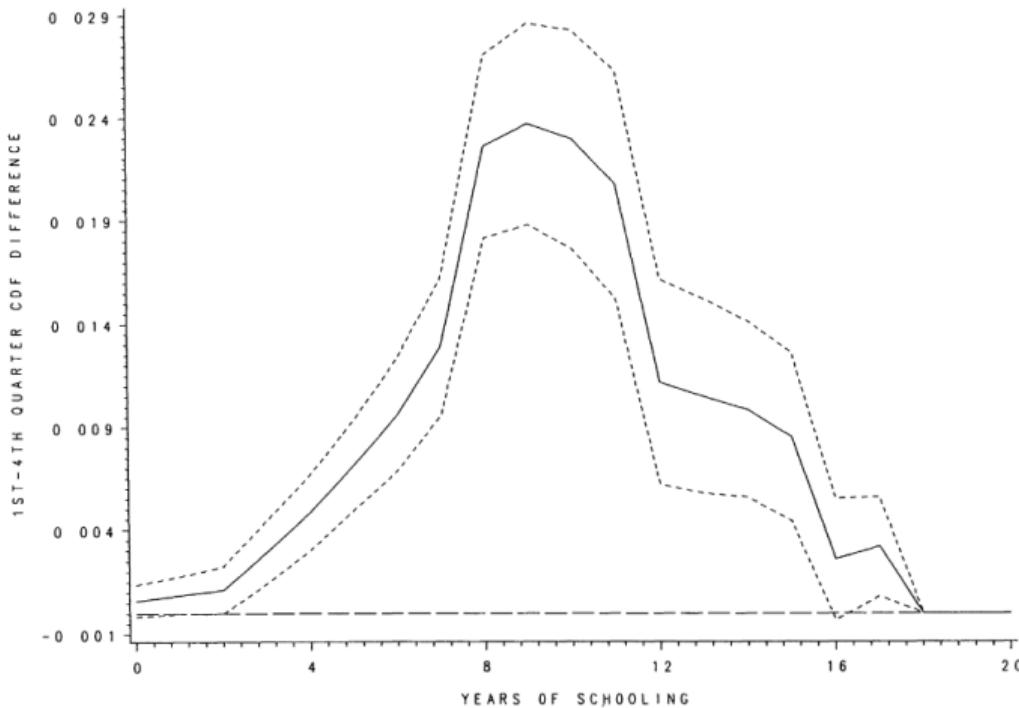
$$\frac{\text{Cov}[Y_i, Z_i]}{\text{Cov}[D_i, Z_i]} = \sum_{q=1}^Q \omega_q \mathbb{E} \left[\frac{Y_i(d_q) - Y_i(d_{q-1})}{d_q - d_{q-1}} \mid D_i(1) \geq d_q > D_i(0) \right]$$

with weights that can be computed:

$$\omega_q = \frac{Pr(D_i(1) \geq d_q > D_i(0))}{\sum_{\ell=1}^Q Pr(D_i(1) \geq d_\ell > D_i(0))}.$$

- Result generalizes immediately to continuous D_i
- We cannot isolate individual margins by conditioning on $D_i \in \{d_{q-1}, d_q\}$

Weights in Angrist and Krueger (1991)



(From Angrist and Imbens (1995), Figure 3)

Multiple instruments

- E.g. Angrist and Evans (1998): $Z_1 = \text{two boys}$, $Z_2 = \text{two girls}$
- 2SLS identifies a weighted average of individual IVs ρ_k/π_k
 - ▶ Weight on Z_k = slope of regression of $\pi_k Z_{ki}$ on the first-stage fitted value $\hat{D}_i = \sum_k \pi_k Z_{ki}$; typically convex but not guaranteed
 - ▶ Lee (2018): conventional SE of 2SLS are wrong!
 - ★ Moments $\mathbb{E}[(Y_i - \tau_{2SLS} D_i) Z_i] = 0$ do not hold with heterogeneous effects!
 - ★ But the correction tends to be small

Multiple instruments (2)

Joint monotonicity is trickier to define:

- Imbens and Angrist (1994): same ranking of $D_i(\vec{z})$ across $\vec{z} = (z_1, \dots, z_K)$ for all i
 - ▶ Very restrictive: essentially \vec{Z} is one multi-valued instrument
 - ▶ Can't have one family respond to two boys only and another to two girls only
- Mogstad, Torgovitsky, and Walters (2021) “partial monotonicity”: holding z_{-k} fixed, same ranking of $D_i(z_k; z_{-k})$ across z_k for all i
 - ▶ Binary $D, Z_1, Z_2 \implies$ 6 groups: e.g. “eager compliers”: $D_i = \mathbf{1}[Z_{1i} = 1 \text{ or } Z_{2i} = 1]$
 - ▶ 2SLS = weighted average of LATEs but some weights can be negative

Multiple treatments

- Literature has focused on exclusive treatments
- Kline and Walters (2016): D_{1i} = early childhood education program (Head Start),
 D_{2i} = alternative public programs, $D_{0i} = 1 - D_{1i} - D_{2i}$
 - ▶ Only one IV: Z_{1i} = randomized offer for the focal program
 - ▶ Monotonicity: no defiers & Z_{1i} doesn't affect choice between D_2 and D_0
 - ▶ Instrumenting D_{1i} with Z_{1i} identifies a weighted average of $Y_i(1) - Y_i(0)$ and $Y_i(1) - Y_i(2)$ among compliers at these margins
 - ▶ Weights are identified: how many kids switch to D_{1i} from each alternative
 - ★ \propto causal effects of D_{1i} on D_{i0} and D_{i2} (estimated by IV)
 - ▶ But “subLATEs” are not identified

Multiple treatments (2)

- Mountjoy (2022): D_{1i} = enrolling in a community college, D_{2i} = enrolling in a 4-year college
 - ▶ Z_{1i}, Z_{2i} = distance from nearest community college and 4-year college
 - ▶ IV with two endogenous vars does not produce causal estimands with heterogeneous effects (Kirkeboen et al. (2016))
 - ▶ Mountjoy develops a nonparametric IV approach to separate LATEs at each margin
 - ▶ See Bhuller and Sigstad (2024) for a different approach

Including necessary covariates

- Suppose Z_i is randomly assigned only conditionally on X_i
- If Z_i is scalar, pscore $\mathbb{E}[Z_i | X_i]$ is linear ("rich controls," e.g. X_i is saturated), and monotonicity holds unconditionally:
 - ⇒ IV that controls for X_i identifies a convex average of conditional-on- X_i LATEs (see MHE Ch. 4.5.1)
- Otherwise, the estimand need not be causal
 - ▶ Misspecified $\mathbb{E}[Z_i | X_i]$: Blandhol, Bonney, Mogstad, and Torgovitsky (2025)
 - ▶ Multiple IVs (or OLS with multiple treatments): Goldsmith-Pinkham, Hull, and Kolesár (2024) "contamination bias"

Non-causal first stage

- We introduced monotonicity in a causal way but it needn't be (Small et al. (2017))
- Independence + linear pscore: $\mathbb{E}[Z_i | Y_i(0), Y_i(1), X_i] = \lambda' X_i$ (no $D_i(z)$!)
- Stochastic monotonicity: $Pr(D_i = 1 | Z_i = z, Y_i(0), Y_i(1), X_i)$ weakly increases in z
- Borusyak and Hull (2024) simplified to binary D_i : for IV with X_i controls,

$$\tau_{IV} = \frac{\mathbb{E}[\phi_i \cdot (Y_i(1) - Y_i(0))]}{\mathbb{E}[\phi_i]} \quad \text{for } \phi_i = \text{Cov}[Z_i, D_i | Y_i(0), Y_i(1), X_i] \geq 0$$

- ▶ “Negative weights are no concern in design-based specifications”
- ▶ But compliers should not be interpreted as those whose treatment causally responds to the IV

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