

# Part D: Panel Data Methods

## D1: Linear Panel Data Methods

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# Panel data methods: Outline

1. Linear panel data methods with constant effects
2. Canonical DiD and event studies
3. DiD with staggered adoption
4. DiD extensions
5. Synthetic control methods and factor models

# D1 outline

- 1 Estimation, efficiency, inference
- 2 Extensions
- 3 Application: Mover designs and the AKM model

Readings: IW Lecture 3 (also CT Ch.21-22 and JW Ch. 10-11)

# Motivation

- Selection on observables is rarely convincing in cross-sectional data
  - ▶ Self-selection is complex, too many unknown confounders
- And we don't always have a convincing instrument
- To allow for selection on (some) unobservables, leverage repeated observations for the same unit over time — **panel data**
  - ▶ How do outcomes change when treatment changes?
  - ▶ This doesn't resolve the fundamental problem of causal inference but helps control for unobserved confounders that are time-invariant
- Panel data are also helpful to evaluate effect dynamics

# Linear panel model

- For  $i = 1, \dots, I$  and  $t = 1, \dots, T$ , consider a constant-effects model

$$Y_{it} = \beta' X_{it} + \alpha_i + \varepsilon_{it}$$

where  $\beta$  is of interest;  $\alpha_i$  is additive “unobserved heterogeneity”

- Denote  $\mathbf{X}_i = (X_{i1}, \dots, X_{iT})$ ,  $\mathbf{Y}_i = (Y_{i1}, \dots, Y_{iT})$ ,  $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, \dots, \varepsilon_{iT})$
- Every  $(i, t)$  is observed  $\implies$  **balanced panel**; with missing data: unbalanced panel

# Asymptotic sequences

- Conventional asymptotic (our focus): **short panel**
  - ▶ A growing sample of  $I$  units for a fixed number of periods,  $T$
  - ▶ Appropriate when  $I \gg T$
- Alternative asymptotic: **long panel**
  - ▶ The sample grows by increasing both  $I$  and  $T$  (at the same or different rates)
  - ▶ More appropriate when  $I \approx T$

# Notions of exogeneity

- **Contemporaneous exogeneity:**  $\mathbb{E}[\varepsilon_{it} \mid X_{it}, \alpha_i] = 0$ 
  - ▶ Interpretation: no time-varying confounders
  - ▶ A natural assumption... but insufficient to identify  $\beta$
- (Main focus) **Strict exogeneity:**  $\mathbb{E}[\varepsilon_{it} \mid X_{i1}, \dots, X_{iT}, \alpha_i] = 0$ 
  - ▶ For  $h > 0$ ,  $\text{Cov}[\varepsilon_{it}, X_{i,t-h}] = 0$ : precludes unmodeled dynamic effects (but solved by including lags of RHS variables)
  - ▶  $\text{Cov}[\varepsilon_{it}, X_{i,t+h}] = 0$ : precludes “**feedback effects**” from  $Y_{it}$  to  $X_{i,t+h}$ 
    - ★ Also precludes  $X_{it} \equiv Y_{i,t-1}$  as one of the RHS variables — “**dynamic panel**” models
- (For later) **Sequential exogeneity:**  $\mathbb{E}[\varepsilon_{it} \mid X_{i1}, \dots, X_{it}, \alpha_i] = 0$ 
  - ▶ Allows feedback effects and dynamic panels models

# Random and fixed effects

- If selection on  $\alpha_i$  is allowed,  $\mathbb{E}[\alpha_i | \mathbf{X}_i] \neq \text{const}$ ,  $\alpha_i$  is called **fixed effect**
  - ▶ If  $\mathbb{E}[\alpha_i | \mathbf{X}_i] = \text{const}$  (no selection on unobservables)  $\implies \alpha_i$  is a **random effect**
  - ▶ These labels are not about whether  $\alpha_i$  is stochastic (always think random sample of  $(\alpha_i, \mathbf{X}_i, \varepsilon_i, \mathbf{Y}_i)_{i=1}^N$ )
- To estimate  $\beta$  in the FE model, remove  $\alpha_i$  in different ways:
  - ▶ “FE regression”: Dummy variable regression = within transformation
  - ▶ First differences
  - ▶ Long differences
- *Note:* random effects model allows for more estimation methods
  - ▶ “Pooled OLS” regression of  $Y_{it}$  on  $X_{it}$  across  $i, t$  identifies  $\beta$
  - ▶ “Random effects” Generalized Least Squares estimator



# Estimation by dummy variables (FE) regression

- View  $\{\alpha_i\}$  as a set of nuisance parameters multiplying dummies for each unit:

$$Y_{it} = \beta' X_{it} + \sum_{j=1}^I \alpha_j W_{ji} + \varepsilon_{it}, \quad W_{ji} \equiv \mathbf{1} [i = j] \quad (*)$$

- ▶ *Note:* don't write e.g.  $Y_{it} = \beta' X_{it} + \sum_j \alpha_j + \varepsilon_{it}$

# Estimation by dummy variables (FE) regression

- By FWL, OLS estimation of (\*) = simple OLS after “**within transformation**”:

$$\dot{Y}_{it} = \beta' \dot{X}_{it} + \dot{\varepsilon}_{it}, \quad \dot{V}_{it} \equiv V_{it} - \bar{V}_i = V_{it} - \frac{1}{T} \sum_{s=1}^T V_{is}$$

- ▶ *Exercise*: prove this
- ▶ This is much faster than using  $I$  dummies! Use *reghdfe* in Stata, *fixest* in R
- For consistency of  $\hat{\beta}$ , we need

$$\mathbb{E} \left[ \dot{X}_{it} \dot{\varepsilon}_{it} \right] = \mathbb{E} \left[ \left( X_{it} - \frac{1}{T} \sum_{s=1}^T X_{is} \right) \left( \varepsilon_{it} - \frac{1}{T} \sum_{s=1}^T \varepsilon_{is} \right) \right] = 0$$

- ▶ Holds under strict exogeneity but not contemporaneous/sequential exogeneity
- ▶ *Question*: why is strict exogeneity necessary for (\*)?

# Estimation by first & long differencing

- The FE model also implies the first-difference (FD) specification:

$$\Delta Y_{it} = \beta' \Delta X_{it} + \Delta \varepsilon_{it}, \quad \Delta V_{it} \equiv V_{it} - V_{i,t-1}, \quad t = 2, \dots, T$$

- ▶  $\mathbb{E} [\Delta X_{it} \cdot \Delta \varepsilon_{it}] = 0$  under strict exogeneity (but not contemporaneous/sequential)
  - ▶ *Exercise:* with  $T = 2$ , OLS estimators of FE and FD equations are identical (even in unbalanced panels) — but not otherwise
- And a long-difference specification: for  $h > 1$ ,

$$Y_{it} - Y_{i,t-h} = \beta' (X_{it} - X_{i,t-h}) + (\varepsilon_{it} - \varepsilon_{i,t-h}), \quad t = h + 1, \dots, T$$

with  $\mathbb{E} [(X_{it} - X_{i,t-h}) \cdot (\varepsilon_{it} - \varepsilon_{i,t-h})] = 0$  under strict exogeneity

# Choosing between FE and FD estimators

- FE and FD estimate  $\beta$  from the same model under the same assumption (strict exogeneity). So how do we choose?

## 1. Efficiency:

- ▶ FE estimator is efficient when  $\varepsilon_{it}$  are serially uncorrelated
- ▶ FD estimator is efficient when  $\varepsilon_{it}$  follow a random walk (i.e.,  $\Delta\varepsilon_{it}$  are serially uncorrelated)
- ▶ It's not about persistence in the outcome (which could come from  $\alpha_i$ ) but about differential persistence of observations close in time
- ▶ FD can lose a lot of data in unbalanced panels

# Choosing between FE and FD estimators (2)

## 2. Robustness to violations of assumptions:

- ▶ When contemporaneous or sequential exogeneity holds but strict exogeneity doesn't,  $\text{bias}(\hat{\beta}_{FE}) = O(1/T)$  while  $\text{bias}(\hat{\beta}_{FD}) = O(1)$  (for weakly dependent data)
- ▶ Differencing can exacerbate measurement error in  $X_{it}$

## 3. Fads: FE estimation is more popular; it *appears* that you've controlled for more

- ▶ False, plus FD & long-diffs are more transparent (especially in more complex situations)
- ▶ If you use a FE specification, always rewrite it in FD



<https://x.com/KhoaVuUmn/status/1630576551325495296>

# Inference

- In short panels, always cluster SE by  $i$ 
  - ▶ See Bertrand, Duflo, and Mullainathan (2004) for an illustration
- In FD regressions, non-clustered (heteroskedasticity-robust) SEs require

$$\text{Cov} [\Delta X_{it} \Delta \varepsilon_{it}, \Delta X_{is} \Delta \varepsilon_{is}] = 0$$

- ▶ Unless  $X_{it}$  or  $\varepsilon_{it}$  is a random walk,  $\Delta X_{it}$  and  $\Delta \varepsilon_{it}$  are serially correlated
- In FE regressions, need  $\text{Cov} [\dot{X}_{it} \dot{\varepsilon}_{it}, \dot{X}_{is} \dot{\varepsilon}_{is}] = 0$ 
  - ▶ But  $X_{it}$  and  $\varepsilon_{it}$  are often serially correlated
  - ▶ And even if  $\varepsilon_{it}$  are uncorrelated,  $\dot{\varepsilon}_{it}$  are *negatively* serially correlated in short panels (although DoF correction by  $\frac{IT}{IT-I-\dim(X)}$  can correct for that)

# Remarks

- FE model cannot estimate the effects of time-invariant variables
  - ▶ They get killed by both within transformation and differencing
- There is no intercept in the FE model
  - ▶ But can estimate  $\hat{\alpha}_i = \frac{1}{T} \sum_t \left( Y_{it} - \hat{\beta}' X_{it} \right)$ : unbiased for  $\alpha_i$  but not consistent in short panels
  - ▶ And report  $\frac{1}{T} \sum_i \hat{\alpha}_i$  that is consistent for  $\mathbb{E}[\alpha_i]$
- With FEs,  $R^2$  is often very high and not very informative
  - ▶ Instead, can use “**within**  $R^2$ ”
  - ▶ I.e.,  $R^2$  after within-transformation: % of  $\text{Var} \left[ \dot{Y}_i \right]$  explained by  $\text{Var} \left[ \dot{X}_i \right]$



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# Extensions

1. Random effects estimation
2. Two-way fixed effects
3. Models with random slopes
4. Modeling dynamic effects
5. Nonlinear panel models
6. Fixed effects beyond panel data

# Random effects estimation

- In addition to  $\mathbb{E}[\varepsilon_{it} \mid \mathbf{X}_i, \alpha_i] = 0$ , assume  $\mathbb{E}[\alpha_i \mid \mathbf{X}_i] = 0$
- Then **Pooled OLS** estimator is consistent:

$$Y_{it} = \beta' \mathbf{X}_{it} + v_{it}, \quad v_{it} = \alpha_i + \varepsilon_{it}, \quad \text{Cov}[v_{it}, \mathbf{X}_{it}] = 0$$

- ▶ But inefficient:  $v_{it}$  are correlated across all  $t$
- RE estimator: feasible GLS for  $\text{Var}[\varepsilon_i \mid \mathbf{X}_i, \alpha_i] = \sigma_\varepsilon^2 \mathbb{I}_T$  and  $\text{Var}[\alpha_i \mid \mathbf{X}_i] = \sigma_\alpha^2$ :

$$\hat{\beta}_{RE} = \left( \sum_i \mathbf{X}_i' \hat{\Omega}^{-1} \mathbf{X}_i \right)^{-1} \left( \sum_i \mathbf{X}_i' \hat{\Omega}^{-1} \mathbf{Y}_i \right), \quad \hat{\Omega} = \begin{pmatrix} \hat{\sigma}_\alpha^2 + \hat{\sigma}_\varepsilon^2 & \hat{\sigma}_\alpha^2 & \dots & \hat{\sigma}_\alpha^2 \\ \hat{\sigma}_\alpha^2 & \hat{\sigma}_\alpha^2 + \hat{\sigma}_\varepsilon^2 & \dots & \hat{\sigma}_\alpha^2 \\ \dots & \dots & \dots & \dots \\ \hat{\sigma}_\alpha^2 & \hat{\sigma}_\alpha^2 & \dots & \hat{\sigma}_\alpha^2 + \hat{\sigma}_\varepsilon^2 \end{pmatrix}$$

## Random effects estimation (2)

- Consistent even without homoskedastic and serially uncorrelated  $\varepsilon_{it}$
- Allows for time-invariant  $X_{it}$
- Can be obtained via **quasi-differencing**: OLS for

$$\left(Y_{it} - \hat{\lambda} \bar{Y}_i\right) = \beta' \left(\mathbf{X}_{it} - \hat{\lambda} \bar{\mathbf{X}}_i\right) + \left(v_{it} - \hat{\lambda} \bar{v}_i\right)$$

where  $\hat{\lambda} = 1 - (1 + T\hat{\sigma}_\alpha^2/\hat{\sigma}_\varepsilon^2)^{-1/2}$

- ▶  $\hat{\lambda} \approx 1$ , so  $\hat{\beta}_{RE} \approx \hat{\beta}_{FE}$ , when  $T$  is large or  $\text{Var}[\alpha_i] \gg \text{Var}[\varepsilon_{it}]$

## Two-way fixed effects (TWFE)

- Besides  $\alpha_i$ , we may want to include (additive) period effects  $\gamma_t$  to capture shocks that affect all units:

$$Y_{it} = \beta' X_{it} + \alpha_i + \gamma_t + \varepsilon_{it} \quad (\#)$$

- Unlike  $\alpha_i$ , period FEs are non-stochastic parameters (on period dummies)
- ( $\#$ ) requires an innocuous normalization on  $\{\alpha_i\}$  or  $\{\gamma_t\}$
- FE estimator: in balanced panels, OLS from **double-differenced** specification:

$$\ddot{Y}_{it} = \beta' \ddot{X}_{it} + \ddot{\varepsilon}_{it}, \quad \ddot{V}_{it} \equiv (V_{it} - \bar{V}_i) - (\bar{V}_t - \bar{V})$$

- ▶ Follows from FWL, because unit and period dummies are in-sample orthogonal
- FD estimator: OLS from  $\Delta Y_{it} = \beta' \Delta X_{it} + \Delta \gamma_t + \Delta \varepsilon_{it}$  where  $\Delta \gamma_t$  are period FEs

# Models with random slopes

- Consider  $Y_{it} = \alpha_i + \beta' X_{it} + \gamma_i' Z_{it} + \varepsilon_{it}$

- ▶ E.g. unit-specific linear trends:  $Y_{it} = \alpha_i + \beta' X_{it} + \gamma_i' t + \varepsilon_{it}$

- Dummy variable representation and estimation:

$$Y_{it} = \sum_{j=1}^I \alpha_j W_{ji} + \beta' X_{it} + \sum_{j=1}^I \gamma_j' Z_{jt} W_{ji} + \varepsilon_{it}$$

- FWL: for each  $i$  separately, residualize  $Y_{it}, X_{it}$  on  $(1, Z_{it})$  in the time series

# Modeling dynamic effects

- **Distributed lags** model can be accommodated with no change:

$$Y_{it} = \beta'_0 X_{it} + \beta'_1 X_{i,t-1} + \dots \beta'_L X_{i,t-L} + \alpha_i + \varepsilon_{it}$$

- **Lagged dependent variable** on the RHS: dynamic panel model

$$Y_{it} = \rho Y_{i,t-1} + \beta' X_{it} + \alpha_i + \varepsilon_{it}$$

- ▶ Violates strict exogeneity:  $\varepsilon_{it}$  can't be mean-independent of  $Y_{i,s-1}$  for  $s = t + 1$
- ▶ At best, we can hope for sequential exogeneity of  $Y_{i,t-1}$ :

$$\mathbb{E} [\varepsilon_{it} \mid Y_{i1}, \dots, Y_{i,t-1}, \mathbf{X}_i, \alpha_i] = 0$$

- ▶ FE estimator is not consistent in short panels: “Nickell bias”  $\propto 1/T$
- ▶ FD estimator has bias  $O(1)$ . But FD specification can be estimated by IV...

# Arellano-Bond estimator

- FD specification:  $\Delta Y_{it} = \rho \Delta Y_{i,t-1} + \beta' \Delta X_{it} + \Delta \varepsilon_{i,t}$
- OLS doesn't work:  $\text{Cov} [\Delta Y_{i,t-1}, \Delta \varepsilon_{it}] = \text{Cov} [Y_{i,t-1} - Y_{i,t-2}, \varepsilon_{it} - \varepsilon_{i,t-1}] \neq 0$
- But can use  $Y_{i1}, \dots, Y_{i,t-2}$  as IVs for  $\Delta Y_{i,t-1} \Rightarrow$  Arellano and Bond (1991) estimator: GMM for moment conditions collected across  $t$

$$\mathbb{E} [(\Delta Y_{it} - \rho \Delta Y_{i,t-1} - \beta' \Delta X_{it}) \cdot (Y_{i1}, \dots, Y_{i,t-2}, \Delta X'_{it})'] = 0, \quad t = 3, \dots, T$$

- These IVs can be weak, especially when  $\rho$  is close to 1
  - ▶ Improvements are available, e.g. Blundell and Bond (1998)
- Similar ideas can be applied with sequentially exogenous  $X_{it}$  other than  $Y_{i,t-1}$  (with feedback effects of  $\varepsilon_{i,t-1}$  on  $X_{it}$ )



# Nonlinear panel data models

Nonlinear models with fixed effects are much more complicated. Consider binary choice models:

$$Pr(Y_{it} = 1 \mid X_{it}, \alpha_i) = F(\beta' X_{it} + \alpha_i), \quad F = \text{probit or logistic}$$

- Likelihood estimation of  $\beta$  along with  $\{\alpha_i\}$  results in the **incidental parameter problem**:  $\hat{\beta}$  is inconsistent in short panels
  - ▶ The problem doesn't arise with linear  $F$  because the within transformation kills  $\alpha_i$
- For logistic regression (but not probit), “conditional logit” estimator eliminates  $\alpha_i$  yielding consistent estimates for  $\beta$ :
  - ▶ But not for average partial effects which depend on  $\alpha_i$
- More progress with long panels; see Fernández-Val and Weidner (2018)

# FEs beyond panel data

There are other types of data with repeated observations:

1. Twin studies = repeated observations in the same family
  - ▶ E.g. Ashenfelter and Rouse (1998) estimate returns to schooling for twins
    - ★  $X_i$  = years of schooling
    - ★ Family FEs control for genetic differences
  - ▶ *Warning*: why does education vary between twins?
    - ★ Need to explain why confounders are the same between the twins while  $X_{it}$  is not
    - ★ Bound and Solon (1999): first-borns have higher weight, IQ, schooling



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Your honor, those are my emotional support fixed effects.

## FEs beyond panel data (2)

2. County-level cross-section = repeated observations for the same state

- ▶ State FEs control for additive state-level unobservables
- ▶ E.g. state-level policies with constant effects

3. In a county-level panel, can include state-by-year FEs:

$$Y_{it} = \beta' X_{it} + \alpha_i + \gamma_{s(i),t} + \varepsilon_{it}$$

- ▶ Including  $\gamma_{s(i),t}$  removes state-year-specific means from  $Y_{it}$  and  $X_{it}$
- ▶ *Note:*  $\sum_{s',t'} \gamma_{s't'} \mathbf{1}[s(i) = s'] \times \mathbf{1}[t = t']$  is correct notation;  $\gamma_{s(i)} \times \delta_t$  is wrong

## FEs beyond panel data (3)

4. Repeated cross-sections: in each year a new random sample from each state
  - ▶ Can't control for individual heterogeneity
  - ▶ But state FEs control for additive state-level unobservables
  - ▶ Cluster at the state-level if  $X_{it}$  only varies by state
5. Dyadic data: e.g. how does distance  $X_{ij}$  between exporting country  $i$  and importing country  $j$  affect log trade flow  $Y_{ij}$ ? (Gravity equation)
  - ▶ *Note:* by construction, it's like a long panel,  $I \times I$
  - ▶ *Exercise:* Which FE would you include? How would you cluster SE?

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## Application: Are there good firms?

- Abowd, Kramarz, and Margolis (1999): Are there good firms that pay higher wages to the same workers?
  - ▶ How much variation in wages is explained by worker characteristics? By firm characteristics?
  - ▶ Do “better” workers tend to work for “better” firms?
- Use 10 years of employer-employee matched data for France
  - ▶ A panel of workers  $i$ : observe employer ID  $j(it)$ , experience, wages

# AKM model

Model of log-wages  $Y_{it}$ :

$$Y_{it} = \beta' X_{it} + \alpha_i + \gamma_{j(it)} + \varepsilon_{it}$$

- $X_{it}$  are time-varying observables, e.g. experience — **not** of interest
- $\{\alpha_i\}$  are worker FEs;  $\{\gamma_j\}$  are firm FEs
- $\varepsilon_{it}$  captures match-specific wage premium



# Identification

- Relative firm FEs are identified by **movers**:

$$\Delta Y_{it} = \beta' \Delta X_{it} + (\gamma_{\mathbf{j}(it)} - \gamma_{\mathbf{j}(i,t-1)}) + \Delta \varepsilon_{it}$$

- ▶ (actual estimation via dummy variable regression, not in first-differences)
- ▶ Identification only within a connected component of the worker-firm graph
- Requires strict exogeneity:  $\mathbb{E}[\varepsilon_{it} \mid X_{i1}, \dots, X_{iT}, \alpha_i, \mathbf{j}(i1), \dots, \mathbf{j}(iT)] = 0$ 
  - ▶ Matching of firms and workers can depend on FEs but not on match quality  $\varepsilon_{it}$  (“**exogenous mobility**”)

# Testing exogenous mobility (Card, Heining, and Kline (2013))

Do movers from high- $\gamma$  to low- $\gamma$  firms lose *less* than movers from low- $\gamma$  to high- $\gamma$  gain?

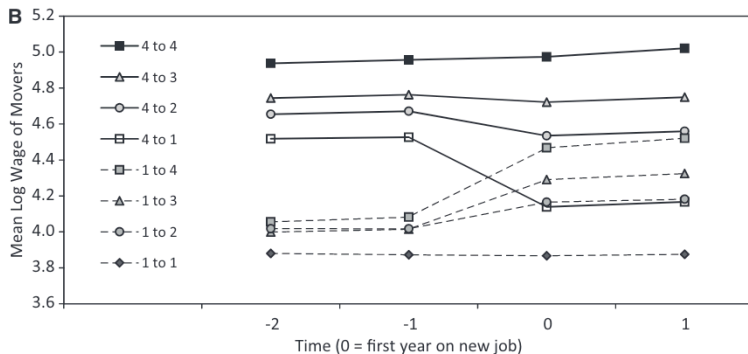


FIGURE V

(use quartiles of average wages paid to other workers; German employer-employee data)

# Estimation issues

- Card et al. (2013) compute:
  - ▶ Variances of  $\hat{\alpha}_i$  and  $\hat{\gamma}_{j(it)}$  as worker and firm contributions to wage inequality
  - ▶ Covariance of  $\hat{\alpha}_i$  and  $\hat{\gamma}_{j(it)}$  as a measure of sorting
- But FEs are not estimated consistently!
  - ▶ For all workers, at most 8 wage observations  $\Rightarrow \text{Var} [\hat{\alpha}_i]$  biased  $\uparrow$
  - ▶ For many firms, there are only a few movers  $\Rightarrow \text{Var} [\hat{\gamma}_{j(it)}]$  biased  $\uparrow$  and  $\text{Cov} [\hat{\alpha}_i, \hat{\gamma}_{j(it)}]$  biased  $\downarrow$
- Kline, Saggio, and Solvsten (2020) provide a bias correction
  - ▶ Consistent estimates of  $\text{Var} [\alpha_i]$ ,  $\text{Var} [\gamma_{j(it)}]$ ,  $\text{Cov} [\alpha_i, \gamma_{j(it)}]$  without consistent estimates of the FEs

# Kline et al. (2020) findings (for Veneto region in Italy)

TABLE II  
VARIANCE DECOMPOSITION<sup>a</sup>

	Pooled	Younger Workers	Older Workers
<i>Variance of Firm Effects</i>			
Plug in (PI)	0.0358	0.0368	0.0415
Homoscedasticity Only (HO)	0.0295	0.0270	0.0350
Leave Out (KSS)	0.0240	0.0218	0.0204
<i>Variance of Person Effects</i>			
Plug in (PI)	0.1321	0.0843	0.2180
Homoscedasticity Only (HO)	0.1173	0.0647	0.2046
Leave Out (KSS)	0.1119	0.0596	0.1910
<i>Covariance of Firm, Person Effects</i>			
Plug in (PI)	0.0039	-0.0058	-0.0032
Homoscedasticity Only (HO)	0.0097	0.0030	0.0040
Leave Out (KSS)	0.0147	0.0075	0.0171
<i>Correlation of Firm, Person Effects</i>			
Plug in (PI)	0.0565	-0.1040	-0.0334
Homoscedasticity Only (HO)	0.1649	0.0726	0.0475
Leave Out (KSS)	0.2830	0.2092	0.2744
<i>Coefficient of Determination (<math>R^2</math>)</i>			
Plug in (PI)	0.9546	0.9183	0.9774
Homoscedasticity Only (HO)	0.9029	0.8184	0.9524
Leave Out (KSS)	0.8976	0.8091	0.9489

# Extensions

- See Kline (2024) for a modern treatment
- Hull (2018) interprets mover designs under heterogeneous effects
- Versions of the AKM model have been applied in other settings:
  - ▶ Wages depend on worker FE and city FE (Glaeser and Mare (2001))
  - ▶ Log healthcare utilization depends on person FE (“demand”) and location FE (“supply”) (Finkelstein, Gentzkow, and Williams (2016))
  - ▶ Changes in credit depends on client firm FE (demand) and bank FE (supply) (Amiti and Weinstein (2018), in a cross-section)

# References I

- ABOWD, J. M., F. KRAMARZ, AND D. N. MARGOLIS (1999): "High wage workers and high wage firms," *Econometrica*, 67, 251–333.
- AMITI, M. AND D. E. WEINSTEIN (2018): "How Much Do Idiosyncratic Bank Shocks Affect Investment? Evidence from Matched Bank-Firm Loan Data," *Journal of Political Economy*, 126, 525–587.
- ARELLANO, M. AND S. BOND (1991): "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations," *The Review of Economic Studies*, 58, 277.
- ASHENFELTER, O. AND C. ROUSE (1998): "Income, Schooling, and Ability: Evidence from a New Sample of Identical Twins," *Quarterly Journal of Economics*, 113, 253–284.
- BERTRAND, M., E. DUFLO, AND S. MULLAINATHAN (2004): "How Much Should We Trust Differences-in-Differences Estimates?" *The Quarterly Journal of Economics*, 119, 249–275.
- BLUNDELL, R. AND S. BOND (1998): "Initial conditions and moment restrictions in dynamic panel data models," *Journal of Econometrics*, 87, 115–143.
- BOUND, J. AND G. SOLON (1999): "Double trouble: on the value of twins-based estimation of the return to schooling," *Economics of Education Review*, 18, 169–182.

## References II

- CARD, D., J. HEINING, AND P. KLINE (2013): "Workplace Heterogeneity and the Rise of West German Wage Inequality," *Quarterly Journal of Economics*, 128, 967–1015.
- FERNÁNDEZ-VAL, I. AND M. WEIDNER (2018): "Fixed Effects Estimation of Large-  $T$  Panel Data Models," *Annual Review of Economics*, 10, 109–138.
- FINKELSTEIN, A., M. GENTZKOW, AND H. WILLIAMS (2016): "Sources of Geographic Variation in Health Care: Evidence From Patient Migration," *The Quarterly Journal of Economics*, 131, 1681–1726.
- GLAESER, E. L. AND D. C. MARE (2001): "Cities and Skills," *Journal of Labor Economics*, 19, 316–342.
- HULL, P. (2018): "Estimating Treatment Effects in Mover Designs," *Working Paper*.
- KLINE, P. (2024): "Firm wage effects," in *Handbook of Labor Economics*, Elsevier, vol. 5, 115–181.
- KLINE, P., R. SAGGIO, AND M. SOLVSTEN (2020): "Leave-Out Estimation of Variance Components," *Econometrica*, 88, 1859–1898.