Zadanie 1:

Znaleźć dziedziny funkcji:

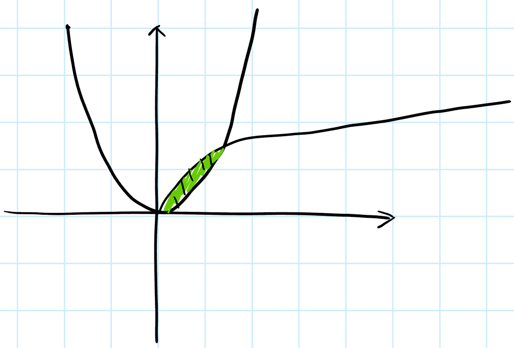
a) $f(x,y) = \sqrt{x} \sin\frac{x}{y}$

$D: \; x \geqslant 0 \quad y \neq 0$

b) $f(x,y) = \sqrt{y - x^2} + \sqrt{x - y^2}$

$y - x^2 \geqslant 0 \qquad \boxed{y \geqslant x^2}$

$x - y^2 \geqslant 0 \qquad x \geqslant y^2 \qquad \boxed{\sqrt{x} \geqslant |y|}$



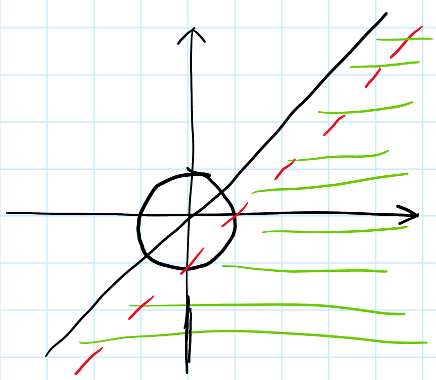
c) $f(x,y) = \dfrac{\sqrt{x^2+y^2-1}}{\ln(x-y)}$

$x^2+y^2-1 \geqslant 0 \rightarrow x^2+y^2 \geqslant 1$

$\ln(x-y) \neq 0 \qquad x-y > 0 \qquad x > y$

$x - y \neq 1$

$y \neq x - 1$



POCHODNE:

Zadanie 2:

Obliczyć pochodne cząstkowe funkcji:

a) $f(x,y) = x^2 + 2x + y^2 - 6y + 3$

$$\frac{df}{dx} = 2x + 2$$

$$\frac{df}{dy} = 2y - 6$$

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b) $f(x,y) = (x+y)e^{-x-2y} = xe^{-x-2y} + ye^{-x-2y}$

$$\frac{df}{dx} = (x+y)' e^{-x-2y} + (x+y)\left(e^{-x-2y}\right)' =$$

$$= e^{-x-2y} + (x+y)e^{-x-2y} \cdot (-1) =$$

$$(1-x-y)e^{-x-2y}$$

$$\frac{df}{dy} = e^{-x-2y} + (x+y)e^{-x-2y} \cdot (-2) =$$

$$= (1-2x-2y)e^{-x-2y}$$

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c) $f(x,y) = \sqrt{x}\,\sin\frac{y}{x}$

$$\frac{df}{dx} = \left(\sqrt{x}\right)' \sin\frac{y}{x} + \sqrt{x}\left(\sin\frac{y}{x}\right)' =$$

$$= \frac{1}{2\sqrt{x}} \sin\frac{x}{y} + \sqrt{x} \cdot \cos\frac{x}{y} \cdot y \cdot \left(\frac{-1}{x^2}\right)$$

$$\frac{df}{dy} = 0 + \sqrt{x} \cdot \left(\sin\frac{y}{x}\right)' = \sqrt{x} \cdot \cos\frac{y}{x} \cdot \frac{1}{x}$$

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d) $g(x,y,z) = \frac{y}{\sqrt{1+x^2+z^2}}$

$$\frac{dg}{dx} = \frac{-y \cdot \frac{1}{2\sqrt{1+x^2+z^2}} \cdot 2x}{1+x^2+z^2}$$

$$\frac{dg}{dy} = \frac{\sqrt{1+x^2+z^2} - y \cdot \frac{1}{2\sqrt{1+x^2+z^2}} \cdot 0}{1+x^2+z^2} = \frac{\sqrt{1+x^2+z^2}}{1+x^2+z^2}$$

$$\frac{df}{dz} = \frac{-y \cdot \frac{1}{2\sqrt{1+x^2+z^2}} \cdot 2z}{1+x^2+z^2}$$

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e) $g(x,y,z) = \sqrt{x+\sqrt{y+\sqrt{z}}}$

$$\frac{dg}{dx} = \frac{1}{2\sqrt{x+\sqrt{y+\sqrt{z}}}}$$

$$\frac{dg}{dy} = \frac{1}{2\sqrt{x+\sqrt{y+\sqrt{z}}}} \cdot \frac{1}{2\sqrt{1+\sqrt{z}}}$$

$$\frac{dg}{dz} = \frac{1}{2\sqrt{x+\sqrt{y+\sqrt{z}}}} \cdot \frac{1}{2\sqrt{y+\sqrt{z}}} \bullet \frac{1}{2\sqrt{z}} =$$

$$= \frac{1}{8} \cdot \frac{1}{\sqrt{x+\sqrt{y+\sqrt{z}}}} \cdot \frac{1}{\sqrt{y+\sqrt{z}}} \cdot \frac{1}{\sqrt{z}}$$

Zadanie 3:

Znaleźć wzory na płaszczyzny styczne do funkcji we wskazanym punkcie.

a) $f(x,y) = x^2\sqrt{y+1}$ $\qquad (1,3,z_0)$

$$z - z_0 = \frac{df}{dx}(x_0,y_0)(x-x_0) + \frac{df}{dy}(x_0,y_0)(y-y_0)$$

$$z_0 = f(x_0,y_0) = 1 \cdot \sqrt{4} = 2$$

$$z_0 = 2$$

$$\frac{df}{dx} = (x^2)'\sqrt{y+1} + x^2(\sqrt{y+1})' = 2x\sqrt{y+1}$$

$$\frac{df}{dy} = (x^2)'\sqrt{y+1} + x^2(\sqrt{y+1})' = \frac{x^2}{2\sqrt{y+1}}$$

$$\frac{df}{dx}(1,3) = 4$$

$$\frac{df}{dy}(1,3) = \frac{1}{4}$$

b) $f(x,y) = y - x^2 + 2\ln(xy)$ $\qquad (-1,-1,z_0)$

$$z_0 = f(-1,-1) = -1-1+2\ln(1) = -2$$

$$\frac{df}{dx} = 0-2x+2\cdot\frac{1}{xy}\cdot y = -2x+\frac{2}{x}$$

$$\frac{df}{dy} = 1-0+2\cdot\frac{1}{xy}\cdot x = 1+\frac{2}{y}$$

$$\frac{df}{dx}(-1,-1) = 2+\frac{2}{-1} = 0$$

$$\frac{df}{dy}(-1,-1) = 1+\frac{2}{-1} = -1$$

$$z+2 = -1(y+1) \qquad z+2 = -y-1$$

c) $f(x,y) = (x^2-y)e^{2x-y} \qquad (2,1,-z_0)$

$$\frac{df}{dx} = (x^2-y)'e^{2x-y} + (x^2-y)(e^{2x-y})' = 2xe^{2x-y}+(x^2-y)e^{2x-y}\cdot 2$$

$$= 2e^{2x-y}(x^2+x-y)$$

$$\frac{df}{dy} = -e^{2x-y}+(x^2-y)e^{2x-y}\cdot(-1) = -e^{2x-y}(x^2-y+1)$$

$$\frac{df}{dx}(2,1) = 2\cdot e^3(5) = 6e^3$$

$$\frac{df}{dy}(2,1) =$$

d) $f(x,y) = 1-2\sqrt{x^2+y^2} \qquad (0,5,z_0)$

$$\frac{df}{dx} = \frac{2}{2\sqrt{x^2+y^2}}\cdot 2x = \frac{2x}{\sqrt{x^2+y^2}}$$

$$\frac{df}{dy} = \frac{2y}{\sqrt{x^2+y^2}}$$

$$z_0 = 1-2\sqrt{25} = -9$$

$$\frac{df}{dx}(0,5) = 0 \qquad \frac{df}{dy}(0,5) = \frac{10}{5} = 2$$

$$z+9 = -2(y-5)$$

e) $f(x,y) = e^{x+y} - e^{4-y}$ $\qquad (x_0, 0, 0)$

$0 = e^{x} - e^{4}$ $\qquad x = 4$

$\frac{df}{dx} = e^{x+y} \cdot 1 - e^{4-y} \cdot 0 = e^{x+y} \cdot y$

$\frac{df}{dy} = e^{x+y} \cdot 1 - e^{4-y} \cdot (-1) = xe^{x+y}$