PODSTAWY AUTOMATYKI

LISTA 1

$\boxed{\text{ZADANIE 1}}$ Znaleźć Laplace'a z definicji

a) $y(t) = 2t + 3$

$$\mathcal{L}\{y(t)\} = \int_0^\infty y(t)e^{-st}\,dt = \int_0^\infty (2t+3)e^{-st}\,dt = \int_0^\infty 2te^{-st}\,dt + \int_0^\infty 3e^{-st}\,dt =$$

$$= 2\int_0^\infty te^{-st}\,dt + 3\int_0^\infty e^{-st}\,dt =$$

$$3\int_0^\infty e^{-st}\,dt = 3\left[\frac{1}{-s}e^{-st}\right]_0^\infty = 3\left[\frac{1}{-s}e^{-\infty} + \frac{1}{+s}e^{-s0}\right] = 3\left[\frac{1}{s}\right] = \frac{3}{s}$$

$$2\int_0^\infty te^{-st}\,dt = \begin{vmatrix} f = t & g = -\frac{1}{s}e^{-st} \\ f' = 1 & g' = e^{-st} \end{vmatrix} = 2\left[-\frac{t}{s}e^{-st} + \frac{1}{s}\int_0^\infty e^{-st}\,dt\right] =$$

$$= 2\left[-\frac{t}{s}e^{-st} - \frac{1}{s^2}e^{-st}\right]_0^\infty = \left[e^{-st}\left(-\frac{t}{s} - \frac{1}{s^2}\right)\right]_0^\infty =$$

$$= 2\left(0 + \frac{1}{s^2}\right) = \frac{2}{s^2}$$

$$F(s) = \frac{2}{s^2} + \frac{3}{s} = \frac{2+3s}{s^2}$$

$$2\left(-\frac{t}{s}e^{-st}\Big|_0^\infty + \frac{1}{s}\int_0^\infty e^{-st}\,dt\right) =$$

b) $y(t) = -t+2$

$$\mathcal{L}\{y(t)\} = \int_0^\infty (-t+2)e^{-st}dt = \int_0^\infty -te^{-st}dt + \int_0^\infty 2e^{-st}dt = -\frac{1}{s^2}+\frac{2}{s} = \frac{2s-1}{s^2}$$

$$\int_0^\infty -te^{-st}dt = \left| \begin{matrix} f = t & g = \frac{-1}{s}e^{-st} \\ f' = 1 & g' = e^{-st} \end{matrix} \right| = -\left(\underbrace{\frac{-t}{s}e^{-st}\Big|_0^\infty}_{0-0} + \underbrace{\frac{1}{s}\int_0^\infty e^{-st}dt}_{\frac{-1}{s^2}\cdot 0 + \frac{1}{s^2}\cdot 1} \right) = -\frac{1}{s^2}$$

$$\int_0^\infty 2e^{-st}dt = 2\int_0^\infty e^{-st}dt = 2\left[\frac{-1}{s}e^{-st}\right]_0^\infty = 2\cdot\frac{1}{s} = \frac{2}{s}$$

c) $y(t) = t^2$

$$\mathcal{L}\{y(t)\} = \int_0^\infty t^2 e^{-st}dt = \left| \begin{matrix} f = t^2 & g = \frac{-1}{s}e^{-st} \\ f' = 2t & g' = e^{-st} \end{matrix} \right| = \frac{-t^2}{s}e^{-st}\Big|_0^\infty - \frac{1}{s}\int_0^\infty 2te^{-st}dt =$$

$$= -\frac{2}{s}\int_0^\infty te^{-st}dt = \left| \begin{matrix} f = t & g = -\frac{1}{s}e^{-st} \\ f' = 1 & g' = e^{-st} \end{matrix} \right| = -\frac{2}{s}\left(\underbrace{\frac{-t}{s}e^{-st}\Big|_0^\infty}_{-\frac{1}{s}\cdot 0} + \frac{1}{s}\int_0^\infty e^{-st}dt \right) =$$

$$= -\frac{2}{s}\left(\frac{1}{s}\cdot\left[-\frac{1}{s}e^{-st}\right]_0^\infty \right) = -\frac{2}{s^2}\left(-0 + \frac{1}{s}\right) = \frac{3}{s^2}$$

$-\frac{1}{s}\cdot 1$

LISTA 1

ZADANIE 2 DANE: $g(t)$ SZUKANE: $G(s)$ WZORY: $G(s)=\mathcal{L}\{g(t)\}$

ZNALEŹĆ $G(s)$

$\mathcal{L}\{e^{-\alpha t}\}=\frac{1}{s+\alpha}$

a) $g(t)=(-2e^{-3t}+3e^{-4t})1(t)$

$G(s)=\mathcal{L}\{g(t)\}=\mathcal{L}\{-2e^{-3t}1(t)+3e^{-4t}1(t)\}=$

$=-2\mathcal{L}\{e^{-3t}1(t)\}+3\mathcal{L}\{e^{-4t}1(t)\}=\frac{-2}{s+3}+\frac{3}{s+4}$

b) $g(t)=(3e^{-3t}+2e^{-2t}+e^{-t})1(t)$

$G(s)=\mathcal{L}\{3e^{-3t}1(t)+2e^{-2t}1(t)+e^{-t}1(t)\}=$

$=3\mathcal{L}\{e^{-3t}1(t)\}+2\mathcal{L}\{e^{-2t}1(t)\}+\mathcal{L}\{e^{-t}1(t)\}=$

$=\frac{3}{s+3}+\frac{2}{s+2}+\frac{1}{s+1}$

$t^n e^{\alpha t}\rightarrow\frac{n!}{(s-\alpha)^{n+1}}$

c) $g(t)=(2t^2e^{-t}+3e^{-2t})1(t)$

$G(s)=\mathcal{L}\{2t^2e^{-t}1(t)+3e^{-2t}1(t)\}=2\mathcal{L}\{t^2e^{-t}1(t)\}+3\mathcal{L}\{e^{-2t}1(t)\}=$

$=2\frac{2!}{(s+1)^3}+3\frac{1}{s+2}=\frac{4}{(s+1)^3}+\frac{3}{s+2}$

ZADANIE 3 Dane: $y_1(t)$ Szukane: $G(s)$

Wzory: $G(s)=s\cdot\mathcal{L}\{y_1(t)\}$

a) $y_1(t)=(2+2e^{-2t}-4e^{-t})1(t)$

$G(s)=s\cdot\mathcal{L}\{2\cdot1(t)+2e^{-2t}1(t)-4e^{-t}1(t)\}=$

$=s\cdot2\mathcal{L}\{1(t)\}+2s\mathcal{L}\{e^{-2t}1(t)\}-4s\mathcal{L}\{e^{-t}1(t)\}=$

$=2\cancel{s}\cdot\frac{2}{\cancel{s}}+2s\cdot\frac{1}{s+2}-4s\cdot\frac{1}{s+1}=$

$=4+\frac{2s}{s+2}-\frac{4s}{s+1}$

b) $y_1(t)=(2te^{-2t})1(t)$

$G(s)=2s\cdot\mathcal{L}\{te^{-2t}1(t)\}=2s\cdot\frac{1}{(s+2)^2}=\frac{2s}{(s+2)^2}$

c) $y_1(t)=(e^{-2t}+(t-1)e^{-t})1(t)$

$G(s)=s\cdot\mathcal{L}\{e^{-2t}1(t)+te^{-t}1(t)-e^{-t}1(t)\}=$

$=s\cdot\frac{1}{s+2}+s\cdot\frac{1}{(s+1)^2}-s\cdot\frac{1}{s+1}=$

$=\frac{s}{s+2}+\frac{s}{(s+1)^2}-\frac{s}{s+1}$

$\frac{e^{-2s}}{s^2+1}$

d) $y_1(t) = \sin(t-2)1(t-2)$

$G(s) = s \cdot \mathcal{L}\{\sin(t-2)1(t-2)\} = s \cdot$???

ZADANIE 4:

DANE: $G(s)$ SZUKANE: $g(t)$ WZORY: $g(t) = \mathcal{L}^{-1}\{G(s)\}$

a) $G(s) = \frac{5s+2}{s^2+6s+8} = \frac{A}{s+2} + \frac{B}{s+4} = \frac{-4}{s+2} + \frac{9}{s+4}$

$s^2+6s+8 \to \Delta = 36-32 = 4 \to \sqrt{\Delta} = 2$

$s_1 = \frac{-6-2}{2} = \frac{-8}{2} = -4$

$s_2 = \frac{-6+2}{2} = \frac{-4}{2} = -2$

$A(s+4) + B(s+2) = 5s+2$

$As + 4A + Bs + 2B = 5s+2$

$(A+B)s = 5s$

$(4A+2B) = 2$

$A+B = 5$

$2A+B = 1$

$A = 5-B$

$A = 5-9 = -4$

$10 - 2B + B = 1 \to 9 = B$

$g(t) = \mathcal{L}^{-1}\left\{\frac{-4}{s+2} + \frac{9}{s+4}\right\} = (-4e^{-2t} + 9e^{-4t})1(t)$

b) $G(s) = \frac{2s+3}{s^2+9s+20} = \frac{A}{s+4} + \frac{B}{s+5} = \frac{-5}{s+4} + \frac{7}{s+5}$

$\Delta = 81-80 = 1 \quad \sqrt{\Delta} = 1 \quad s_1 = \frac{-9-1}{2} = -5 \quad s_2 = \frac{-9+1}{2} = -4$

$A(s+5) + B(s+4) = 2s+3$

$(A+B)x = 2x \quad A = 2-B$

$5A+4B = 3$

$10 - 5B + 4B = 3$

$B = 7 \quad A = 2-B = 2-7 = -5$

$g(t) = \mathcal{L}^{-1}\left\{\frac{-5}{s+4} + \frac{7}{s+5}\right\} = (-5e^{-4t} + 7e^{-5t})1(t)$

c) $G(s) = \frac{2s+1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \frac{1}{s} + \frac{1}{s+1}$

$A(s+1) + Bs = 2s+1$

$A+B = 2$

$A = 1 \quad B = 1$

$g(t) = \mathcal{L}^{-1}\{G(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{1}{s+1}\right\} =$

$= 1(t) + e^{-t}1(t)$

d) $G(s) = \frac{s+1}{(s+2)^2} = \cancel{\frac{A}{(s+2)^2} + \frac{B}{s+2}} = \frac{s+2}{(s+2)^2} - \frac{1}{(s+2)^2}$

$g(t) = \mathcal{L}^{-1}\left\{\frac{1}{s+2} - \frac{1}{(s+2)^2}\right\} =$

$= (e^{-2t} - te^{-2t})1(t)$

$B(s+2)^2 + A(s+2) = s+1$

$B(s^2+2s+4) + A(s+2) = s+1$

$\cancel{Bs^2 = 0 \quad B = 0}$

$\cancel{(2B+A)s = 1 \quad A = 1}$

$Bs^2 + 2Bs + 4B + As + 2A = s+1$

$Bs^2 = 0s^2$

$2B+A = 1$

$4B+2A = 1$

e) $G(s)=\frac{s^2+s+1}{(s+3)(s^2+5s+6)} = \frac{s^2+s+1}{(s+3)^2(s+2)}$

$\Delta = 25-24 = 1 \quad \sqrt{\Delta}=1 \quad s_1 = \frac{-5-1}{2} = -3$

$s_2 = \frac{-5+1}{2} = -2$

$\frac{4-2+1}{1^2}e^{-2t} = 3e^{-2t}$

Metoda residuów:

$\cancel{\frac{d}{ds}\left[\frac{s^2+s+1}{s+2}\right]\Big|_{s=-3}} \; 1(t)\left(\frac{d}{ds}\left[\frac{s^2+s+1}{s+2}e^{st}\right]\Big|_{s=-3} + \frac{s^2+s+1}{(s+3)^2}e^{st}\Big|_{s=-2}\right) =$

$=$

$\frac{f}{g}\left(\frac{s^2+s+1}{s+2}\right)' = \frac{f'g-fg'}{g^2} = \frac{(2s+1)(s+2)-(s^2+s+1)(1)}{(s+2)^2} = \frac{s^2+4s+1}{(s+2)^2} =$

$(uv)' = u'v+uv' = \frac{s^2+4s+1}{(s+2)^2}e^{st} + \frac{s^2+s+1}{s+2}\cdot t e^{st}\Big|_{s=-3} =$

$= \frac{9-12+1}{(-1)^2}e^{-3t} + \frac{9-3+1}{-1}\cdot t e^{-3t} = -2e^{-3t} - 7te^{-3t} = \cancel{19e^{-3t}}$

$\cancel{g(t) = (19e^{-3t}+3e^{-2t})1(t)} \quad g(t) = (-2e^{-3t} - 7te^{-3t} + 3e^{-2t})1(t)$

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f) $G(s) = \frac{2s+3}{(s+1)^3} = \frac{A}{(s+1)^3} + \frac{B(s+1)}{(s+1)^2} + \frac{C(s+1)^2}{(s+1)} = \frac{1}{(s+1)^3} + \frac{2}{(s+1)^2}$

$A + B(s+1) + C(s+1)^2 = 2s+3$

$A + Bs + B + Cs^2 + 2Cs + C = 2s+3$

$2! = 2$

$(A+B+C)=3$	$A=1$
$B+2C=2$	$B=2$
$C=0$	$C=0$

$g(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^3} + \frac{2}{(s+1)^2}\right\} = \left(\frac{1}{2}t^2e^{-t} + 2te^{-t}\right)1(t)$

ZADANIE 5:

DANE: $Y(s), U(s)$ SZUKANE: $G(s)$, $g(t)$

WZORY: $\frac{Y(s)}{U(s)} = G(s)$ $\quad g(t) = \mathcal{L}^{-1}\{G(s)\}$

a) $2y'' + 12y' + 10y = 2u' + 8u$

$$2s^2 Y(s) + 12s\,Y(s) + 10\,Y(s) = 2s\,U(s) + 8\,U(s)$$

$$Y(s)(2s^2+12s+10) = U(s)(2s+8)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s+8}{2s^2+12s+10} = \frac{2s+8}{2(s+1)(s+5)} = \frac{s+4}{(s+1)(s+5)}$$

$\Delta = 144 - 80 = 64$ $\quad \sqrt{\Delta} = 8$ $\quad s_1 = \frac{-12-8}{4} = -5$

$s_2 = \frac{-12+8}{4} = \frac{-4}{4} = -1$

RESIDUA:

$$g(t) = 1(t)\left(\frac{s+4}{s+5}e^{st}\Big|_{s=-1} + \frac{s+4}{s+1}e^{st}\Big|_{s=-5}\right) =$$

$\cancel{\frac{-2+8}{-4+5} = \frac{6}{4} = 1{,}5e^{-t}}$ $\quad \cancel{\frac{-10+8}{-5+1} = \frac{-2}{-4} = 0{,}5e^{-5t}}$

$\cancel{= (1{,}5e^{-t} + 0{,}5e^{-5t})1(t)}$

$\frac{-3}{-4}e^{-t} = \frac{3}{4}e^{-t}$ $\quad + \frac{-1}{-4}e^{-5t}$

$$= \left(\frac{3}{4}e^{-t} + \frac{1}{4}e^{-5t}\right)1(t)$$

b) $2y'' + 12y' + 16y = 8u' + 4u$

$$Y(s)(2s^2+12s+16) = U(s)(8s+4)$$

$$G(s) = \frac{8s+4}{2s^2+12s+16} = \frac{4s+2}{s^2+6s+8} = \frac{4s+2}{(s+4)(s+2)}$$

$\Delta = 36 - 32$ $\quad \sqrt{\Delta} = 2$ $\quad s_1 = \frac{-6-2}{2} = -4$

$s_2 = \frac{-6+2}{2} = -2$

$$g(t) = 1(t)\left(\frac{4s+2}{s+2}e^{st}\Big|_{s=-4} + \frac{4s+2}{s+4}e^{st}\Big|_{s=-2}\right) = (7e^{-4t} - 3e^{-2t})1(t)$$

$= 1(t)\big($

$\frac{\cancel{-16+2}\ -14}{-2}e^{-4t} = 7e^{-4t}$ $\quad \frac{\cancel{-8+2}\ -6}{\cancel{-2+4}\ 2}e^{-2t} = -3e^{-2t}$

c) $3y'' + 15y' + 12y = 9u' + 6u$

$Y(s)(3s^2+15s+12) = U(s)(9s+6)$

$G(s) = \frac{9s+6}{3s^2+15s+12} = \frac{3s+2}{s^2+5s+4} = \frac{3s+2}{(s+4)(s+1)}$

$\Delta = 25-16 = 9 \quad \sqrt{\Delta} = 3 \quad s_1 = \frac{-5-3}{2} = -4 \quad s_2 = \frac{-5+3}{2} = -1$

$g(t) = 1(t)\left(\frac{3s+2}{s+1}e^{st}\Big|_{s=-4} + \frac{3s+2}{s+4}e^{st}\Big|_{s=-1}\right) = \left(3\frac{1}{3}e^{-4t} - \frac{1}{3}e^{-t}\right)1(t)$

$\frac{-12+2}{-3}e^{-4t} = \frac{-10}{-3}e^{-4t} \qquad \frac{-3+2}{3}e^{-t} = \frac{-1}{3}e^{-t}$

ZADANIE 6:

DANE: $Y(s), U(s)$ SZUKANE: $G(s)$, $y_1(t)$

WZORY: $G(s) = s \cdot \mathcal{L}\{y_1(t)\} \rightarrow \frac{G(s)}{s} = \mathcal{L}\{y_1(t)\}$

$y_1(t) = \mathcal{L}^{-1}\left\{\frac{G(s)}{s}\right\} \qquad G(s) = \frac{Y(s)}{U(s)}$

a) $y'' + 4y' + 3y = u' + u$

$Y(s)(s^2+4s+3) = U(s)(s+1)$

$G(s) = \frac{s+1}{s^2+4s+3} \qquad \cancel{\frac{G(s)}{s} = \frac{s+1}{s^3+4s^2+3s}}$

$\frac{G(s)}{s} = \frac{s+1}{s(s^2+4s+3)} = \frac{s+1}{s(s+1)(s+3)} = \frac{1}{s(s+3)}$

$\Delta = 16-12 = 4 \quad \sqrt{\Delta} = 2 \quad s_1 = \frac{-4-2}{2} = -3 \quad s_2 = \frac{-4+2}{2} = -1$

$y_1(t) = 1(t)\left(\frac{1}{s+3}e^{st}\Big|_{s=0} + \frac{1}{s}e^{st}\Big|_{s=-3}\right) = \left(\frac{1}{3} - \frac{1}{3}e^{-3t}\right)1(t)$

b) $y''' + 5y'' + 6y' = 2u'' + 6u'$

$Y(s)(s^3+5s^2+6s) = U(s)(2s^2+6s)$

$G(s) = \frac{2s^2+6s}{s^3+5s^2+6s} = \frac{2s+6}{s^2+5s+6} = \frac{2s+6}{(s+3)(s+2)} = \frac{2(s+3)}{(s+3)(s+2)} = \frac{2}{s+2}$

$\Delta = 25-24 \quad \sqrt{\Delta} = 1 \quad s_1 = \frac{-5-1}{2} = -3 \quad s_2 = \frac{-5+1}{2} = -2$

$G(s)/s = \frac{2}{s(s+2)}$

$y_1(t) = 1(t)\left(\frac{2}{s+2}e^{st}\Big|_{s=0} + \frac{2}{s}e^{st}\Big|_{s=-2}\right) = \left(1 - e^{-2t}\right)1(t)$

$\frac{2}{2}e^{0} = 1 \qquad \frac{2}{-2}e^{-2t} = -e^{-2t}$

c) $y'' + 5y' + 4y = u'' + u' + u$

$Y(s)(s^2+5s+4) = U(s)(s^2+s+1)$

$G(s) = \frac{s^2+s+1}{s^2+5s+4} =$

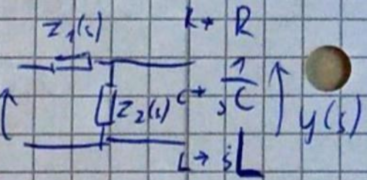
$\frac{G(s)}{s} = \frac{s^2+s+1}{s(s^2+5s+4)} = \frac{s^2+s+1}{s(s+4)(s+1)}$

$\Delta = 25-16 = 9 \quad \sqrt{\Delta} = 3 \quad s_1 = \frac{-5-3}{2} = -4 \quad s_2 = \frac{-2}{2} = -1$

$y_1(t) = 1(t)\left(\frac{s^2+s+1}{(s+4)(s+1)} e^{st}\Big|_{s=0} + \frac{s^2+s+1}{s(s+1)} e^{st}\Big|_{s=-4} + \frac{s^2+s+1}{s(s+4)} e^{st}\Big|_{s=-1} \right)$

$= 1(t)\left(\frac{1}{4} + \frac{13}{12} e^{-4t} - \frac{1}{3} e^{-t} \right)$

$\frac{16-4+1}{-4\cdot(-3)} = \frac{13}{12} \qquad \frac{1-1+1}{-1(+3)} = \frac{1}{-3}$

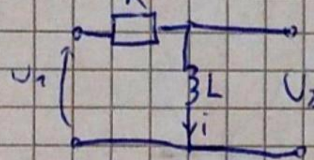


ZADANIE 7

WZORY: $G(s) = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$

DANE: U, Y

SZUKANE: $G(s)$



$U_1 = u(t) \qquad U_2 = L\frac{di(t)}{dt}$

$U_2 = y(t)$

$U_1 - Ri(t) - L\frac{di(t)}{dt} = 0$

$U_2 = L\frac{di(t)}{dt} \rightarrow \frac{U_2}{L} = \frac{di(t)}{dt}$

$U_1 - i(t)(R+Ls) = 0$

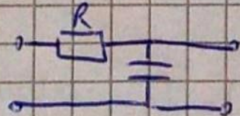
$U_2 = Ls\, i(t) \rightarrow i(t) = \frac{U_2}{Ls}$

$U_1 - \frac{U_2}{Ls}(R+Ls) = 0$

$U_1 = \frac{U_2}{Ls}(R+Ls) = U_2\frac{R}{Ls} - U_2 = U_2\left(\frac{R}{Ls}+1\right)$

$\frac{1}{G(s)} = \frac{U_2}{U_1} = \frac{R-Ls}{Ls} \rightarrow G(s) = \frac{Ls}{R+Ls} = Ts+1 \qquad T = \frac{L}{R}$

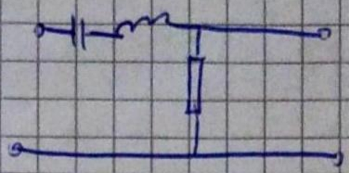
b)



$G(s) = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{\frac{1}{sC}}{\frac{RsC+1}{sC}} = \frac{1}{\cancel{sC}} \cdot \frac{\cancel{sC}}{RsC+1} =$

$= \frac{1}{sRC+1} = \frac{1}{Ts+1}$

c)



$$G(s) = \frac{R}{R + Ls + \frac{1}{sC}} = \frac{R}{\frac{RsC + Ls^2C + 1}{sC}} =$$

$$= \frac{sCR}{sRC + Ls^2 + 1} = \frac{sRC}{Ls^2 + sRC + 1}$$

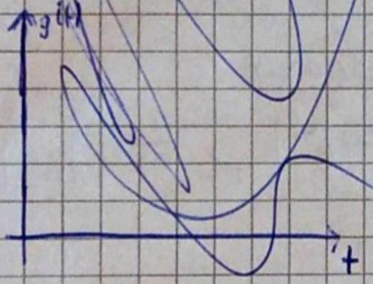
LISTA 2

ZADANIE 1, 2, 3, 4:

1) $G(s) = 5$

CHAR. IMPULSOWA

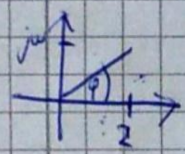
$g(t) = \mathcal{L}^{-1}\{5\} = 5\delta(t)$



63,43
26,56

CHAR. AMPLITUDOWO-FAZOWA NA KARCIE NICHOLSA

$\varphi = \text{arctg}\left(\frac{\omega}{2}\right)$

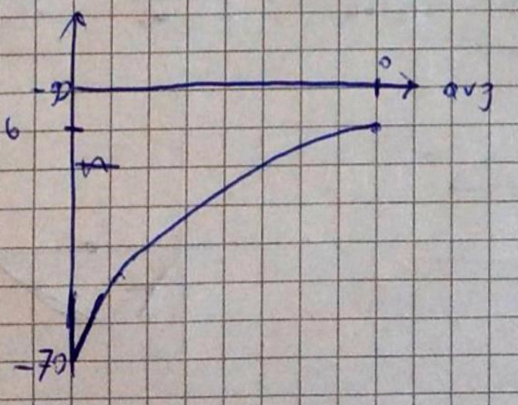


$G(s) = \frac{1}{s+2}$ $\quad G(j\omega) = \frac{1}{j\omega+2}$

$L(\omega) = |G(j\omega)|_{dB} = 20\log\left|\frac{1}{j\omega+2}\right| = 20\log\left(\frac{1}{\sqrt{\omega^2+4}}\right)$

$\arg\{G(j\omega)\} = \arg\left\{\frac{1}{j\omega+2}\right\} = -\text{arctg}\left(\frac{\omega}{2}\right)$

ω	0	5	10	∞
$L(\omega)$	-6,02	-14,62	-20,17	-70
$\arg(\omega)$	0	-68,2	-78,69	-90



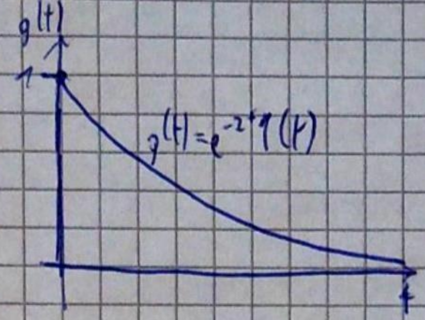
ZADANIE 1.2.3.6

b) $G(s) = \frac{1}{s+2}$

$g(t) = e^{-2t} 1(t)$

CHAR. IMPULSOWA:

t	0	1	∞
$g(t)$	1	e^{-2}	0



CHAR. ODP. NA SKOK JEDNOSTKOWY

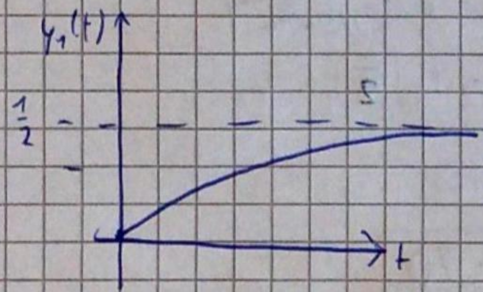
$y_1(t) = \mathcal{L}^{-1}\left\{\frac{G(s)}{s}\right\} = \frac{1}{s(s+2)} = 1(t)\left(\frac{1}{s} e^{st}\Big|_{s=-2} + \frac{1}{s+2} e^{st}\Big|_{s=0}\right) =$

$y_1(t) = 1(t)\left(\frac{-1}{2} e^{-2t} + \frac{1}{2}\right)$

t	0	∞
$y_1(t)$	0	$\frac{1}{2}$

$-\frac{1}{2} \cdot e^{0} = -\frac{1}{2} + \frac{1}{2} = 0$

$-\frac{1}{2} \cdot \underbrace{e^{-\infty}}_{\to 0} + \frac{1}{2} = \frac{1}{2}$

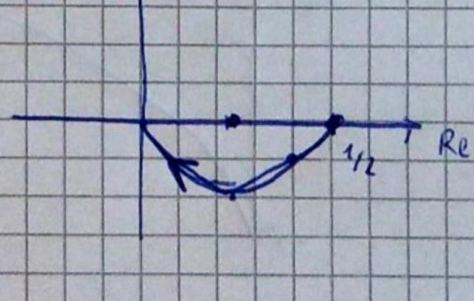


CHAR. NYQUISTA (AMPLITUDOWO - FAZOWA)

$G(j\omega) = \frac{1}{j\omega+2} = \frac{j\omega-2}{(j\omega+2)(j\omega-2)} = \frac{j\omega-2}{-\omega^2-4} = \frac{j\omega-2}{-(\omega^2+4)} = \frac{(2-j\omega)}{(\omega^2+4)} =$

$= \frac{2}{\omega^2+4} - j\frac{\omega}{\omega^2+4}$

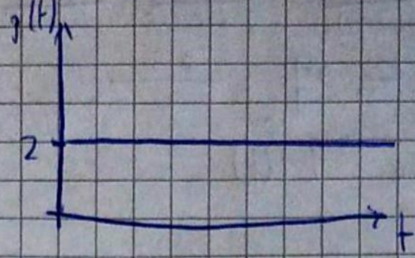
ω	0	1	2	∞
$Re\{G(j\omega)\}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{4}$	0
$Im\{G(j\omega)\}$	0	$-\frac{1}{5}$	$-\frac{1}{4}$	0



c) $G(s) = \frac{2}{s} = 2 \cdot \frac{1}{s}$

$g(t) = 2 \cdot 1(t)$

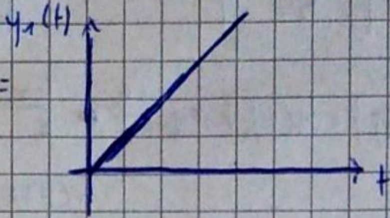
CHAR. IMPULSOWA



$y_1(t) = \mathcal{L}^{-1}\left\{\frac{2}{s^2}\right\} = 2\,\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} =$

$= 2t\,1(t)$

CHAR. ODP. NA SKOK JEDN.

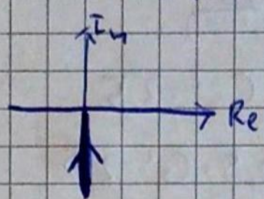


$G(j\omega) = \frac{1}{j\omega} = \frac{j\omega}{j\omega \cdot j\omega} = \frac{j\omega}{-\omega^2} =$

$= \frac{-j}{\omega}$

ω	0	1	∞
$Re\{G(j\omega)\}$	0	0	0
$Im\{G(j\omega)\}$	$-\infty$	-1	0

CHAR. NYQUISTA

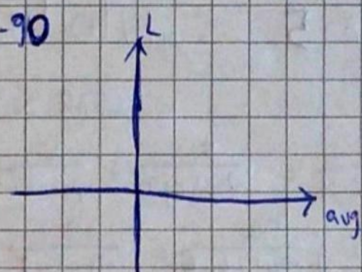


$L(\omega) = 20\log\left|\frac{-j}{\omega}\right| = 20\log\left(\frac{1}{\omega}\right)$

$arg\{G(j\omega)\} = -arctg(-j\ \cancel{\omega}) = -90$

ω	0
$L(\omega)$	-100
$arg(G(j\omega))$	-90

CHAR. AMPLITUDOWO-FAZOWA NA KARCIE NICHOLSA



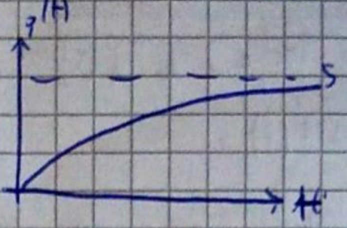
d) $G(s)=\frac{5}{s(2s+1)}=\frac{5}{2\cdot s\cdot(s+0,5)}=\frac{2,5}{s(s+0,5)}$

$g(t)=1(t)\left(\frac{2,5}{s+0,5}e^{st}\Big|_{s=0}+\frac{2,5}{s}e^{st}\Big|_{s=-0,5}\right)=\left(5+(-5)e^{-0,5t}\right)1(t)$

$g(t)=\left(5-5e^{-0,5t}\right)1(t)$

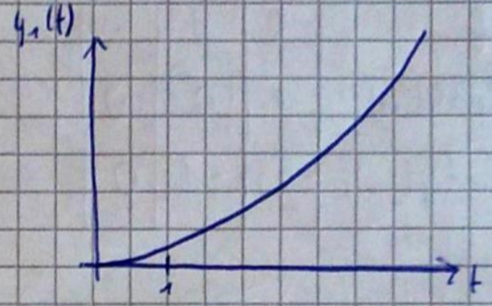
CHAR. IMPULSOWA

t	0	1	∞
$g(t)$	0	1,97	5



$y_1(t)=\mathcal{L}^{-1}\left\{\frac{2,5}{s^2(s+0,5)}\right\}=\mathcal{L}^{-1}\left(\frac{5}{s^2}-\frac{10}{s}+\frac{10}{s+0,5}\right)=\left(5t-10+10e^{-0,5t}\right)1(t)$

CHAR. ODP. NA SK. JEDN.



$\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+0,5}=\frac{2,5}{s^2(s+0,5)}$

$A(s+0,5)+Bs(s+0,5)+Cs^2=2,5$

$As+0,5A+Bs^2+Bs\,0,5+Cs^2=2,5$

$B+C=0$ | $C=10$

$A+0,5B=0$ | $B=-10$

$0,5A=2,5$ | $A=5$

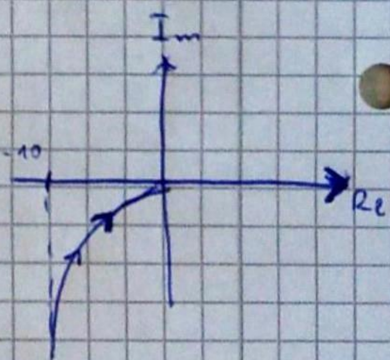
t	0	1	10	∞
$y(t)$	0	1,06	40,06	∞

CHAR. NYQUISTA

$G(j\omega)=\frac{5}{j\omega(2j\omega+1)}=\frac{5}{-2\omega^2+j\omega}=\frac{5}{j\omega-2\omega^2}\cdot\frac{(j\omega+2\omega^2)}{j\omega+2\omega^2}=\frac{5(j\omega+2\omega^2)}{-\omega^2-4\omega^4}=$

$=\frac{10\omega^2}{-\omega^2-4\omega^4}+j\frac{5\omega}{-\omega^2-4\omega^4}=\frac{10}{-1-2\omega^2}+j\frac{5}{-\omega-4\omega^3}$

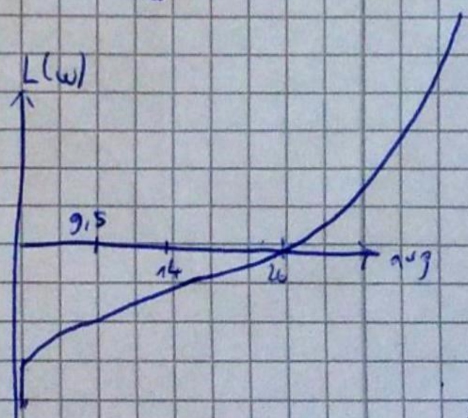
ω	0	1	2	∞
$Re\{G(j\omega)\}$	-10	-3,33	-1,11	0
$Im\{G(j\omega)\}$	$-\infty$	-1	-0,15	0



$L(\omega)=20\log\left|\frac{5}{j\omega(2j\omega+1)}\right|=20\log\left|\frac{5}{(-2\omega^2+j\omega)}\right|=20\log\frac{5}{\sqrt{\omega^2+4\omega^4}}$

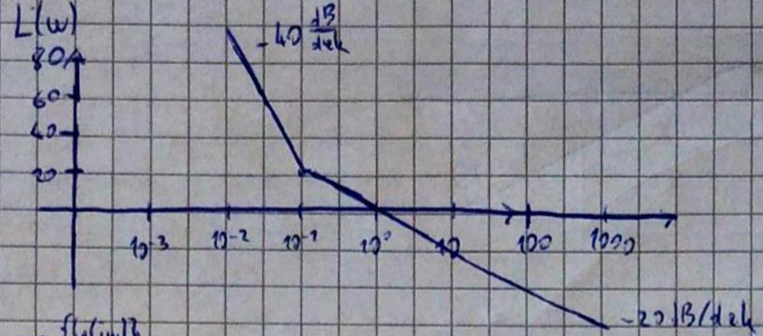
$\arg\{G(j\omega)\}=-\mathrm{arctg}\left(\frac{\omega}{-2\omega^2}\right)=-\mathrm{arctg}\left(\frac{-1}{2\omega}\right)$

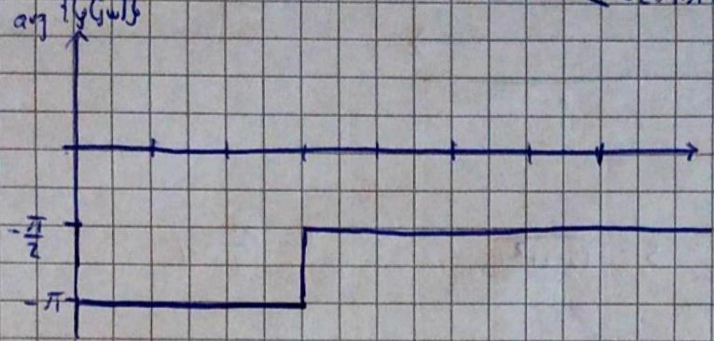
ω	0	1	2	3	∞
arg	90	26,56	14	9,46	0
$L(\omega)$	∞	6,99	-4,35	-11,25	$-\infty$



ZADANIE 5, WYKRESY BODEGO

a) $G(s) = \frac{10s+1}{s^2} = \frac{1}{s^2}\cdot(10s+1)$ $\qquad \frac{1}{10} \to 0,1$

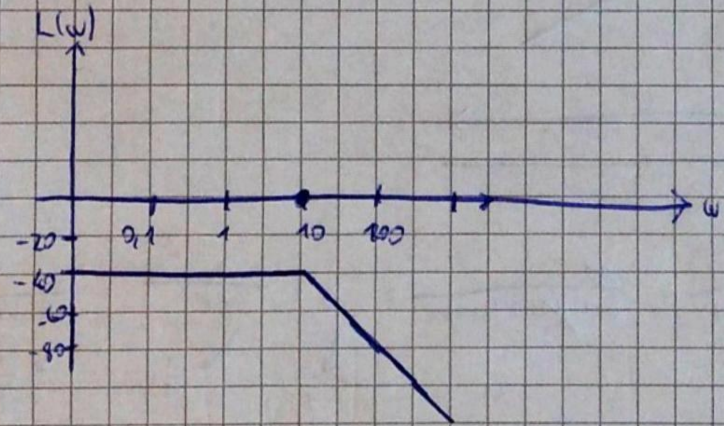


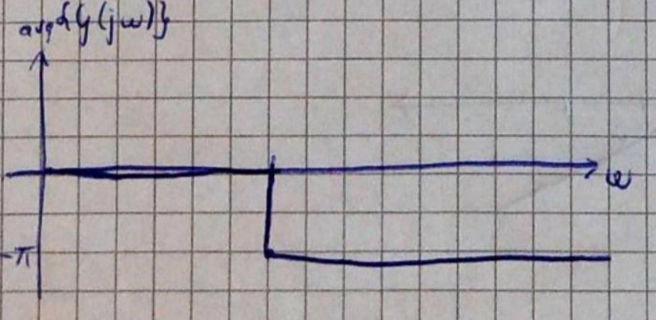


b) $G(s) = \frac{1}{(s+10)^2} = \frac{1}{((0,1s+1)(10))^2}$

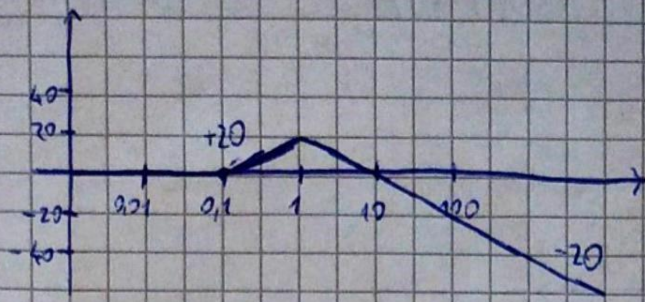
$s^2+20s+100 \qquad 100(0,01s^2+0,2s+1)$

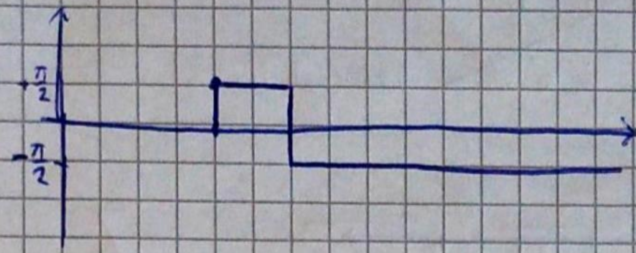
$G(s) = \frac{1}{100\cdot(0,1s+1)^2} = \frac{1}{100}\cdot\frac{1}{(0,1s+1)^2}$ $\qquad k = 0,01$



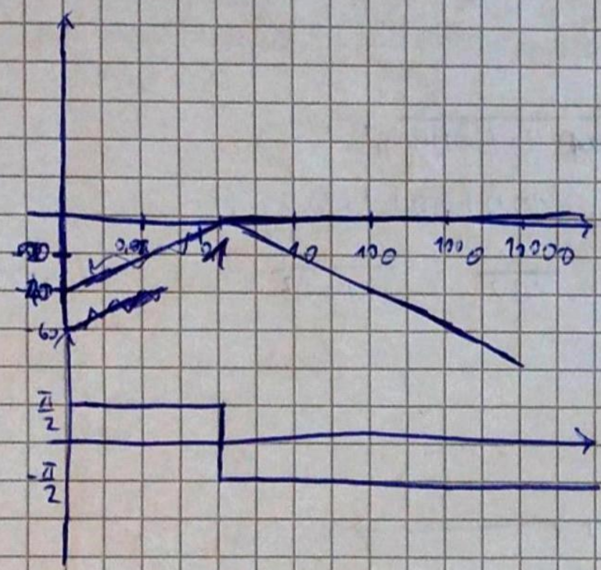


c) $G(s) = \frac{s+1}{(0{,}1s+1)^2} = (s+1) \cdot \frac{1}{(0{,}1s+1)^2}$



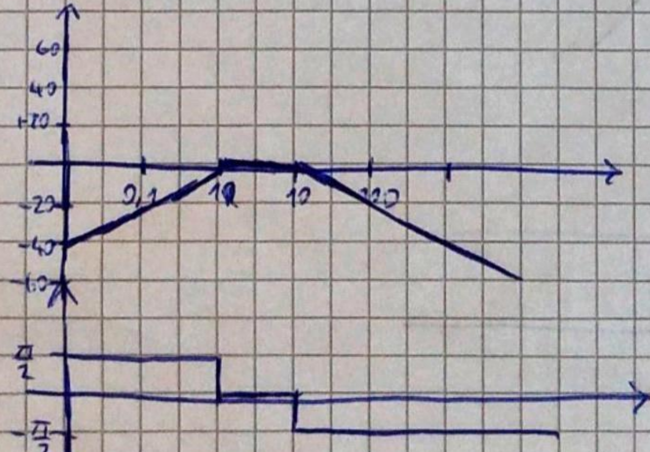


d) $G(s) = \frac{s}{(s+1)^2} = s \cdot \frac{1}{(s+1)^2}$ $\quad T = \frac{1}{1} = 1$



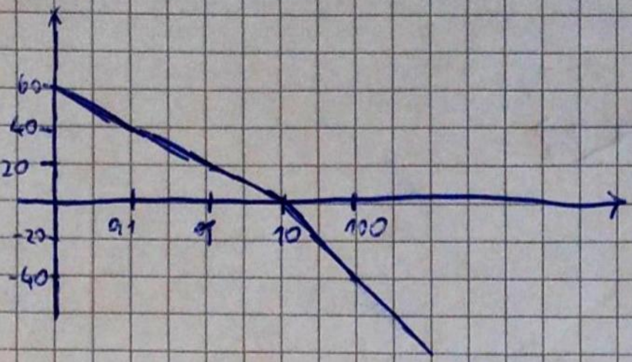
e) $G(s) = \frac{10s}{(s+1)(s+10)} = \frac{\cancel{10}s}{(s+1)\cancel{10}(0{,}1s+1)} = s \cdot \frac{1}{s+1} \cdot \frac{1}{0{,}1s+1}$

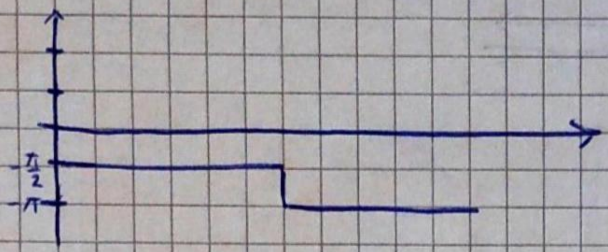
$s \rightarrow \frac{1}{1}$ $\quad \frac{1}{s+1} \rightarrow \frac{1}{1}$ $\quad \frac{1}{0{,}1s+1} \rightarrow \frac{1}{0{,}1} = 10$



f) $G(s) = \frac{100}{s(s+10)} = \frac{10}{s(0{,}1s+1)} = 10 \cdot \frac{1}{s} \cdot \frac{1}{(0{,}1s+1)}$

$\frac{1}{1} = 1$ $\quad \frac{1}{0{,}1} = 10$

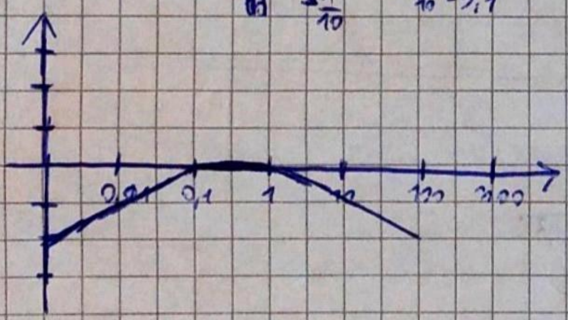


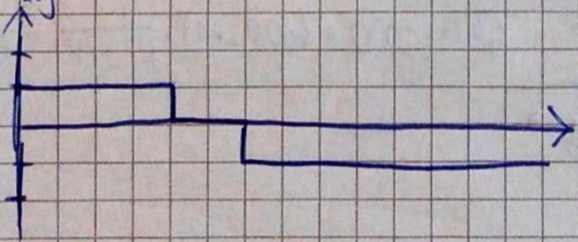


g) $G(s) = \frac{s}{(s+1)(s+0{,}1)} = \frac{s}{(s+1)0{,}1(10s+1)} = 10s \cdot \frac{1}{s+1} \cdot \frac{1}{10s+1} =$

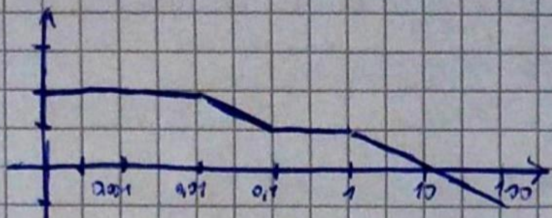
$G(s) = 10s \cdot \frac{1}{10s+1} \cdot \frac{1}{s+1}$

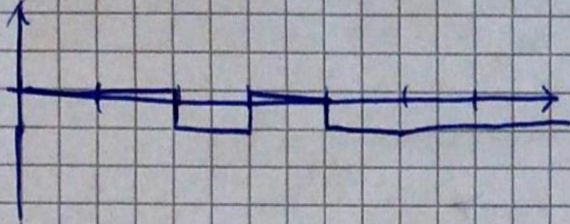
$\frac{1}{10}$ $\quad \frac{1}{10} = 0{,}1$ $\quad \frac{1}{1} = 1$





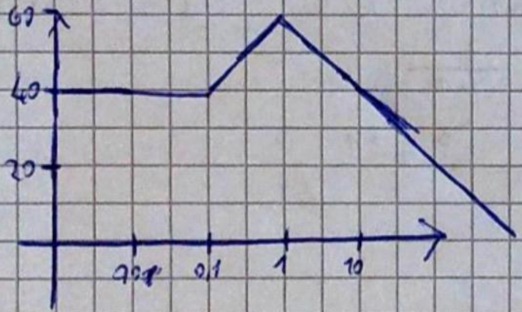
j) $G(s) = \frac{10(10s+1)}{(0{,}1s+0{,}1)(100s+1)} = \frac{10(10s+1)}{0{,}1(s+1)(100s+1)} = 100 \cdot \frac{1}{100s+1} \cdot (10s+1) \cdot \frac{1}{s+1}$





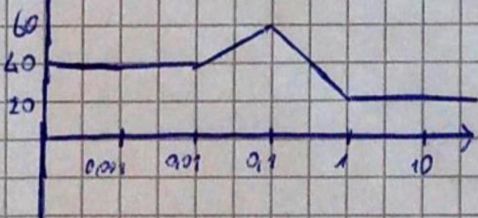
ZADANIE 6:

a)



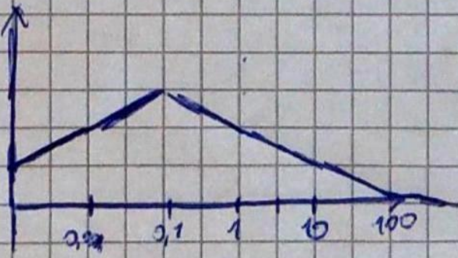
$G(s) = 100 \cdot (10s+1) \cdot \frac{1}{(s+1)^2}$

b)



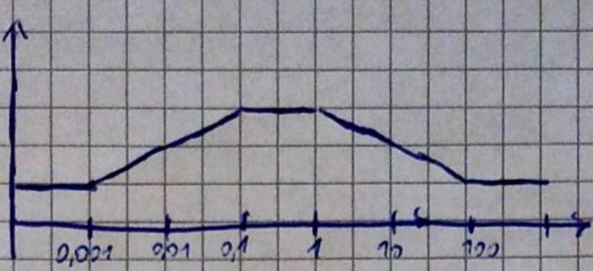
$G(s) = 100 \cdot (100s+1) \cdot \frac{1}{(10s+1)^3} \cdot \frac{1}{(s+1)^2}$

c)



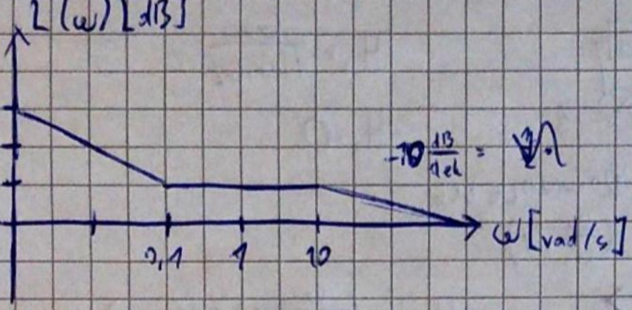
$G(s) = 10000s \cdot \frac{1}{(10s+1)^2}$

d)



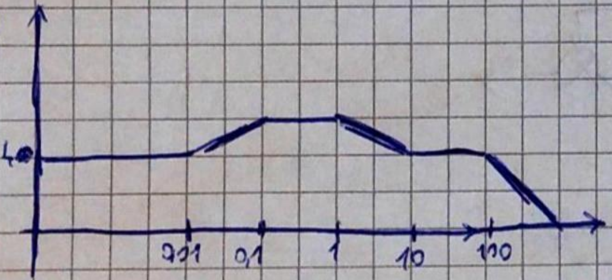
$$G(s) = 10 \cdot (1000s+1) \cdot \left(\frac{1}{10s+1}\right) \cdot \left(\frac{1}{s+1}\right) \cdot (0{,}01s+1)$$

e)



$$G(s) = \frac{1}{s} \cdot (10s+1) \cdot \left(\frac{1}{(0{,}1s+1)^{0{,}5}}\right)$$

f)

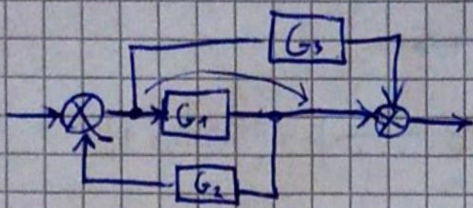


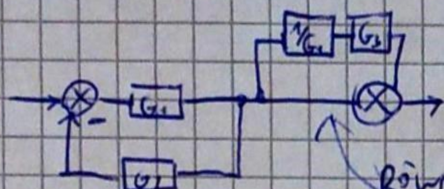
$$G(s) = 100 \cdot (100s+1)\left(\frac{1}{10s+1}\right)\left(\frac{1}{s+1}\right)(0{,}1s+1)\left(\frac{1}{(0{,}01s+1)^2}\right)$$

LISTA 3

ZADANIE 1

a)

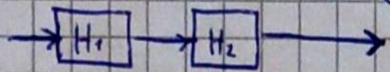




RÓWNOLEGLE $\left(\frac{G_3}{G_1}+1\right)$

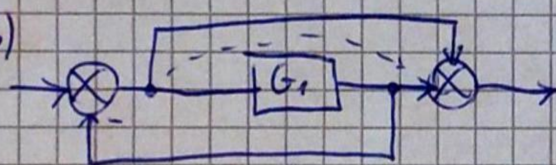
$$H_1 = \frac{G_1}{1+G_1G_2}$$

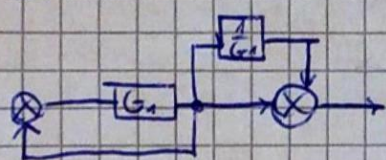
$$H_2 = 0$$



$$H = \left(\frac{G_1}{1+G_1G_2}\right)\left(\frac{G_2}{G_1}+1\right)$$

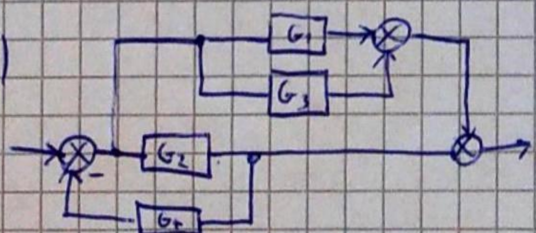
b)

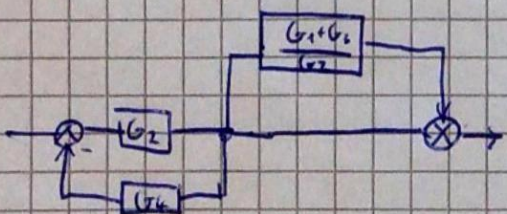




$$H = \left(\frac{G_1}{1+G_1}\right)\left(\frac{1}{G_1}+1\right) = \frac{G_1}{1+G_1} \cdot \frac{G_1+1}{G_1} = 1$$

c)



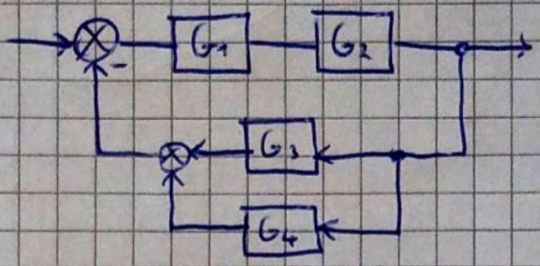


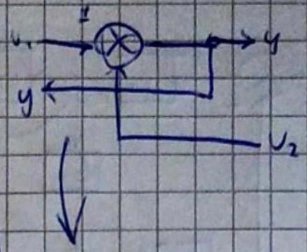
$$H(s) = \left(\frac{G_2}{1+G_2G_4}\right)\left(1+\frac{G_1+G_3}{G_2}\right) = \left(\frac{\cancel{G_2}}{1+G_2G_4}\right)\left(\frac{G_2+G_1+G_3}{\cancel{G_2}}\right) =$$

$$\frac{G_1+G_2+G_3}{1+G_2G_4}$$

ZADANIE 1 / L3

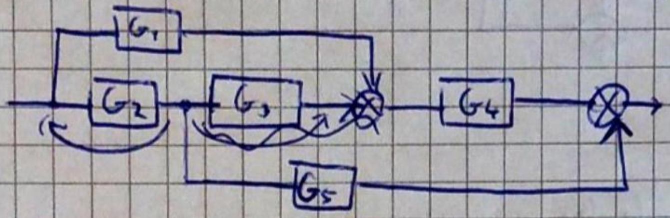
d)

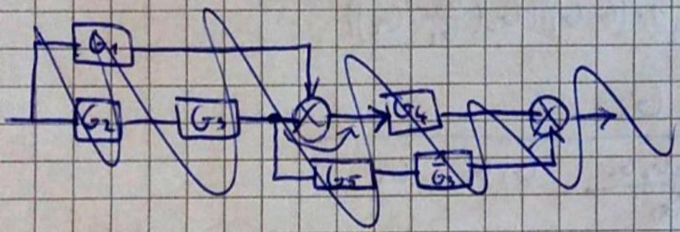


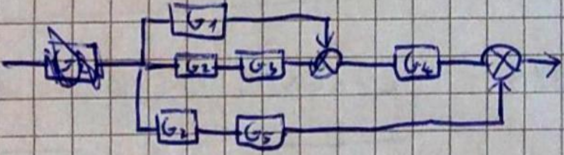


$$H(s) = \frac{G_1 G_2}{1 + G_1 G_2 (G_3 + G_4)}$$

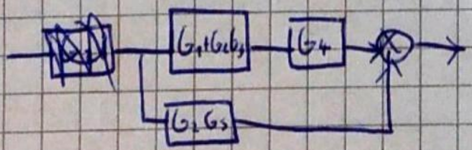
e)







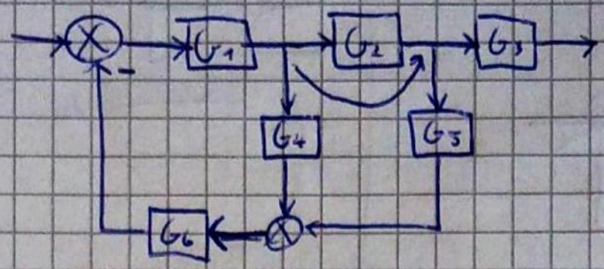
$$\cancel{H(s) = G_2 \cdot \frac{(G_1 + G_2 G_3) G_4}{1 + (G_1 + G_2 G_3) G_4 G_2 G_5} =}$$

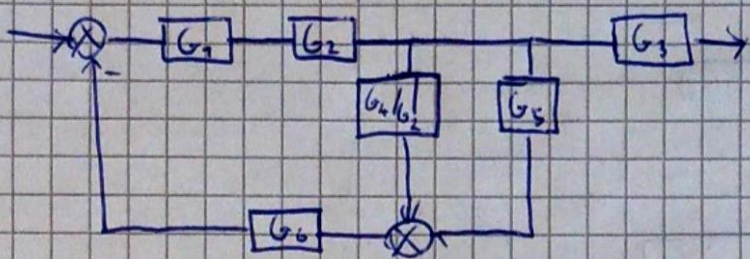


$$H(s) = \cancel{G_2} \left((G_1 + G_2 G_3) G_4 + G_2 G_5 \right) =$$

$$= G_1 G_4 + G_2 G_3 G_4 + G_2 G_5$$

f)





$$H(s) = G_3 \cdot \frac{G_1G_2}{1+(G_1G_2)\left(G_6\left(\frac{G_4}{G_2}+G_5\right)\right)} =$$

$$= \frac{G_1G_2G_3}{1+\frac{G_1\cancel{G_2}G_6G_4}{\cancel{G_2}}+G_1G_2G_6G_5}$$

ZADANIE 2/L3: UCHYB POŁOŻENIA, PRĘDKOŚCI, PRZYSPIESZENIA

WZORY: $e_p = \frac{1}{1+\lim\limits_{s\to 0} G_{12}(s)}$ $\quad e_v = \frac{1}{\lim\limits_{s\to 0} s\,G_{12}(s)}$ $\quad e_a = \frac{1}{\lim\limits_{s\to 0} s^2 G_{12}(s)}$

a) $G_{12}(s) = 4$

$e_p = \frac{1}{1+\lim\limits_{s\to 0} G_{12}(s)} = \frac{1}{5}$ $\quad e_v = \frac{1}{\lim\limits_{s\to 0} s\cdot G_{12}(s)} = \infty$ $\quad e_a = \infty$

b) $G_{12}(s) = \frac{5}{s}$

$e_p = \frac{1}{1+\lim\limits_{s\to 0}\frac{5}{s}} = 0$

$e_v = \frac{1}{\lim\limits_{s\to 0}\cancel{s}\frac{5}{\cancel{s}}} = \frac{1}{5}$

$\frac{s+5}{s} = \frac{\cancel{s}(1+\frac{5}{s})}{\cancel{s}}$

$e_a = \frac{1}{\lim\limits_{s\to 0} s^2\frac{5}{s}} = \frac{1}{\lim\limits_{s\to 0} 5s} = \infty$

c) $G_{12}(s) = \frac{s+5}{s^2}$ $\quad e_p = \frac{1}{1+\lim\limits_{s\to 0}\frac{s+5}{s^2}} = 0$

$e_v = \frac{1}{\lim\limits_{s\to 0} s\frac{s+5}{s^2}} = 0$ $\quad e_a = \frac{1}{\lim\limits_{s\to 0}\cancel{s^2}\frac{s+5}{\cancel{s^2}}} = \frac{1}{5}$

ZADANIE 2/L3:

d) $G_{+z}(s) = \frac{5}{s+5}$ $\qquad e_p = \frac{1}{1+\lim\limits_{s\to 0}\frac{5}{s+5}} = \frac{1}{2}$

$e_v = \frac{1}{\lim\limits_{s\to 0} s\frac{5}{s+5}} = \frac{1}{\frac{0}{5}} = \infty$

$e_a = \frac{1}{\lim\limits_{s\to 0}\frac{5s^2}{s+5}} = \infty$

h) $G_{+z}(s) = \frac{4s^4+3s^2+2s+0,5}{s^4+2s^3}$ *

$\lim\limits_{s\to 0} G_{+z}(s) = \frac{\cancel{s^4}(4+3\frac{1}{s^2}+2\frac{1}{s^3}+0,5\frac{1}{s^4})}{\cancel{s^4}(1+\frac{1}{s})} = \infty$

$e_p = \frac{1}{1+\lim\limits_{s\to 0} G_{+z}(s)} = 0$

$e_v = \frac{1}{\lim\limits_{s\to 0} sG_{+z}(s)} = 0$

$e_a = 0$

i) $G_{+z}(s) = \frac{4}{(s+1)^3}$

$e_p = \frac{1}{1+\lim\limits_{s\to 0}\frac{4}{(s+1)^3}} = \frac{1}{1+\frac{4}{1}} = \frac{1}{5}$

$e_v = \frac{1}{\lim\limits_{s\to 0} s\cdot\frac{4}{(s+1)^3}} = \infty$

$e_a = \frac{1}{\lim\limits_{s\to 0} s^2\cdot\frac{4}{(s+1)^3}} = \infty$

LISTA 4

ZADANIE 1: KRYTERIUM ROUTHA

~~DANE~~ SZUKANE: BIEGUNY Z PRAWEJ I LEWEJ, STABILNOŚĆ

a) $G(s) = \frac{10s+1}{5s^4+4s^3+3s^2+2s+1}$ rzędu: 5

$$\left|\begin{matrix} 5 & 3 & 1 \\ 4 & 2 & 0 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{matrix}\right|$$

$b_1 = \frac{\begin{vmatrix} 5 & 3 \\ 4 & 2 \end{vmatrix}}{-4} = \frac{-2}{-4} = 0{,}5$

$b_2 = \frac{\begin{vmatrix} 5 & 1 \\ 4 & 0 \end{vmatrix}}{-4} = \frac{-4}{-4} = 1$

$b_3 = \frac{\begin{vmatrix} 2 & 0 \\ 4 & 0 \end{vmatrix}}{-4} = 0$

$$\left|\begin{matrix} 5 & 3 & 1 \\ 4 & 2 & 0 \\ 0{,}5 & 1 & 0 \\ -6 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}\right|$$

$c_1 = \frac{\begin{vmatrix} 4 & 2 \\ 0{,}5 & 1 \end{vmatrix}}{-0{,}5} = \frac{4-\cancel{1}^{\,3}}{-0{,}5} = -6$

$c_2 = \frac{\begin{vmatrix} 4 & 0 \\ 0{,}5 & 0 \end{vmatrix}}{-0{,}5} = 0$

$c_3 = 0$

$[5 \quad 4 \quad 0{,}5 \quad -6 \quad 0] \rightarrow$ 4 bieguny w lewej

NIESTABILNY

b) $G(s) = \frac{1}{s^4+4s^3+3s^2+2s+1}$

$$\left|\begin{matrix} 1 & 3 & 1 \\ 4 & 2 & 0 \\ 2{,}5 & 1 & 0 \\ 0{,}4 & 0 & 0 \\ 1 & 0 & 0 \end{matrix}\right|$$

$b_1 = \frac{\begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix}}{-4} = \frac{2-12}{-4} = 2{,}5$

$b_2 = \frac{\begin{vmatrix} 1 & 1 \\ 4 & 0 \end{vmatrix}}{-4} = \frac{0-4}{-4} = 1$

$c_1 = \frac{\begin{vmatrix} 4 & 2 \\ 2{,}5 & 1 \end{vmatrix}}{-2{,}5} = \frac{4-5}{-2{,}5} = \frac{-1}{-2{,}5} = 0{,}4$

STABILNY

$c_2 = 0$

4 z lewej

$d_1 = \frac{\begin{vmatrix} 2{,}5 & 1 \\ 0{,}4 & 0 \end{vmatrix}}{-0{,}4} = \frac{-0{,}4}{-0{,}4} = 1$

c) $G(s) = \frac{3s+1}{s^4+5s^3+5s^2-5s-6}$

$$\left|\begin{matrix} 1 & 5 & -6 \\ 5 & -5 & 0 \\ 6 & -\frac{31}{5} & 0 \\ -\frac{1}{6} & 0 & 0 \\ -\frac{31}{5} & 0 & 0 \end{matrix}\right|$$

$b_1 = \frac{\begin{vmatrix} 1 & 5 \\ 5 & -5 \end{vmatrix}}{-5} = \frac{-5-25}{-5} = \frac{-30}{-5} = 6$

$b_2 = \frac{\begin{vmatrix} 1 & -6 \\ 5 & 0 \end{vmatrix}}{-5} = \frac{1+30}{-5} = \frac{-31}{5}$

$c_1 = \frac{\begin{vmatrix} 5 & -5 \\ 6 & -\frac{31}{5} \end{vmatrix}}{-6} = \frac{-31+30}{-6} = \frac{-1}{6}$

NIESTABILNY

$d_1 = \frac{\begin{vmatrix} 6 & -\frac{31}{5} \\ -\frac{1}{6} & 0 \end{vmatrix}}{\frac{1}{6}} = \frac{-31}{\cancel{30}\,5} \cdot \cancel{6}^{\,1} = \frac{-31}{5}$

1 po PRAWEJ

3 po LEWEJ

ZADANIE 1/L4

d) $G(s)=\frac{s}{3s^3+2s^2+s+1}$

$$\begin{vmatrix} 3 & 1 & 0 \\ 2 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$b_1=\frac{\begin{vmatrix}3 & 1\\ 2 & 1\end{vmatrix}}{-2}=\frac{3-2}{-2}=\frac{1}{-2}$

$b_2=0$

$c_1=\frac{\begin{vmatrix}2 & 1\\ -0{,}5 & 0\end{vmatrix}}{0{,}5}=\frac{0{,}5}{0{,}5}=1$

NIESTABILNY
2 PRAWE, 1 LEWE

e) $G(s)=\frac{5}{s^3+2s^2+3s+4}$

$$\begin{vmatrix} 1 & 3 \\ 2 & 4 \\ 1 & 0 \\ 2 & 0 \end{vmatrix}$$

$b_1=\frac{\begin{vmatrix}1 & 3\\ 2 & 4\end{vmatrix}}{-2}=\frac{4-6}{-2}=1$

$c_1=\frac{\begin{vmatrix}2 & 4\\ 1 & 0\end{vmatrix}}{-1}=\frac{2-4}{-1}=2$

STABILNY, 3 Z LEWEJ

f) $G(s)=\frac{s+10}{5s^3+s^2+s+1}$

$$\begin{vmatrix} 5 & 1 \\ 1 & 1 \\ -4 & 0 \\ 1 & 0 \end{vmatrix}$$

$b_1=\frac{\begin{vmatrix}5 & 1\\ 1 & 1\end{vmatrix}}{-1}=\frac{5-1}{-1}=-4$

$c_1=\frac{\begin{vmatrix}1 & 1\\ -4 & 1\end{vmatrix}}{4}=\frac{4}{4}=1$

NIESTABILNY,
2 Z PRAWEJ 1L

ZADANIE 2, KRYT. MICHAJŁOWA $\Delta Arg\, M(j\omega)=n\frac{\pi}{2}$

a) $G(s)=\frac{2}{s^3+s^2+s+1}$ $n=3$

$K(s)=\frac{G(s)}{1+G(s)}=\frac{\frac{2}{s^3+s^2+s+1}}{1+\frac{2}{s^3+s^2+s+1}}=\frac{\frac{2}{s^3+s^2+s+1}}{\frac{s^3+s^2+s+3}{s^3+s^2+s+1}}=\frac{2}{s^3+s^2+s+3}$

$M(j\omega)=-j\omega^3-\omega^2+j\omega+3=(-\omega^2+3)+j(\omega-\omega^3)$

ω	0	1	2	10	∞
R_e	3	2	-1	-97	$-\infty$
Im	0	0	-6	-990	$-\infty$

NIESTABILNY

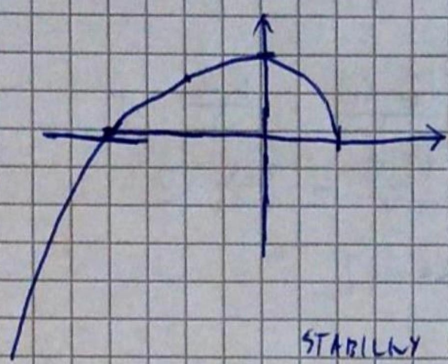
B) $G(s) = \dfrac{1}{s^3+2s^2+3s+1}$

$$K(s) = \frac{\frac{1}{s^3+2s^2+3s+1}}{1+\frac{1}{s^3+2s^2+3s+1}} = \frac{\frac{1}{s^3+2s^2+3s+1}}{\frac{s^3+2s^2+3s+2}{s^3+2s^2+3s+1}} = \frac{1}{s^3+2s^2+3s+2}$$

$$M(j\omega) = -j\omega^3 - 2\omega^2 + 3j\omega + 2 = (2-2\omega^2) + j(3\omega - \omega^3)$$

$$\omega(3-\omega^2) = \omega(\sqrt{3}-\omega)(\sqrt{3}+\omega)$$

ω	0	1	$\sqrt{2}$	$\sqrt{3}$	10	∞
Re	2	0	-2	-4		$-\infty$
Im	0	2	$\sqrt{2}$	0		$-\infty$



STABILNY

d) $\dfrac{e^{-s}}{s^2+s+1}$ $\quad K(s) = \dfrac{\frac{e^{-s}}{s^2+s+1}}{\frac{e^{-s}+s^2+s+1}{s^2+s+1}} = \dfrac{e^{-s}}{e^{-s}+s^2+s+1}$?

c) $G(s) = \dfrac{1}{3s^3+2s^2+3s+1}$

$$K(s) = \frac{\frac{1}{3s^3+2s^2+3s+1}}{\frac{3s^3+2s^2+3s+2}{3s^3+2s^2+3s+1}} = \frac{1}{3s^3+2s^2+3s+2}$$

$$M(j\omega) = -3j\omega^3 - 2\omega^2 + 3j\omega + 2 = (2-2\omega^2) + j(3\omega - 3\omega^3)$$

$$3\omega(1-\omega^2) = (3\omega)(1-\omega)(1+\omega)$$

ω	0	$\sqrt{2}$	1	$\sqrt{3}$	∞
Re	2	-2	0		$-\infty$
Im	0	$-3\sqrt{2}$	0		$-\infty$

STABILNY GRANICZNIE

LISTA 4, ZADANIE 3

KRYTERIUM NYQUISTA

a) $G_{12}(s) = \dfrac{4}{(s+1)(s^2+1{,}5s-1)}$

$s^2 + 1{,}5s - 1 \rightarrow \Delta = 2{,}25 + 4 = 6{,}25 \quad \sqrt{\Delta} = 2{,}5$

$s_1 = \dfrac{-1{,}5-2{,}5}{2} = \dfrac{-4}{2} = -2$

$s_2 = \dfrac{-1{,}5+2{,}5}{2} = \dfrac{1}{2}$

$G_{12}(s) = \dfrac{4}{(s+1)(s+2)(s-\frac{1}{2})}$ $\rightarrow$ 1 b. w prawej

$\Delta \arg(1+G(j\omega)) = 180^\circ$

$$G(j\omega) = \frac{4}{(j\omega)(-\omega^2+1{,}5j\omega-1)} = \frac{4}{-j\omega^3-1{,}5\omega^2-j\omega} = \frac{4}{-1{,}5\omega^2-(j\omega^3+j\omega)} =$$

$$= \frac{4(-1{,}5\omega^2+j\omega^3+j\omega)}{2{,}25\omega^4+(\omega^3+\omega)^2} = \frac{-6\omega^2+4j\omega^3+4j\omega}{2{,}25\omega^4+\omega^6+2\omega^4+\omega^2} =$$

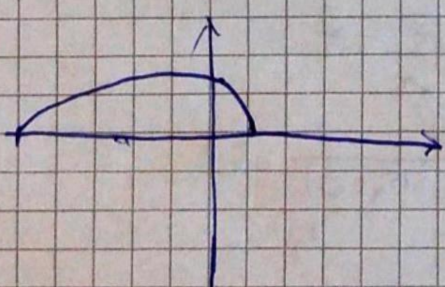
$$(-1{,}5\omega^2 - j(\omega^3+\omega))(-1{,}5\omega^2 + j(\omega^3+\omega)) =$$

$$= 2{,}25\omega^2 + (\omega^3+\omega)^2$$

$1 = \frac{6}{6{,}25}$

$$= 1 + \frac{-6\cancel{\omega}}{\omega^5+4{,}25\omega^3+\cancel{\omega}} + j\,\frac{4\omega^2+4}{4{,}25\omega^3+\omega^5+\omega} = 1 + \frac{-6}{\omega^4+4{,}25\omega^2+1} + j\,\frac{4\omega^2+4}{4{,}25\omega^3+\omega^5+\omega}$$

ω	0	1	∞
Re	-5	0,04	1
Im	∞	1,28	0



ZAD. 3 a: IL4

$$G(s)=\frac{4}{(s+1)(s^2+1,5s-1)} \qquad \Delta=2,25+4=6,25 \quad \sqrt{\Delta}=2,5$$

$$s_1=\frac{-1,5-2,5}{2}=\frac{-4}{2}=-2 \qquad s_2=\frac{1}{2}=0,5$$

$$G(s)=\frac{4}{(s+1)(s+2)(s-0,5)}$$ jeden biegun po prawej, niestabilny

$$G(j\omega)=\frac{4}{(j\omega+1)(j\omega+2)(j\omega-0,5)}=\frac{4}{(-\omega^2+2j\omega+j\omega+2)(j\omega-0,5)}=\frac{4}{(-\omega^2+3j\omega+2)(j\omega-0,5)}=$$

$(-2,5\omega^2+1)(-2,5\omega^2+1)=$
$=6,25\omega^4+2,5\omega^2+2,5\omega^2+1$

$$=\frac{4}{-j\omega^3-3\omega^2+2j\omega+0,5\omega^2-1,5j\omega-1}=\frac{4}{-j\omega^3-2,5\omega^2+0,5j\omega-1}=\frac{4}{(-2,5\omega^2-1)+j(0,5\omega-\omega^3)}$$

$$=\frac{4((-2,5\omega^2-1)-j(0,5\omega-\omega^3))}{(6,25\omega^4+5\omega^2+1)+(0,25\omega^2-\omega^4+\omega^6)}=\frac{-10\omega^2-4-j2\omega+j4\omega^3}{5,25\omega^4+\omega^6+5,25\omega^2+1}=$$

$$=\cancel{\frac{-10\omega^2-10-j2\omega+j4\omega^3}{}}\ \frac{-10\omega^2-4}{\omega^6+5,25\omega^4+5,25\omega^2+1}+j\frac{4\omega^3-2\omega}{5,25\omega^4+\omega^6+5,25\omega^2+1}$$

$$\frac{-4}{1}=-4$$

ω	0	1	∞
Re	-3	-0,12	1
Im	0	0,16	∞

ω	0	1	$\frac{\sqrt{2}}{2}$	∞	2
Re	-2	-1,12	-1,78	0	
Im	0	0,16	0	0	

$$1+\frac{-10-4}{1+13,5+1}=\frac{-14}{12,5} \qquad 1+\frac{-\omega-4}{}$$

$$\frac{4-2}{12,5} \qquad \frac{-40-4}{64}$$

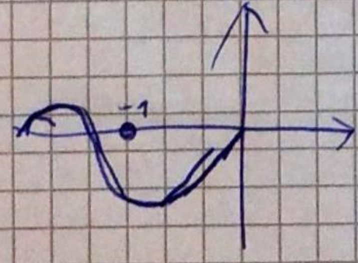
$$-10\omega^2-4=0 \rightarrow \cancel{\omega^2=-\frac{4}{10}} \quad -10\omega^2=4 \quad \omega^2=\sqrt{\frac{4}{-10}}=\sqrt{\frac{2}{-5}}$$

$$4\omega^3-2\omega=0$$

$$4\omega^2=2 \quad \omega=\sqrt{\frac{2}{4}}=\sqrt{\frac{1}{2}}=\frac{\sqrt{2}}{2}$$

$$\frac{-10\omega^2-4}{\omega^6+5,25\omega^4+5,25\omega^2+1} \rightarrow \frac{\omega^2(-10-\frac{4}{\omega^2}\to 0)}{\omega^2(\omega^4+5,25\omega^2+5,25+\frac{1}{\omega^2}\to 0)}=0$$

$$\frac{4\omega^3-2\omega}{\omega^6+5,25\omega^4+5,25\omega^2+1}=\frac{\omega^3(4-2/\omega^2\to 0)}{\omega^3(\omega^3+5,25\omega+\frac{5,25}{\omega}\to 0+\frac{1}{\omega^3}\to 0)}=\frac{4}{\infty}=0$$



3b) L4:

$$G_{12}(s) = \frac{1}{(s^2+2s+1)(s-1)} = \frac{1}{(s+1)^2(s-1)}$$

$$\Delta \arg \{ G_{12}(s)+1 \} = 180°$$

$(-\omega^2-1)(-\omega^2-1) =$

$= \omega^4 + \omega^2 + \omega^2 + 1$

$$G_{12}(j\omega) = \frac{1}{(-\omega^2+2j\omega+1)(j\omega-1)} = \frac{1}{-j\omega^3+\omega^2-2\omega^2+2j\omega+j\omega-1} = \frac{1}{-j\omega^3-\omega^2-j\omega-1} =$$

$$= \frac{1}{(-\omega^2-1)+j(-\omega^3-\omega)} = \frac{(-\omega^2-1) - j(-\omega^3-\omega)}{(\omega^4+2\omega^2+1)+(\omega^6+2\omega^4+\omega^2)} = \frac{(-\omega^2-1)+j(\omega^3+\omega)}{\omega^6+3\omega^4+3\omega^2+1}$$

$$= \frac{-\omega^2-1}{\omega^6+3\omega^4+3\omega^2+1} + j\,\frac{\omega^3+\omega}{\omega^6+3\omega^4+3\omega^2+1}$$

$-\omega^2-1=0$

$-\omega^2=1$

$\omega^2=-1$

$\omega(\omega^2+1)=0$

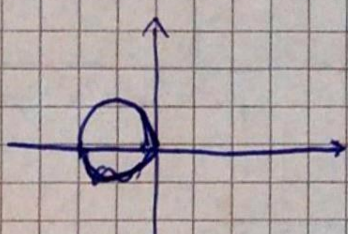
ω	0	1	2	∞
Re	-1	$-\frac{1}{4}$	$-\frac{1}{25}$	0
Im	0	$\frac{1}{4}$	$\frac{2}{25}$	0

$\frac{-2}{8}$

$\frac{2}{8} =$

$\frac{-4-1}{64+48+12+1} =$

$\underline{-5}$

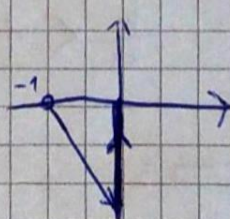


c) $G_{12}(s) = \frac{1}{s^3-s} = \frac{1}{s(s^2-1)} = \frac{1}{s(s+1)(s-1)}$

$\arg\{G_{12}(j\omega)+1\} = 270°$

$$G_{12}(j\omega) = \frac{1}{-j\omega^3-j\omega} = \frac{1}{-j(\omega^3+\omega)} = \frac{j(\omega^3+\omega)}{\omega^6+2\omega^4+\omega^2} = \frac{j(\omega^2+1)}{\omega^5+2\omega^3+\omega}$$

ω	0	1	∞
Re	0	0	0
Im	∞	$\frac{1}{2}$	0



ZADANIE 4 / L4 WARUNEK: $\boxed{Re\{G_{12}(j\omega)\} > -1 \quad \text{jeśli } Im\{G_{12}(j\omega)\} = 0}$

a) $G_{12}(s) = \frac{k}{(s+4)^3}$

potrójny pierw. $\Rightarrow$ lewej $\Rightarrow$ stabilny
kryt. lewej strony (?)

$$G_{12}(j\omega) = \frac{k}{(j\omega+4)^3} = \frac{k}{(-\omega^2+8j\omega+16)(j\omega+4)} = \frac{k}{-j\omega^3-4\omega^2-8\omega^2+32j\omega+16j\omega+64} =$$

$$= \frac{k}{-j\omega^3-12\omega^2+48j\omega+64} = \frac{k}{-12\omega^2+64+j(-\omega^3+48)}$$

$$-\omega^2+48=0 \rightarrow \omega = \sqrt{48} = 4\sqrt{3} \qquad -4\sqrt{3}<0$$

$$Re\{G_{12}(j\omega_0)\} = \frac{k}{-12(4\sqrt{3})^2+64} > -1 \rightarrow \frac{k}{512} < 1$$

$$\frac{k}{-512} > -1$$

$$k > 512$$

$$k < 512$$

$$(j\omega+1)(j\omega+1) = -\omega^2+j\omega+j\omega+1$$

b) $G_{12}(s) = \frac{k}{(s+1)^2(s+3)}$

$$G_{12}(j\omega) = \frac{k}{(j\omega+1)^2(j\omega+3)} = \frac{k}{(-\omega^2+2j\omega+1)(j\omega+3)} = \frac{k}{-j\omega^3-3\omega^2-2\omega^2+6j\omega+j\omega+3} =$$

$$= \frac{k}{-j\omega^3-5\omega^2+7j\omega+3} = \frac{k}{(-5\omega^2+3)+j(-\omega^3+7\omega)}$$

$$-\omega^3+7\omega=0 \qquad 7=\omega^2 \qquad \omega = \pm\sqrt{7}$$

$$Re\{G_{12}(j\omega_0)\} = \frac{k}{-35+3} > -1 \rightarrow \frac{k}{-32} > -1$$

$$\frac{k}{32} < 1 \qquad k < 32$$

c) $G_{12}(s) = \frac{2k}{(s+2)(s+4)(s+6)} =$

$$G_{12}(j\omega) = \frac{2k}{(j\omega+2)(j\omega+4)(j\omega+6)} = \frac{2k}{\cancel{(-\omega^2+4j\omega+2j\omega+8)(j\omega+6)}\;(-\omega^2+6j\omega+8)(j\omega+6)} = \frac{2k}{-j\omega^3-6\omega^2-6\omega^2+36j\omega+8j\omega+48} =$$

$$= \frac{1}{-j\omega^3-12\omega^2+44j\omega+48} = \frac{1}{-12\omega^2+48+j(-\omega^3+44\omega)}$$

$$-\omega^3+44\omega=0$$

$$\omega^2=44 \qquad \omega=\sqrt{44}$$

$$Re\{G_{12}(j\omega_1)\} = \frac{2k}{-12\omega^2+48} > -1$$

$$\frac{2k}{-480} > -1$$

$$\frac{2k}{480} < 1$$

$$k < 240$$

LISTA 5

ZADANIE 1: KRYTERIUM LOGARYTMICZNE

a) $G_{12}(s) = \frac{7}{(s+1)^3}$

$L(\omega) < 0$ dla $\omega_1 \to \arg\{G_{12}(j\omega)\} = -\pi$

$\arg\{G_{12}(j\omega_1)\} = -\pi$ dla $\omega_2 \to |G_{12}(j\omega)| = 1$

$\arg\{G_{12}(j\omega)\} = -\pi$

$\arg\{G_{12}(j\omega)\} = -\operatorname{arctg}$

$L(\omega) = 20 \log |G(j\omega)|$

$$G_{12}(j\omega) = \frac{7}{(j\omega+1)^3} = \frac{7}{(\omega^2+2j\omega+1)(j\omega+1)} = \frac{7}{-j\omega^3-\omega^2-2\omega^2+2j\omega+j\omega+1} =$$

$$= \frac{7}{-j\omega^3-3\omega^2+3j\omega+1} = \frac{7}{-3\omega^2+1+j(-\omega^3+3\omega)}$$

$$\sqrt{(3\omega^2+1)^2+(-\omega^3+3\omega)^2} = \sqrt{(3\cdot 3+1)^2+(-3\sqrt{3}+3\sqrt{3})^2} = \sqrt{100} = 10$$

$$\arg = -\operatorname{arctg}\frac{-\omega_3+3\omega}{-3\omega^2+1} = -\pi$$

$$\operatorname{arctg}\frac{-\omega^3+3\omega}{-3\omega^2+1} = \pi$$

$$\operatorname{tg}\pi = \frac{-\omega^3+3\omega}{-3\omega^2+1} \qquad -\omega^3+3\omega = 0$$

$\omega(3-\omega^2) = 0$ N/D

$\omega(\sqrt{3}-\omega)(\sqrt{3}+\omega) = 0$

$\downarrow$ N/D $\quad \downarrow$ K

$$L(\omega_1) = 20\log\frac{7}{(\sqrt{3}+1)^3} = -9{,}28$$