Wykład 4

sobota, 16 grudnia 2017 11:12

Obwód prostokąta: $2a+2b$

$$u(X) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial x}{\partial x_i}\right)^2 u^2(X_i)}$$

$$u(Ob_\square) = \sqrt{\left(\frac{\partial Ob}{\partial a}\right)^2 u^2(a) + \left(\frac{\partial Ob_\square}{\partial b}\right)^2 + u^2(b)} = \sqrt{2^2 u^2(a) + 2^2 u^2(b)} = 2\sqrt{u^2(a) + u^2(b)}$$

$$\frac{\partial Ob_\square}{\partial a} = 2 \qquad \frac{\partial Ob_\square}{\partial b} = 2$$

Niepewność względna:

$$U_r(Ob_\square) = \frac{u(Ob_\square)}{Ob_\square} \cdot 100 = \cancel{2}\sqrt{\frac{u^2(a)}{(\cancel{2}a+2b)^2} + \frac{u^2(b)}{(\cancel{2}a+2b)^2}}$$

ZADANIE II:

POLE PROSTOKĄTA

$\partial \to$ „d" $\to$ pochodna

$$P_\square = a \cdot b$$

$$u(P_\square) = \sqrt{\left(\frac{\partial P_\square}{\partial a}\right)^2 u^2(a) + \left(\frac{\partial P_\square}{\partial b}\right)^2 u^2(b)} = \sqrt{b^2 u^2(a) + a^2 u^2(b)}$$

$$\frac{\partial P_\square}{\partial a} = b \qquad \frac{d P_\square}{d b} = a$$

$$U_r(P_\square) = \frac{u(P_\square)}{P_\square} \cdot 100 = \sqrt{\frac{b^2 u^2(a)}{a^2 b^2} + \frac{a^2 u^2(b)}{a^2 b^2}} \cdot 100 = \sqrt{U_r^2(a) + U_r^2(b)}$$

Kolokwium z notatkami, mało czasu

PRZYKŁAD III:

$$F = ma + mg = m(a+g)$$

$$u(F) = \sqrt{\left(\frac{dF}{dm}\right)^2 u^2(m) + \left(\frac{dF}{da}\right)^2 u^2(a) + \left(\frac{dF}{dg}\right)^2 u^2(g)}$$

} Prawo propagacji niepewności

$$\frac{dF}{dm} = a+g \qquad \frac{dF}{da} = m \qquad \frac{dF}{dg} = m$$

} pochodne cząstkowe

$$\frac{dF}{dm} = a+g \qquad \frac{dF}{da} = m \qquad \frac{dF}{dg} = m$$

$$u(F) = \sqrt{(a+g)^2 u^2(m) + m^2 u^2(a) + m^2 u^2(g)}$$

niepewność bezwzględna

$$u_r(F) = \frac{u(F)}{F} \cdot 100 =$$

$$= \sqrt{\frac{\cancel{(a+g)}^2 u^2(m)}{m^2 \cancel{(a+g)}^2} + \frac{\cancel{m}^2 u^2(g) g^2}{\cancel{m}^2 (a+g)^2} + \frac{\cancel{m}^2 u^2(a) a^2}{\cancel{m}^2 (a+g)^2}} =$$

$$= \sqrt{u_r^2(m) + \left(\frac{a}{a+g}\right)^2 u_r^2(a) + \left(\frac{g}{a+g}\right)^2 + \left(\frac{g}{a+g}\right)^2 u_r^2(g)}$$

niepewność względna

$$\Delta X \qquad dX = \frac{\Delta X}{X} \cdot 100$$

$$u(X) \qquad u_r(X) = \frac{u(X)}{X} \cdot 100$$

PRZYKŁAD IV:

$$V = \frac{s}{t}$$

$$u(V) = \sqrt{\left(\frac{dV}{ds}\right)^2 u^2(s) + \left(\frac{dV}{dt}\right)^2 u^2(t)}$$

$$\frac{dV}{ds} = \frac{1}{t} \qquad \frac{dV}{dt} = -\frac{s}{t^2}$$

$$u(V) = \sqrt{\frac{1}{t^2} u^2(s) + \frac{s^2}{t^4} u^2(t)}$$

$$u_r(V) = \frac{u(V)}{V} \cdot 100 = \sqrt{\frac{\frac{1}{t^2}}{\frac{s^2}{t^2}} u^2(s) + \frac{\frac{s^2}{t^4}}{\frac{s^2}{t^4}} u^2(t)} = \sqrt{u_r^2(s) + u_r^2(t)}$$

PRZYKŁAD V

$$a = \frac{s}{t^2}$$

$$u(a) = \sqrt{\left(\frac{da}{ds}\right)^2 u^2(s) + \left(\frac{da}{dt}\right)^2 u^2(t)}$$

$$\frac{da}{ds} = \frac{1}{t^2} \qquad \frac{da}{dt} = -\frac{2s}{t^3}$$

$$u(a) = \sqrt{\frac{1}{t^4} u^2(s) + \frac{4s^2}{t^6} u^2(t)}$$

$$u_r(a) = \sqrt{\frac{\frac{1}{t^4}}{\frac{s^2}{t^4}} u^2(s) + \frac{\frac{4s^2}{t^6}}{\frac{s^2}{t^4}} u^2(t)} = \sqrt{u_r^2(s) + 4u_r^2(t)}$$

PRZYKŁAD VI

$$U_r = U_1 - U_2$$

$$u(U_r) = \sqrt{\left(\frac{dU_r}{dU_1}\right)^2 u^2(U_1) + \left(\frac{dU_r}{dU_2}\right)^2 u^2(U_2)}$$

$$\frac{dU_r}{dU_1} = 1 \qquad \frac{dU_r}{dU_2} = -1$$

$$u(U_r) = \sqrt{u^2(U_1) + u^2(U_2)}$$

$$u_r(U_r) = \sqrt{\frac{u^2(U_1)}{(U_1-U_2)^2} + \frac{u^2(U_2)}{(U_1-U_2)^2}} = \sqrt{\left(\frac{U_1}{U_1-U_2}\right)^2 u_r^2(U_1) + \left(\frac{U_2}{U_1-U_2}\right)^2 u_r^2(U_2)}$$

PRZYKŁAD VII

$$R_{źR} = R_0 \frac{U_1-U_2}{U_2} = R_0 \frac{U_1}{U_2} - R_0$$

$$u(R_{źR}) = \sqrt{\left(\frac{dR_{źR}}{dR_0}\right)^2 u^2(R_0) + \left(\frac{dR_{źR}}{dU_1}\right)^2 u^2(U_1) + \left(\frac{dR_{źR}}{dU_2}\right)^2 u^2(U_2)}$$

$$\frac{dR_{źR}}{dR_0} = \frac{U_1-U_2}{U_2} \qquad \frac{dR_{źR}}{dU_1} = \frac{R_0}{U_2} \qquad \frac{dR_{źR}}{dU_2} = -\frac{R_0 U_1}{U_2^2}$$

$$u(R_{źR}) = \sqrt{\left(\frac{U_1-U_2}{U_2}\right)^2 u^2(R_0) + \left(\frac{R_0}{U_2}\right)^2 u^2(U_1) + \left(\frac{R_0 U_1}{U_2^2}\right)^2 u^2(U_2)}$$

$$u_r(R_{źR}) = \frac{u(R_{źR})}{R_{źR}} \cdot 100 = \sqrt{\frac{\cancel{\left(\frac{U_1-U_2}{U_2}\right)^2}}{R_0^2 \cancel{\left(\frac{U_1-U_2}{U_2}\right)^2}} u^2(R_0) + \frac{\left(\frac{R_0}{U_2}\right)^2}{R_0^2\left(\frac{U_1-U_2}{U_2}\right)^2} u^2(U_1) + \frac{\left(\frac{R_0 U_1}{U_2^2}\right)^2}{R_0^2\left(\frac{U_1-U_2}{U_2}\right)^2} u^2(U_2)}$$

$$= \sqrt{u_r^2(R_0) + \left(\frac{U_1}{U_1-U_2}\right)^2 u_r^2(U_1) + \left(\frac{U_1}{U_1-U_2}\right)^2 u_r^2(U_2)}$$

$$= \sqrt{u_r^2(R_0) + \left(\frac{U_1}{U_1-U_2}\right)^2 \left(u_r^2(U_1) + u_r^2(U_2)\right)}$$

PRZYKŁAD VIII

BŁĄD POMIARU PRZYRZĄDU →

$$\Delta U = \frac{0{,}05\,U_x + 0{,}05\,U_n}{100} =$$

niep. stand. bezwzględna →

$$u(U) = \frac{\Delta U}{\sqrt{3}} =$$

niep. rozszerzona bezwzględna →

$$U(U) = k_p \cdot u(U) = \sqrt{3} \cdot p \cdot \frac{\Delta U}{\sqrt{3}} = \Delta U \cdot p$$

NA KOLOKWIUM:
- KALKULATOR
- WYNIK ZOSTAJE Z $\sqrt{3}$
- 5 min. na zadanie

ZAKRES:
- PRZ. ANALOGOWY
- PRZ. CYFROWY

niep. rozszerzona
względna

$k_p = \sqrt{3} \cdot p$

$U \pm U(U) = (14,592 \pm 0,008)\,V$

$$\begin{cases} U = 14,592\,V \\ U(U) = 0,00773\,V \end{cases}$$

$U_r(U) =$