Ćwiczenia 5

wtorek, 13 listopada 2018 11:16

1

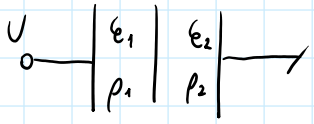
$\varepsilon_{r_1} = 3 \qquad \varepsilon_{r_2} = 6$

$\rho_1 = 10^8\,\Omega m \qquad \rho_2 = 0{,}5 \cdot 10^8\,\Omega m$

$d_1 = d_2 = 3\,mm = 0{,}003\,m = 0{,}3\,cm$

$S = 50\,cm^2$

$U = 1{,}2\,kV$



$R_1 = \rho_1 \frac{d}{S} = 10^8 \cdot \frac{0{,}3}{50} = 0{,}006 \cdot 10^8 = 6 \cdot 10^7\,\Omega$

$R_2 = \rho_2 \frac{d}{S} = 10^8 \cdot \frac{1}{2} \cdot \frac{0{,}3}{50} = 3 \cdot 10^7\,\Omega$

$I = \frac{U}{R} = \frac{U}{R_1 + R_2} = \frac{1200\,V}{9 \cdot 10^7\,\Omega} = 1{,}33 \cdot 10^{-5}\,A$

$j = \frac{I}{S} = \frac{1{,}33 \cdot 10^{-5}}{5 \cdot 10^{-3}} = 2{,}67 \cdot 10^{-3}\,\frac{A}{m}$

$\Delta P = I^2 (R_1 + R_2) = (1{,}33 \cdot 10^{-5})^2 \cdot 9 \cdot 10^7 = 0{,}016\,W$

$\mathcal{P} = \frac{\Delta P}{V} = \frac{\Delta P}{S \cdot d} = \frac{0{,}016}{5 \cdot 10^{-3} \cdot 3 \cdot 10^{-3}} = 1067\,\frac{W}{m^3}$

2

$l = 1\,km \qquad U = 5\,kV \qquad \gamma = 5 \cdot 10^{-9}\,Sm^{-1}$

$r_1 = 10\,mm \qquad r_2 = 27{,}3\,mm \qquad \varepsilon_r = 3{,}5$

$C = \frac{Q}{U} = \frac{2\pi\varepsilon l}{\ln \frac{r_2}{r_1}}$

$C = \frac{6{,}28 \cdot 3{,}5 \cdot 1{,}16}{\ln \frac{0{,}0273}{0{,}01}} = \frac{21{,}940}{1{,}004} = 21892{,}4$

$\Delta P = \frac{U^2}{R} =$

$R = \frac{\rho}{2\pi l} \ln \frac{r_2}{r_1} \qquad R = \frac{1}{\gamma 2\pi l} \ln \frac{r_2}{r_1} = \frac{1}{5 \cdot 10^{-9} \frac{S}{m} \cdot 2\pi \cdot 6{,}28 \cdot 1000} \cdot 1{,}004 =$

$= 31984{,}1\,\Omega$

$\Delta P = \frac{U^2}{R} = \frac{5000 \cdot 5000}{31984{,}1\,\Omega} = 781{,}6\,W$

$I = \frac{U}{R} = \frac{5000\,V}{31984{,}1\,\Omega} = 0{,}16\,A$

$\vec{j} = \gamma \cdot \vec{E} \qquad j = \gamma E = \gamma \cdot \frac{Q}{2\pi\varepsilon hr}$

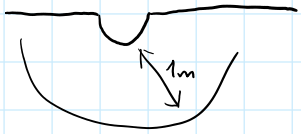
$\vec{j} = \gamma \cdot \vec{E}$ $\quad j = \gamma E = \gamma \cdot \frac{U}{2\pi\varepsilon hr}$

$\mathcal{P} = jE = \gamma E^2 \quad / E = \frac{U}{r \ln \frac{r_2}{r_1}}$

$P = \int_V P dV = \int \gamma \frac{U^2}{r^2 \ln^2 \frac{r_2}{r_1}} r\,dr\,d\varphi\,dz = \int_0^L \int_0^{2\pi} \int_{r_1}^{r_2} \gamma \frac{U^2}{r^2 \ln^2 \frac{r_2}{r_1}} r\,dr\,d\varphi\,dz =$

$= \gamma \frac{U^2}{r \ln \frac{r_2}{r_1}} 2\pi = \frac{U^2}{R}$

(3) $I = 100A \qquad r_0 = 2{,}5\,cm = 0{,}025\,m \qquad \rho = 10\,\Omega m$



$j = \frac{I}{S} = \frac{I}{\frac{4\pi r^2}{2}} = \frac{2I}{\pi r^2} \qquad E = j \cdot \rho = \frac{I\rho}{2\pi r^2}$

$j = \frac{100}{2\pi (2{,}5 \cdot 10^{-2})^2} =$

$\varphi = \int_r^\infty \vec{E} \cdot d\vec{r} = \int_r^\infty \frac{I\rho}{2\pi r^2} d\vec{r} = \frac{I\rho}{2\pi} \int_r^\infty \frac{1}{r^2} dr =$

$\frac{I\rho}{2\pi} \left[-\frac{1}{r} \right]_r^\infty = \frac{I\rho}{2\pi} \left(\lim_{r \to \infty} \left(-\frac{1}{r}\right) - \left(-\frac{1}{r}\right) \right) = \frac{I\rho}{2\pi r}$

$\varphi = \frac{I\rho}{2\pi r} \qquad \varphi_E = \frac{100 \cdot 10}{2\pi \cdot 2{,}5 \cdot 10^{-2}}$

$\frac{4}{\cancel{1000}} = 40$

$R = \frac{\varphi_0}{3} = \frac{P}{2\pi r} = \frac{10}{2\pi \cdot 2{,}5 \cdot 10^{-2}}$

$\int_{r_0}^{r} \vec{E} \cdot d\vec{r} = \frac{I\rho}{2\pi} \left[-\frac{1}{r} \right]_{0{,}025m}^{1m} = \frac{I\rho}{2\pi} \left(-\frac{1}{1} + \frac{1}{0{,}025} \right) = \frac{100 \cdot 10}{2\pi} \cdot (-39)$

(4) $r = 10^{-6}\,\Omega m$

$r_1 = 2\,cm = 0{,}02\,m$

$r_2 = 5\,cm = 0{,}05\,m$

$h = 10\,cm = 0{,}1\,m$

Opcja A $\rho \cdot \frac{d}{S} = 10^{-6} \cdot \frac{2\pi \cdot 0{,}015\,m}{0{,}1\,m \cdot 0{,}03\,m} =$

$S = h(r_2 - r_1)$ $\quad d = \pi\left(\frac{r_1+r_2}{2}\right) = 3{,}5\,cm$

Opcja C: $S = \frac{\pi r_2^2 - \pi r_1^2}{2} = \frac{\pi}{2}(r_2^2 - r_1^2) = 3{,}3 \cdot 10^{-3}\,m^2$

$R = \rho \frac{h}{S} = 10^{-6} \cdot \frac{0{,}1}{3{,}3 \cdot 10^{-3}} = 3{,}03 \cdot 10^{-5}\,\Omega$

$R_{ab} = \int_\alpha \int_S \frac{1}{\frac{r\,ds}{dl}} = \int_\alpha \frac{1}{\int \frac{r\,r\,d\varphi\,dz}{dr}} = \int_\alpha \frac{dr}{\int \gamma r\,d\varphi\,dz} = \int_{r_1}^{r_2} \frac{dr}{r\,h\pi\gamma} = \frac{1}{h\pi\gamma} \ln\frac{r_2}{r_1}$

$\int_S \gamma r\,d\varphi\,dz = \gamma \int_0^h \int_0^\pi r\,d\varphi\,dz = r h \pi \gamma$

$R_{ab} = \frac{1}{10^6 \cdot \pi \cdot 21} \cdot \ln\frac{0{,}05}{0{,}02} = 2{,}91 \cdot 10^{-6}$

$I = \int_S \vec{j} \circ d\vec{S} = j(r) \int_0^h \int_0^\pi r\,d\varphi\,dz = j\pi 2r$

$j = \frac{I}{\pi h r}$ $\quad j = \gamma E$ $\quad E = \frac{j}{\gamma} = \frac{I}{\pi h r \gamma}$

$U = \int_L \vec{E} \circ d\vec{l} = \int_{r_1}^{r_2}$

$R = \frac{U}{I} = \frac{1}{\pi h r} \ln\frac{r_2}{r_1}$

(5) $\rho = 10^6$ $\quad r_1 = 5\,cm$ $\quad r_2 = 8\,cm$ $\quad h = 10\,cm$

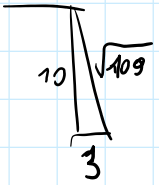
ω - szerokość

$R = \rho \frac{l}{w} = \frac{10{,}4 \cdot 10^{-2}}{6{,}5 \cdot 10^{-2}} = 1{,}6 \cdot 10^8\,\Omega$

$$K = \rho \frac{w}{S} = \frac{w}{6{,}5 \cdot 10^{-2}} = 1{,}6 \cdot 10^{-1}\ \Omega$$

$$w_{3r} = \frac{r_1 + r_2}{2} = 6{,}5\ cm = 6{,}5 \cdot 10^{-2}\ m$$

$$l = \sqrt{109} \approx 10{,}4$$



$$R = \frac{\rho}{2\pi} \frac{\sqrt{(r_2 - r_1)^2 + h^2}}{r_2 - r_1} \ln \frac{r_2}{r_1}$$

(6) $\gamma = (56\ m\Omega^{-1}\ mm^{-2})$ $\quad d = 2\ mm \quad (1\ mm)$

Pole jednorodne $\quad I = 10\ A \quad P = ? \quad l = 1\ m$

$$P = RI^2 = \frac{l}{\gamma S} \cdot I^2 = \frac{1}{56 \cdot \pi \cdot 1^2} \cdot 10\ A =$$

$$= 0{,}57\ W$$

Pole $j = ar^k$

$$P = jE = j^2 \rho \qquad P = \int_V p\, dV$$

$$P = \int j^2 \rho\, dV = \int_V j^2 \rho\, dV =$$

$$P = \rho \int_V j^2\, dV = \rho a^2 \int_V r^{2k} = \rho a^2 \int_0^r \int_0^{2\pi} \int_0^l r^{2k+1}\, dr\, d\varphi\, dz =$$

$$= 2\pi l \int_0^r r^{2k+1}\, dr = 2\pi l \rho a^2 \frac{r^{2k+2}}{2k+2}$$

$$I = \int_S j\, d\vec{S} = \int_S a r^k\, dS = a \iint r^{k+1}\, dr\, d\varphi = 2a\pi \frac{r^{k+2}}{k+2} = 10$$

$a = ?$