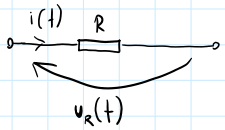
Ćwiczenia 1: zadania 2-4

poniedziałek, 19 lutego 2018 20:32

Lista 1

REZYSTOR:

$i(t) = I_m \sin(\omega t + \psi)$ $\quad i(t) = \frac{1}{R} u_0(t)$



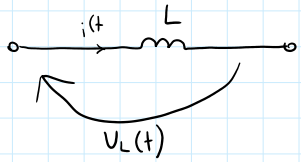
$u_R(t) = R i(t) = R I_m \sin(\omega t + \psi)$

$= U_m \sin(\omega t + \psi)$

$U_m = R I_m \Rightarrow R = \frac{U_m}{I_m} = \frac{\sqrt{2}U}{\sqrt{2}I} = \frac{U}{I}$

$\psi_u = \psi_i \Rightarrow \varphi = \psi_u - \psi_i$

ZWOJNICA:



$\Psi = L i(t)$

$u_L(t) = \frac{d\Psi}{dt} = L \frac{d i(t)}{dt} =$

$= L \frac{d}{dt}\left[I_m \sin(\omega t + \psi)\right] =$

$= \omega L I_m \cos(\omega t + \psi) =$

$= \omega L I_m \cos(\omega t + \psi_i + \frac{\pi}{2}) =$

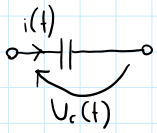
$= U_m \sin(\omega t + \psi_u)$

$U_m = L I_m \Rightarrow \omega L = X_L = \frac{U_m}{I_m}$

L [H] $\quad X_L$ [Ω]

$\varphi = \psi_u - \psi_i = \frac{\pi}{2}$

KONDENSATOR:



C [F]

$q = C u(t)$

$i(t) = \frac{dq}{dt} = C \frac{du_c(t)}{dt} \Rightarrow u_c(t) = \frac{1}{C} \int i(t)\, dt =$

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$= \frac{1}{C}\int I_m \sin\omega t\,dt = \frac{I_m}{\omega C}\left(-\cos(\omega t + \psi_i)\right) = \frac{I_m}{\omega C}\sin\left(\omega t + \psi_i - \frac{\pi}{2}\right)$

$\varphi = \psi_u - \psi_i = -\frac{\pi}{2}$

$U_m = \frac{I_m}{\omega C} \Rightarrow \frac{1}{\omega C} = X_c = \frac{U_m}{I_m}$

(2)

a) $u(t) = 150\sin\left(\omega t + \frac{\pi}{6}\right)$

$i(t) = 5\sin(\omega t + 30°)$

$\varphi = \psi_u - \psi_i = \frac{\pi}{6} - \frac{\pi}{6} = 0 \rightarrow$ Rezystor

$R = \frac{U_m}{I_m} = \frac{150}{5} = 30\,\Omega$

b) $u(t) = 125\sin\left(157t - \frac{\pi}{6}\right)$

$i(t) = 2{,}5\cos\left(157t - \frac{\pi}{6}\right) = 2{,}5\sin\left(157t - \frac{\pi}{6} + \frac{\pi}{2}\right)$

$\varphi = \frac{-\pi}{6} - \left(-\frac{\pi}{6} + \frac{\pi}{2}\right) = -\frac{\pi}{2}$

Prąd wyprzedza napięcie: kondensator

$\frac{1}{\omega C} = T = \frac{U_m}{I_m} = \frac{125}{2{,}5} = 50 = \frac{1}{157C}$

$C = \frac{1}{157 \cdot 50}$

c) $u(t) = 220\sqrt{2}\sin\left(314t - \frac{\pi}{4}\right)$

$i(t) = -\sqrt{2}\cos\left(314t - \frac{\pi}{4}\right) = -\sqrt{2}\sin\left(314t + \frac{\pi}{4}\right)$

$\psi_u - \psi_i = -\frac{\pi}{4} - \left(-\frac{\pi}{4} + \frac{\pi}{2}\right) = \frac{\pi}{2}$

Cewka

$\omega L = \frac{U_m}{I_m} = \frac{220\sqrt{2}}{\sqrt{2}} = 220\,[\Omega]$

$220 = 314 \cdot L$

$L = \frac{220}{314}\,[H]$

d) $u(t) = 120\sqrt{2}\cos(314t - 30°)$

$i(t) = 6\sqrt{2}\sin(314t + 60°) = 6\sqrt{2}\cos(314t - 30°)$

$\psi_u - \psi_i = -30° - (-30°) = 0$

Rezystor

$$R = \frac{U_m}{I_m} = \frac{120\sqrt{2}}{6\sqrt{2}} = 20\,\Omega$$

(3)

$$RI^2T = R\int_0^T i^2(t)\,dt \Rightarrow$$

$$I = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

$$I_{śR} = \frac{1}{T}\int i(t)\,dt$$

$$i(t) = \frac{I_m}{T}t \qquad 0 < t < T$$

a)

$$I = \sqrt{\frac{1}{T}\int_0^T \left(\frac{I_m}{T}t\right)^2 dt} =$$

$$= \sqrt{\frac{1}{T}\frac{I_m^2}{T^2}\int_0^T t^2\,dt} = \sqrt{\frac{I_m^2}{T^3}\int_0^T t^2\,dt} =$$

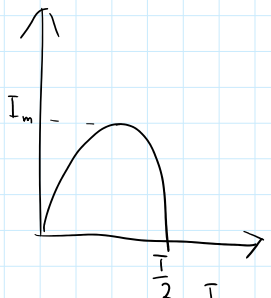
$$= \sqrt{\frac{I_m^2}{T^3}\frac{t^3}{3}\Big|_0^T} = \sqrt{\frac{I_m^2}{T^3}\frac{T^3}{3}} = \frac{I_m}{\sqrt{3}}$$

$$I_{śR} = \frac{1}{T}\cdot\frac{1}{2}\cdot T\cdot I = \frac{I_m}{2}$$

$$k_{sz} = \frac{\frac{I_m}{\sqrt{3}}}{\frac{I_m}{2}} = \sqrt{3}$$

$$k_{kształtu} = \frac{\frac{I_m}{\sqrt{3}}}{\frac{I_m}{2}} = \frac{2}{\sqrt{3}}$$

b:



$$i(t) = I_m \sin(\omega t) \qquad 0 < t < \tfrac{T}{2}$$

$$i(t) = 0 \qquad \tfrac{T}{2} < t < T$$

$$\omega = \frac{2\pi}{T}$$

$$I_{śR} = \frac{1}{T}\int_0^{\frac{T}{2}} I_m \sin(\omega t)\,dt = \frac{I_m}{T}\int_0^{\frac{\pi}{2}} \sin(\omega t)\,dt =$$

$$= \frac{I_m}{T}\left(-\cos\omega t\right)\Big|_0^{\frac{T}{2}} = \frac{I_m}{\frac{2\pi}{T}\cdot T}\Big(\underbrace{-\cos\frac{2\pi}{T}\cdot\frac{T}{2}}_{-1} + \underbrace{\cos\frac{2\pi}{T}\cdot 0}_{1}\Big) =$$

$$= \frac{I_m}{2\pi}\cdot 2 = \frac{I_m}{\pi}$$

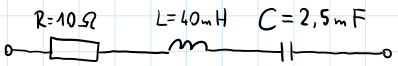
$$I_{sk} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} I_m^2 \sin^2(\omega t)\,dt} = \sqrt{\frac{I_m^2}{T}\int_0^{\frac{T}{2}} \sin^2(\omega t)\,dt} =$$

$$= \sqrt{\frac{I_m^2}{T}\int_0^{\frac{T}{2}} \frac{1}{2}(1-\cos 2\omega t)\,dt} = \frac{I_m^2}{2T}\int_0^{\frac{T}{2}} dt - \frac{I_m^2}{2T}\int \cos 2\omega t\,dt =$$

$$= \sqrt{\frac{I_m^2}{2T}\left(\frac{T}{2}\right) - \frac{I_m^2}{2T}\,\frac{\sin 2\omega t}{2\omega}\Big|_0^{\frac{T}{2}}} = \sqrt{\frac{I_m^2}{4} - \frac{I_m^2}{2T}\left(\frac{\sin 2\cdot\frac{2\pi}{T}\cdot\frac{T}{2}}{2\omega} - \frac{\sin(0)}{2\omega}\right)} =$$

$$= \sqrt{\frac{I_m^2}{4}} = \frac{I_m}{2}$$

(4) c) $i(t) = 10\sin(100t)$



$$u(t) = u_R(t) + u_L(t) + u_c(t)$$

$$u_R(t) = R\cdot i(t) = 10\cdot 10\sin(100t) = 100\sin 100t$$

$$U_R = \frac{U_M}{\sqrt{2}} = \frac{100}{\sqrt{2}}$$

$$u_L(t) = L\frac{di(t)}{dt} = 40\cdot 10^{-3}\cdot 10\cos(100t)\cdot 100 =$$

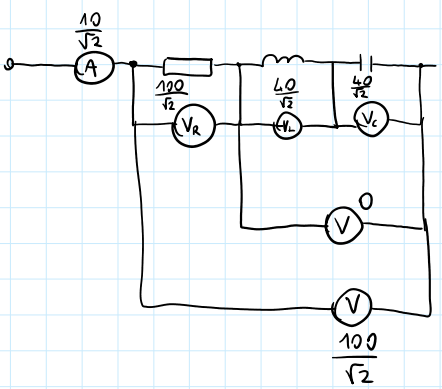
$$= 40\sin\left(100t + \frac{\pi}{2}\right)$$

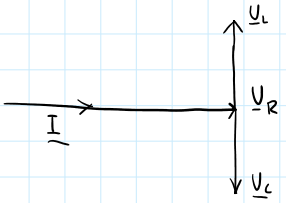
$$U_L = \frac{40}{\sqrt{2}}$$

$$u_c(t) = \frac{1}{C}\int i(t)\,dt = \frac{1}{2{,}5\cdot 10^{-3}}\int 10\sin(100t)\,dt =$$

$$= \frac{10}{2{,}5\cdot 10^{-3}\cdot 100}\cdot\sin\left(100t - \frac{\pi}{2}\right) = 40\sin\left(10\sin 100t - \frac{\pi}{2}\right)$$

$$U_c = \frac{40}{\sqrt{2}}$$





$$u(t) = u_R(t)$$

$$U = U_R$$