60c (walcowe): x^2+y^2 $\quad f(x,y,z)=r^2$

U: $x^2+y^2\leq 4$, $1-x\leq z\leq 2-x$ $\qquad 1-x\leq 2-x$, $1\leq 2$

$\iiint_U (x^2+y^2)\,dx\,dy\,dz$ $\qquad x=r\cos\varphi \quad y=r\sin\varphi$

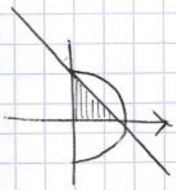
$r^2\leq 4 \qquad \varphi\in\langle 0,2\pi\rangle \qquad r\in\langle 0,2\rangle$

$$\int_0^{2\pi} d\varphi \int_0^2 dr \int_{1-r\cos\varphi}^{2-r\cos\varphi} (r^2\cos^2\varphi + r^2\sin^2\varphi)\,r\,dh = \int_0^{2\pi} d\varphi\int_0^2 r^3\,dr \int_{1-r\cos\varphi}^{2-r\cos\varphi} dh = \int_0^{2\pi}d\varphi\int_0^2 r^3\,dr\,[h]_{1-r\cos\varphi}^{2-r\cos\varphi}=$$

$x\geq 0$

$$=\int_0^{2\pi}d\varphi\int_0^2 r^3(2-r\cos\varphi-(1-r\cos\varphi))\,dr = \int_0^{2\pi}d\varphi\int_0^2 r^3\,dr = \frac14\int_0^{2\pi}[r^4]_0^2\, d\varphi=$$

$$=\frac{2^4}{2^2}\int_0^{2\pi}d\varphi = 2^2\cdot 2\pi = 2^3\pi = 8\pi$$



61 d) $\iiint_U (x+y+z)\,dx\,dy\,dz$ $\qquad U: x^2+y^2\leq 1 \quad 0\leq z\leq 2-x-y \qquad x\geq 0$

$r^2\leq 1 \qquad 0\leq r\leq 1 \qquad \varphi\in\langle -\frac{\pi}{2},\frac{\pi}{2}\rangle$

$x=r\cos\varphi \quad y=r\sin\varphi \quad z=h \qquad 0\leq 2-x-y$

$0\leq h\leq 2-r\cos\varphi-r\sin\varphi \qquad 0\leq 2-x-y \quad y\leq 2-x$

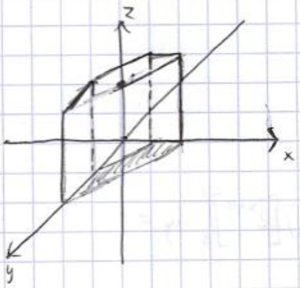
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi\int_0^1 dr\int_0^{2-r\cos\varphi-r\sin\varphi} r(r^2\cos^2\varphi + r^2\sin^2\varphi + h)\,dh =$$

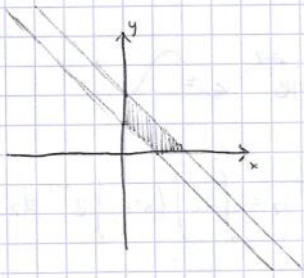
$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\int_0^1 r\,dr \int \left[r\cos\varphi\, h + r\sin\varphi\, h + \frac{h^2}{2}\right]_0^2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\int_0^1$$

S.12/C3:

$f(x,y,z) = z$ $\quad$ V: $x+y=1$ $\quad$ $x+y=2$ $\quad$ $x=0$ $\quad$ $y=0$ $\quad$ $z=0$ $\quad$ $z=1$

$\qquad\qquad\qquad\qquad$ $y=1-x$ $\quad$ $y=2-x$ $\qquad\qquad$ $y=0$





$$\cancel{\iiint_V dx\,dy\,dz = \int_0^1 dz \int_0^2 dy \int_0^2 z\,dx = \int_0^1 dz \int_0^2 dy \left[xz\right]_0^2 = \int_0^1 dz \int_0^2 2xz\,dy =}$$

$$\cancel{= \int_0^1 dz \left[2xyz\right]_0^2 = \int_0^1 4xyz\,dz = \int_0^1 4xy\,z\,dz = \left[\frac{4xyz^2}{2}\right]_0^1 = \left[2xyz^2\right]_0^1 = 2xy}$$

$$\iiint_V z\,dx\,dy\,dz = \int_0^2 dy \int_0^{2-y} dx \int_0^1 z\,dz - \int_0^2 dy \int_0^{1-y} dx \int_0^1 z\,dz =$$

$$= \int_0^2 dy \int_0^{2-y} dx \left[\frac{z^2}{2}\right]_0^1 - \int_0^2 dy \int_0^{1-y} dx \left[\frac{x^2}{2}\right]_0^1 =$$

60d) $g(x,y,z) = x^2y^2$ $\qquad$ U: $0 \leqslant x \leqslant y \leqslant z \leqslant 1$

$\qquad\qquad\qquad\qquad$ $0 \leqslant x \leqslant 1$ $\quad$ $0 \leqslant y \leqslant 1$ $\quad$ $0 \leqslant z \leqslant 1$

$$\iiint_U g(x,y,z)\,dx\,dy\,dz = \int_0^1 dx \int_x^1 dy \int_y^1 x^2y^2\,dz = \int_0^1 x^2\,dx \int_x^1 y^2\,dy \left[z\right]_y^1 =$$

$$= \int_0^1 x^2\,dx \int_x^1 y^2(1-y)\,dy = \int_0^1 (x^2)\,dx \int_x^1 y^2 - y^3\,dy = \int_0^1 x^2\,dx \left[\frac{y^3}{3} - \frac{y^4}{4}\right]_x^1 = \int_0^1 x^2\left(\frac{1}{3} - \frac{1}{4} + \frac{x^3}{3} + \frac{x^4}{4}\right)dx =$$

$$= \int_0^1 \frac{x^2}{3} - \frac{x^2}{4} - \frac{x^5}{3} + \frac{x^6}{4}\Big)\,dx = \left[\frac{x^3}{9} - \frac{x^3}{12} - \frac{x^6}{18} + \frac{x^7}{28}\right]_0^1 = \frac{1}{9} - \frac{1}{12} - \frac{1}{18} + \frac{1}{28} = \frac{1}{126}$$

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57:

d) $\iiint\limits_U (x+y)e^{x+z}\,dx\,dy\,dz \qquad U=\overset{x}{[0,1]}\times\overset{y}{[0,1]}\times\overset{z}{[0,1]}$

$$\int_0^1 dx\int_0^1 (x+y)\,dy\int_0^1 e^{x+z}\,dz=\int_0^1 dx\int_0^1 (x+y)\int_0^1 e^{x+z}\,dz=\int_0^1 dx\int_0^1 (x+y)\left[e^{x+z}\right]_0^1 dy=$$

$$=\int_0^1 dx\int_0^1 (x+y)(e^{x+1}-e^x)\,dy=\int_0^1 (e^{x+1}-e^x)\,dx\int_0^1 x+y\,dy=$$

$$=\int_0^1 (e^{x+1}-e^x)\left[xy+\frac{y^2}{2}\right]_0^1 dx=\int_0^1 (e^{x+1}-e^x)\left(x+\frac{1}{2}-0\right)dx=$$

$$=\int_0^1 (e^{x+1}-e^x)\left(x+\frac{1}{2}\right)dx=\iint=(e-1)\int_0^1 e^x\left(x+\frac{1}{2}\right)dx=*$$

$$=\left|\begin{matrix} u=x+\frac{1}{2} & v'=e^x \\ u'=1 & v=e^x \end{matrix}\right|=$$

$$=e^x\left(x+\frac{1}{2}\right)-\int e^x dx=e^x\left(x+\frac{1}{2}\right)-\int e^x dx=e^x\left(x+\frac{1}{2}-1\right)=$$

$$*=(e-1)\left(e-\frac{1}{2}-\left(1-\frac{1}{2}\right)\right)=(e-1)^2=e^2-2e+1$$