Ćwiczenia 13

niedziela, 18 marca 2018 12:43

44 a) $\iint_D (x+xy-x^2-2y)\,dx\,dy \qquad D: [0,1]\times[0,1]$

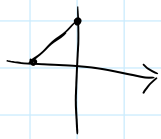
$\int_0^1 dx \int_0^1 x+xy-x^2-2y\,dy = \int x\,dy + \int xy\,dy - \int x^2\,dy - \int 2y\,dy =$

$= xy + \frac{1}{2}xy^2 - x^2y - y^2$

$\int_0^1 dx \left[xy + \frac{1}{2}xy^2 - x^2y - y^2\right]_0^1 = \int_0^1 \left(x + \frac{1}{2}x - x^2 - 1\right)dx =$

$= \left[\frac{x^2}{2} + \frac{x^2}{4} - \frac{x^3}{3} - x\right]_0^1 = \frac{1}{2} + \frac{1}{4} - \frac{1}{3} - 1 = -\frac{7}{12}$

e) $\iint_D e^{2x-y}\,dx\,dy \qquad D: [0,1]\times[-1,0]$



$e^{2x} = g(x)$
$e^{-y} = h(y)$

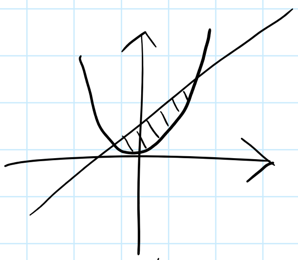
$\iint_D g(x)h(y)\,dx\,dy = \int_a^b g(x)\,dx \int_a^b h(y)\,dy =$

$= \int_0^1 e^{2x}\,dx \int_{-1}^0 e^{-y}\,dy = \left[\frac{1}{2}e^{2x}\right]_0^1 \times \left[-e^{-y}\right]_{-1}^0 =$

$= \left(\frac{1}{2}e^2 - \frac{1}{2}\right)\cdot\left(-1 + \frac{1}{e}\right)$

45 $\iint f(x,y)\,dx\,dy$

a) $y = x^2 \qquad y = x+2$



$x^2 = x+2$

$x^2 - x - 2 = 0$

$\Delta = 1+8 = 9 \quad \sqrt{\Delta} = 3$

$x_1 = \frac{1-3}{2} = -1 \qquad x_2 = 2$

$D: \int_{-1}^{2} dx \int_1^4 f(x,y)\,dy \qquad x^2 \leqslant y \leqslant x+2$

$1 \leqslant y \leqslant 4$

$D: \int_{-1} dx \int_{1} f(x,y)\,dy$ $\qquad x \leqslant y \leqslant x+2$

$1 \leqslant y \leqslant 4$

$$\iint_D f(x,y)\,dx\,dy = \int_{-1}^{2} dx \int_{x^2}^{x+2} dy = \int_{-1}^{2} dx \int_{x^2}^{x+2} f(x,y)\,dy$$

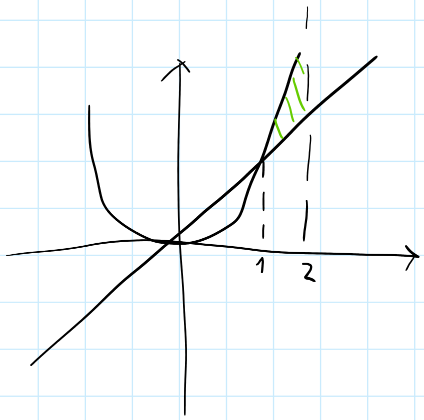
$*$ D) $y^2 = 4x \qquad x+y=3 \qquad y=0 \qquad (y \geqslant 0)$

(46)

a) $$\int_1^2 dx \int_x^{x^2} \frac{y}{x^2}\,dy = \int_1^2 dx \cdot \frac{1}{x^2} \cdot \int_x^{x^2} \frac{y^2}{2}\,dy = \int_1^2 \frac{1}{x^2}\left[\frac{x^4}{2} - \frac{x^2}{2}\right] dx =$$

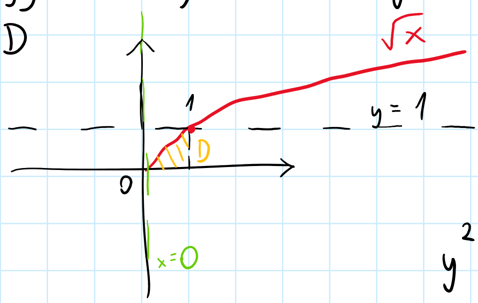
$$= \int_1^2 \frac{x^2}{2} - \frac{1}{2}\,dx = \left[\frac{x^3}{6} - \frac{x}{2}\right]_1^2 = \left(\frac{8}{6} - 1\right) - \left(\frac{1}{6} - \frac{1}{2}\right) = \frac{8}{6} - \frac{6}{6} - \frac{1}{6} + \frac{3}{6} =$$

$$= \frac{2}{3}$$



(47) $\star$) $$\int_0^4 dx \int_{\sqrt{4x-x^2}}^{2\sqrt{x}} xy\,dy$$

(48) c) $$\iint_D e^{\frac{x}{y}}\,dx\,dy \qquad D: y = \sqrt{x} \qquad x = 0 \qquad y = 1$$



$$y^2 \to \int_{-1}^{0}$$

$$\iint_D e^{\frac{x}{y}}\,dx\,dy = \int_a^b dy \int_c^d e^{\frac{x}{y}}\,dx \to \int_a^b \frac{1}{x}\,dy \int_c^d \frac{e^{\frac{x}{y}}}{\cancel{x}}$$

$$\int_0^1 dy \int_0^{y^2} e^{\frac{x}{y}}\,dx = \int_0^1 dy \left[y e^{\frac{x}{y}}\right]_0^{y^2} = \int_0^1 \left[y e^{y} - y\right] dy =$$

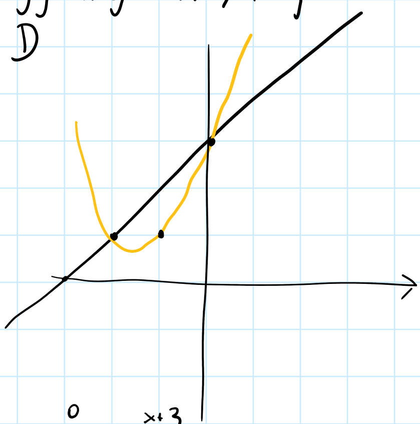
$$\int_0^1 ye^y - \left[\frac{y^2}{2}\right]_0^1$$

$$\int f(x)g(x) = f(x)gx - \int f'(x)g(x) = \left| \begin{matrix} f=y & g=e^y \\ f'=1 & g'=e^y \end{matrix} \right| =$$

$$= \int ye^y = ye^y - \int e^y dy = (y-1)e^y =$$

$$= \left[(y+1)e^y\right]_0^1 - \left[\frac{y^2}{2}\right]_0^1 = +1 - \left(\frac{1}{2}\right) = \frac{1}{2}$$

d) $\iint\limits_D (xy + 4x^2)\,dxdy$ D: $y = x+3$ $y = x^2 + 3x + 3$



$x+3 = x^2 + 3x + 3$

$x^2 + 2x = 0$

$x(x+2)$ $x = -2$ $x = 0$

$$\int_{-2}^{0} dx \int_{x^2+3x+3}^{x+3} xy + 4x^2\, dy =$$