KACPER BORUCKI 245365 - LISTA 5

ZADANIE 1C:

$G(s) = \frac{2s+1}{2s^2+4s+6} = \frac{s+0,5}{s^2+2s+3}$

$G(s) = \frac{Y(s)}{U(s)}$ $G(s) = \frac{L(s)}{M(s)}$ $E(s) = \frac{U(s)}{M(s)}$ $Y(s) = E(s)L(s)$

Z powyższych równań:

$U(s) = E(s)M(s)$

$U(s) = (s^2+2s+3)E(s)$

$s^2E(s) = U(s) - 2sE(s) - 3E(s)$

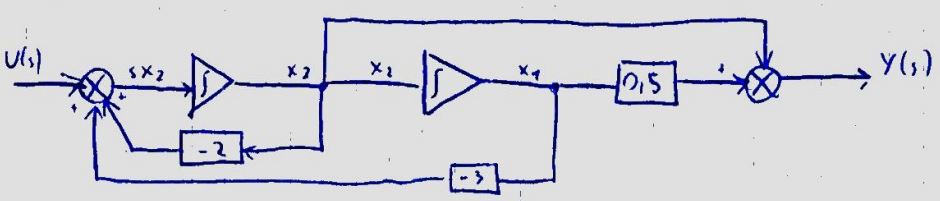
$Y(s) = E(s)L(s)$

$Y(s) = E(s)(s+0,5)$

$Y(s) = 0,5x_1 + x_2$

Zmienne stanu: $x_1 = E(s)$ $x_2 = sE(s)$

$sx_2 = -3x_1 - 2x_2 + U(s)$



$sx_1 = x_2$
$sx_2 = -3x_1 - 2x_2 + U(s)$
$Y(s) = x_1 + 2x_2$

$\rightarrow$ $s\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}U$

$Y = \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]U$

ZADANIE 2C:

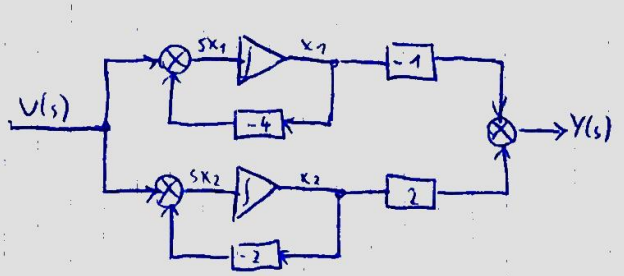
$G(s) = \frac{2,5s+3}{2,5s^2+3s+4} = \frac{s+6}{s^2+6s+8}$

$M(s) = s^2+6s+8 = (s+4)(s+2)$

$\Delta = 36-32 = 4$ $\sqrt{\Delta} = 2$ $s_1 = \frac{-6-2}{2} = -4$ $s_2 = \frac{-6+2}{2} = -2$

$M'(s) = 2s+6$ $h_1 = \frac{s+6}{2s+6}\Big|_{s=-4} = \frac{2}{-2} = -1$ $h_2 = \frac{s+6}{2s+6}\Big|_{s=-2} = \frac{4}{2} = 2$

$G(s) = \frac{-1}{s+4} + \frac{2}{s+2}$



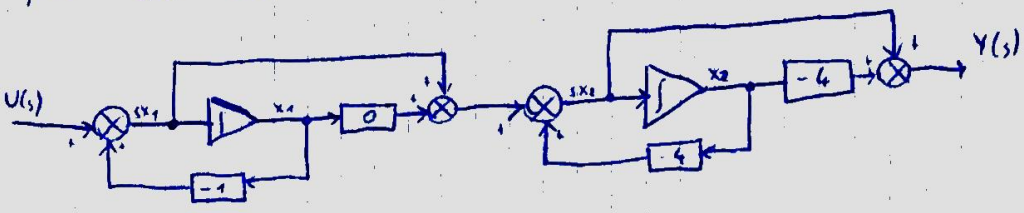
$sx_1 = -4x_1 + U(s)$
$sx_2 = -2x_2 + U(s)$
$Y(s) = x_1 + 2x_2$

$\rightarrow$ $s\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}U$

$Y = \begin{bmatrix} -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0[U]$

ZADANIE 3C: $G(s) = \frac{s(s-4)}{(s+1)(s+4)}$

$G(s) = \frac{s}{s+1} \cdot \frac{s-4}{s+4}$



$$sx_1 = -x_1 + U(s)$$
$$sx_2 = -4x_2 + sx_1 = -x_1 - 4x_2 + U(s)$$
$$Y(s) = -4x_2 + sx_2 = -4x_2 - 4x_2 - x_1 + U(s) = -x_1 - 8x_2 + U(s)$$

$$s\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U$$
$$Y = \begin{bmatrix} -1 & -8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [1]\,U$$

ZADANIE 4C

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = [1 \;\; 0] \quad D = 0$$

$$G(s) = C(sI - A)^{-1}B + D$$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} s-2 & -1 \\ -1 & s-2 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s-2)^2 - 1}\begin{bmatrix} s-2 & -1 \\ -1 & s-2 \end{bmatrix}$$

$$C(sI - A)^{-1} = [1 \;\; 0]\,\frac{1}{s^2 - 4s + 3}\begin{bmatrix} s-2 & -1 \\ -1 & s-2 \end{bmatrix} = \frac{1}{s^2 - 4s + 3}[s-2 \quad 1]$$

$$C(sI - A)^{-1}B = \frac{1}{s^2 - 4s + 3}[s-2 \quad 1]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{s-2}{s^2 - 4s + 3}$$

$$D = 0$$

$$G(s) = \frac{s-2}{s^2 - 4s + 3}$$