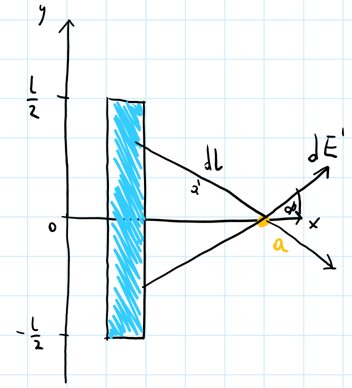
Lista 2:

6

$Q_1 = \frac{Q}{l}$

$Q = Q_1 l$

$F = \frac{q \cdot Q_1 l}{4\pi\varepsilon_0 Q^2}$



$dE = \frac{dQ}{4\pi\varepsilon_0 (a')^2} = \frac{Q_l\, dl}{4\pi\varepsilon_0 (a')^2}$

$\left(\frac{l}{2}\right)^2 + Q^2 = (a')^2$

$Q = \sqrt{\left(\frac{l}{2}\right)^2 + a^2}$

$dE = 2dE'\cos\alpha$

$dE = 2dE'\frac{a}{Q}$

$dE = 2dE'\frac{Q}{\sqrt{\frac{L^2}{4}+a^2}}$ $\to dl = v\,dx$

$2\frac{dQ}{4\pi\varepsilon_0\left(\frac{x^2}{4}+a^2\right)} \cdot \frac{a}{\sqrt{\frac{x^2}{4}+a^2}} = dE$

$E = \int_0^{\frac{l}{2}} \frac{2Q_l a\, dx}{4\pi\varepsilon_0\left(\frac{x^2}{4}+a^2\right)\sqrt{\frac{x^2}{4}+a^2}} = \frac{2Q_l a}{4\pi\varepsilon_0}\int_0^{\frac{l}{2}}\frac{1}{\left(\frac{x^2}{4}+a^2\right)\sqrt{\frac{x^2}{4}+a^2}}dx = \frac{2Q_l a}{4\pi\varepsilon_0}\int_0^{\frac{l}{2}}\frac{1}{\left(\frac{x^2}{4}+a^2\right)^{\frac{3}{2}}}dx =$

$= \left| \begin{array}{l} t = \left(\frac{x^2}{4}+a^2\right) \\ dt = \frac{2}{4}x\,dx \\ \frac{4}{2}dt = x\,dx\end{array}\right| = \int_0^{\frac{l}{2}}\frac{2\,dt}{t^{\frac{3}{2}}} = \int_0^{\frac{l}{2}} 2t^{-\frac{3}{2}}\,dt = 2\int_0^{\frac{l}{2}} t^{-\frac{3}{2}}\,dt = 2\cdot\frac{1}{\sqrt{t}}\Big|_0^{\frac{l}{2}} =$

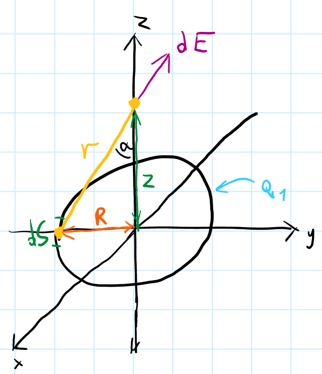
$= \frac{2Q_l a}{4\pi\varepsilon_0}\cdot 2\cdot\frac{1}{\sqrt{\frac{x^2}{4}+a^2}}\Big|_0^{\frac{l}{2}} = \frac{4Q_l a}{4\pi\varepsilon_0\sqrt{\frac{l^2}{16}+a^2}} - \frac{4Q_l a}{4\pi\varepsilon_0\sqrt{a^2}}$

8

$P(0,0,z)$

$x^2+y^2=R^2$

Q_1



$r = \sqrt{R^2+z^2}$

$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dQ}{r^2}$

$\cos\alpha = \frac{z}{r}$

$dQ = Q_L\cdot dS$

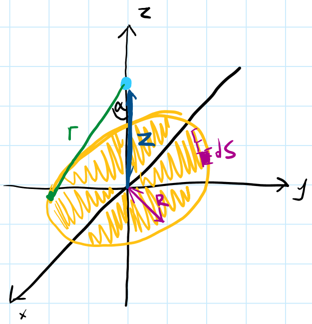
$\int_0^{2\pi r} dE\cos\alpha = \int_0^{2\pi r}\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_L\,dS}{r^2}\cdot\frac{z}{r}$

$= \frac{Q_L z}{4\pi\varepsilon_0(R^2+z^2)^{\frac{3}{2}}}\int_0^{2\pi r} dS =$

$= \frac{2\pi R Q_L z}{4\pi\varepsilon_0(R^2+z^2)^{\frac{3}{2}}} = \frac{Qz}{4\pi\varepsilon_0(R^2+z^2)^{\frac{3}{2}}}$

$\int dV = \int\frac{dQ}{4\pi\varepsilon_0 r} = \int\frac{Q_L\cdot ds}{4\pi\varepsilon_0 r} = \frac{Q_L\cdot 2\pi R}{4\pi\varepsilon_0(R^2+z^2)^{\frac{1}{2}}} = \frac{Q_L R}{2\varepsilon_0(R^2+z^2)^{\frac{1}{2}}}$

(9) $dS = \varrho\, d\varrho\, d\varphi$



$$\int dE \cos\alpha = \iint \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r} \rho\, d\rho\, d\varphi =$$

$$= \frac{q_1}{4\pi\varepsilon_0} \int_0^{2\pi} d\varphi \cdot \int_0^R \frac{z}{\left(\sqrt{\rho^2+z^2}\right)^3} \rho\, d\rho = \frac{2\pi q_1 z}{4\pi\varepsilon_0} \int_0^{2\pi} d\varphi \int_0^R \frac{\rho}{\left(\sqrt{\rho^2+z^2}\right)^3} d\rho =$$

$$= \left| \begin{array}{l} \rho^2 + x^2 = t \\ dt = 2\rho\, d\rho \\ d\rho = \frac{dt}{2\rho} \end{array} \right| = \int \frac{dt}{2\sqrt{t^3}} = \frac{1}{2} \int t^{-\frac{3}{2}} dt = -t^{-\frac{1}{2}} = -\frac{1}{\sqrt{t}} = -\frac{1}{\sqrt{\rho^2+x^2}}$$

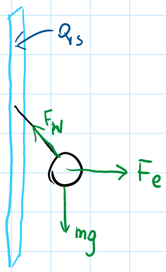
$$E \cos\alpha = \frac{q_1 z}{2\varepsilon_0} \cdot \left(-\frac{1}{\sqrt{R^2+z^2}} + \frac{1}{z} \right)$$

$$\int dV = \int \frac{dq}{4\pi\varepsilon_0 r} = \int \frac{q_S\, dS}{4\pi\varepsilon_0 \sqrt{R^2+z^2}} = \iint \frac{q_S R\, dR\, d\varphi}{4\pi\varepsilon_0 \sqrt{R^2+z^2}} = \frac{q_S}{4\pi\varepsilon_0} \int_0^{2\pi} d\varphi \int_0^R \frac{R}{\sqrt{R^2+z^2}} dR =$$

$dS = R\, dR\, d\varphi$

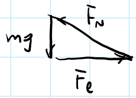
$$= \left| \begin{array}{l} R^2 + z^2 = t \\ dt = 2R\, dR \\ dR = \frac{dt}{2R} \end{array} \right| = \int_0^R \frac{dt}{2\sqrt{t}} = -\sqrt{t} \Big|_0^R = -\sqrt{R^2+z^2} \Big|_0^R = -\sqrt{R^2+z^2} + z$$

(10)



$$E = \frac{q_s}{2\varepsilon_0}$$

$$F_e = F \cdot q$$



$$mg = 4 \cdot 10^{-4} N$$

$$F_N = 4{,}9 \cdot 10^{-4} N$$

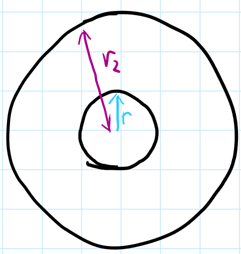
$$F_E = \sqrt{4{,}9^2 - 4^2} \approx 2{,}83 \cdot 10^{-4} N$$

$$q \frac{q_S}{2\varepsilon_0} = F_e \quad \rightarrow \quad q_s = \frac{F_e}{q} 2\varepsilon_0$$

$$q_s = \frac{2{,}85 \cdot 10^{-4} N \cdot 2 \cdot 8{,}85 \cdot 10^{-12}}{6{,}67 \cdot 10^{-10}} = 7{,}56 \cdot 10^{-6} \frac{C}{m^2}$$

(11) $E = 10 \frac{kV}{m}$ $\quad r_1 = 5 cm = 0{,}05 m$

$U = 100 V$



$$V = k_e \frac{q}{r}$$

$$E = k_e \frac{q}{r^2} \rightarrow q = 4\pi\varepsilon_0 E r_1^2 =$$

$$= \frac{4\pi}{36\pi} \cdot 10^{-9} \cdot 10^4 \cdot 0{,}05^2$$

$$q = 2{,}78 \cdot 10^{-9}$$

$$q = 2{,}78 \cdot 10^{-9}$$

$$V_1 - V_2 = 100$$

$$kq\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 100$$

$$\frac{1}{0{,}05} - \frac{1}{r^2} = \frac{100}{kq}$$

$$20 - \frac{100}{kq} = \frac{1}{r^2}$$

$$\frac{1}{r_2^2} = \frac{20}{1} - \frac{100}{\frac{10^{-9}}{36\pi}} = 20 - \frac{3600\pi}{10^{-9}}$$

$$\frac{1}{r_2} = \sqrt{}$$