Ćwiczenia 8

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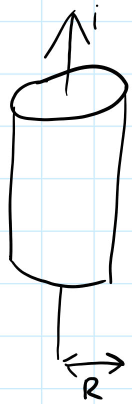
LISTA 7

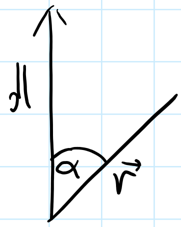
(1)



$B(r) = ?$

$r \geqslant R$

$0 \leqslant r \leqslant \infty$



Prawo Ampera:

$$\oint B \cdot dl = \mu_0 \cdot I_r$$

$$B \cdot 2\pi r = \mu_0 I_r$$

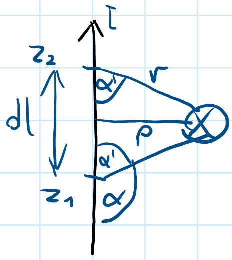
$$B = \frac{\mu_0 I_r}{2\pi r}$$

Prawo Biota-Savarta-Laplace'a:

$$d B = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$B = \frac{\mu_0}{4\pi} I \cdot \frac{\sin\alpha}{r^3}$$

$$H = \frac{I}{4\pi\rho}(\cos\alpha_1 + \cos\alpha_2)\,\vec{1}_\varphi$$



$$\vec{dH} = \frac{I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$r = \sqrt{\rho^2 + z^2}$$

$$dl = dz \cdot \hat{k}$$

$$\vec{r} = \vec{\rho} + \vec{z} = -z\hat{k} + \rho\cos\varphi\, d\varphi\,\hat{i} + \rho\sin\varphi\,\hat{j}$$

$$\vec{dl} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & dz \\ \rho\cos\rho & \rho\sin\varphi & -z \end{vmatrix} = \rho\, dz \sqrt{-\sin\varphi\,\hat{i} + \cos\varphi\,\hat{j}}$$

(2) $H(r) = ?$

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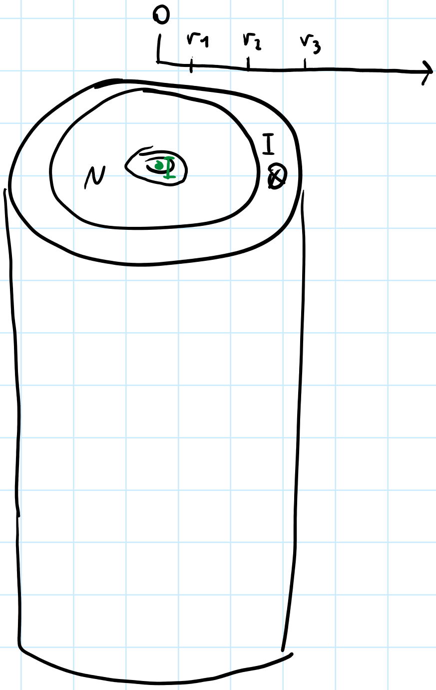
$r_1 = 5\,mm \qquad r_2 = 8\,mm \qquad r_3 = 10\,mm \qquad I = 62{,}8\,A$



$$\int \vec{j} \circ d\vec{S} = \oint \vec{H} \circ d\vec{l}$$

$$\int_0^{r_1} \frac{I}{S}\, dS = \oint \vec{H} \cdot d\vec{l}$$

$$\frac{I}{\pi r_1^2} \int_0^{r} dS = \oint H \cdot d\vec{l}$$

$$\frac{I \pi r^2}{\pi r_1^2} = \oint H \cdot dl$$

$$I\left(\frac{r}{r_1}\right)^2 = H \cdot 2\pi r$$

$$H = \frac{I\left(\frac{r}{r_1}\right)^2}{2\pi r}$$

$$\int_0^{r} \frac{I}{S}\, dS = \oint H_2 \cdot dl$$

$$\frac{I \cdot \pi r^2}{\pi (r_2 - r_1)^2} = H_2\, 2\pi r$$

$$H_2 = \frac{I r}{2\pi (r_2 - r_1)^2}$$

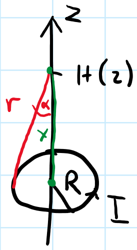
$$\int_0^{r} \frac{I}{S}\, dS = \oint H_3\, dl$$

$$\frac{I \pi r^2}{\pi (r_3 - r_2)} = H_3 \cdot 2\pi r$$

$$\frac{I \cancel{\pi} r^{\cancel{2}}}{\cancel{\pi} (r_2 - r_1)^2} - \frac{I \cancel{\pi} r^{\cancel{2}}}{\cancel{\pi} (r_3 - r_2)^2} = H_3 \cdot 2\pi \cancel{r}$$

$$\frac{I r}{2\pi} \left(\frac{1}{(r_2 - r_1)^2} - \frac{1}{(r_3 - r_2)^2} \right) = H_3$$

(3) $H(z) = ? \qquad I = N\,[A]$



$$dH = \frac{I}{4\pi}\frac{dl\sin 90^\circ}{r^2} = \frac{I}{4\pi}\frac{dl}{r^2}$$

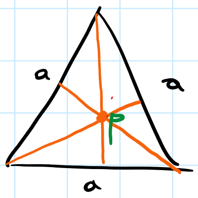
$$dH_z = \frac{I\,dl\cos\alpha}{4\pi r^2} = \frac{I\,dl\,x}{4\pi r^3} \qquad r = \sqrt{R^2+x^2}$$

$$\cos\alpha = \frac{x}{r} \qquad dH_z = \frac{I\,dl\,x}{4\pi\sqrt{R^2+x^2}}$$

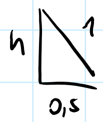
$$\frac{Ix}{4\pi\sqrt{R^2+x^2}}\int_0^{2\pi R} dl = \frac{Ix\,2\cancel{\pi}R}{\cancel{4\pi}_{2}\sqrt{R^2+x^2}} = \frac{IxR}{2\sqrt{R^2+x^2}}$$

4 $\quad l = 3\,m \qquad I = 20\,A$

KOLOKWIUM



$x = \frac{2}{3}h \qquad h = \sqrt{1-0{,}25} = 0{,}87$

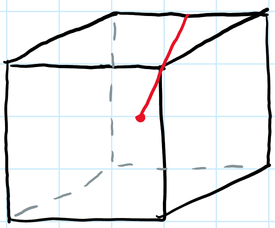


$\mu_0 = 4\pi\cdot 10^{-7}$

$$H_1 = \frac{I}{2\pi\rho}$$

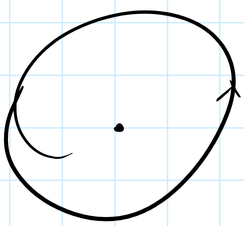
$$H = 3H_1 = \frac{3I}{2\pi\rho} = \frac{3\cdot 20}{2\pi\cdot 0{,}3} = 31{,}9$$

$$B = \mu_0 H = 4\pi\, 10^{-7}\cdot 31{,}9 = 4\cdot 10^{-5}\,H$$



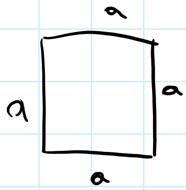
$a = 0{,}5\,m$

Odp: $H = \frac{6I}{4\pi\rho}$



$$H = \frac{1}{4\pi r^2}\cdot 2\pi r = \frac{1}{2\pi r} \qquad H = \frac{I\pi}{l}$$

$$l = 2\pi r \qquad r = \frac{R}{2\pi}$$



$$\rho = \frac{a}{2}$$

$$H = 4H_I = \frac{4I}{2\pi\rho} = \frac{4I}{\pi a}$$

(5)

$$H = \frac{I}{4\pi\rho}\underbrace{\left(\cos\alpha_1 + \cos\alpha_2\right)}_{2\cos\alpha_1}$$

$$\rho = R\sin\alpha_1$$

$$\lim_{n\to\infty} H(n) = ?$$

$$H = n \cdot \frac{I}{4\pi R\sin\alpha_1} \cdot 2\cos\alpha_1 = \frac{I \cdot n}{2\pi R}\,\frac{\cos\alpha_1}{\sin\alpha_1} = \frac{I n}{2\pi R}\,\mathrm{ctg}\left(\frac{180 - \frac{360}{n}}{2}\right)$$

Odpowiedź

$$\frac{I}{2\pi R}\lim_{n\to\infty}\mathrm{ctg}\left(\frac{180 - \frac{360}{n}}{2}\right)\cdot n = \lim_{n\to\infty} n\,\mathrm{ctg}\left(90 - \boxed{\frac{180}{n}}\right) = \frac{I}{2\pi R}\cdot\pi = \frac{I}{2R}$$

$$\boxed{\frac{180}{n}} \to 0$$

$$= \lim_{n\to\infty} n \cdot \frac{\cos(90^\circ - \frac{180}{n})}{\sin(90^\circ - \frac{180}{n})} = \frac{\lim_{n\to\infty} n\cos\left(90 - \frac{180}{n}\right)}{\lim_{n\to\infty}\sin\left(90 - \frac{180}{n}\right)} = \lim_{n\to\infty}\frac{\cos\left(90 - \frac{180}{n}\right)}{\frac{1}{n}} \Rightarrow$$

$$n\cos\left(90 - \frac{180}{n}\right) \to 0, \qquad \sin\left(90 - \frac{180}{n}\right) \to 1$$

$$\lim_{n\to\infty}\frac{-\sin\left(90 - \frac{180}{n}\right)}{-\frac{1}{n^2}} = \pi$$

(6)

$H = ?$ $\quad I = N\,A$ $\quad R = I\ \mathrm{cm}$

$$d\vec{H} = \frac{I}{4\pi}\cdot\frac{dl \times \vec{r}}{r^3}$$

$$dH = \frac{I}{4\pi r^2}\,dl \qquad H_2 = \frac{I}{4\pi r^2}\cdot\pi r = \frac{I}{4r}$$

$$H = H_1 + H_2 + H_3 = \frac{2I}{2\pi r} + \frac{I}{4r} = \frac{I}{\pi r} + \frac{I}{4r}$$

$$H_1 = \frac{I}{2\pi r}$$