LISTA 2

1. $Q_1 = 1 mC \quad (3, 2, -1)$ $\qquad \varepsilon_0 = \frac{10^{-9}}{36\pi}$

$Q_2 = -2 mC \quad (-1, -1, 4)$ $\qquad E = ?$

$Q_3 = 10 nC \quad P(0, 3, 1)$ $\qquad F = ?$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$\overline{Q_1P} = [-3, 1, 2]$ $\qquad \overline{Q_2P} = [1, 4, -3]$

$|\overline{Q_1P}| = \sqrt{19}$ $\qquad |\overline{Q_2P}| = \sqrt{26}$

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1 \cdot 10^{-3}}{\sqrt{19}^3} \cdot [-3, 1, 2]$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-2 \cdot 10^{-3}}{\sqrt{26}^3} \cdot [1, 4, -3]$$

$$\vec{E}_{1x} = \left[\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{1 \cdot 10^{-3}}{\sqrt{19}^3} \cdot (-3)\right)\right]$$

$\vec{E}_{1y} =$ — " —

$\vec{E}_{1z} =$ — " —

$$\vec{E} = [E_{1x} + E_{2x},\ E_{1y} + E_{2y},\ E_{1z} + E_{2z}]$$

$$\vec{F} = \vec{E} Q$$

2. $Q_1: 5 nC \qquad P(2, 0, 4)$

$Q_2: -2 nC \qquad Q(-3, 0, 5)$

$Q_3: 1 nC \qquad R(1, -3, 7)$

$\overline{Q_1R} = (-1, -3, 3)$ $\qquad |\overline{Q_1R}| = \sqrt{19}$

$\overline{Q_2R} = (4, -3, 2)$ $\qquad |\overline{Q_2R}| = \sqrt{29}$

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q}{|\vec{r}|^3} \vec{r} = \frac{1}{\frac{\cancel{10^{-9}}}{9}} \cdot \frac{5 \cdot 10^{\cancel{-18}}}{\sqrt{19}^3} \cdot [1, -3, 3] =$$

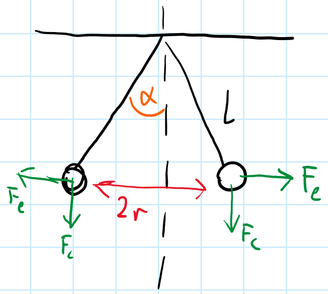
$$= \frac{45 \cdot 10^{-9}}{19\sqrt{19}}[-1,-3,3]$$

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2 Q_1}{|\vec{r}|^3}\vec{r} = \frac{1}{\cancel{4\pi} 9 \cdot \frac{10^{-9}}{\cancel{36\pi}}} \cdot \frac{-2 \cdot \cancel{10^{-9}} \cdot 1 \cdot 10^{-9}}{(\sqrt{29})^3} \cdot [4,-3,2] =$$

$$= \frac{-2 \cdot 10^{-9}}{29\sqrt{29}} \cdot [4,-3,2]$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = [\vec{F}_{1x} + \vec{F}_{2x},\ \vec{F}_{1y} + \vec{F}_{2y},\ \vec{F}_{1z} + \vec{F}_{2z}]$$

(3)



$$Q^2 = 16\pi\varepsilon_0 m g l^2 \sin^2\alpha \, tg\alpha$$

$$F_e = k\frac{Q^2}{(2r)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{4r^2}$$

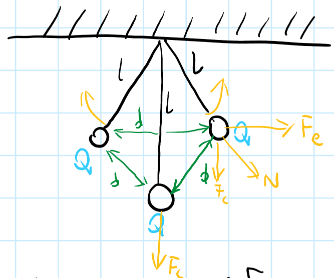
$$Q^2 = 16 F_e r^2 \varepsilon_0 \pi =$$

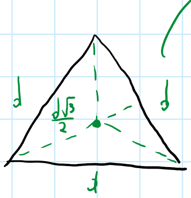
$$\sin\alpha = \frac{r}{L} \qquad r = l\sin\alpha$$

$$Fe = mg\,tg\alpha$$

$$Q^2 = 16\, mg\,tg\alpha\, l^2 \sin^2\alpha\, \varepsilon_0 \pi$$

(4)



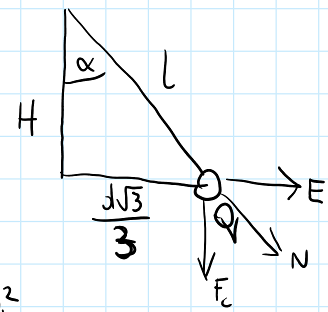


$$\frac{d\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{3} = \frac{d\sqrt{3}}{3}$$

$$Q^2 = \frac{4}{3}\pi\varepsilon_0 m g d^3 \underbrace{\left[l^2 - \frac{d^2}{3}\right]^{-\frac{1}{2}}}$$

$$H = \sqrt{l^2 - \left(\frac{d\sqrt{3}}{3}\right)^2} = \sqrt{l^2 - \frac{d^2}{3}}$$

$$= \left(l^2 - \frac{d^2}{3}\right)^{\frac{1}{2}}$$



$$F_{e_{13}} = k\frac{Q_1 Q_2}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{d^2} = \frac{Q^2}{4\pi\varepsilon_0 d^2} \cdot \cos 30^\circ$$

$$F_{e_{13}} = k \frac{\cdot\cdot}{r^2} = \frac{}{4\pi\varepsilon_0} \cdot \frac{}{d^2} = \frac{}{4\pi\varepsilon_0 d^2} \cdot \cos 30^\circ$$

$$Q^2 = F_e \cdot 4\pi\varepsilon_0 d^2 \cdot$$

$$F_e =$$

$$H F_e = mg\frac{2}{3}h$$

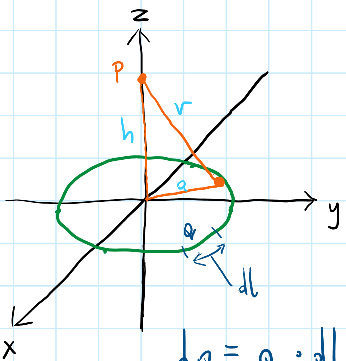
$$\frac{\cancel{\sqrt{3}}\, Q^2}{2\cdot 4\pi\varepsilon_0 d^2}\sqrt{l^2-\left(\frac{1}{3}\sqrt{3}\,d^2\right)} = mg\frac{1}{3}\cancel{\sqrt{3}}\,\cancel{d}$$

$$Q^2 = \frac{8}{3}\varepsilon_0 \pi d\, mg\left(\sqrt{l^2-\frac{d^2}{3}}\right)^{-1}$$

5

$$Q_1 \frac{C}{m}$$

$$E(0,0,h) = \frac{Q_1 a h}{2\varepsilon_0 [h^2+a^2]^{\frac{3}{2}}} 1_z$$



$$dQ = Q_1 \cdot dl$$

$$r = \sqrt{a^2+h^2}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 \vec{r}^3} \cdot \vec{r}$$

$$dE = \frac{dQ}{4\pi\varepsilon_0 r^3} \cdot \vec{r} = \frac{Q_1 dl}{4\pi\varepsilon_0 r^3} \cdot \vec{r} =$$

$$= \frac{Q_1 dl}{4\pi\varepsilon_0 \sqrt{a^2+h^2}} \cdot \vec{r} = \frac{Q_1 dl}{4\pi\varepsilon_0\sqrt{a^2+h^2}}\left[\cos\varphi\hat{i}, \sin\varphi\hat{j}, z\hat{k}\right]$$

$$\vec{r} = \left[\cos\varphi\hat{i}, \sin\varphi\hat{j}, h\hat{k}\right]$$

$$dE = \frac{Q_1 a\, d\varphi}{4\pi\varepsilon_0\sqrt{a^2+h^2}}\left[\cos\varphi\hat{i}, \sin\varphi\hat{j}, h\hat{k}\right]$$

$$\vec{E} = \oint d\vec{E}$$

$$E_x = \int_0^{2\pi} \frac{Q_1 a\, d\varphi}{4\pi\varepsilon_0\sqrt{a^2+h^2}^3} = 0$$

$\int$ okresu $\cos\varphi = 0$

$$E_y = \int_0^{2\pi} \frac{Q_1 a\, d\varphi}{4\pi\varepsilon_0\sqrt{a^2+h^2}^3} = 0$$

$\int$ okres $\sin\varphi = 0$

$$E_z = \int_0^{2\pi} \frac{Q_1 a\, d\varphi}{4\pi\varepsilon_0\sqrt{a^2+h^2}^3} h = \frac{Q_1 a}{4\pi\varepsilon_0\sqrt{a^2+h^2}^3} h \int_0^{2\pi} d\varphi = \frac{2\pi Q_1 a}{4\pi\varepsilon_0\sqrt{a^2+h^2}} h$$

$$= \frac{Q_1 a h}{2\varepsilon_0\sqrt{a^2+h^2}}$$

b) $$\frac{d}{dh}\frac{Q_1 a h}{2\varepsilon_0\sqrt{a^2+h^2}^3} = \frac{\cancel{Q_1 a}\,\cancel{2}\varepsilon_0\sqrt{a^2+h^2}^3 - \cancel{Q_1 a}h\cdot\cancel{2}\varepsilon_0\frac{3}{2}\cdot 2h\cdot\sqrt{a^2+h^2}}{4\varepsilon_0^2(a^2+h^2)^3} = 0$$

b) $\frac{\ldots}{dh}\ \frac{\ldots}{2\varepsilon_0\sqrt{a^2+h^2}^3} = \frac{\ldots}{4\varepsilon_0^2(a^2+h^2)^3} = 0$

$$3h^2\sqrt{a^2+h^2} = \sqrt{a^2+h^2}^3$$

$$3h^2 = a^2 + h^2$$

$$2h^2 = a^2 \qquad h = \frac{a}{\sqrt{2}}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 z^3}\cdot\vec{r}$$