KACPER BORUCKI 245365

LISTA 4, 3G:

$$G_{12}(z) = \frac{2z^2-0{,}96z-0{,}12}{z^3-1{,}9z^2+0{,}8z+0{,}1} = \frac{2z^2-0{,}96z-0{,}12}{(z-1)^2(z+0{,}1)} \qquad T_p=1$$

$$M(z) = z^3-1{,}9z^2+0{,}8z+0{,}1 = z(z^2-1{,}9z+0{,}8)+0{,}1 =$$

$$= z(z^2-2z+1) + z(0{,}1z-0{,}2)+0{,}1 = z(z-1)^2 + 0{,}1z^2-0{,}2z+0{,}1 =$$

$$= z(z-1)^2 + 0{,}1(z^2-\cancel{0{,}}2z+1) = z(z-1)^2+0{,}1(z-1)^2 = (z-1)^2(z+0{,}1)$$

$$e_p = \frac{T_p}{1+\lim\limits_{z\to1}G_{12}(z)} = \frac{1}{1+\lim\limits_{z\to1}\frac{2z^2-0{,}96z-0{,}12}{(z-1)^2(z+0{,}1)}} = \frac{1}{1+\frac{0{,}92}{0}} = \left[\frac{1}{\infty}\right] = 0$$

$$e_v = \frac{1}{\lim\limits_{z\to1}(z-1)\frac{2z^2-0{,}96z-0{,}12}{(z-1)^2(z+0{,}1)}} = \frac{1}{\left[\frac{0{,}92}{0}\right]} = 0$$

$$e_a = \frac{1}{\lim\limits_{z\to1}\cancel{(z-1)^2}\frac{2z^2-0{,}96z-0{,}12}{\cancel{(z-1)^2}(z+0{,}1)}} = \frac{1}{\frac{0{,}92}{1{,}1}} = \frac{1{,}1}{0{,}92} = 1{,}196$$

LISTA 4, 1B

$$G(z) = \frac{2}{z^2-0{,}4z-1{,}92}$$

$$G_z(z) = \frac{\frac{2}{z^2-0{,}4z-1{,}92}}{1+\frac{2}{z^2-0{,}4z-1{,}92}} = \frac{\frac{2}{z^2-0{,}4z-1{,}92}}{\frac{z^2-0{,}4z+0{,}08}{z^2-0{,}4z-1{,}92}} = \frac{2}{z^2-0{,}4z+0{,}08}$$

$$\Delta = 0{,}4^2-4\cdot0{,}08 = -0{,}16 \qquad \sqrt{\Delta} = j0{,}4$$

$$z_1 = \frac{0{,}4-j0{,}4}{2} = 0{,}2-j0{,}2 \qquad |z_1| = 0{,}283 < 1$$

$$z_2 = \frac{0{,}4+j0{,}4}{2} = 0{,}2+j0{,}2 \qquad |z_2| = 0{,}283 < 1$$

UKŁAD STABILNY

LISTA 4, 2E:

$$G(z) = \frac{2z^2+5z+1}{4z^4+2z^3+2z^2+2z+2}$$

1) $M(1) = 4+2+2+2+2 > 0$ ok
2) $(-1)^4 M(-1) = 4-2+2-2+2 = 4 > 0$ ok
3) $|a_0| < |a_n| \rightarrow 1 < 2$ ok
4) $|b_0| > |b_{n-1}| \rightarrow 12 > 4$ ok
5) $|c_0| > |c_{n-2}| \rightarrow 128 > 32$ ok

$$\begin{vmatrix} 2 & 2 & 2 & 2 & 4 \\ 4 & 2 & 2 & 2 & 2 \\ -12 & -4 & -4 & -4 & 0 \\ -4 & -4 & -4 & -12 & 0 \\ 128 & 32 & 32 & 0 & 0 \end{vmatrix}$$

$$b_0 = \begin{vmatrix} 2 & 4 \\ 4 & 2 \end{vmatrix} = 4-16 = -12$$

$$b_1 = \begin{vmatrix} 2 & 2 \\ 4 & 2 \end{vmatrix} = 4-8 = -4$$

$$b_2 = \begin{vmatrix} 2 & 2 \\ 4 & 2 \end{vmatrix} = -4$$

$$b_3 = \begin{vmatrix} 2 & 2 \\ 4 & 2 \end{vmatrix} = -4$$

$$c_0 = \begin{vmatrix} -12 & -4 \\ -4 & -12 \end{vmatrix} = 144-16 = 128$$

$$c_1 = \begin{vmatrix} -12 & -4 \\ -4 & -4 \end{vmatrix} = 48-16 = 32$$

$$c_2 = \begin{vmatrix} -12 & -4 \\ -4 & -4 \end{vmatrix} = 32$$

UKŁAD STABILNY

LISTA 4, 3C

$$G_{12}(z) = \frac{k}{z+0,8}$$

$$G_{12}(s) = \frac{k}{e^{sTp}+0,8} \xrightarrow{Tp=1} G_{12}(j\omega) = \frac{k}{\cos\omega + 0,8 + j\sin\omega}$$

$Im\{G_{12}(j\omega)\} = 0$ dla $\omega_1 = 0$ i $\omega_1 = \pi$

$$Re\{G(j\omega_1)\} = \frac{k}{\cos\pi + 0,8} = \frac{k}{-0,2}$$

Z kryterium lewej strony układ będzie niestabilny jeśli $\frac{k}{-0,2} < -1$

$$\frac{k}{-0,2} < -1 \rightarrow \frac{k}{0,2} > 1 \rightarrow k > 0,2$$

LISTA 4, 4C:

$$G(z) = \frac{1}{5z^3 - 2z^2 + 3z + 1}$$

$$G(w) = \frac{1}{5\left(\frac{1+w}{1-w}\right)^3 - 2\left(\frac{1+w}{1-w}\right)^2 + 3\left(\frac{1+w}{1-w}\right) + 1} = \frac{1}{\frac{5(1+w)^3 - 2(1+w)^2(1-w) + 3(1+w)(1-w)^2 + (1-w)^3}{(1-w)^3}}$$

$$= \frac{(1-w)^3}{5(1+3w+3w^2+w^3) - 2(1+2w+w^2)(1-w) + 3(1+w)(1-w)^2 + (1-w)^3} =$$

$$= \frac{(1-w)^3}{5+15w+15w^2+5w^3 - 2(1-w+2w-2w^2+w^2-w^3) + 3(1-2w+w^2+w-2w^2+w^3) + 1 - 3w + 3w^2 - w^3} =$$

$$= \frac{(1-w)^3}{9w^3+17w^2+7w+7}$$

$$\begin{bmatrix} 9 & 7 \\ 17 & 7 \\ 3,3 & 0 \\ 7 & 0 \end{bmatrix}$$

PIERWSZY WARUNEK KRYTERIUM ROUTHA SPEŁNIONY

$$b_1 = \frac{\begin{vmatrix} 9 & 7 \\ 17 & 7 \end{vmatrix}}{-17} = \frac{63-119}{-17} = \frac{-36}{-17} \approx 3,3$$

$$b_2 = 0$$

$$c_1 = \frac{\begin{vmatrix} 17 & 7 \\ 3,3 & 0 \end{vmatrix}}{-3,3} = \frac{-3,3 \cdot 7}{-3,3} = +7$$

Brak zmian znaku w pierwszej kolumnie → układ stabilny