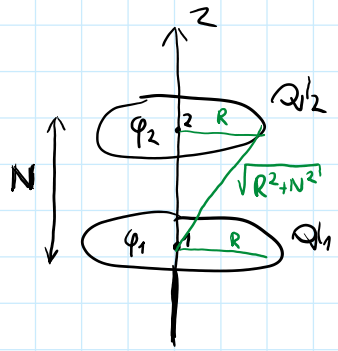
Ćwiczenia 7

wtorek, 20 listopada 2018 11:16

LISTA 6

(1)



$\varphi_1 = I \qquad \varphi_2 = N$

Pkt. 1 od 1. kółka:

$$d\varphi_{1.1} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_{l_1}dl}{r} =$$

$$\varphi_{1.1} = \frac{q_{l_1}}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{dl}{R} = \frac{q_{l_1}}{4\pi\varepsilon_0 R}\int dl = \frac{Q_{l_1}2\pi R}{4\pi\varepsilon_0 R} = \frac{q_{l_1}}{2\varepsilon_0}$$

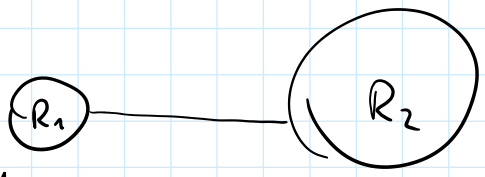
$$\varphi_{1.2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{\sqrt{R^2+N^2}} = \frac{1}{4\pi\varepsilon_0\sqrt{R^2+N^2}}\cdot\int Q_{l_2}\,dl =$$

$$= \frac{Q_{l_2}2\pi R}{4\pi\varepsilon_0\sqrt{R^2+N^2}} = \frac{Q_{l_2}}{2\varepsilon_0}\,\frac{R}{\sqrt{R^2+N^2}}$$

$$\varphi_1 = \varphi_{1.1} + \varphi_{1.2} = \frac{Q_{l_1}}{2\varepsilon_0} + \frac{Q_{l_2}R}{2\varepsilon_0\sqrt{R^2+N^2}}$$

$$\varphi_2 = \frac{1}{2\varepsilon_0}\cdot\left(Q_{l_2} + \frac{Q_{l_1}R}{\sqrt{R^2+N^2}}\right)$$

(2)



1mm
9,1cm

10cm

$Q = 10\,\mu C$

$Q_1 + Q_2 = 10\,\mu C$

$\sigma_S = ? = \frac{Q}{S} \qquad S = 4\pi r^2$

$10\cdot10^{-6} \qquad 1\cdot10^{-3}$

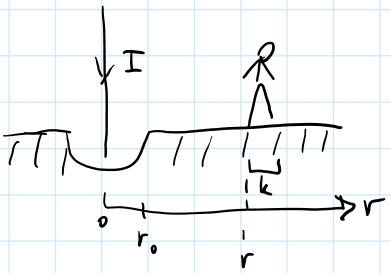
$\varphi_S = \frac{Q}{S}$ $\quad S = 4\pi r^2$

ODP: $Q_{S_1} = \frac{10 \cdot 10^{-6}}{101 \cdot 4 \cdot 3{,}14 \cdot 10^{-6}}$ $\quad Q_{S_2} = \frac{1 \cdot 10^{-3}}{101 \cdot 3{,}14 \cdot 4 \cdot 100 \cdot 10^{-4}}$

Założenia do rozwiązania: $\varphi_1 \cong \varphi_2$

(3) $I = 1V \cdot 10A$ $\quad r_0 = 4\,cm$ $\quad \rho = 100\,\Omega m$

$k = 0{,}8\,m$ $\quad \rho = \frac{1}{\gamma}$



$$U_{kr} = \int_{r_A}^{r_B} E\,dr = \frac{I}{2\pi\gamma} \int_{r_A}^{r_B} \frac{1}{r^2}\,dr = \frac{I\rho}{2\pi}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$r_B = r_A + 0{,}8$

$$100 = \frac{N \cdot 10 \cdot 100}{2\pi}\left(\frac{1}{r_A} - \frac{1}{r_A + 0{,}8}\right)$$

$$100 = \frac{N1000}{2\pi}\left(\frac{0{,}8}{r_A^2 + 0{,}8 r_A}\right)$$

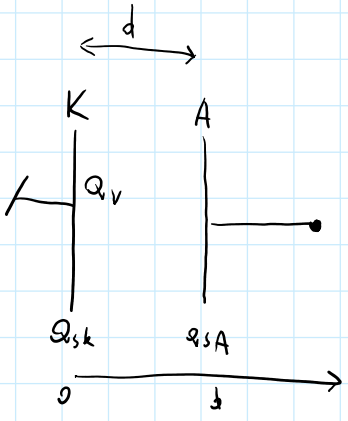
$$\frac{100 \cdot 6{,}28}{N1000} = \frac{0{,}8}{r_A^2 + 0{,}8 r_A}$$

$$\left(r_A^2 + 0{,}8 r_A\right) 6{,}28 = 8N$$

$$r_A^2 + 0{,}8 r_A = \frac{8N}{6{,}28} \qquad r_A^2 + 0{,}8 r_A - \frac{8N}{6{,}28} = 0$$

$U = \varphi_A - \varphi_B$ $\quad \varphi = \frac{I\rho}{2\pi r}$

(14)



$Q_S = ?$
$Q_V = ?$

$\varphi(x) = k \cdot x^{\frac{4}{3}}$

$-\frac{Q_V}{\varepsilon} = \varphi''(x)$

$$-\frac{Q_V}{\varepsilon} = \varphi''(x)$$

$$\varphi = k \cdot x^{\frac{4}{3}} \qquad \varphi' = \frac{4}{3} k x^{\frac{1}{3}} \qquad \varphi'' = \frac{4}{9} k x^{-\frac{2}{3}}$$

$$Q_V = -\varepsilon \frac{4}{9} k \frac{1}{\sqrt[3]{x^2}}$$

$$D_{2N} = Q_S \qquad E = -\text{grad} V = -\frac{d}{dx} V(x)$$

$$Q_S = \varepsilon E = -\frac{4}{3} k x^{\frac{1}{3}} \varepsilon$$

$$Q_{SL} = 0$$

$$Q_{SA} = -\frac{4}{3} k \varepsilon \sqrt[3]{d}$$

(5)

(b) $$\vec{D} = \varepsilon \vec{E} = \varepsilon \frac{Q}{4\pi r^2 \varepsilon} = \frac{Q}{4\pi r^2}$$

Nie zmieni się → przy rozwarciu klucza k ładunek jest stały, natomiast przenikalność elektryczna otoczenia nie wpływa na wektor indukcji elektrycznej

(a) Zmieni się, ponieważ przy zamknięciu klucza k zmienia się ładunek na kuli

$$\vec{D} = \varepsilon \vec{E} = \frac{Q}{4\pi r^2}$$

$$\vec{E} = \frac{Q}{4\pi r^2 \varepsilon} \qquad U = \int_{r_1}^{r_2} E \, dr$$

$$\int_{r_1}^{r_2} E \, dr = \int_{r_1}^{r_2} \frac{Q}{4\pi r^2 \varepsilon} = \frac{Q}{4\pi \varepsilon} \int_{r_1}^{r_2} \frac{1}{r^2} dr =$$

$$= -\frac{1}{r}\Big|_{r_1}^{r_2} = -\frac{1}{r_2} + \frac{1}{r_1} = \frac{1}{r_1} - \frac{1}{r_2}$$

$$U = \frac{Q}{4\pi \varepsilon} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{Q}{r_1 4\pi \varepsilon}$$

$\frac{1}{r_2} \to 0$ gdy $r_2 \to \infty$

$$C = \frac{Q}{\frac{Q}{4\pi \varepsilon r_1}} = 4\pi \varepsilon r_1$$

$$\vec{D} = \varepsilon \vec{E} = \frac{Q}{4\pi r^2} = \frac{4\pi \varepsilon r_1 U}{4\pi r^2} = \frac{\varepsilon U}{r}$$

(6) $r_1 = 20\,mm \quad r_2 = 30\,mm \quad r_3 = 40\,mm$

$\varepsilon_{r_1} = 2{,}5 \quad \varepsilon_{r_2} = 4$

a)

$$C_2 = \frac{C_1 C_2}{C_1 + C_2}$$

$$U = \frac{q}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$C_1 = \frac{4\pi\varepsilon_1}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$C_2 = \frac{4\pi\varepsilon_2}{\left(\frac{1}{r_2} - \frac{1}{r_3}\right)}$$

$$C_2 = \frac{\frac{4\pi\varepsilon_1}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \cdot \frac{4\pi\varepsilon_2}{\left(\frac{1}{r_2} - \frac{1}{r_3}\right)}}{\frac{4\pi\varepsilon_1}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} + \frac{4\pi\varepsilon_2}{\left(\frac{1}{r_2} - \frac{1}{r_3}\right)}} =$$

b)

$$C_2 = \frac{4\pi\varepsilon_1}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} + \frac{4\pi\varepsilon_2}{\left(\frac{1}{r_2} - \frac{1}{r_3}\right)} = \frac{4\pi\varepsilon_1\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + 4\pi\varepsilon_2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\left(\frac{1}{r_2} + \frac{1}{r_3}\right)} =$$