Supervised classification - improving capacity learning

0. Import library

Import library

```
# Import libraries

# math library
import numpy as np

# visualization library
%matplotlib inline
from IPython.display import set_matplotlib_formats
set_matplotlib_formats('png2x','pdf')
import matplotlib.pyplot as plt

# machine learning library
from sklearn.linear_model import LogisticRegression

# 3d visualization
from mpl_toolkits.mplot3d import axes3d

# computational time
import time

import math
```

1. Load and plot the dataset (dataset-noise-01.txt)

The data features for each data i are $x_i = (x_{i(1)}, x_{i(2)})$.

The data label/target, y_i, indicates two classes with value 0 or 1.

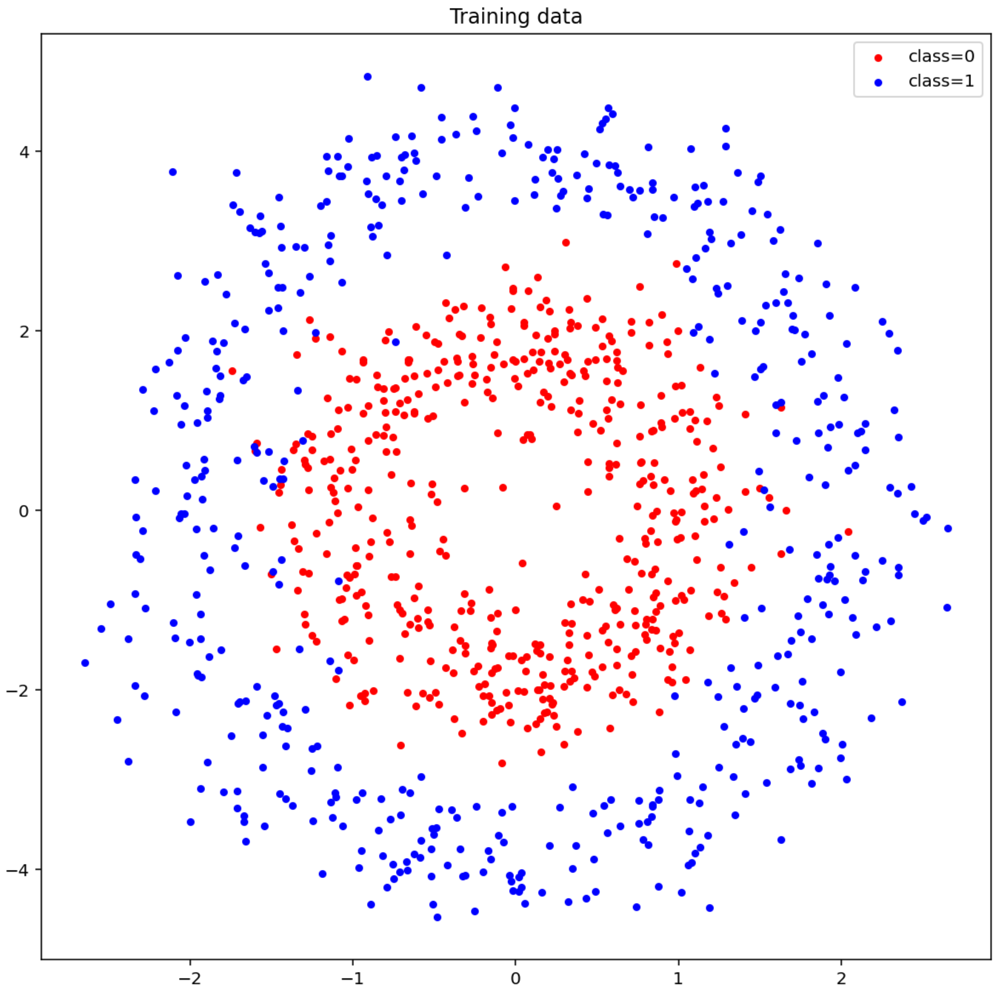
Plot the data points.

You may use matplotlib function `scatter(x,y)`.

```
# import data with numpy
data = np.loadtxt('/content/drive/My Drive/Colab Notebooks/MachineLearningProject/05/datase

# number of training data
n = data.shape[0]
print('Number of the data = {}'.format(n))
print('Shape of the data = {}'.format(data.shape))
print('Data type of the data = {}'.format(data.dtype))

# plot
x1 = data[:,0].astype(np.float64) # feature 1
x2 = data[:,1].astype(np.float64) # feature 2
idx = data[:,2].astype(np.float64) # label

x1_idx0     = x1[idx == 0]
x1_idx1     = x1[idx == 1]

x2_idx0     = x2[idx == 0]
x2_idx1     = x2[idx == 1]
```

```
plt.figure(1,figsize=(10,10))
plt.scatter(x1_idx0, x2_idx0 , s=50, c='r', marker='.', label='class=0')
plt.scatter(x1_idx1, x2_idx1 , s=50, c='b', marker='.', label='class=1')
plt.title('Training data')
plt.legend()
plt.show()
```

```
Number of the data = 1000
Shape of the data = (1000, 3)
Data type of the data = float64
```



2. Define a logistic regression loss function and its gradient

```
# sigmoid function
def sigmoid(z):
    try:
        return 1 / (1 + np.exp(-z))
    except OverflowError:
        return 1e-9

# predictive function definition
def f_pred(X,w):
    p = np.dot(X,w)
    return p

# loss function definition
```

```
def loss_logreg(y_pred,y):
    n = len(y)
    #loss = (np.dot((sigmoid(y_pred) - y).T, (sigmoid(y_pred) - y))) / n
    loss = -(np.dot(y.T, np.log(sigmoid(y_pred))) + np.dot((1-y).T, np.log(1-sigmoid(y_pred
    return loss

# gradient function definition
def grad_loss(y_pred, y, X):
    n = len(y)
    #grad = 2 * np.dot(X.T, np.dot((sigmoid(y_pred)-y), np.dot(sigmoid(y_pred).T, (1-sigmoi
    grad = 2 * np.dot(X.T, (sigmoid(y_pred) - y)) / n
    return grad

# gradient descent function definition
def grad_desc(X, y , w_init, tau, max_iter):

    L_iters = np.zeros([max_iter]) # record the loss values
    w = w_init # initialization
    for i in range(max_iter): # loop over the iterations
        y_pred = f_pred(X,w) # linear predicition function
        grad_f = grad_loss(y_pred,y,X) # gradient of the loss
        w = w - tau* grad_f # update rule of gradient descent
        L_iters[i] = loss_logreg(y_pred,y) # save the current loss value

    return w, L_iters
```

3. define a prediction function and run a gradient descent algorithm

The logistic regression/classification predictive function is defined as:

$$p_w(x) = \sigma(Xw)$$

The prediction function can be defined in terms of the following feature functions f_i as follows:

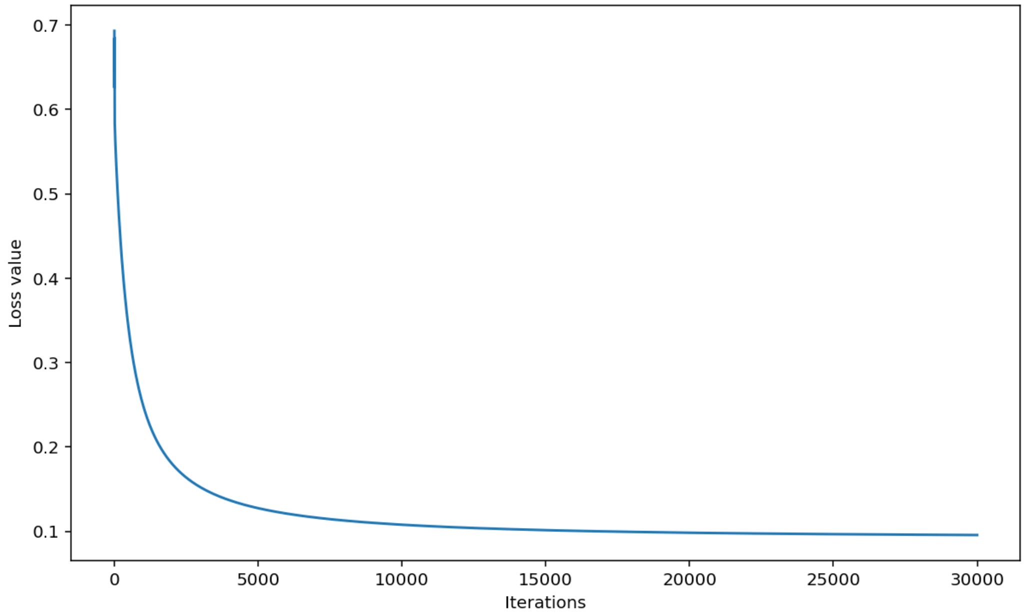
$$X = \begin{bmatrix} f_0(x_1) & f_1(x_1) & f_2(x_1) & f_3(x_1) & f_4(x_1) & f_5(x_1) & f_6(x_1) & f_7(x_1) & f_8(x_1) & f_9(x_1) \\ f_0(x_2) & f_1(x_2) & f_2(x_2) & f_3(x_2) & f_4(x_2) & f_5(x_2) & f_6(x_2) & f_7(x_2) & f_8(x_2) & f_9(x_2) \\ \vdots & & & & & & & & & \\ f_0(x_n) & f_1(x_n) & f_2(x_n) & f_3(x_n) & f_4(x_n) & f_5(x_n) & f_6(x_n) & f_7(x_n) & f_8(x_n) & f_9(x_n) \end{bmatrix} \quad \text{and} \quad w =$$

where $x_i = (x_i(1), x_i(2))$ and you can define a feature function f_i as you want.

You can use at most 10 feature functions $f_i, i = 0, 1, 2, \cdots, 9$ in such a way that the classification accuracy is maximized. You are allowed to use less than 10 feature functions.

Implement the logistic regression function with gradient descent using a vectorization scheme.

```
import math
# construct the data matrix X, and label vector y
def poly(X1, X2, degree):
    func = np.ones(len(X1))
    for i in range(1, degree+1):
        for j in range(0, i+1):
            func = np.column_stack((func, (X1**(i-j)) * (X2**j)))
    return func

n = data.shape[0]
X = poly(x1, x2, 3)
y = data[:,2][:,None] # label

# run gradient descent algorithm
start = time.time()
w_init = np.array([0,0,0,0,0,0,0,0,0,0,0])[:,None]
tau = 1e-2; max_iter = 30000
w, L_iters = grad_desc(X,y,w_init,tau,max_iter)

# plot
plt.figure(3, figsize=(10,6))
plt.plot(np.array(range(max_iter)), L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```

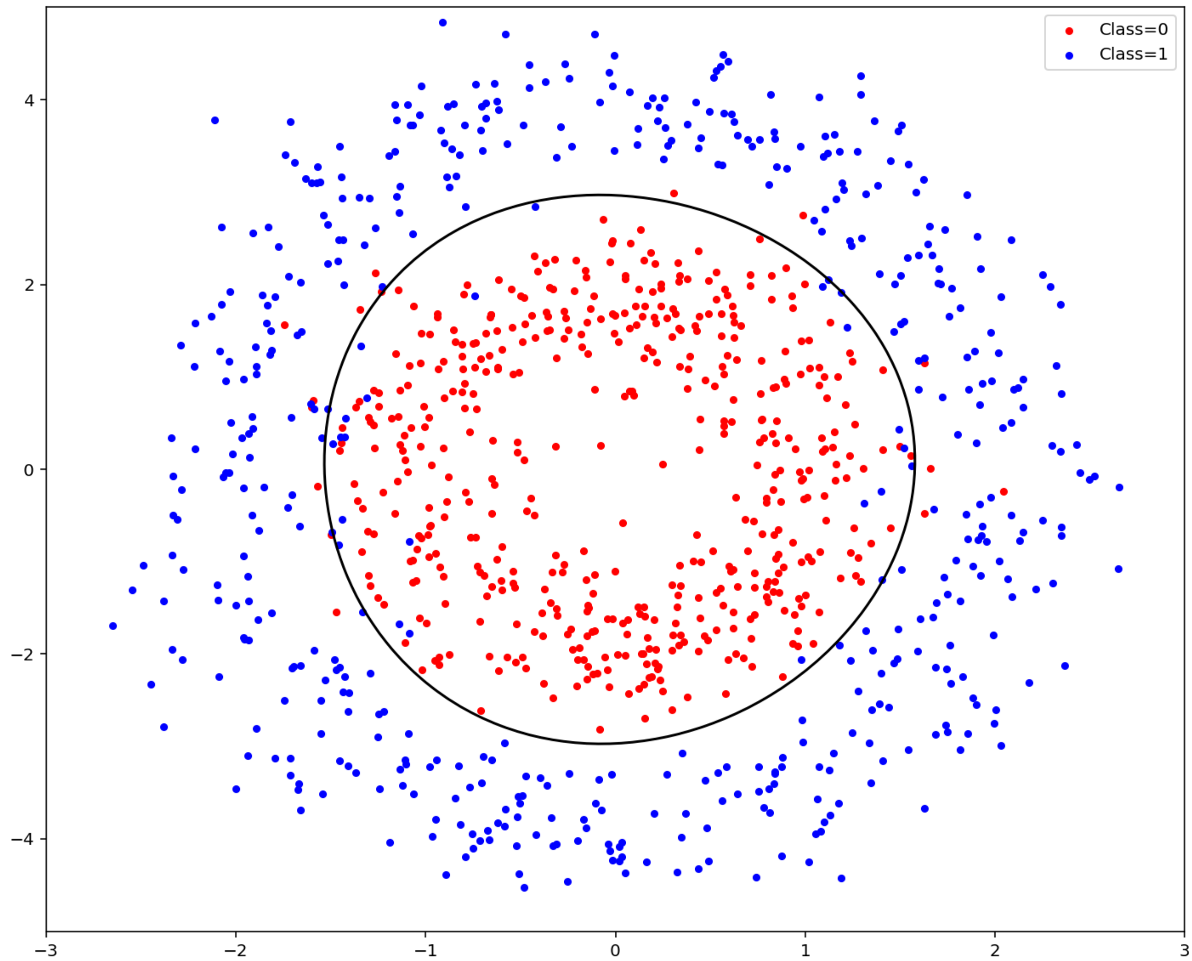


4. Plot the decisoin boundary

```
def boundary(x1_0, x2_0, x1_1, x2_1, w, degree):
    plt.figure(figsize=(12, 10))
    plt.scatter(x1_0, x2_0, s=50, c='r', marker='.', label='Class=0')
    plt.scatter(x1_1, x2_1, s=50, c='b', marker='.', label='Class=1')

    X = np.linspace(-3, 3, 100)
    Y = np.linspace(-5, 5, 100)
    XX, YY = np.meshgrid(X,Y)
    XX = np.ravel(XX)
    YY = np.ravel(YY)

```

```
    Z = np.zeros((len(X)*len(Y)))
    poly_line = poly(XX, YY, degree)
    Z = poly_line.dot(w)

    XX = XX.reshape((len(X), len(Y)))
    YY = YY.reshape((len(X), len(Y)))
    Z = Z.reshape((len(X), len(Y)))
    plt.contour(XX, YY, Z, levels=[0], colors='k')
    plt.legend()
    plt.show()
```

```
boundary(x1_idx0, x2_idx0, x1_idx1, x2_idx1, w, 3)
```



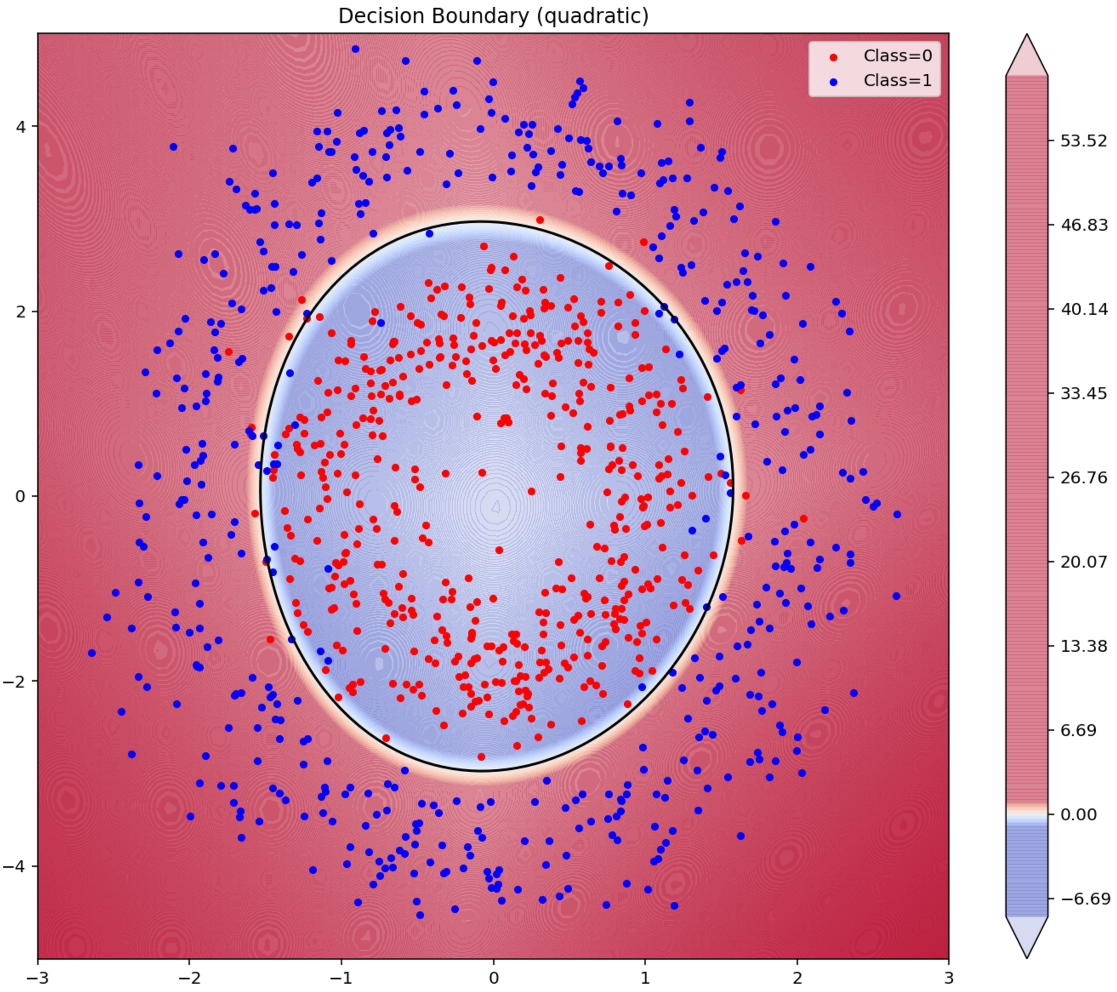
5. Plot the probability map

```
def boundary_map(x1_0, x2_0, x1_1, x2_1, w, degree):
    plt.figure(4, figsize=(12, 10))

    X = np.linspace(-3, 3, 100)
    Y = np.linspace(-5, 5, 100)
    XX, YY = np.meshgrid(X,Y)
    XX = np.ravel(XX)
    YY = np.ravel(YY)

    Z = np.zeros((len(X)*len(Y)))
    poly_line = poly(XX, YY, degree)
    Z = poly_line.dot(w)

    XX = XX.reshape((len(X), len(Y)))
```

```
    YY = YY.reshape((len(X), len(Y)))
    Z = Z.reshape((len(X), len(Y)))

    ax = plt.contourf(XX,YY,Z,2500,vmin=-1,vmax=1,cmap='coolwarm', alpha=0.2,extend='both')
    cbar = plt.colorbar(ax)
    cbar.update_ticks()

    plt.scatter(x1_0, x2_0, s=50, c='r', marker='.', label='Class=0')
    plt.scatter(x1_1, x2_1, s=50, c='b', marker='.', label='Class=1')
    plt.contour(XX, YY, Z, levels=[0], colors='k')
    plt.legend()
    plt.title('Decision Boundary (quadratic)')
    plt.show()
```

```
boundary_map(x1_idx0, x2_idx0, x1_idx1, x2_idx1, w, 3)
```

```
Locator attempting to generate 2229 ticks ([-8.16, ..., 58.68000000000001]), which exceeds Locator.MAXT
```



6. Compute the classification accuracy

The accuracy is computed by:

$$\text{accuracy} = \frac{\text{number of correctly classified data}}{\text{total number of data}}$$

```
# compute the accuracy of the classifier
n = data.shape[0]

```

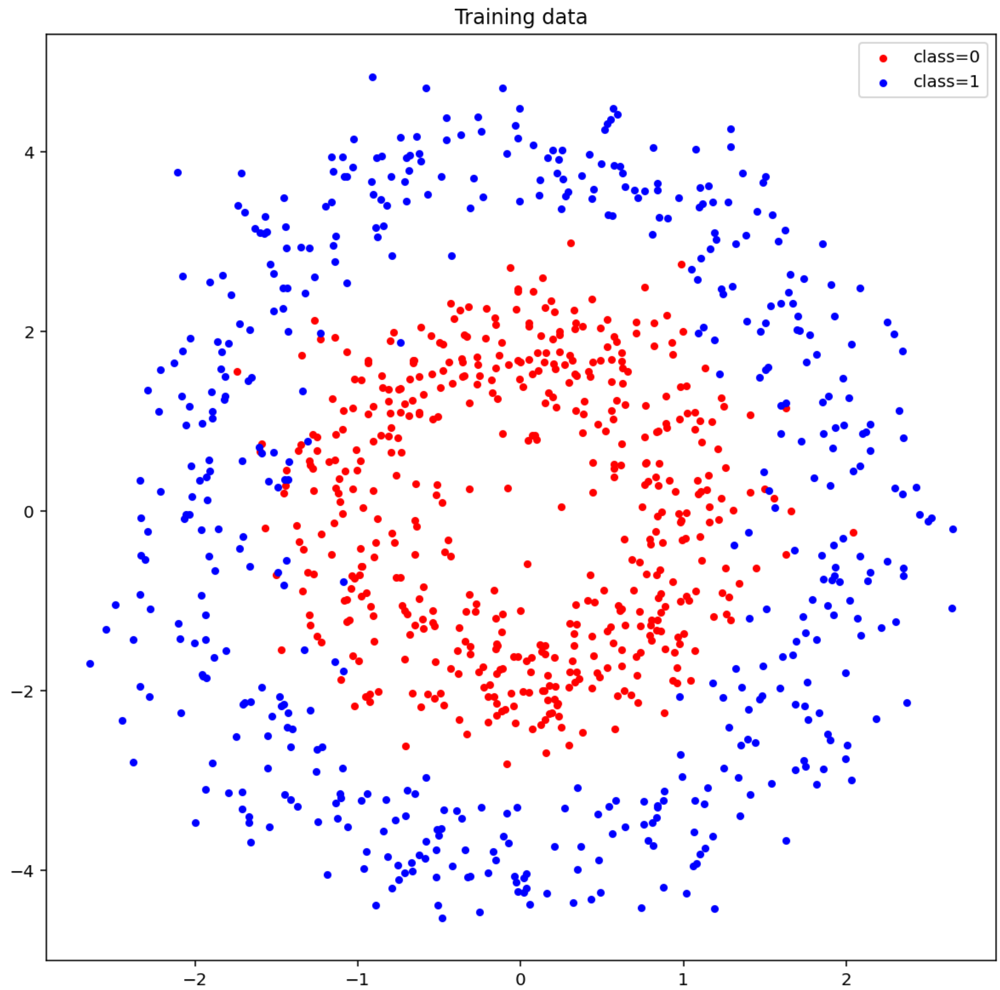
```
# plot
x1 = data[:,0].astype(np.float64) # feature 1
x2 = data[:,1].astype(np.float64) # feature 2

X = poly(x1, x2, 3)
y = data[:,2][:,None] # label
p = f_pred(X,w)

tmp = []
for i, j in zip(p, y):
    if np.round(sigmoid(i)) == j:
        tmp.append(1)

print('total number of data = {}'.format(n))
print('total number of correctly classified data = ', len(tmp))
print('accuracy(%) = ',  100*len(tmp) / len(data))
```

```
total number of data = 1000
total number of correctly classified data =  961
accuracy(%) =  96.1
```

Output using the dataset (dataset-noise-01.txt)

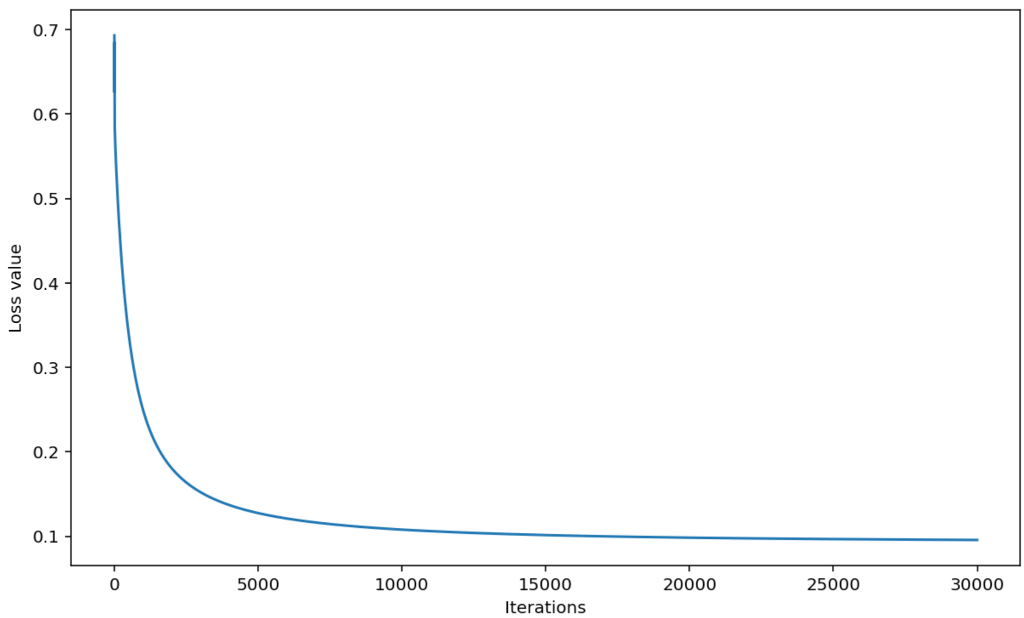
1. Visualize the data [1pt]

```
plt.figure(1,figsize=(10,10))
plt.scatter(x1_idx0, x2_idx0 , s=50, c='r', marker='.', label='class=0')
plt.scatter(x1_idx1, x2_idx1 , s=50, c='b', marker='.', label='class=1')
plt.title('Training data')
plt.legend()
plt.show()
```



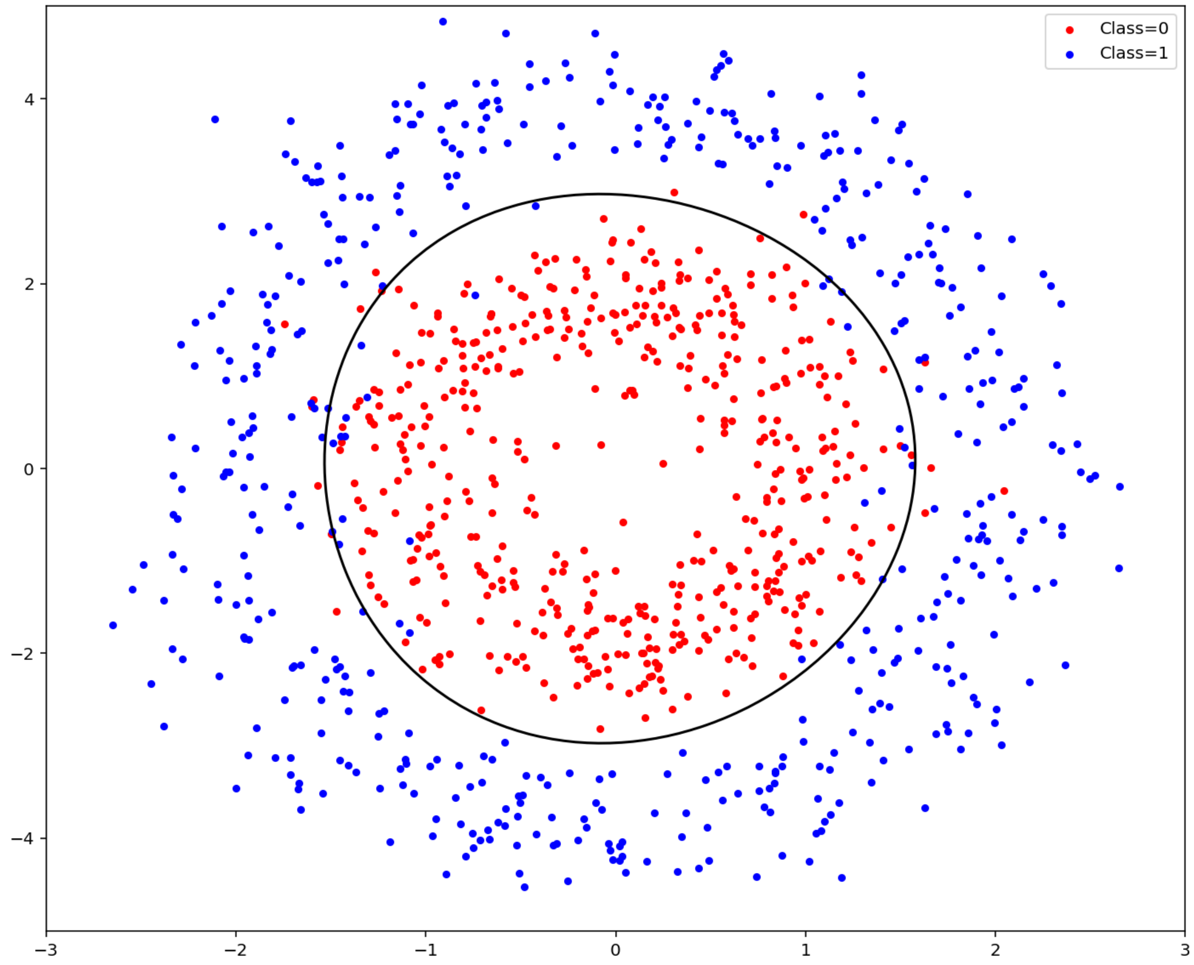
2. Plot the loss curve obtained by the gradient descent until the convergence [2pt]

```
plt.figure(3, figsize=(10,6))
plt.plot(np.array(range(max_iter)), L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```



3. Plot the decisoin boundary of the obtained classifier [2pt]

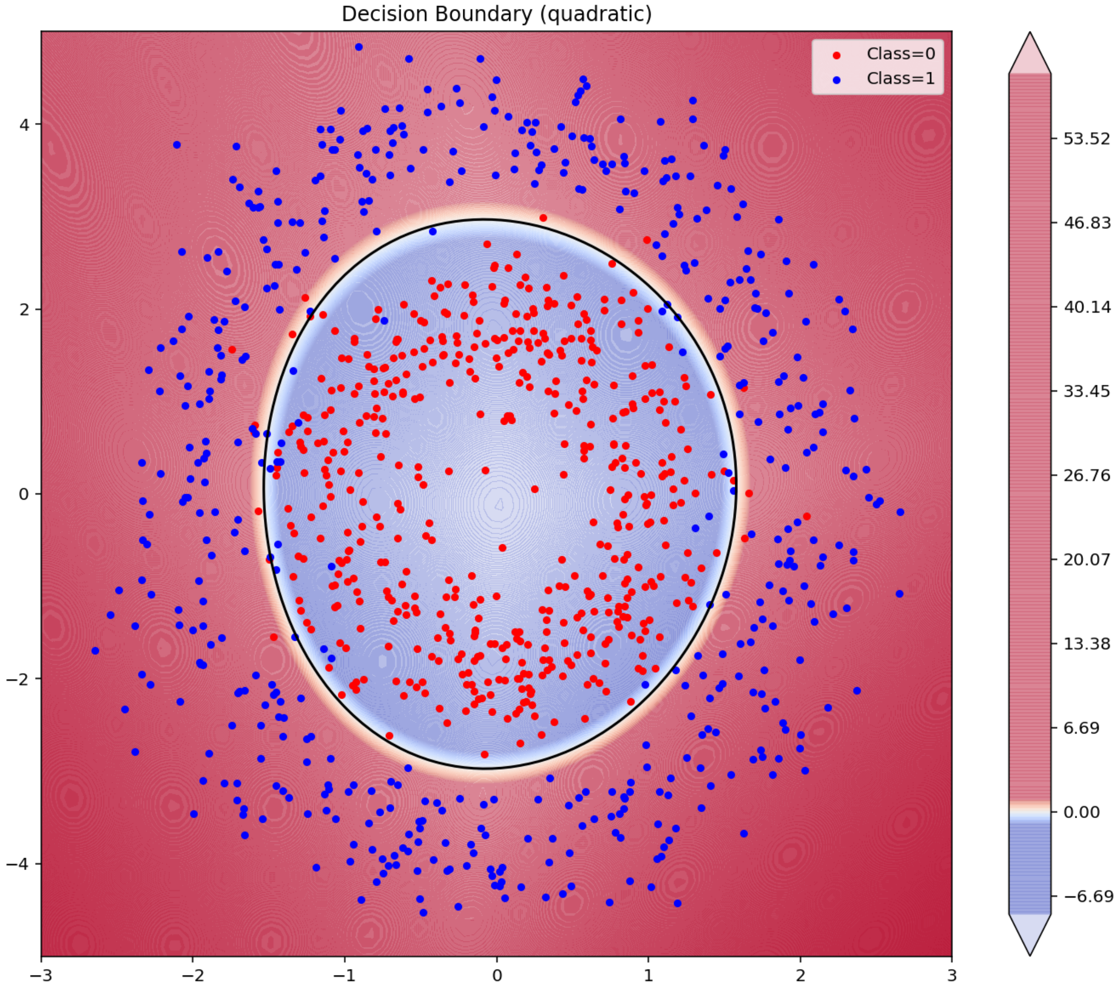
```
1 boundary(x1_idx0, x2_idx0, x1_idx1, x2_idx1, w, 3)
```



4. Plot the probability map of the obtained classifier [2pt]

```
1 boundary_map(x1_idx0, x2_idx0, x1_idx1, x2_idx1, w, 3)
```

```
Locator attempting to generate 2229 ticks ([-8.16, ..., 58.68000000000001]), which exceeds Locator.MAXT
```



5. Compute the classification accuracy [1pt]

```
1 print('total number of data = {}'.format(n))
2 print('total number of correctly classified data = ', len(tmp))
3 print('accuracy(%) = ',  100*len(tmp) / len(data))
```

```
total number of data = 1000
total number of correctly classified data =  961
accuracy(%) =  96.1
```