

HOMEWORK 2 – From Wireless Communication Principles and Practice: Problems 2.1, 2.3 & 2.7**Question 2.1**

Prove that for hexagonal geometry, the co-channel reuse ratio is given by $Q = \sqrt{3N}$, where $N = i^2 + ij + j^2$.

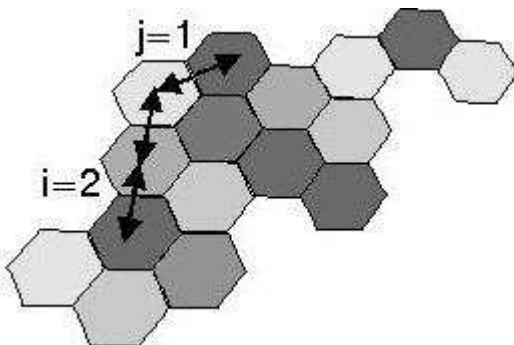
Solution 2.1

First consideration, when the size of each cell is approximately the same and the base stations transmit the same power, co-channel interference becomes only a function of the cell radius, R and the distance between centres of nearest co-channel cells, D . This defines a parameter called co-channel re-use ratio, Q :

$$Q = D / R$$

(1)

Let us now derive the relationship between Q (co-channel re-use ratio) and N (Cluster size). We first make reference to figure 1 below. The parameters i and j are non-negative integers that are used to locate the nearest co-channel cells to a particular cell. Therefore moving from the centre of the desired cell, one needs to move through i cell centres and then makes a 60° counterclockwise turn and then moves through j cell centres up to the destination cell. This takes you to the co-channel neighbours. These movements of i and j are through the flats of the hexagons as shown below.

**Fig.1**

Let us denote the distance from the centre of a hexagon to its side as R' and from the centre to a corner as R . From hexagon geometry, R' is given by

$$R' = R \frac{\sqrt{3}}{2}$$

The distance from one cell centre to another = $2R' = R\sqrt{3}$. From this it therefore follows that the movement from the cell centre up to the 60° turn is equal to $i * R' = i * R\sqrt{3}$ and the

movement from the 60° turn to the destination cell is equal to $jR' = jR\sqrt{3}$. From the above, we obtain a scalene triangle shown below where a is the reference cell, b is the cell where we make the 60° turn and c is the co-channel cell. Since we turned 60° away from a straight line, the total angle ABC is equal to 120° . D is the distance between the reference cell, a and the destination co-channel cell, c . This is depicted in figure two below:

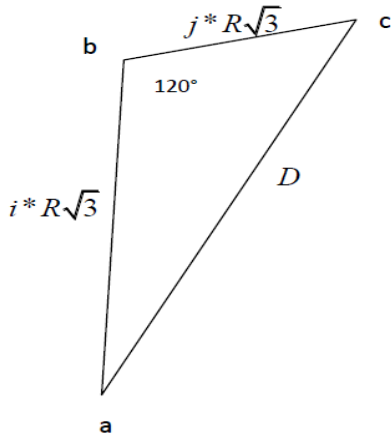


Fig2.

Using the cosine law:

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$D^2 = (iR\sqrt{3})^2 + (jR\sqrt{3})^2 - 2(iR\sqrt{3})(jR\sqrt{3})\cos 120^\circ$$

$$D^2 = 3i^2R^2 + 3j^2R^2 - 6ijR^2(-0.5) = 3i^2R^2 + 3j^2R^2 + 3ijR^2$$

$$D = \sqrt{3i^2R^2 + 3j^2R^2 + 3ijR^2} = \sqrt{3R^2(i^2 + ij + j^2)}$$

But from the quantitative definition of cluster size, we know that $i^2 + ij + j^2 = N$

$$\text{Therefore } D = \sqrt{3R^2(i^2 + ij + j^2)} = \sqrt{3R^2N} = R\sqrt{3N}$$

(2)

Using equation (1) and (2)

$$Q = D/R = \frac{R\sqrt{3N}}{R} = \sqrt{3N}$$

$$D = \sqrt{3N}$$

Hence Proved.

Question 2.3

A cellular service provider decides to use a digital TDMA scheme which can tolerate a SIR of 15dB in the worst case. Find the optimal value of N for (a) omnidirectional antennas, (b) 120° sectoring, and (c) 60° sectoring. Should the sectoring be used? If so, which case (60° or 120°) should be used? (Assume a path loss exponent of $n=4$ and considering trunking efficiency.)

Solution 2.3

From the empirical path loss formula, power, P_r at a receiver at a distance d from a transmitter is given by

$P_r = P_o \left(\frac{d}{d_o}\right)^{-n}$ where P_o is the transmitted power and d_o is the distance for a reference receiver

in the far-field. n is the path loss exponent. When we are comparing signals from the desired base station and the co-channel interfering cells, we use the ratio of the desired signal, S to the sum of signals from co-channel interference I . For i_o equidistant co-channel cells at D (i.e. $D_1 = D_2 = \dots = D_{i_o}$ and the mobile located at cell boarder R , this gives

$$S / I = \frac{P_o \left(\frac{R}{d_o}\right)^{-n}}{\frac{P_o}{d_o^{-n}} \sum_{i=1}^{i_o} D^{-n}} = \frac{R^{-n}}{i_o D^{-n}} = \frac{(D/R)^n}{i_o} = \frac{(\sqrt{3N})^n}{i_o}$$

In this problem, we are dealing with S/I greater than 15dB=31.623. Therefore

$$S / I = \frac{(\sqrt{3N})^n}{i_o} > 31.623$$

$N > (i_o 31.623)^{2/n} / 3$ but since $n = 4$ for this problem, then

$$N > (\sqrt{i_o 31.623}) / 3$$

(3)

(a) For Omni-directional antenna, the number of interfering co-channel cells $i_o=6$, therefore using equation (3) above gives

$$N > (\sqrt{i_o 31.623}) / 3 = (\sqrt{6 * 31.623}) / 3 = 4.59$$

Since values of N are 3, 7, 12, etc. So we choose our optimal value as

$$N = 7$$

- (b) For 120° sectoring, the centre cell only has 2 cells pointing in its direction with the same frequency, therefore $i_o = 2$. Using this in equation (3) above,

$$N > (\sqrt{(i_o 31.623)}) / 3 = (\sqrt{(2 * 31.623)}) / 3$$

$$N > 2.65$$

So we choose an optimal value of $N = 3$

- (c) For 60° sectoring, the centre cell only has 1 cell pointing in its direction with the same frequency, therefore $i_o = 1$. Using this in equation (3) above,

$$N > (\sqrt{(i_o 31.623)}) / 3 = (\sqrt{(1 * 31.623)}) / 3$$

$$N > 1.87$$

So we choose an optimal value of $N = 3$ again.

Yes, sectoring should be used as it will reduce the cluster size N from $N=7$ to $N=3$. This will increase the number of times the frequencies are re-used to cover the same area therefore increasing the capacity. But sectoring decreases the trunking efficiency in that the available channels in a pool are subdivided into smaller pools. Therefore 120° sectoring should be used because 60° will further decrease the trunking efficiency.

Question 2.7

Exercises in trunking (queueing) theory:

- What is the maximum system capacity (total and per channel) in Erlangs when providing a 2% blocking probability with four channels, with 20 channels, with 40 channels?
- How many users can be supported with 40 channels at 2% blocking? Assume $H=105$ seconds, $\lambda = 1$ call/hour.
- Using the traffic intensity calculated in (a), find the GoS in a lost call delayed system for the case of delays being greater than 20 seconds. Assume that $H=105s$, and determine the GoS for four channels, for 20 channels, for 40 channels.
- Comparing part (a) and part (c), does a lost call delayed system with a 20 second queue perform better than a system that clears blocked calls?

Solution 2.7

(a) Using Erlang-B Table:

4 Channels at 2% Blocking Probability

- all channels grouped can carry **1.092 Erlangs**
- each channel can carry 0.02041 Erlangs, therefore
traffic carried by each of the 4 channels combined = $4 \times 0.02041 =$ **0.0816 Erlangs**

20 Channels at 2% Blocking Probability

- all channels grouped can carry **13.182 Erlangs**
- each channel can carry 0.02041 Erlangs, therefore
traffic carried by each of the 20 channels combined = $20 \times 0.02041 =$ **0.408 Erlangs**

40 Channels at 2% Blocking Probability

- all channels grouped can carry **30.997 Erlangs**
- each channel can carry 0.02041 Erlangs, therefore
traffic carried by each of the 40 channels combined = $40 \times 0.02041 =$ **0.816 Erlangs**

(b) The total offered traffic A , is equal to traffic intensity per user, A_u multiplied by total number of users, U

$$A = A_u U \quad (4)$$

But $A_u = \lambda H$ where λ is the call-request rate and H is the mean holding time.

From above given data, $\lambda = 1$ call/hour = $1/3600$ calls/second, $H = 105$ seconds.

$$A_u = (1/3600) \times 105 = 0.0291 \text{ Erlangs per user}$$

We then obtain the total traffic offered traffic by 40 channels at 2% GoS. Using Erlang-B table, we get $A = 30.997$ Erlangs

Using Equation (4) above,

$$U = A / A_u = 30.997 / 0.0291$$

$$U = 1061 \text{ Users}$$

- (c) The probability that a delayed call is forced to wait for t seconds is given by the probability that a call is delayed, multiplied by the conditional probability that the delay is greater than t seconds. The GoS of a lost-call-delayed system is thus given as:

$$P_r[\text{delay} > t] = P_r[\text{delay} > 0] P_r[\text{delay} > t \mid \text{delay} > 0] \quad (5)$$

$$\text{Where } P_r[\text{delay} > 0] = \frac{A^C}{A^C + C! \left(1 - \frac{A}{C}\right) \sum_{k=0}^{C-1} \frac{A^k}{k!}} \quad (6)$$

$$\text{And } P_r[\text{delay} > t \mid \text{delay} > 0] = \exp[-(C - A)t / H] \quad (7)$$

Let us now use the traffic intensities in part (a) and get the GoS for each number of channels asked for:

C = 4 Channels, A = 1.092 Erlangs

$$\text{Using equation (6) } P_r[\text{delay} > 0] = \frac{1.092^4}{1.092^4 + 4! \left(1 - \frac{1.092}{4}\right) \sum_{k=0}^3 \frac{1.092^k}{k!}}$$

Using Matlab to evaluate, $P_r[\text{delay} > 0] = 0.0273$ or 2.73%

Evaluating equation (7) $P_r[\text{delay} > 20 \mid \text{delay} > 0] = \exp[-(4 - 1.092) * 20 / 105] = 0.5747$

From equation (5) GoS = $P_r[\text{delay} > t] = P_r[\text{delay} > 0] P_r[\text{delay} > t \mid \text{delay} > 0]$

$$P_r[\text{delay} > 20] = 0.0273 * 0.5747 = \underline{0.016 \text{ or } 1.6\%}$$

C= 20 Channels, A = 13.182 Erlangs

$$\text{Using equation (6) } P_r[\text{delay} > 0] = \frac{13.182^{20}}{13.182^{20} + 20! \left(1 - \frac{13.182}{20}\right) \sum_{k=0}^{19} \frac{13.182^k}{k!}}$$

Using Matlab to evaluate, $P_r[\text{delay} > 0] = 0.0565$ or 5.65%

Evaluating equation (7) $P_r[\text{delay} > 20 | \text{delay} > 0] = \exp [-(20-13.182)*20/105] = 0.273$

From equation (5) GoS= $P_r[\text{delay} > t] = P_r[\text{delay} > 0]P_r[\text{delay} > t | \text{delay} > 0]$

$$P_r[\text{delay} > 20] = 0.0565 * 0.273 = \mathbf{0.015 \text{ or } 1.5\%}$$

C= 40 Channels, A = 30.997 Erlangs

$$\text{Using equation (6) } P_r[\text{delay} > 0] = \frac{30.997^{40}}{30.997^{40} + 40! \left(1 - \frac{30.997}{40}\right) \sum_{k=0}^{39} \frac{30.997^k}{k!}}$$

Using Matlab to evaluate, $P_r[\text{delay} > 0] = 0.0831$ or 8.31%

Evaluating equation (7) $P_r[\text{delay} > 20 | \text{delay} > 0] = \exp [-(40-30.997)*20/105] = 0.18$

From equation (5) GoS= $P_r[\text{delay} > t] = P_r[\text{delay} > 0]P_r[\text{delay} > t | \text{delay} > 0]$

$$P_r[\text{delay} > 20] = 0.0831 * 0.18 = \mathbf{0.015 \text{ or } 1.5\%}$$

- (d) By comparing the GoS for a lost-call-cleared (LCC) system in part(a) and the GoS for the lost-call-delayed (LCD) in part (c), we can note that the probability of a call being denied access at all is 2% while the probability of being put in a queue of 20 seconds is about 1.5%. This shows that the LCD system would perform better than the LCC system in terms of comforting the users when all trunks are in use. This is so because in the 2% LCC system, the users will be cleared at a probability of 2% and will not have 'hope' of connecting, but on the other hand, the LCD system at the same traffic level will make users wait for 20 seconds at a lower probability, 1.5%.