Calculation attenuation for cone object using a rotating source

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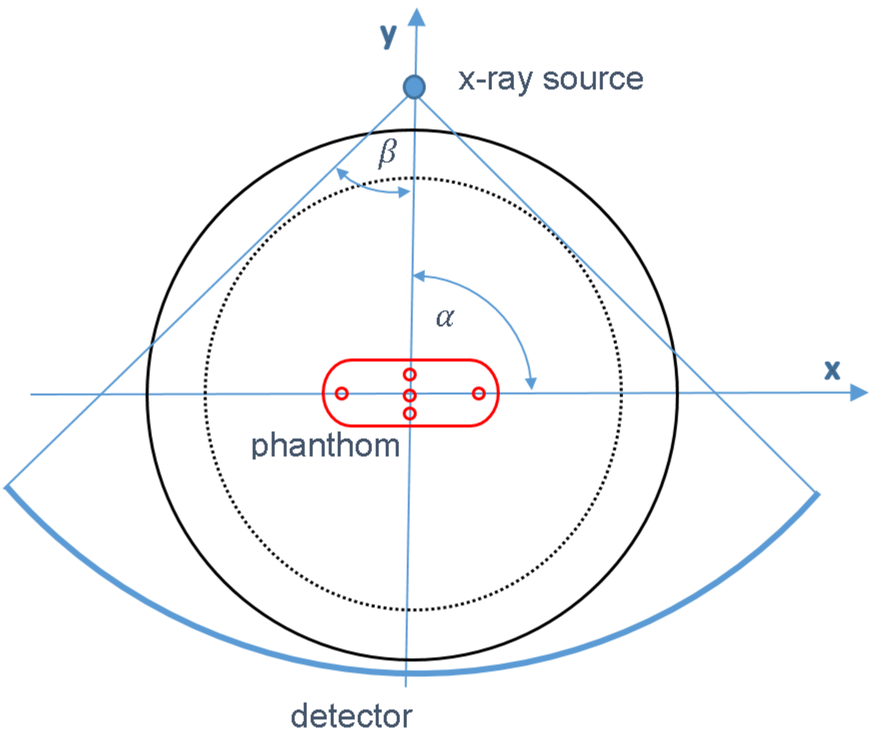
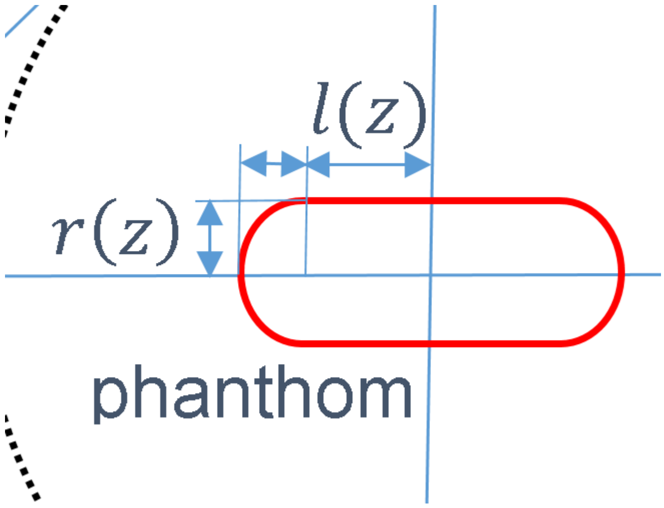
1 Introduction

The all geometry calculations depends of two parameters α and z



The diameter of scanned area is 500 mm. (Radius is 250 mm) Distance from the isocenter to the x-ray source is 595 mm. We can calculate the angle β.
$\sin(\beta) = \frac{250}{595}$
$\beta = \arcsin \frac{250}{595} =$
$\beta = \arcsin \frac{250}{595} = \arcsin 0.420168 = 24.85°$
The scanning area is from $-\beta = -24.85°$ to $+\beta = +24.85°$

Now we can express dimensions of the phantom as a function of z.

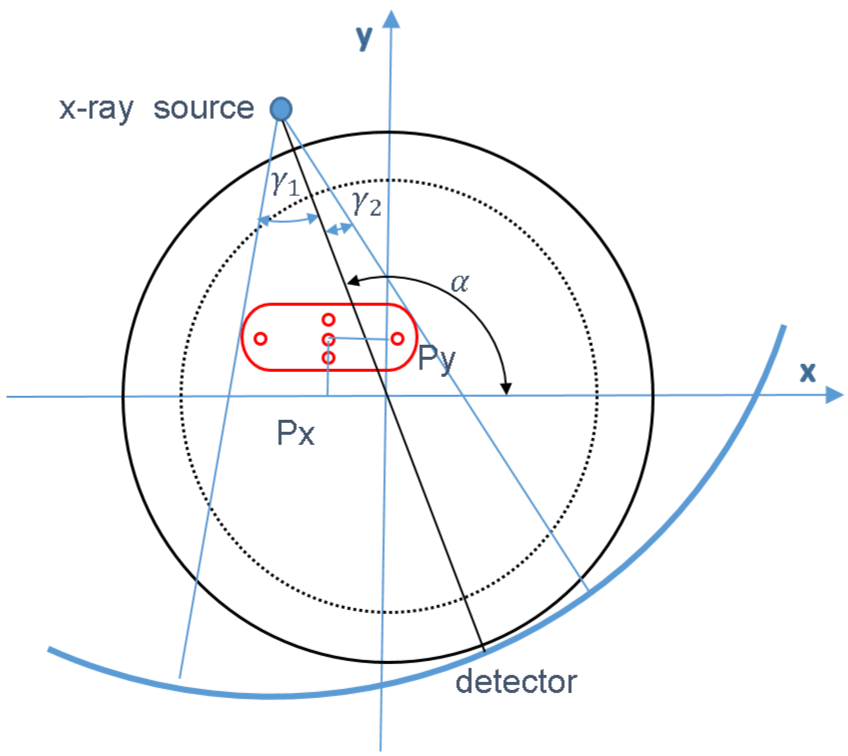


Because $r(0) = 40$ and $r(300) = 80$ then $r(z) = 40 + \frac{80-40}{300}z = 40 + \frac{40}{300}z = 40 + \frac{4}{30}z$.
And from $l(0) = 20$ and $l(300) = 100$ follows $l(z) = 20 + \frac{100-20}{300}z = 20 + \frac{80}{300}z = 20 + \frac{8}{30}z$.

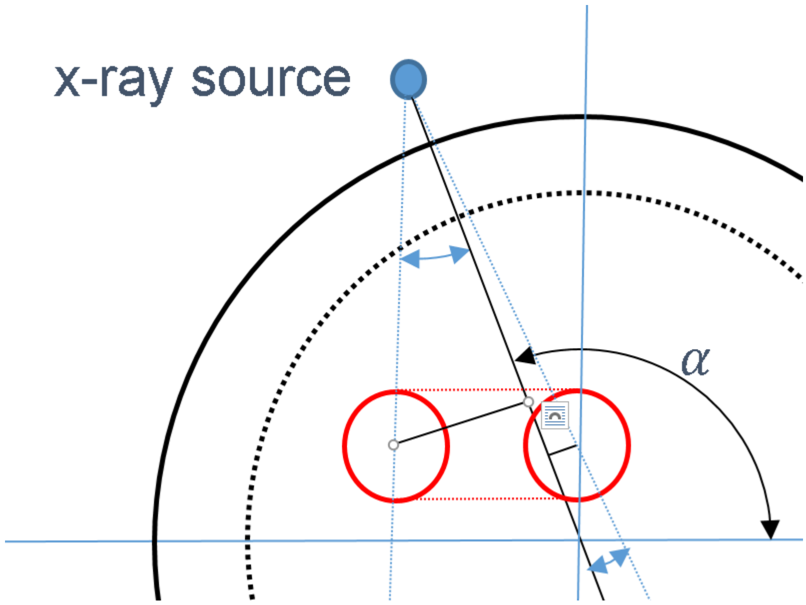
The width of the phantom is $2l(z) + 2r(z)$ for any z form 0 to 300.
The height of the phantom is $2r(z)$ for any z form 0 to 300.

2 Field of view

The (x, y) position of the x-ray source is determined by angle α.
$Sx(\alpha) = 595 \sin \alpha$
$Sy(\alpha) = 595 \cos \alpha$



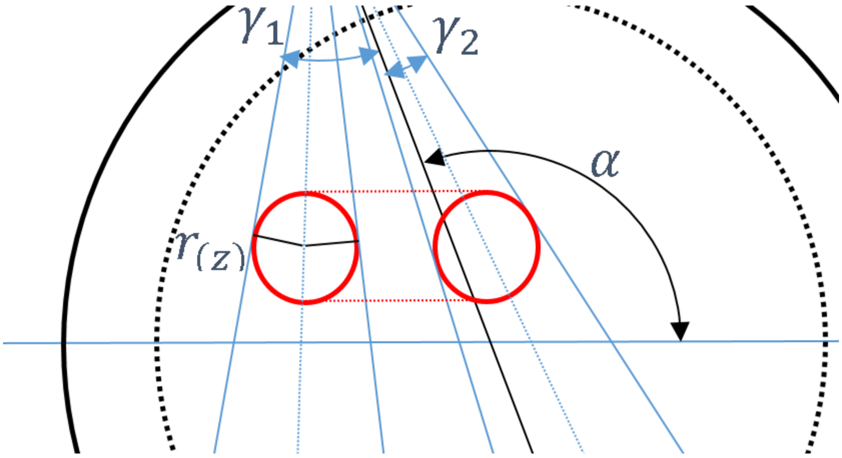
Constants are $Px = 0, Py = 0, Rs = 595mm$



What are the (x, y) coordinates of the center of the two circles.
One is $(Px - l(z), Py)$ and another one center is at $(Px + l(z), Py)$.

The straight line going thought "x-ray source" (Sx, Sy) and isocenter $(0, 0)$.
Direction vector is $(0, 0) - (Sx, Sy) = (-Sx, -Sy) = (-595 \sin \alpha, -595 \cos \alpha)$.
The unit direction vector is $(-\sin \alpha, -\cos \alpha)$.
Rotating this vector by 90° counter clockwise (rotation formula (x,y) to (-y,x)) we will get the normal vector.
the unit normal vector $(\cos \alpha, -\sin \alpha)$

By doing scalar product of the normal vector with vectors from "x-ray source" to the center of the circle we will get distance of the center to the line. By dividing those distances with distances from "x-ray source" to the center of the circles we will get sin of the angles. Calculating arcsin of the sin of the angles we will get angles.



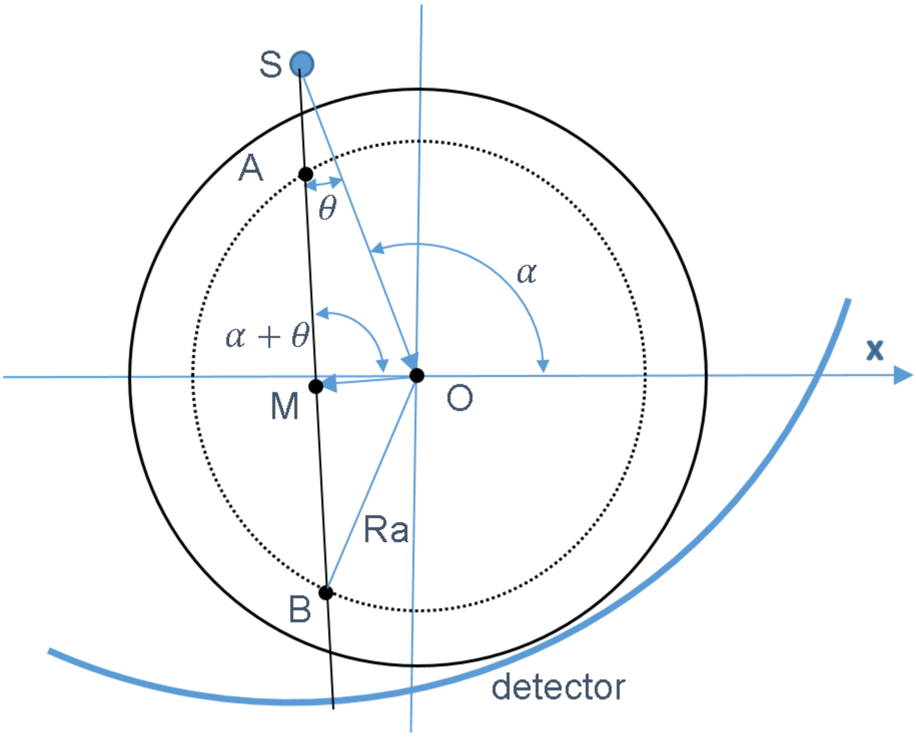
By using two angles from the previous step we will get 4 angles.

Minimum of those 4 we will named as γ_1 and maximum as γ_2.
If γ_1 is less than $-\beta$ then $\gamma_1 = -\beta$.
If γ_2 is greater than β then $\gamma_2 = \beta$.

3 The length of the intersection

This part is about finding three things . The last is "The length of the intersection"

3.1 Find the intersection with the scanned area



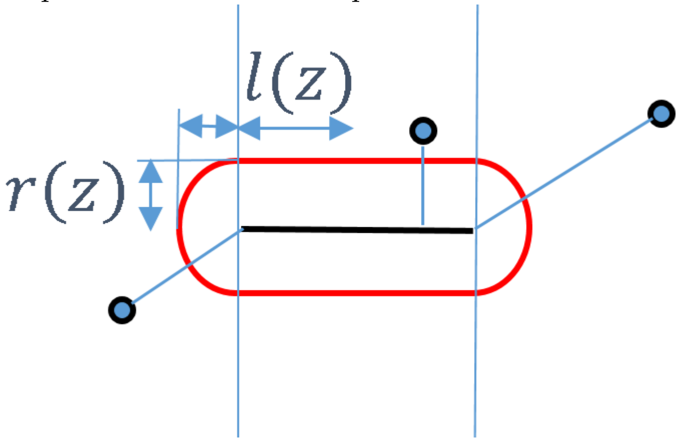
The unit normal vector of the straight line going though the points S, A, M, B is
$\overrightarrow{n} = (\sin(-\alpha - \theta + 90°), \cos(-\alpha - \theta + 90°))$
$|OM| = \left\langle \overrightarrow{n}, \overrightarrow{SO} \right\rangle$

If $|OM|$ is bigger than $Ra = 250mm$ "error"
$M = |OM| \cdot \overrightarrow{n}$
$|MB| = \sqrt{Ra^2 - |OM|^2}$

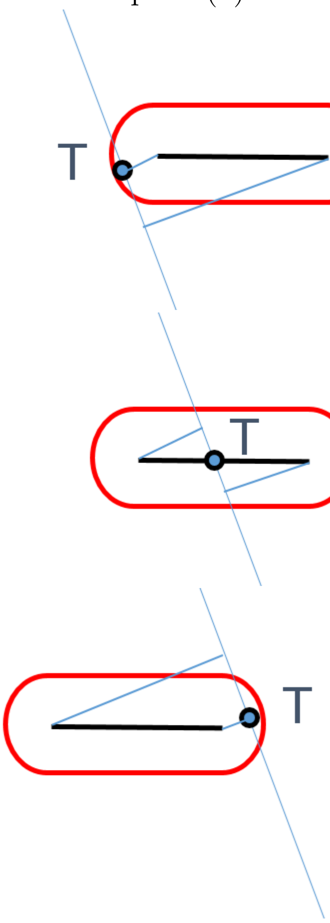
$\overrightarrow{d} = (\sin(-\alpha - \theta), \cos(-\alpha - \theta))$
$B = M + |MB| \cdot \overrightarrow{d}$
$A = M - |MB| \cdot \overrightarrow{d}$

3.2 Find the point inside the phantom

If point A is inside the phantom then C=A. If point B is inside the phantom then D=B.

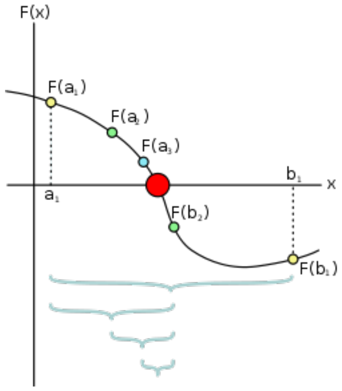


The point is inside the phantom if the distance from the straight line segment from $(Px - l(z), Py)$ to $(Px + l(z), Py)$ is less or equal $r(z)$

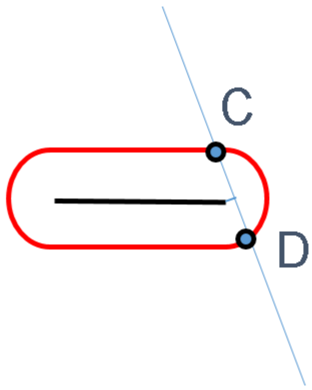


3.3 Find the points on the edge of the phantom

By using Bisection method ...



-for points T and A we will get point C.
-for points T and B we will get point D.



4 Scanning the phantom

The phantom is divided on 60 slices
1. from 0 to 5 mm ... this slice should be scanned at z=2.5 mm
2. from 5 to 10 mm ... this slice should be scanned at z=7.5 mm
3. from 10 to 15 mm ... this slice should be scanned at z=12.5 mm
...
60. from 295 to 300 mm ... this slice should be scanned at z=297.5 mm