

Solutions to Section 1.1

Variables

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1 Exercise Set 1.1

1.

- a. Is there a real number x such that $x^2 = -1$?
- b. Does there exist a real number x such that $x^2 = -1$?

2.

- a. Is there an integer n such that n has a remainder of 2 when it is divided by 5 and a remainder of 3 when it is divided by 6?
- b. Does there exist an integer n such that if n is divided by 5 the remainder is 2 and if n is divided by 6 the remainder is 3?

3.

- a. Given any two distinct real numbers a and b , there is a real number c such that c is between a and b .
- b. For any two distinct real numbers a and b , there is a real number c such that c is between a and b .

4.

- a. Given any real number r , there is a real number s such that s is greater than r .
- b. For any real number r , there is a real number s such that $s > r$

5.

- a. Given any positive real number r , the reciprocal of r is positive.
- b. For any real number r , if r is positive, then its reciprocal is positive.
- c. If a real number r is positive, then the reciprocal of r is positive.

6.

- a. Given any negative real number s , the cube root of s is negative.
- b. For any real number s , if s is negative then the cube root of s is negative.
- c. If a real number s is negative, then the cube root of s is negative.

7.

- a. False: There are two real numbers such that their sum is less than their difference.
- b. False: There is a real number whose greater than its square.
- c. True: If any integer is positive, it is less than its square.
- d. True: The absolute value of the sum of two real numbers is less than or equal to the sum of the absolute value of two real numbers.

8.

- a. All squares have four sides.
- b. Every square has four sides.
- c. If an object is a square, then it has four sides.
- d. If J is a square, then J had four sides.
- e. For every square J , J has four sides.

9.

- a. All quadratic equations have at most two real solutions.
- b. Every quadratic equation has at most two real solutions.
- c. If an equation is quadratic, then it has at most two real solutions.
- d. If E is a quadratic equation, then E has at most two real solutions.
- e. For every quadratic equation E , E has at most two real solutions.

10.

- a. All nonzero real numbers have a reciprocal.
- b. For every nonzero real number r , there is a reciprocal for r .
- c. For every nonzero real number r , there is a real number s such that s is a reciprocal of r .

11.

- a. All positive numbers have a positive square root.
- b. For every positive number e , there is a positive square root for e .
- c. For every positive number e , there is a positive number r such that r is the square root of e .

12.

- a. Some real number has the property that its product with every number leaves the number unchanged.
- b. There is a real number r such that the product of r and every number leaves the number unchanged.
- c. There is a real number r with the property that for every real number s , the product of r and s leaves s unchanged.

13.

- a. Some real number has the property that its product with every real number equals zero.
- b. There is a real number a such that the product of a and every real number equals zero.
- c. There is a real number a with the property that for every real number b , the product of a and b is zero.