Solutions to Section 1.1
Logical Form and Logical Equivalence

bosnianbyte

February 2025

1 Exercise Set 1.1

1. If all algebraic expressions can be written in prefix notation, then $(a+2b)(a^2-b)$ can be written in prefix notation.

All algebraic expressions can be written in prefix notation.

Therefore $(a+2b)(a^2-b)$ can be written in prefix notation.

2. If all prime numbers are odd, then 2 is odd.

2 is not odd.

Therefore, it is not the case that all prime numbers are odd.

3. My mind is shot or logic is confusing.

My mind is not shot.

Therefore, logic is confusing.

4. If x = 0, then the guard condition for the while loop is false.

If the guard condition for the while loop is false, then program execution moves to the next instruction following the loop.

Therefore, if x = 0, then the guard condition evaluates to 'false', and program execution moves to the next instruction following the loop.

5.

a. Statement

b. Statement

c. Statement

d. Not a statement

6.

a. $s \wedge i$

b. $\neg s \wedge \neg i$

7. $m \wedge \neg c$

8.

a. $(h \wedge w) \wedge \neg s$

b. $\neg h \wedge (w \wedge s)$

c. $\neg h \wedge \neg w \wedge \neg s$

d. $(\neg w \wedge \neg s) \wedge h$

e. $h \wedge (\neg w \wedge \neg s)$

9. $(n \vee k) \wedge \neg(n \wedge k)$

10.

a. $p \wedge q \wedge r$

b. $p \wedge \neg q$

c. $p \wedge (\neg q \vee \neg r)$

d. $(\neg p \wedge q) \wedge \neg r$

e. $\neg p \vee (q \wedge r)$

11. Inclusive OR

12. "United States President" AND (14th OR fourteenth) AND NOT amendment

13. (jaguar AND cheetah) AND (speed or fastest) AND NOT (car or automobile or auto)

14.

p	q	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

15.

p	q	$\neg(p \wedge q)$	$p \vee q$	$\neg(p \wedge q) \vee (p \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	T	T	T
F	F	T	F	T

16.

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

17.

p	q	r	$\neg q$	$\neg q \vee r$	$p \wedge (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

18.

p	q	r	$\neg p$	$\neg r$	$\neg p \vee q$	$q \wedge \neg r$	$p \vee (\neg p \vee q)$	$\neg(q \vee \neg r)$	$(p \vee (\neg p \vee q)) \wedge \neg(q \vee \neg r)$
T	T	T	F	F	T	F	T	F	F
T	T	F	F	T	T	T	T	F	F
T	F	T	F	F	F	F	T	T	T
T	F	F	F	T	F	F	T	F	F
F	T	T	T	F	T	F	T	F	F
F	T	F	T	T	T	T	T	F	F
F	F	T	T	F	T	F	T	T	T
F	F	F	T	T	T	F	T	F	F

19. The statements 'p' and '$p \vee (p \wedge q)$' always have the same truth values, so they are logically equivalent.

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

20. The statements '$\neg(p \wedge q)$' and '$\neg p \wedge \neg q$' do not have the same truth values, so they aren't logically equivalent.

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

21. The statements '$p \vee t$' and 't' always have the same truth values, so they are logically equivalent.

p	t	$p \vee t$
T	T	T
T	T	T
F	T	T
F	T	T

22. The statements '$p \wedge t$' and 't' do not have the same truth values, so they aren't logically equivalent.

p	t	$p \wedge t$
T	T	T
T	T	T
F	T	F
F	T	F

23. The statements '$(p \wedge q) \wedge r$' and '$p \wedge (q \wedge r)$' have the same truth values, so they are logically equivalent.

p	q	r	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

24. The statement '$p \wedge (q \vee r)$' and '$(p \wedge q) \vee (p \wedge r)$' have the same truth values, so they are logically equivalent.

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

25. The statements '$(p \wedge q) \vee r$' and '$p \wedge (q \vee r)$' have different truth values, so they aren't logically equivalent.

p	q	r	$q \vee r$	$p \wedge q$	$(p \wedge q) \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	F	T	T
T	F	F	F	F	F	F
F	T	T	T	F	T	F
F	T	F	T	F	F	F
F	F	T	T	F	T	F
F	F	F	F	F	F	F

26. The statements '$(p \vee q) \vee (p \wedge r)$' and '$(p \vee q) \wedge r$' have the different truth values, so they aren't logically equivalent.

p	q	r	$p \vee q$	$p \wedge r$	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

27. The statements '$((\neg p \vee q) \wedge (p \vee \neg r)) \wedge (\neg p \vee \neg q)$' and '$\neg(p \vee r)$' have the same truth values, so they are logically equivalent.

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg p \vee q$	$p \vee \neg r$	$\neg p \vee \neg q$	$p \vee r$
T	T	T	F	F	F	T	T	F	T
T	T	F	F	F	T	T	T	F	T
T	F	T	F	T	F	F	T	T	T
T	F	F	F	T	T	F	T	T	T
F	T	T	T	F	F	T	F	T	T
F	T	F	T	F	T	T	T	T	F
F	F	T	T	T	F	T	F	T	T
F	F	F	T	T	T	T	T	T	F

$\neg(p \vee r)$	$((\neg p \vee q) \wedge (p \vee \neg r)) \wedge (\neg p \vee \neg q)$
F	F
F	F
F	F
F	F
F	F
T	T
F	F
T	T

28. The statements '$(r \vee p) \wedge ((\neg r \vee (p \wedge q)) \wedge (r \vee q))$' and '$p \wedge q$' have the same truth values, so they are logically equivalent.

p	q	r	$\neg r$	$r \vee p$	$p \wedge q$	$\neg r \vee (p \wedge q)$	$r \vee q$	$p \wedge q$	$(r \vee p) \wedge ((\neg r \vee (p \wedge q)) \wedge (r \vee q))$
T	T	T	F	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T	T
T	F	T	F	T	F	F	T	F	F
T	F	F	T	T	F	T	F	F	F
F	T	T	F	T	F	F	T	F	F
F	T	F	T	F	F	T	T	F	F
F	F	T	F	T	F	F	T	F	F
F	F	F	T	F	F	T	F	F	F

29. Hal is not a math major or Hal's sister is not a computer science major.

30. Sam is not an orange belt or Kate is not a red belt.

31. The connector is not loose and the machine is not unplugged

32. This computer program does not have a logical error in the first ten lines and it is not being run with an incomplete data set

33. The dollar is not at an all-time high or the stock market is not at a record low.

34. The train is not late and my watch is not fast.

35. $-2 \geq x \vee x \geq 7$

36. $-10 \geq x \vee x \geq 2$

37. $1 \leq x \vee x < -3$

38. $0 \leq x \vee x < -7$

39. $(num\_orders \leq 100 \vee num\_instock > 500) \wedge num\_instock \geq 200$

40. $(num\_orders \geq 50 \vee num\_instock \leq 300) \wedge (50 > num\_orders \geq 75 \vee num\_instock \leq 500)$

41. The statement '$(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$' has all T truth values, so it is a tautology.

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \wedge \neg q$	$(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	F	F	T
F	F	T	T	F	F	T

42. The statement '$(p \wedge \neg q) \wedge (\neg p \vee q)$' has all F truth values, so it is a contradiction.

p	q	$\neg p$	$\neg q$	$p \wedge \neg q$	$\neg p \vee q$	$(p \wedge \neg q) \wedge (\neg p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

43. The statement '$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$' has all F truth values, so it is a contradiction.

p	q	r	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge q$	$q \wedge r$	$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$
T	T	T	F	F	T	F	T	F
T	T	F	F	F	T	F	F	F
T	F	T	F	T	F	F	F	F
T	F	F	F	T	F	F	F	F
F	T	T	T	F	F	T	T	F
F	T	F	T	F	F	T	F	F
F	F	T	T	T	F	F	F	F
F	F	F	T	T	F	F	F	F

44. The statement '$(\neg p \vee q) \vee (p \wedge \neg q)$' has all T truth values, so it is a tautology.

p	q	$\neg p$	$\neg q$	$p \vee q$	$p \wedge q$	$\neg p \vee q$	$p \wedge \neg q$	$(\neg p \vee q) \vee (p \wedge \neg q)$
T	T	F	F	T	T	T	F	T
T	F	F	T	T	F	F	T	T
F	T	T	F	T	F	T	F	T
F	F	T	T	F	F	T	F	T

45. $(p \wedge \neg q) \vee (p \wedge q) \equiv p \vee (\neg q \vee q)$
$\equiv p \wedge (q \vee \neg q)$
$\equiv p \wedge t$
$\equiv p$

a. Distributive law
b. Commutative law
c. Negation law
d. Identity law

46. $(p \vee \neg q) \wedge (\neg p \vee \neg q) \equiv (\neg q \vee p) \wedge (\neg q \vee \neg p)$
$\equiv \neg q \vee (p \wedge \neg p)$
$\equiv \neg q \vee c$
$\equiv \neg q$

a. Commutative law
b. Distributive law
c. Negation law
d. Identity law

47. $(p \wedge \neg q) \vee p \equiv p$

1. Absorption law

48. $p \wedge (\neg q \vee p) \equiv p$

1. Absorption law

49. $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$

1. De Morgan's law: $(\neg p \wedge q) \vee (\neg p \wedge \neg q)$
2. Distributive law: $\neg p \wedge (q \vee \neg q)$
3. Negation law: $\neg p \wedge t$
4. Identity law: $\neg p$

50. $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$

1. Distributive law: $\neg(\neg p \wedge (q \vee \neg q)) \vee (p \wedge q)$
2. Negation law: $\neg(\neg p \wedge t) \vee (p \wedge q)$
3. Identity law: $\neg(\neg p) \vee (p \wedge q)$
4. Double negative law: $p \vee (p \wedge q)$
5. Absorption law: p

51. $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$

1. De Morgan's law: $(p \wedge (p \wedge \neg q)) \vee (p \wedge q)$
2. Associative law: $((p \wedge p) \wedge \neg q) \vee (p \wedge q)$
3. Idempotent law: $(p \wedge \neg q) \vee (p \wedge q)$
4. Distributive law: $p \wedge (\neg q \vee q)$
5. Negation law: $p \wedge t$
6. Identity law: p

52. a. $p \oplus p \equiv c$

b. Yes because of the Associative laws.

c. Yes because of the Distributive laws.

53. Sarcasm is an example of a double positive being equivalent to a negative in standard English.

54. $p \vee (q \wedge (\neg r \vee (\neg s \vee \neg t)))$