

1. NOTATIONS

(u, v) : Current edge

T : Set of nodes creating triangles with edge (u, v)

N_u : Set of nodes only neighbor of u

N_v : Set of nodes only neighbor of v

$N_{Ti} = \text{Neighbor}(i) \cap T$ For any node $i \in (T \cup N_u \cup N_v)$

$N_{ui} = \text{Neighbor}(i) \cap N_u$ For any node $i \in (T \cup N_u \cup N_v)$

$N_{vi} = \text{Neighbor}(i) \cap N_v$ For any node $i \in (T \cup N_u \cup N_v)$

Algorithm 1: $i, j \in T, k \in T, k \in (N_u \cup N_v)$

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1: for all  $i \in T$  do
2:   for all  $j \in N_{Ti}$  do
3:     //  $k \in T$ 
4:      $N_{Tij} \leftarrow N_{Ti} \cap N_{Tj}$ 
5:      $f_{67ij} \leftarrow f_{67ij} + |N_{Tij}|$ 
6:      $f_{66ij} \leftarrow f_{66ij} + (|N_{Tj}| - |N_{Tij}| - 1)$  // 1 is deducted for node  $i$ 
7:      $f_{60ij} \leftarrow f_{60ij} + (|T| - |N_{Ti}| - |N_{Tj}| + |N_{Tij}|)$ 
8:     //  $k \in N_u$ 
9:      $N_{uij} \leftarrow N_{ui} \cap N_{uj}$ 
10:     $f_{50uij} \leftarrow f_{50uij} + (|N_u| - |N_{ui}| - |N_{uj}| + |N_{uij}|)$ 
11:    //  $k \in N_v$ 
12:     $N_{vij} \leftarrow N_{vi} \cap N_{vj}$ 
13:     $f_{50vij} \leftarrow f_{50vij} + (|N_v| - |N_{vi}| - |N_{vj}| + |N_{vij}|)$ 
14:  end for
15:   $f_{67i} \leftarrow f_{67i} + f_{67ij}/2$ 
16:   $f_{66i} \leftarrow f_{66i} + f_{66ij}$ 
17:   $f_{60i} \leftarrow f_{60i} + f_{60ij}$ 
18:   $f_{50i} \leftarrow f_{50i} + f_{50uij} + f_{50vij}$ 
19: end for
20:  $f_{67} \leftarrow f_{67} + f_{67i}/3$ 
21:  $f_{66} \leftarrow f_{66} + f_{66i}/2$ 
22:  $f_{60} \leftarrow f_{60} + f_{60i}/2$ 
23:  $f_{50} \leftarrow f_{50} + f_{50i}/2$ 

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Algorithm 2: $i \in T, j \in N_u, k \in T$

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1: for all  $i \in T$  do
2:   for all  $j \in N_{ui}$  do
3:      $N_{Tij} \leftarrow N_{Ti} \cap N_{Tj}$ 
4:      $f_{65_{ij}} \leftarrow f_{65_{ij}} + |N_{Tij}|$ 
5:      $f_{64_{ij}} \leftarrow f_{64_{ij}} + (|N_{Tj}| - |N_{Tij}| - 1)$  // 1 is deducted for node  $i$ 
6:      $f_{61_{ij}} \leftarrow f_{61_{ij}} + (|N_{Ti}| - |N_{Tij}|)$ 
7:      $f_{54_{ij}} \leftarrow f_{54_{ij}} + (|T| - |N_{Ti}| - |N_{Tj}| + |N_{Tij}|)$ 
8:   end for
9:    $f_{65_i} \leftarrow f_{65_i} + f_{65_{ij}}$ 
10:   $f_{64_i} \leftarrow f_{64_i} + f_{64_{ij}}$ 
11:   $f_{61_i} \leftarrow f_{61_i} + f_{61_{ij}}$ 
12:   $f_{54_i} \leftarrow f_{54_i} + f_{54_{ij}}$ 
13: end for
14:  $f_{65} \leftarrow f_{65} + f_{65_i}/2$ 
15:  $f_{64} \leftarrow f_{64} + f_{64_i}/2$ 
16:  $f_{61} \leftarrow f_{61} + f_{61_i}$ 
17:  $f_{54} \leftarrow f_{54} + f_{54_i}$ 

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Algorithm 3: $i \in T, j \in N_u, k \in (N_u \cup N_v)$

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1: for all  $i \in T$  do
2:   for all  $j \in N_{ui}$  do
3:      $// k \in N_u$ 
4:      $N_{uij} \leftarrow N_{ui} \cap N_{uj}$ 
5:      $f_{59_{ij}} \leftarrow f_{59_{ij}} + |N_{uij}|$ 
6:      $f_{52_{ij}} \leftarrow f_{52_{ij}} + (|N_{uj}| - |N_{uij}|)$ 
7:      $f_{47_{ij}} \leftarrow f_{47_{ij}} + (|N_{ui}| - |N_{uij}|)$ 
8:      $f_{34_{ij}} \leftarrow f_{34_{ij}} + (|N_u| - |N_{ui}| - |N_{uj}| + |N_{uij}|)$ 
9:      $// k \in N_v$ 
10:     $N_{vij} \leftarrow N_{vi} \cap N_{vj}$ 
11:     $f_{63_{ij}} \leftarrow f_{63_{ij}} + |N_{vij}|$   $//$  to avoid duplicate counting, dont count for  $j \in N_{vi}$ 
12:     $f_{56_{ij}} \leftarrow f_{56_{ij}} + (|N_{vj}| - |N_{vij}|)$ 
13:     $f_{55_{ij}} \leftarrow f_{55_{ij}} + (|N_{vi}| - |N_{vij}|)$   $//$  to avoid duplicate counting, dont count for
         $j \in N_{vi}$ 
14:     $f_{40_{ij}} \leftarrow f_{40_{ij}} + (|N_v| - |N_{vi}| - |N_{vj}| + |N_{vij}|)$ 
15:  end for
16:   $f_{59_i} \leftarrow f_{59_i} + f_{59_{ij}}/2$ 
17:   $f_{52_i} \leftarrow f_{52_i} + f_{52_{ij}}$ 
18:   $f_{47_i} \leftarrow f_{47_i} + f_{47_{ij}}/2$ 
19:   $f_{34_i} \leftarrow f_{34_i} + f_{34_{ij}}$ 
20:   $f_{63_i} \leftarrow f_{63_i} + f_{63_{ij}}$ 
21:   $f_{56_i} \leftarrow f_{56_i} + f_{56_{ij}}$ 
22:   $f_{55_i} \leftarrow f_{55_i} + f_{55_{ij}}$ 
23:   $f_{40_i} \leftarrow f_{40_i} + f_{40_{ij}}$ 
24: end for
25:  $f_{59} \leftarrow f_{59} + f_{59_i}$ 
26:  $f_{52} \leftarrow f_{52} + f_{52_i}$ 
27:  $f_{47} \leftarrow f_{47} + f_{47_i}$ 
28:  $f_{34} \leftarrow f_{34} + f_{34_i}$ 
29:  $f_{63} \leftarrow f_{63} + f_{63_i}$ 
30:  $f_{56} \leftarrow f_{56} + f_{56_i}$ 
31:  $f_{55} \leftarrow f_{55} + f_{55_i}$ 
32:  $f_{40} \leftarrow f_{40} + f_{40_i}$ 

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Algorithm 4: $i, j \in N_u, k \in (N_u \cup N_v \cup T)$

```

1: for all  $i \in N_u$  do
2:   for all  $j \in N_{ui}$  do
3:     //  $k \in N_u$ 
4:      $N_{uij} \leftarrow N_{ui} \cap N_{uj}$ 
5:      $f_{51_{ij}} \leftarrow f_{51_{ij}} + |N_{uij}|$ 
6:      $f_{35_{ij}} \leftarrow f_{35_{ij}} + (|N_{uj}| - |N_{uij}| - 1)$  // deduct 1 for node  $i$ 
7:      $f_{27_{ij}} \leftarrow f_{27_{ij}} + (|N_u| - |N_{ui}| - |N_{uj}| + |N_{uij}|)$ 
8:     //  $k \in N_v$ 
9:      $N_{vij} \leftarrow N_{vi} \cap N_{vj}$ 
10:     $f_{57_{ij}} \leftarrow f_{57_{ij}} + |N_{vij}|$ 
11:     $f_{45_{ij}} \leftarrow f_{45_{ij}} + (|N_{vj}| - |N_{vij}|)$ 
12:     $f_{23_{ij}} \leftarrow f_{23_{ij}} + (|N_v| - |N_{vi}| - |N_{vj}| + |N_{vij}|)$ 
13:    //  $k \in T$ 
14:     $N_{Tij} \leftarrow N_{Ti} \cap N_{Tj}$ 
15:     $f_{37_{ij}} \leftarrow f_{37_{ij}} + (|T| - |N_{Ti}| - |N_{Tj}| + |N_{Tij}|)$ 
16:  end for
17:   $f_{51_i} \leftarrow f_{51_i} + f_{51_{ij}}/2$ 
18:   $f_{35_i} \leftarrow f_{35_i} + f_{35_{ij}}$ 
19:   $f_{27_i} \leftarrow f_{27_i} + f_{27_{ij}}$ 
20:   $f_{57_i} \leftarrow f_{57_i} + f_{57_{ij}}$ 
21:   $f_{45_i} \leftarrow f_{45_i} + f_{45_{ij}}$ 
22:   $f_{23_i} \leftarrow f_{23_i} + f_{23_{ij}}$ 
23:   $f_{37_i} \leftarrow f_{37_i} + f_{37_{ij}}$ 
24: end for
25:  $f_{51} \leftarrow f_{51} + f_{51_i}/3$ 
26:  $f_{35} \leftarrow f_{35} + f_{35_i}/2$ 
27:  $f_{27} \leftarrow f_{27} + f_{27_i}/2$ 
28:  $f_{57} \leftarrow f_{57} + f_{57_i}/2$ 
29:  $f_{45} \leftarrow f_{45} + f_{45_i}$ 
30:  $f_{23} \leftarrow f_{23} + f_{23_i}/2$ 
31:  $f_{37} \leftarrow f_{37} + f_{37_i}/2$ 

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Algorithm 5: $i \in N_u, j \in \overline{N_{ui}}, k \in (N_u \cup N_v)$

```

1: for all  $i \in N_u$  do                                     #  $i \in N_v$ 
2:    $\overline{N_{ui}} \leftarrow N_u - (N_{ui} \cup \{i\})$ 
3:   for all  $j \in \overline{N_{ui}}$  do                               #  $j \in \overline{N_{ui}}$ 
4:     //  $k \in N_u$                                            // #  $k \in N_v$ 
5:      $N_{uij} \leftarrow N_{ui} \cap N_{uj}$ 
6:      $f_{17_{ij}} \leftarrow f_{17_{ij}} + (|N_u| - |N_{ui}| - |N_{uj}| + |N_{uij}| - 2)$  // 2 is deducted for nodes  $i$  &  $j$ 
7:     //  $k \in N_v$                                            // #  $k \in N_u$ 
8:      $N_{vij} \leftarrow N_{vi} \cap N_{vj}$ 
9:      $f_{42_{vij}} \leftarrow f_{42_{vij}} + |N_{vij}|$ 
10:     $f_{30_{vij}} \leftarrow f_{30_{vij}} + (|N_{vj}| - |N_{vij}|)$ 
11:     $f_{15_{vij}} \leftarrow f_{15_{vij}} + (|N_v| - |N_{vi}| - |N_{vj}| + |N_{vij}|)$ 
12:  end for
13:   $f_{17_i} \leftarrow f_{17_i} + f_{17_{ij}}/2$ 
14:   $f_{15_i} \leftarrow f_{15_i} + f_{15_{ij}}$ 
15: end for
16:  $f_{17} \leftarrow f_{17} + f_{17_i}/3$ 
17:  $f_{15} \leftarrow f_{15} + f_{15_i}/2$ 
18:  $f_{42} \leftarrow f_{42} + f_{42_i}/2$ 
19:  $f_{30} \leftarrow f_{30} + f_{30_i}$ 

```

Algorithm 6: $i \in T, j \in (\overline{N_{Ti}} \cup \overline{N_{ui}} \cup \overline{N_{vi}}), k \in (N_u \cup N_v)$

```

1: for all  $i \in T$  do
2:    $\overline{j} \in \overline{N_{Ti}}$ 
3:    $\overline{N_{Ti}} \leftarrow T - (N_{Ti} \cup \{i\})$ 
4:   for all  $j \in \overline{N_{Ti}}$  do
5:      $k \in T$ 
6:      $N_{Tij} \leftarrow N_{Ti} \cap N_{Tj}$ 
7:      $f_{48_{ij}} \leftarrow f_{48_{ij}} + (|T| - |N_{Ti}| - |N_{Tj}| + |N_{Tij}| - 2)$       // deduct counts for  $i$  &  $j$ 
8:      $k \in N_u$ 
9:      $N_{uij} \leftarrow N_{ui} \cap N_{uj}$ 
10:     $f_{33_{uij}} \leftarrow f_{33_{uij}} + (|N_u| - |N_{ui}| - |N_{uj}| + |N_{uij}|)$ 
11:     $k \in N_v$ 
12:     $N_{vij} \leftarrow N_{vi} \cap N_{vj}$ 
13:     $f_{33_{vij}} \leftarrow f_{33_{vij}} + (|N_v| - |N_{vi}| - |N_{vj}| + |N_{vij}|)$ 
14:  end for
15:   $f_{48_i} \leftarrow f_{48_i} + f_{48_{ij}}/2$ 
16:   $f_{33_i} \leftarrow f_{33_i} + f_{33_{uij}} + f_{33_{vij}}$ 
17:   $\overline{j} \in \overline{N_{ui}}$ 
18:   $\overline{N_{ui}} \leftarrow N_u - N_{ui}$ 
19:  for all  $j \in \overline{N_{ui}}$  do
20:     $k \in N_u$ 
21:     $N_{uij} \leftarrow N_{ui} \cap N_{uj}$ 
22:     $f_{26_{uij}} \leftarrow f_{26_{uij}} + (|N_u| - |N_{ui}| - |N_{uj}| + |N_{uij}| - 1)$       // deduct count for  $j$ 
23:     $k \in N_v$ 
24:     $N_{vij} \leftarrow N_{vi} \cap N_{vj}$ 
25:     $f_{44_{ij}} \leftarrow f_{44_{ij}} + (|N_{vj}| - |N_{vij}|)$ 
26:     $f_{19_{ij}} \leftarrow f_{19_{ij}} + (|N_v| - |N_{vi}| - |N_{vj}| + |N_{vij}|)$       // to avoid duplicate counting,
    dont count  $j \in \overline{N_{vi}}$ 
27:  end for
28:   $\overline{j} \in \overline{N_{vi}}$ 
29:   $\overline{N_{vi}} \leftarrow N_v - N_{vi}$ 
30:  for all  $j \in \overline{N_{vi}}$  do
31:     $k \in N_v$ 
32:     $N_{vij} \leftarrow N_{vi} \cap N_{vj}$ 
33:     $f_{26_{vij}} \leftarrow f_{26_{vij}} + (|N_v| - |N_{vi}| - |N_{vj}| + |N_{vij}| - 1)$ 
34:  end for
35:   $f_{26_i} \leftarrow f_{26_i} + f_{26_{uij}}/2 + f_{26_{vij}}/2$ 
36:   $f_{19_i} \leftarrow f_{19_i} + f_{19_{ij}}$ 
37: end for
38:  $f_{48} \leftarrow f_{48} + f_{48_i}/3$ 
39:  $f_{33} \leftarrow f_{33} + f_{33_i}/2$ 
40:  $f_{26} \leftarrow f_{26} + f_{26_i}$ 
41:  $f_{19} \leftarrow f_{19} + f_{19_i}$ 

```

2. TEMPLATE ALGORITHM

$set - ij = \{(T, N_{Ti}), (T, N_{ui}), (T, \overline{N_{Ti}}), (T, \overline{N_{ui}}), (T, \overline{N_{vi}}), (N_u, N_{ui}), (N_u, \overline{N_{ui}})\}$

Algorithm 7: Template Algorithm

```

1: for all  $(S1, S2) \in set - ij$  do
2:   for all  $i \in S1$  do
3:     for all  $j \in S2$  do
4:       //  $k \in T$ 
5:       if  $k\text{-var} == 0$  then
6:          $f1_{ij}, f2_{ij}, f3_{ij}, f4_{ij} \leftarrow \text{get\_freq\_k\_inT}(i, j)$ 
7:       else if  $k\text{-var} == 1$  then
8:          $f1_{ij}, f2_{ij}, f3_{ij}, f4_{ij} \leftarrow \text{get\_freq\_k\_inNu}(i, j)$ 
9:       else
10:         $f1_{ij}, f2_{ij}, f3_{ij}, f4_{ij} \leftarrow \text{get\_freq\_k\_inNv}(i, j)$ 
11:      end if
12:    end for // end of loop for  $j$ 
13:     $f1_i \leftarrow f1_i + f1_{ij}/d1_i$ 
14:     $f2_i \leftarrow f2_i + f2_{ij}/d2_i$ 
15:     $f3_i \leftarrow f3_i + f3_{ij}/d3_i$ 
16:     $f4_i \leftarrow f4_i + f4_{ij}/d4_i$ 
17:  end for // end of loop for  $i$ 
18:   $f1 \leftarrow f1 + f1_i/d1$ 
19:   $f2 \leftarrow f2 + f2_i/d2$ 
20:   $f3 \leftarrow f3 + f3_i/d3$ 
21:   $f4 \leftarrow f4 + f4_i/d4$ 
22: end for

```

Algorithm 8: get_freq_k_inT(i, j)

```

1:  $N_{Tij} \leftarrow N_{Ti} \cap N_{Tj}$ 
2:  $f1_{ij} \leftarrow f1_{ij} + |N_{Tij}|$ 
3:  $f2_{ij} \leftarrow f2_{ij} + (|N_{Ti}| - |N_{Tij}|)$ 
4:  $f3_{ij} \leftarrow f3_{ij} + (|N_{Tj}| - |N_{Tij}|)$ 
5:  $f4_{ij} \leftarrow f4_{ij} + (|T| - |N_{Ti}| - |N_{Tj}| + |N_{Tij}|)$ 
6: return  $f1_{ij}, f2_{ij}, f3_{ij}, f4_{ij}$ 

```

Algorithm 9: get_freq_k_inNu(i, j)

```

1:  $N_{uij} \leftarrow N_{ui} \cap N_{uj}$ 
2:  $f1_{ij} \leftarrow f1_{ij} + |N_{uij}|$ 
3:  $f2_{ij} \leftarrow f2_{ij} + (|N_{ui}| - |N_{uij}|)$ 
4:  $f3_{ij} \leftarrow f3_{ij} + (|N_{uj}| - |N_{uij}|)$ 
5:  $f4_{ij} \leftarrow f4_{ij} + (|N_u| - |N_{ui}| - |N_{uj}| + |N_{uij}|)$ 
6: return  $f1_{ij}, f2_{ij}, f3_{ij}, f4_{ij}$ 

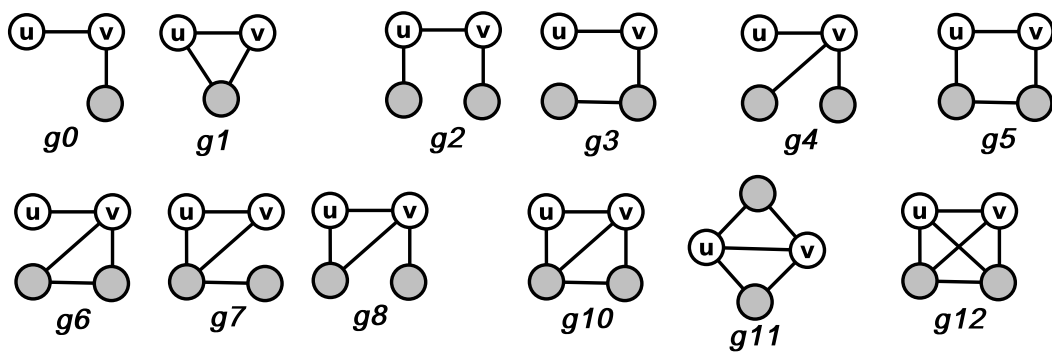
```

Algorithm 10: get_freq_k_inNv(i, j)

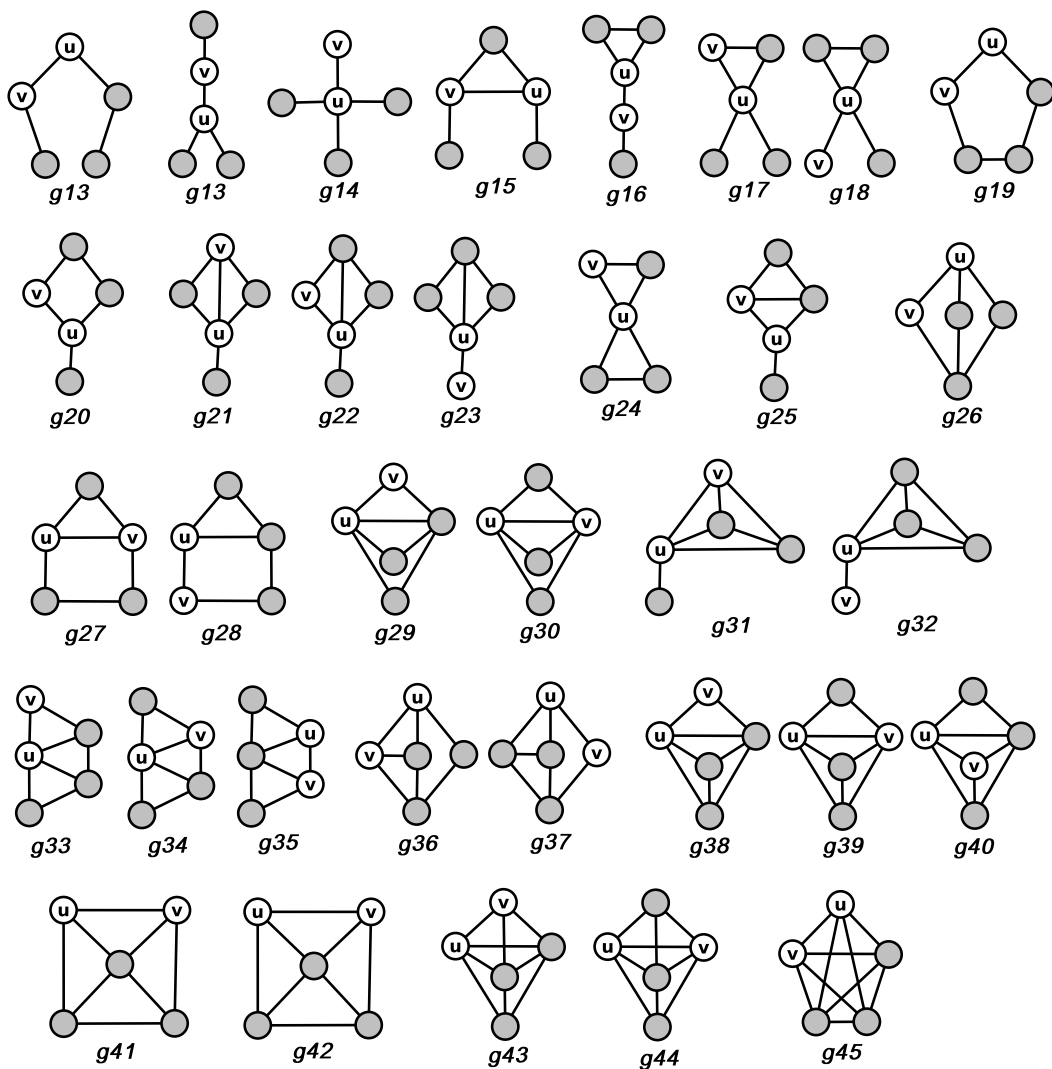
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1:  $N_{vij} \leftarrow N_{vi} \cap N_{vj}$ 
2:  $f1_{ij} \leftarrow f1_{ij} + |N_{vij}|$ 
3:  $f2_{ij} \leftarrow f2_{ij} + (|N_{vi}| - |N_{vij}|)$ 
4:  $f3_{ij} \leftarrow f3_{ij} + (|N_{vj}| - |N_{vij}|)$ 
5:  $f4_{ij} \leftarrow f4_{ij} + (|N_v| - |N_{vi}| - |N_{vj}| + |N_{vij}|)$ 
6: return  $f1_{ij}, f2_{ij}, f3_{ij}, f4_{ij}$ 

```



i: 3-4 size local graphlets



ii: 5 size local graphlets

Fig. 1: Local graphlets