mechanics

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mechanics

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mission

fragment

- effective gravity induced by rotation of facility

  Only second-order effect is present at a perturbative level.
- epicycle model

  Fitting Kepler orbit of binary star system to epicycle system?
- space channel in gravitational fields

  More fictional rather than a realistic one. Few materials are found.

Kepler

- effective potential
- orbit stability

  Bertland Theorem

- Hydrogen and $SO(4)$ symmetry

  Laplace-Runge-Lenz vector does not necessarily commute with angular momentum L in the $\mathcal{L}$ notion. It is conserved given that $\mathcal{H}$ is time-independent. In other words, it is invariant under a different (more strict) variational condition.

- action / time evolution

  Time evolution operators classically (failed)

asymptotic

tidal force

$$\begin{aligned}(1+x)^{-\frac{1}{2}} &= 1 + (-\frac{1}{2})x + (\frac{3}{8})x^2 + o(x^3) \\ (1+x)^{-\frac{3}{2}} &= 1 + (-\frac{3}{2})x + (\frac{15}{8})x^2 + o(x^3)\end{aligned} \tag{taylorM}$$

$$\begin{aligned}\vec{a}_g &= -\frac{k}{|\vec{r}|^3}\vec{r} \\ &= -k(\vec{r}^2)^{-\frac{3}{2}}\vec{r} \\ &= -\frac{k}{r_0^3}(1 + \frac{(\vec{r_0} + \vec{\Delta}r)^2 - \vec{r_0}^2}{\vec{r_0}^2})^{-\frac{3}{2}}(\vec{r_0} + \vec{\Delta}r) \\ &= -\frac{k}{r_0^3}(1 + \frac{2\vec{r_0} \cdot \vec{\Delta}r + (\vec{\Delta}r)^2}{\vec{r_0}^2})^{-\frac{3}{2}}(\vec{r_0} + \vec{\Delta}r) \\ &= -\frac{k}{r_0^3}(1 - \frac{3\vec{r_0} \cdot \vec{\Delta}r}{\vec{r_0}^2} + \frac{15(\vec{r_0} \cdot \vec{\Delta}r)^2}{2\vec{r_0}^4} - \frac{3(\vec{\Delta}r)^2}{2(\vec{r_0})^2})(\vec{r_0} + \vec{\Delta}r)\end{aligned} \tag{taylorG}$$

$$
\begin{aligned}
\vec{a}_c &= -\frac{k}{r_0^3}\vec{r_0} \\
\vec{a}_{Tide} &= \vec{a}_g - \vec{a}_c \\
&= o((\vec{\Delta} r)^3) - \frac{k}{r_0^3}(\vec{\Delta} r - \frac{3\vec{r_0}\cdot\vec{\Delta} r}{\vec{r_0}^2}\vec{r_0}) \\
&- \frac{k}{r_0^3}(\frac{15(\vec{r_0}\cdot\vec{\Delta} r)^2}{2\vec{r_0}^4} - \frac{3(\vec{\Delta} r)^2}{2(\vec{r_0})^2})\vec{r_0} - \frac{k}{r_0^3}(-\frac{3\vec{r_0}\cdot\vec{\Delta} r}{\vec{r_0}^2})\vec{\Delta} r \\
&= \vec{a}^1_{Tide} + \vec{a}^2_{Tide} + o((\vec{\Delta} r)^3) \\
\vec{a}^1_{Tide} &= -\frac{k}{r_0^3}(\vec{\Delta} r_\perp - 2\vec{\Delta} r_\parallel)
\end{aligned}
\tag{taylorTide}
$$

$$
\begin{aligned}
V_g &= -\frac{k}{|\vec{r}|} \\
&= -k(\vec{r}^2)^{-\frac{1}{2}} \\
&= -\frac{k}{r_0}(1 + \frac{(\vec{r_0} + \vec{\Delta} r)^2 - \vec{r_0}^2}{\vec{r_0}^2})^{-\frac{1}{2}} \\
&= -\frac{k}{r_0}(1 + \frac{2\vec{r_0}\cdot\vec{\Delta} r + (\vec{\Delta} r)^2}{\vec{r_0}^2})^{-\frac{1}{2}} \\
&= -\frac{k}{r_0}(1 - \frac{\vec{r_0}\cdot\vec{\Delta} r}{\vec{r_0}^2} + \frac{3(\vec{r_0}\cdot\vec{\Delta} r)^2}{2\vec{r_0}^4} - \frac{(\vec{\Delta} r)^2}{2(\vec{r_0})^2}) + o((\vec{\Delta} r)^3) \\
&= V_0 + V_1 + V_2 + o((\vec{\Delta} r)^3)
\end{aligned}
\tag{taylorV}
$$

Coriolis Force

In system with rotation $(\vec{\omega}, \vec{\beta})$ around point O, Non-inertial forces are

$$
\begin{aligned}
\vec{a}_c &= -2(\vec{\omega} \times \vec{\delta v}) \\
\vec{a}_\beta &= -\beta \times \vec{\delta r}
\end{aligned}
\tag{aRot}
$$

thermal/hydro

- Fluid Roche limit

To be continued.

- Waterball without gravity
  ossilation?

scenarios

wander around space station

Space station O is orbiting $\vec{r}:(r,\theta)$ around the earth and an astronaut is wandering around O with displacement $\vec{\Delta r}$ in non-rotating system and $\vec{\delta r}$ in rorating system.

Dynamic in the rotating system is described in (aDelta)

$$\begin{aligned}\vec{a}_\delta &= \vec{a}_{Tide} + \vec{a}_c + \vec{a}_\beta \\ \vec{a}_{Tide} &= -\frac{k}{r_0^3}(-2\delta r_r \hat{r} + \delta r_\theta \hat{\theta}) \\ \vec{a}_c &= 2\dot{\theta}(\dot{\delta r}_\theta \hat{r} - \dot{\delta r}_r \hat{\theta}) \\ \vec{a}_\beta &= \ddot{\theta}(\delta r_\theta \hat{r} - \delta r_r \hat{\theta})\end{aligned} \tag{aDelta}$$

Assume O is orbiting on a circle $(\dot{\theta},\ddot{\theta}) = (\omega, 0)$, we have $\omega = \frac{k}{r_0^3}$,

then (aDelta) is simplified as (aCircle)

$$\begin{aligned}\frac{d^2}{dt^2}\vec{\delta r} &= \vec{a}_{Tide} + \vec{a}_c \\ &= -\omega^2(-2\delta r_r \hat{r} + \delta r_\theta \hat{\theta}) + 2\omega(\dot{\delta r}_\theta \hat{r} - \dot{\delta r}_r \hat{\theta})\end{aligned} \tag{aCircle}$$

Qualitative analysis of the dynamics: Assume the astronaut orbits around the space station with $\dot{\delta\theta} < 0$ and a period same as the space station orbit T_0

- Averagely, orbit of $\vec{\delta r}$ operates with $\vec{-\omega}$;
- At vertex along the $\hat{r}$ direction, $\vec{a}_{Tide}$ points outwards, so velocity should be large to generate massive $\vec{a}_c$;
- At vertex along the $\hat{\theta}$ direction, $\vec{a}_{Tide}$ points inwards, enough to keep the orbit bound, so velocity should be small;
- Quantitave description of the orbit dynamic remains mystery;

Lagrangian points

We consider the effective dynamics in a Non-inertial system of two-body gravity system.

Consider a two-body system consists of two celestial bodies M_1 and M_2 with distance of D. The two-body effective mass and orbiting angular velocity are

$$\begin{aligned} M &= \frac{M_1 M_2}{M_1 + M_2} \\ \frac{GM_1 M_2}{D^2} &= \frac{M_1 M_2}{\omega}^2 D \\ \omega^2 &= \frac{G(M_1 + M_2)}{D^3} \end{aligned} \tag{motionBin}$$

The two celestial bodies are orbiting around the centroid O. Now we consider the Lagrangian point L_4 locating at the vertex of an equilateral triangle connecting M_1 and M_2, thus we have $\vec{r} = \frac{M_1\vec{r_1} + M_2\vec{r_2}}{M_1 + M_2}$

In the rotating celestial system, the effective potential around L_4 is

$$\begin{aligned} V_{eff} &= V_0(\vec{r_1}) + V_1(\vec{r_1}) + V_2(\vec{r_1}) + V_0(\vec{r_2}) + V_1(\vec{r_2}) + V_2(\vec{r_2}) + o((\vec{\Delta} r)^3) + V_\omega \\ &= V_{eff}^0 + V_{eff}^1 + V_{eff}^2 + o((\vec{\Delta} r)^3) \\ V_\omega &= -\frac{1}{2}\omega^2(\vec{r} + \vec{\Delta} r)^2 \\ V_{eff}^1 &= -\frac{GM_1}{D}(-\frac{\vec{r_1} \cdot \vec{\Delta} r}{|\vec{r_1}|^2}) - \frac{GM_2}{D}(-\frac{\vec{r_2} \cdot \vec{\Delta} r}{|\vec{r_2}|^2}) - \omega^2 \vec{r} \cdot \vec{\Delta} r \\ &= \frac{G}{D^3}(M_1\vec{r_1} + M_2\vec{r_2}) \cdot \vec{\Delta} r - \frac{G}{D^3}(M_1\vec{r_1} + M_2\vec{r_2}) \cdot \vec{\Delta} r \\ &= 0 \\ V_{eff} &= V_{eff}^0 + V_{eff}^2 + o((\vec{\Delta} r)^3) \end{aligned} \tag{taylorBin}$$

In fact V_{eff}^2 is convex around L_4, thus L_4 is a smooth maximum in the rotating system. Celestial bodies around L_4 are bounded by Coriolis force under certain condition (the mass ratio boundary $\frac{M_1}{M_2}$ actually)