Experiment 2: Observing Water dispersion and Solitons

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I. INTRODUCTION

In this experiment we sought to observe the dispersion of water waves in a glass channel it was a aqueous solution but composed mostly water with some photoflo to diminish other effects on the experiment.We wanted to measure the damping of the waves by first sweeping through the waves to capture various modes and through those various modes so we could then drive the channel at the modes we collected so we can record there decay and in essence the composite of frequencies that composed the wave. The last operation we did was measuring the amplitudes of a non-propagating soliton that can be generated, and in effect we learned how to create solitons in our specific channel and observed the motion they undetook.

II. THEORY

The purpose of this experiment was to measure the properties of waves in a predefined channel a such we can create a dispersive spectrum containing the modes that elicit strong amplitudes and dispersion. We had to take into account several physical phenomenon to properly gauge the theoretical modes for the channel. A fairly needed measurement was the Length of the channel and the height as well with the height of the water taken into account also, this was all done to see the theoretical expected frequencies in this experiment. To gather the theoretical low frequency dispersion relation in shallow water is given by equation 1.

$$\omega^2 = gktanh(kh) \qquad [1]$$

In this equation g is grvity, k is the wave number given by the relation of $\frac{n\pi}{L}$, and h is the height of the water in our experiment. We can then take our equation 1 and apply the notion for very long wavelengths $kh << 1$ meaning the tanh function can be eliminated such that $tanh(kh) = kh$ and in this relation the phase veloctiy can be written as seen in equation 2.

$$c_{ph} = \frac{\omega}{k} = \sqrt{gh} \qquad [2]$$

Equation 2 shows a nondispersive phase velocity, though the dispersion relation is incomplete. At higher frequencies the wavelength of our system becomes so large that the capillary length of water which is only several milimeters, and the surface tension given by the walls of the system start to take effect . This can be accounted for and is applied in equation 3 which is given below.

$$\omega^2 = (gk + \frac{\sigma}{p}k^3)tanh(kh) \qquad [3]$$

where σ is the surface tension which can be expressed as $\sigma = \frac{1}{2}pgrh$ and p is the liquid density.

The theoretical nature of solitons is that they are wave packets that are self propagating, they are the combination of removing the nonlinear and dispersive effects. Though there is much mathematical theory associated with them we are not subjected to that rather we only look at the point of creation of solitons the wave number is such that $k = \frac{k}{W}$ where W is the width of the channel because we are creating solitons along the width not the length of the channel.

This gives us a hyphenated and rough method of processing the data and gives us a theoretical access at predicting modes in the channel, again these modes are highly dependent on the dimensions which they occupy which is also why the lengths and widths of the box are accounted for, we now are faced with creating an accurate way of collecting data.

III. APPARATUS AND PROCEDURE

In our experiment we had a glass channel that was constructed such that the water inside the channel is always touching glass and is fully contained. The channel itself has two connections one to an amplifier and driver which sweeps through the frequencies we needed and can sustain a pulse at a frequency at the modes we observed. The other side of the channel is then connected to a full wave rectifier bridge which collected the AC voltage generated by the moving channel, which it does using two leads that are electrically in contact with the channel, the collected DC after going through the rectifier is then sent to a Lock in Amplifier which then sends the translated electricity readings to a computer for recording. We measure the depth of the channel using a ruler and the channel sits on a ball bearing allowing it to move with the driver for frequencies, and the resistance of the two leads collecting the voltage as well oscillates with the channel.

The apparatus schematic is shown below.

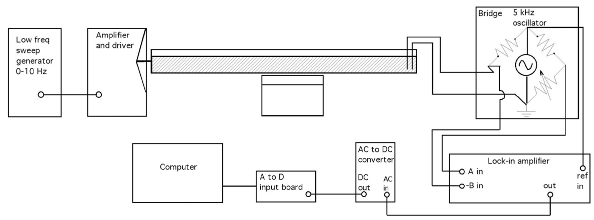

Figure 1: This Diagram shows the setup used to measure the modes of the channel which a driver which is controlled at which frequency it moves and collected with a computer.

For actually carrying out the experiment we followed a fairly simple procedure. The initial tasks were to prepare the setup such that it meets the standards of our theoretical calculations, so first we had to fill the channel to a depth between 2.5 and 3 cm with the sample water, counting salt and photoflo. We then align and level the channel with the speakers such that the motion is unihibited and tune the frequencies to the theoretical expected modes to see if the behave accordingly and we expect only odd numbered to show up in our results.

The next step of action we took was sweeping through all the unnecessary frequencies to find the actual modes that we want to observe in the channel. To do this we adjusted the driver amplifier such that it sweeped through the frequencies of .5-3.5 Hz with a sweep time of 1999s we then repeat for frequencies 3.5-7 Hz and 7-10 Hz.

The next course of action was then to measure the damping coefficient for a factor known as α, to achieve this we went through the first 5 and 6 modes by manually tuning to each resonant frequency we measured from the earlier data. We recorded the time decay of the waves and expect them to follow a pattern of $A(t) = A_0 e^{-\alpha t}$.

The next thing we did was operate with solitons. We had to lay the speaker flat as the soliton is generated along the width of the channel, and we tune to a frequency below twice of the first mode in the direction across the channel which is around 10 kHz. We then use a wooden paddle to gently oscillate along the width to find a soliton and after generating one by swishing width wise back and forth we increase the drive amplitude and map the frequencies which the soliton was stable.

This was a simple experiment to operate though from our data we had to do many calculations.

IV. DATA AND ANALYSIS

In this laboratory the first act that we completed was determining the resonant frequencies of the channel. We had ways of predicting this we used equations 1 and 3, where they basically cover how to calculate the resonant frequencies where equation 3 adds correction term for higher modes. We present the data for the modes in 3 ways first we will run down a calculation, then show the charted modes we measured and the theoretical counterparts, and lastly we shows the sweep measurements we made displayed in mathematica. To start we look at the calculation, first the first mode of equation 1 and the first mode of equation 3.

In both the equations, we are actually presented with angular frequency, so first we break down how to calculate the frequency. Before we do, I present the measurements of the channels length which was 38 cm $\pm$.1 cm and the amount of water depth used which was 2.5 $\pm .1cm$. To start we break down equation 1 starting by rooting both sides, then we divide both sides by 2π. Then we apply numbers and the values each exponent represents.

$$\omega = \sqrt{gktan(kh)}$$

$$f = \frac{\sqrt{gktanh(kh)}}{2\pi}$$

$$f = \frac{\sqrt{(9.8\frac{m}{s})(\frac{1\pi}{.38m}tanh(\frac{1\pi}{.38m}.025m)}}{2\pi}$$

$$f = .647\frac{1}{s}$$

This is a simple run down of how equation 1 yields the theoretical frequencies. Then we begin with equation 3, this is similar to equation 1, except there is a factor of $\frac{\sigma}{p}k^3$, where p is the liquid density of water which is found using texts, and yields a value of $997kg/m^3$. The method we used to find the value of σ was simply using the equation 4.

$$\sigma = \frac{1}{2}pghr \qquad [4]$$

Equation 4 is only significant in that it requires another technique to find the factor we need to properly analyze the channel, since it is made of glass, we simply retrieved a 100 tube, which had a height of $8.8 \pm .1cm$ and we measured the amount of water rise that occurs in the tube in water which was $1.1 \pm .1cm$. So to find the radius of equation 4 we had to use the formula of $V = \pi r^2 h$ and in this case it yielded a value of $.19 \pm .03cm$. We then apply all the necessary values to formula 4 which looks as follows

$$\sigma = \frac{1}{2} * 997\frac{kg}{m^3} * 9.8\frac{m}{s^2} * .011m * .0019m = .051\frac{N}{m}$$

We then again look at equation 3, for our theoretical predictions with the corrected frequencies. All we need to do is plug in our appropriate values which then yields the target frequencies, and in our case the correlation was very strong. We look at the calculation shown below

$$f = \frac{\sqrt{(((9.8\frac{m}{s})(\frac{21\pi}{.38m})) + (\frac{.051\frac{N}{m}*21\pi}{(.38m)*(997\frac{kg}{m^3})}))}}{2\pi}$$

$$with \frac{\sqrt{tanh((\frac{21\pi}{.38m}) * .025m)}}{2\pi}$$

$$f = 7.06\frac{1}{s}$$

This allows us to predict the modes, which we did, and we show the totality of our collected sweep data in a chart of the measured and predicted modes below.

MODES	Theoretical Value		Theoretical Corrected Value		Measured Modes	
1	0.65	±0.09	0.65	±0.09	0.65	±.02
3	1.84	±0.15	1.85	±0.05	1.9	±.01
5	2.82	±0.12	2.83	±0.03	2.88	±.01
7	3.59	±0.07	3.62	±0.01	3.67	±.01
9	4.19	±0.04	4.23	±0.07	4.32	±.02
11	4.7	±0.02	4.8	±0.03	4.9	±.02
13	5.14	±0.02	5.3	±0.01	5.4	±.04
15	5.54	±0.01	5.76	±0.07	5.85	±.01
17	5.9	±0.01	6.2	±0.04	6.26	±.02
19	6.24	±0.01	6.63	±0.02	6.67	±.01
21	6.57	±0.01	7.06	±0.07	7.08	±.02
23	6.87	±0.01	7.49	±0.003	7.5	±.03
25	7.16	±0.01	7.92	±0.02	7.83	±.01
27	7.45	±0.01	8.36	±0.2	8.2	±.02

Table 1: This Table shows the Theoretical expected modes, from dimensions of the glass chamber as well the measured frequencies and corrections

To collect the sweep data we simply sweeped through frequencies, to find the modes we simply found the peaks that occurred in the data. we show graphs of the data we observed below for the three sweeps we did and we indicated the observed modes in table 1. You can see the graphs of the sweeps in figures 2-4.

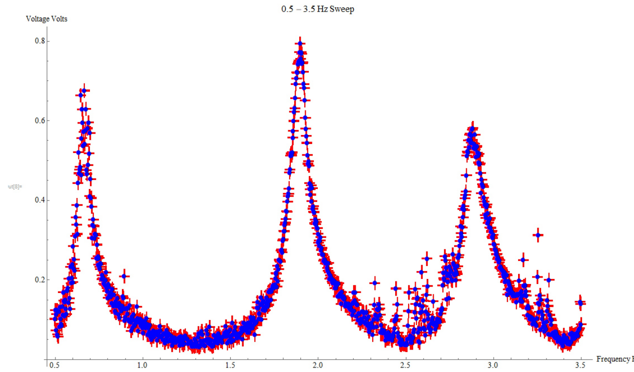

Figure 2: This shows the sweeped frequencies of the glass channel from .5-3.5 Hz.

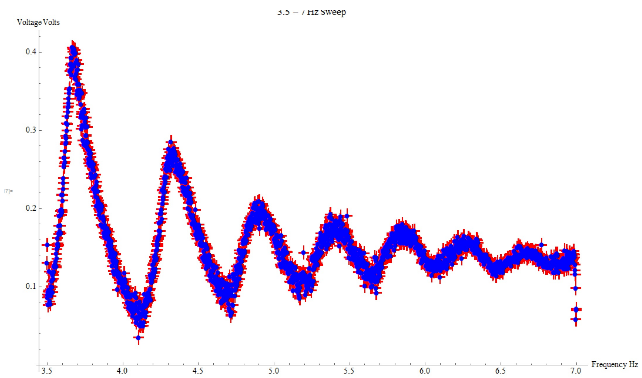

Figure 3: This shows the sweeped frequencies of the glass channel from 3.5-7 Hz.

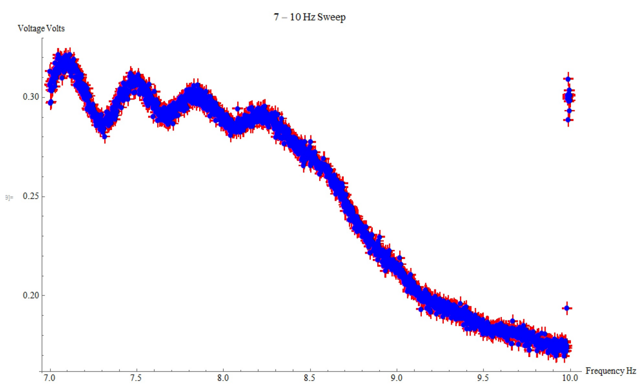

Figure 4: This shows the sweeped frequencies of the glass channel from 7-10 Hz .

The next set of data that we had to go through was basically taking the frequencies we had and converting them into phase velocity. To do this we utilize equation 2. We show a sample calculation below where basically we take the frequency over wave number. What we then do with the phase velocity is plot it against the wave number and in essence the increasing frequency what is then yielded is that it decays with frequency, and furthering, we look at the the theoretical phase velocities besides them and we see that the corrected one is closer to the actual results by a significant margin. You can see a calculation of phase velocity below and the graphs of the phase versus wave number in figures 5-7. In table 2 you can see the data tables of the phase velocity.

$$c_{ph} = \frac{2\pi .65Hz}{8.971\frac{1}{\lambda}} = .455\frac{m}{s}$$

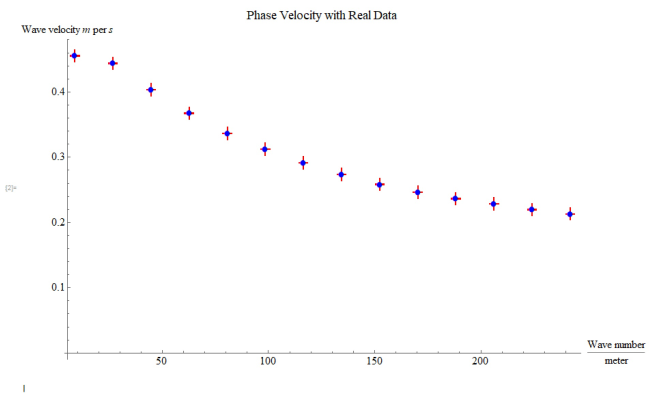

Figure 5: This is the real phase velocity against wave number.

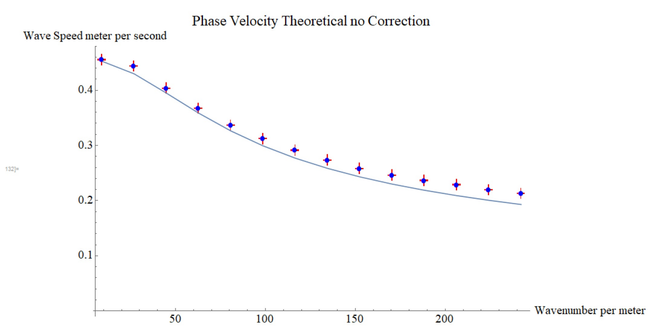

Figure 6: This is the real phase velocity against wave number with the theoretical line for comparison.

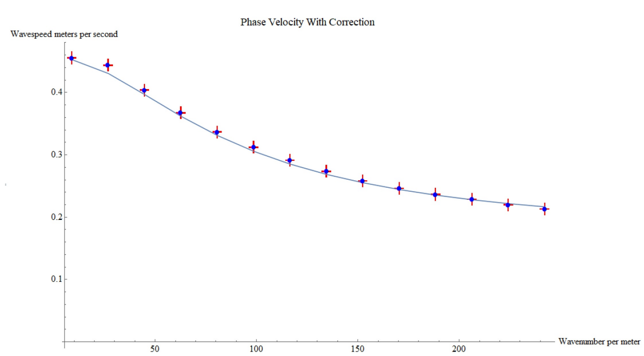

Figure 7: This is the phase velocity against wave number the line is the theoretical wavespeed with corrections.

Wave Number	Phase Velocity Measured	Wave Number	Phase Velocity Theoretical	Wave Number	Phase Velocity corrected
8.97 ±.01	0.46 ±.008	8.97 ±.01	0.45 ±.003	8.97 ±.01	0.45 ±.003
26.9 ±.01	0.44 ±.004	26.9 ±.01	0.43 ±.003	26.9 ±.01	0.43 ±.002
44.9 ±.01	0.4 ±.004	44.9 ±.01	0.39 ±.003	44.9 ±.01	0.39 ±.001
62.8 ±.01	0.37 ±.004	62.8 ±.01	0.36 ±.003	62.8 ±.01	0.36 ±.004
80.7 ±.01	0.34 ±.008	80.7 ±.01	0.33 ±.003	80.7 ±.01	0.33 ±.003
98.7 ±.01	0.31 ±.008	98.7 ±.01	0.29 ±.003	98.7 ±.01	0.31 ±.002
116.6 ±.01	0.29 ±.02	116.6 ±.01	0.28 ±.003	116.6 ±.01	0.29 ±.004
134.6 ±.01	0.27 ±.004	134.6 ±.01	0.26 ±.003	134.6 ±.01	0.27 ±.003
152.5 ±.01	0.26 ±.008	152.5 ±.01	0.24 ±.003	152.5 ±.01	0.26 ±.001
170.5 ±.01	0.25 ±.004	170.5 ±.01	0.23 ±.003	170.5 ±.01	0.24 ±.008
188.4 ±.01	0.24 ±.008	188.4 ±.01	0.22 ±.003	188.4 ±.01	0.24 ±.002
206.3 ±.01	0.23 ±.01	206.3 ±.01	0.21 ±.003	206.3 ±.01	0.23 ±.001
224.3 ±.01	0.22 ±.004	224.3 ±.01	0.2 ±.003	224.3 ±.01	0.22 ±.008
242.2 ±.01	0.21 ±.008	242.2 ±.01	0.19 ±.0003	242.2 ±.01	0.22 ±.03

Table 2: This chart shows the Phase velocities and the theoretical values with the glass channel.

In comparing the theoretical values to the measured values we see that the corrected theoretical values are much better than the ones that were not corrected, and that they all correlate similarly, though the extreme values do not fall off like the ones in figure 6.

The next set of data that we collected was the Q factor of the decay and from the sweeps we preformed. These factors can show us the decay of the wave and the quality of the data that we collected. To collect the sweep Q factor we use a simple relation of $\frac{f_r}{\delta f * \sqrt{2}}$, we show a table of the sweep Q factor in table 3. We also then plot the Q factor against wave number for

Mode	All f in HZ	Frequency	1/2 f left	1/2 f right	delta f	Wave Number	Q factor
1		0.65	0.61	0.73	0.12	8.97	4
3		1.9	1.83	1.96	0.13	26.9	10.3
5		2.88	2.8	3	0.2	44.9	10.2
7		3.67	3.6	3.84	0.24	62.8	10.8
9		4.32	4.22	4.54	0.32	80.7	9.5
11		4.9	4.74	5.1	0.36	98.7	9.6

Table 3: This Table shows Q factor for the sweeped modes.

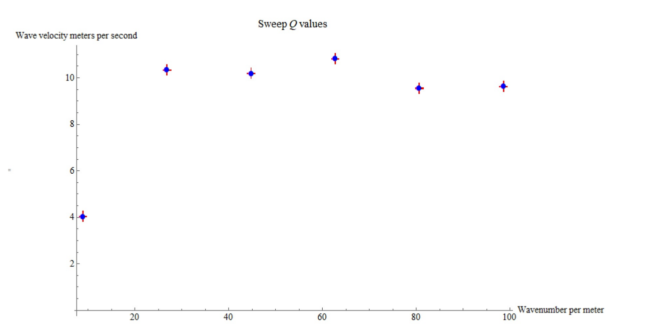

Figure 8: This graph is the Q factor of the sweeped modes versus the wave number .

The next set of data we take is the Q factor of the time decay data. First we present the raw decay data which can be seen in figures 9-14. Then we take the raw data and extrapolate the amplitudes and take the log of e and find the slope of the best fit line which can be seen in figures 15-20 for the various modes. The slope of the loged graph gives is a measurement of α which then can be transferred to a Q value using the relation of $\frac{\omega_0}{2\alpha}$. We show the data table for the Alpha and Q values from the decay in table 4 and the graphed Q factor in figure 21.

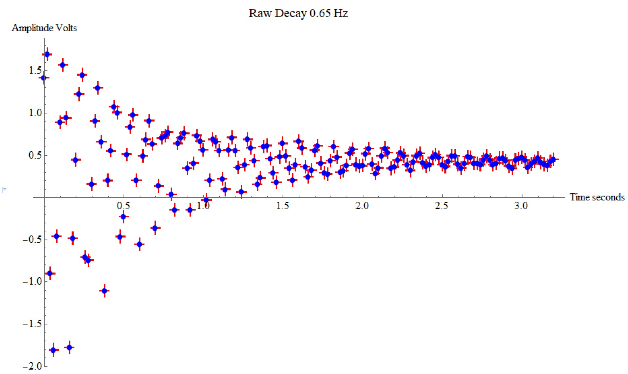

Figure 9: This shows the Raw Decay data for the .65 Hz mode.

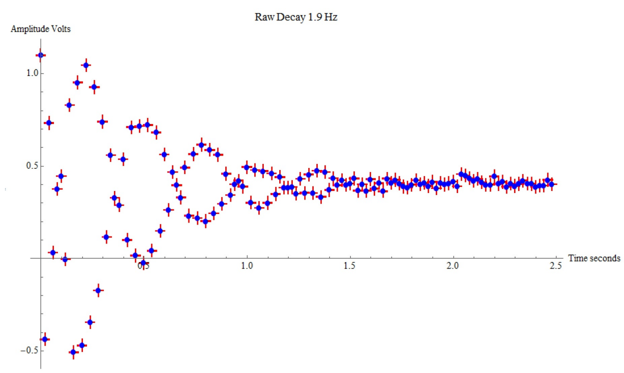

Figure 10: This shows the Raw Decay data for the 1.9 Hz mode.

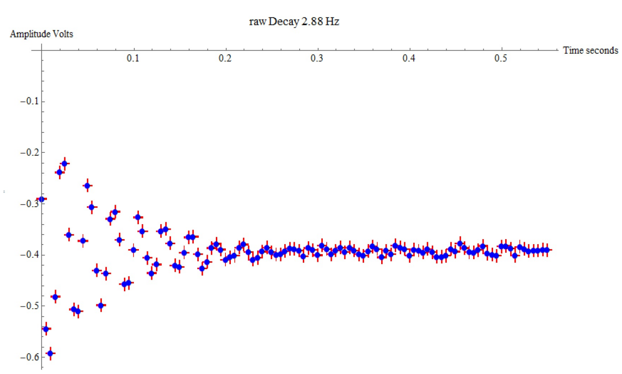

Figure 11: This shows the Raw Decay data for the 2.88 Hz mode.

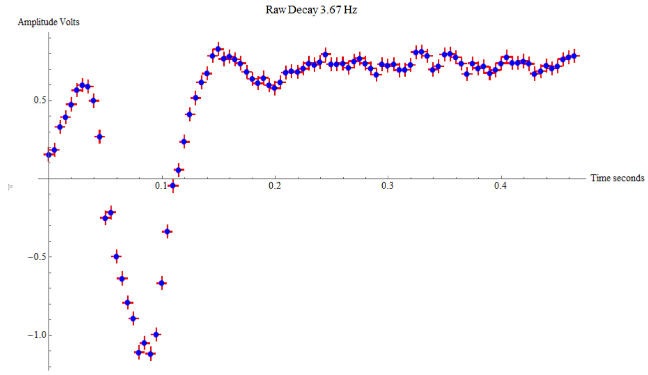

Figure 12: This shows the Raw Decay data for the 3.67 Hz mode.

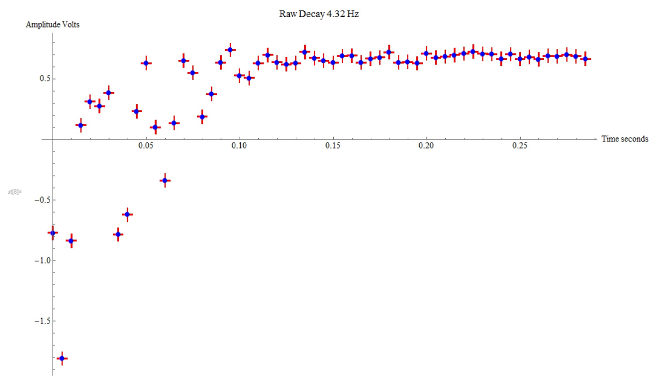

Figure 13:This shows the Raw Decay data for the 4.32 Hz mode.

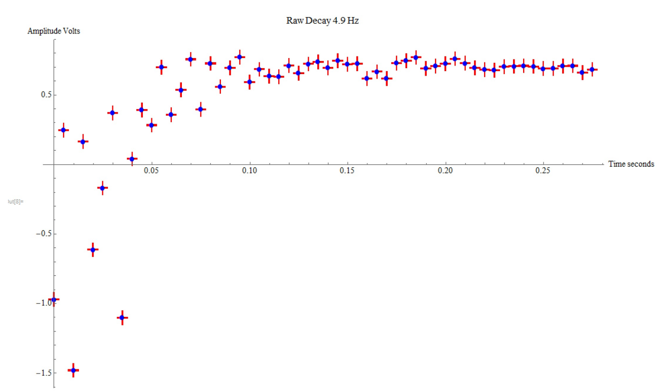

Figure 14: This shows the Raw Decay data for the 4.9 Hz mode.

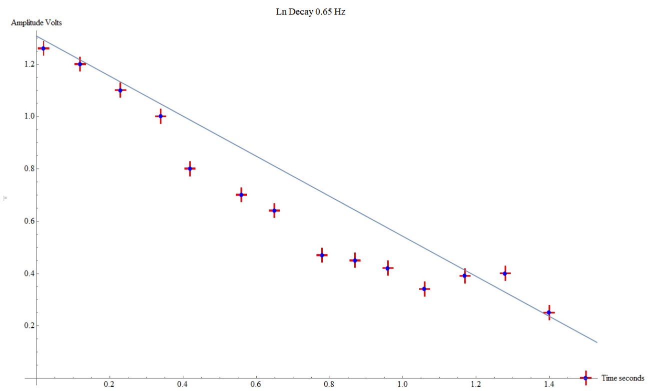

Figure 15: This Graph shows the ln of the amplitudes of the .65 Hz mode giving us the Q value.

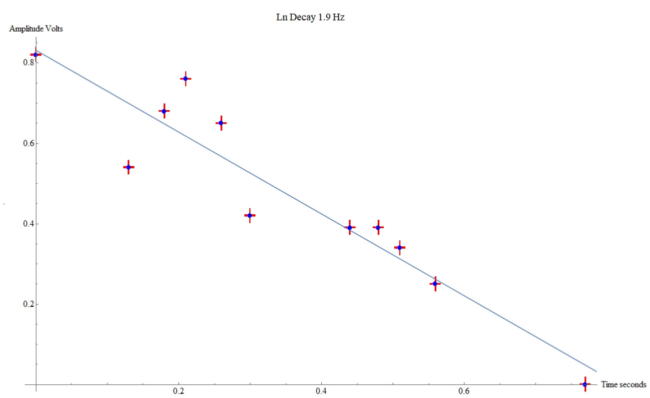

Figure 16: This Graph shows the ln of the amplitudes of the 1.9 Hz mode giving us the Q value.

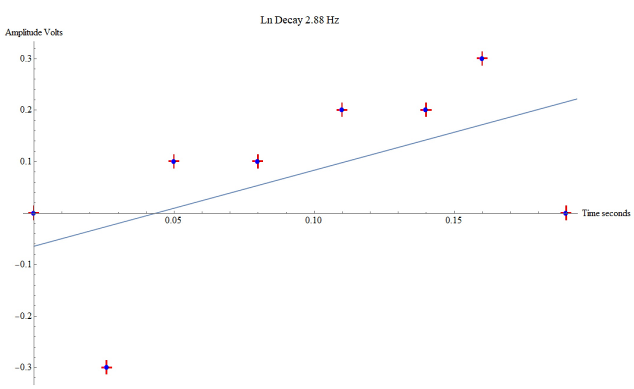

Figure 17: This Graph shows the ln of the amplitudes of the 2.88 Hz mode giving us the Q value.

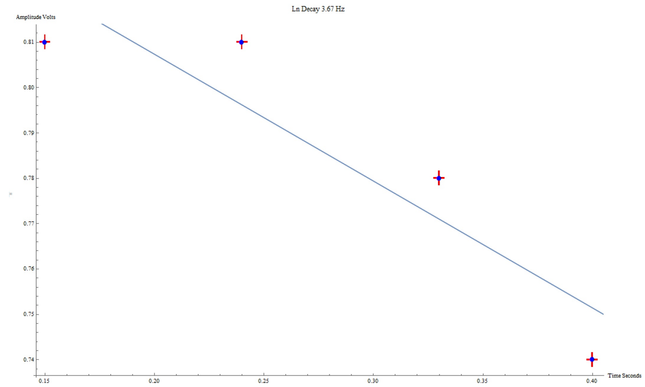

Figure 18: This Graph shows the ln of the amplitudes of the 3.67 Hz mode giving us the Q value.

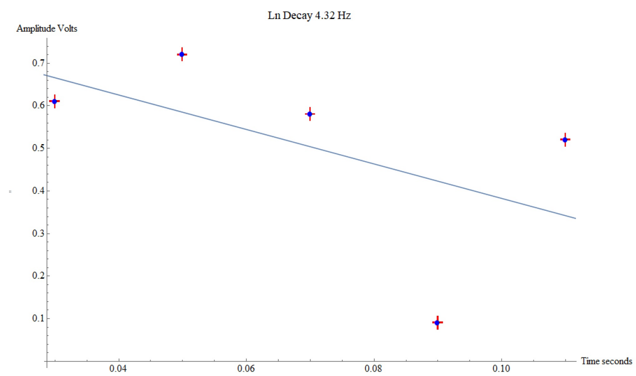

Figure 19: This Graph shows the ln of the amplitudes of the 4.32 Hz mode giving us the Q value.

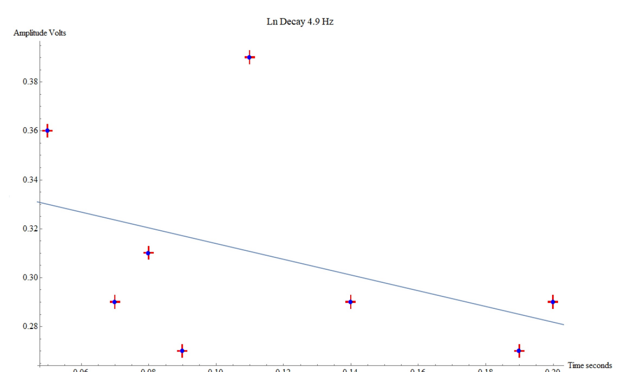

Figure 20: This Graph shows the ln of the amplitudes of the 4.9 Hz mode giving us the Q value.

Amplitude	Time	1 ln(Amp)	Time	Amplitude	Ln(amp)	Time	Amplitude	Ln(amp)	Time	Amplitude	Ln (Amp)	Time	Amplitude	Ln(Amp)	Time	Amplitude	Ln(Amp)	Mode	Slope of Lr	Wave Num	Alpha	Q	Frequency
1.7	0.02	0.53	0	1.09	0.09	0	0.3	-1.2	0.03	0.59	-0.52	0.03	0.4	-0.91	0.05	0.7	-0.36	1	0.3	8.9	0.77	0.42	0.65
1.6	0.12	0.47	0.13	0.82	-0.19	0.026	0.22	-1.5	0.15	0.83	-0.19	0.05	0.62	-0.48	0.07	0.75	-0.29						
1.45	0.23	0.37	0.18	0.95	-0.05	0.05	0.32	-1.1	0.24	0.82	-0.19	0.07	0.66	-0.42	0.08	0.73	-0.31	3	0.8	26.9	1.02	0.93	1.9
1.31	0.34	0.27	0.21	1.03	0.03	0.08	0.33	-1.1	0.33	0.8	-0.22	0.09	0.75	-0.28	0.09	0.76	-0.27						
1.08	0.42	0.07	0.26	0.92	-0.08	0.11	0.35	-1	0.4	0.77	-0.26	0.11	0.68	-0.39	0.11	0.68	-0.39	7	0.77	62.8	0.28	6.6	3.67
0.97	0.56	-0.03	0.3	0.73	-0.31	0.14	0.36	-1							0.14	0.75	-0.29						
0.91	0.65	-0.09	0.44	0.71	-0.34	0.16	0.38	-0.9							0.19	0.766	-0.27	9	5.22	80.7	4.05	0.53	4.32
0.77	0.78	-0.26	0.48	0.71	-0.34	0.19	0.3	-1.2							0.2	0.75	-0.29						
0.75	0.87	-0.28	0.51	0.68	-0.39													11	2.15	98.7	0.32	7.7	4.9
0.73	0.96	-0.31	0.56	0.62	-0.48																		
0.68	1.06	-0.39	0.77	0.48	-0.73																		
0.71	1.17	-0.34																					
0.72	1.28	-0.33																					
0.62	1.4	-0.48																					
0.48	1.5	-0.73																					

Table 4: This Table shows the Q factor for the Decay spectrum for the various modes we analyzed.

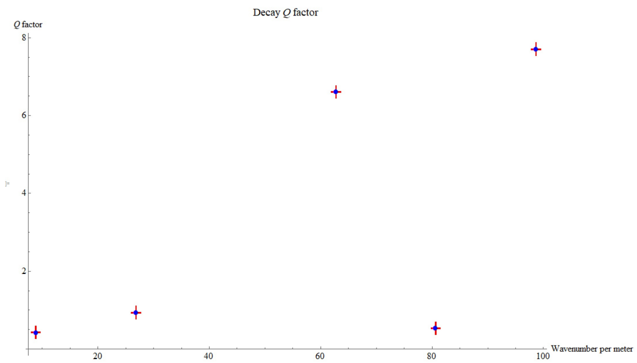

Figure 21: This graph shows the Q values of the Decay modes plotted against the wave number.

The two graphs we got in figures 21 and 8 represent the Q values from the decay and Sweep respectively from analyzing them we can see that the Q values we get from the decay readings are much stronger than the mode analysis. The reason we can assume this is that the Q values are much smaller than the Sweep with the exception of 1 point that is an outlier. The Q value in this case can then show that the decay shows better Q values and thus better data overall for measuring the waves.

The Solitons we observed can be summarized as such. We had to tune to a frequency in the order of $k = \frac{\pi}{W}$ in our experiment this was about $10.7Hz$. This was the frequency at which we began to see Solitons form after moving them with the paddle. We then created more and then tuned the frequency to see where the solitons again began to decay which was at 12 Hz. The solitons we observed were self sustaining once driven properly.

V. ERROR ANALYSIS

For the Error we use equation 5 shown below.

$$\delta F = \sqrt{(\frac{dF}{dx})^2(\delta x)^2 + (\frac{dF}{dy})^2(\delta y)^2} \qquad [5]$$

In our experiment we have error noted already in the tables for the measured modes and the wave number, we also note the error for the lengths and the other factors we measured in the experiment in the Data section of our experiment. So to Go through the Error simply we look at the first 3 equations in our experiment. The first equation is the theoretical measurement without correction of the expected modes of the channel but begins from the angular frequency. To see the propagation of error we take the derivative of the depth of the channel and the length of the channel which comes from the wave number. The errors for them are both $\pm .1cm$ and we show the derivatives below and then a error calculation for the first equation for the 27 mode.

$$\frac{dF}{dL} = gtanh(kh) + (gk - gktanh^2(kh)) = 9.8Hz$$

$$\frac{dF}{dh} = gk - gktanh^2(kh) = .03Hz$$

$$\delta F = \sqrt{(\frac{dF}{dL})^2(\delta h)^2 + (\frac{dF}{dh})^2(\delta L)^2}$$

$$\delta F = \sqrt{(.03)^2(.001m)^2 + (9.8Hz)^2(.001m)^2} = .009Hz$$

The next equation we look at is the wave speed calculation. To calculate the error again we take derivatives but this time of the propagated error from equation 1 and the error of the length from the wave number. The calculations are shown below.

$$\frac{dC}{dL} = \frac{2\pi}{(\frac{27\pi}{L})} = .03m$$

$$\frac{dC}{df} = -\frac{2\pi f}{(\frac{27\pi}{L}^2} = .0009s$$

$$\delta C = \sqrt{(\frac{dC}{df})^2(\delta L)^2 + (\frac{dC}{dL})^2(\delta f)^2}$$

$$\delta C = \sqrt{(.0009)^2(.001)^2 + (.03)^2(.009)^2} = .0003\frac{m}{s}$$

The last equation we see error propagate in is the theoretical mode calculation. This error is the same as the first equation involving the length and the depth of water though the derivative yields a different result due to the factor of k^3. We look at the calculation of the error for the corrected modes shown below.

$$\frac{dF}{dL} = \frac{\pi(h(gk + \frac{k^3q}{p})Sech(hk) + (g + \frac{3k^2q}{p})Tanh(hk))}{4\sqrt{(gk + \frac{k^3q}{p})Tanh(hk)}}$$

$$\frac{dF}{dh} = \frac{k\pi(gk + \frac{k^3q}{p})Sech(hk)^2}{4\sqrt{(gk + \frac{k^3q}{p})Tanh(hk)}}$$

$$\delta F = \sqrt{(\frac{dF}{dL})^2(\delta h)^2 + (\frac{dF}{dh})^2(\delta L)^2}$$

$$\delta F = \sqrt{(.26)^2(.001)^2 + (.23)^2(.001)^2} = .0004Hz$$

Other error in this experiment was the error in the measured modes themselves,and the wave speeds we calculate from them but the calculation is the same as the error in equation 2, the error is shown in the data tables.

VI. CONCLUSION

In this experiment we were faced with many challenges though we moved past them all by careful analysis and checking our results thoroughly. In our experiment we sought to observe the normal modes of water waves in a glass channel, this meant we had to take into account all physical factors, which in this case, meant we had to make sure the glass had photoflo such that the water would not stick to the walls, we take into account the friction of the walls as the surface tension, the density of water and the length of the channel were also useful. In the end what we can conclude is that similarly to acoustical waves the geometrical shape and the materials themselves can work in tandem such that they create points of resonance, at particular modes. In a way these modes are a way of organizing the materials such that they are equidistant and oscillates in continuous loop, this can be achieved artificially, though exciting modes in a cavity they naturally form these patterns and habits and thus can be exploited. We exploited this when we observed the decay and generated solitons because we had such depth of knowledge of the channels geometrical and physical characteristics. In the end we conclude that the data we observed followed theory and sources of error can come from the fact that our driver was not a perfectly operating system and our glass channel may be contaminated overall and does not provide the best room for accurate measurement. In the end our results were highly accurate and provide a thorough analysis of what a water wave looks like physically and mathematically.