NYU CS Bridge to Tandon Course: Homework #6

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Problem 5

Use the definition of θ in order to show the following:

(a) $5n^3 + 2n^2 + 3n = \theta(n^3)$

$0f(n)$ = we will show that $5n^3 + 2n^2 + 3n = 0(n^3)$

Proof. $5n^3 + 2n^2 + 3n = 0(n^3)$ Let c = 10 and $n_0 = 1$, then for any $n \geq n_0$.

Since $5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3$.

$$\begin{aligned} 5n^3 + 2n^2 + 3n \leq &= 5n^3 + 2n^3 + 3n^3 \\ &= 10n^3 \\ c &= 10 \end{aligned}$$

■

Proof. $5n^3 + 2n^2 + 3n = \Omega(n^3)$ Let c = 5 and $n_0 = 1$, then for any $n \geq n_0$.

Since $5n^3 \leq 5n^3 + 2n^2 + 3n$.

$$\begin{aligned} 5n^3 \leq &= 5n^3 + 2n^2 + 3n \\ 5n^3 &= \\ cn^3 &= \end{aligned}$$

Since $5n^3 + 2n^2 + 3n = 0(n^3)$ and $5n^3 + 2n^2 + 3n = \Omega(n^3)$ we can conclude $5n^3 + 2n^2 + 3n = \theta(n^3)$. ■

Problem 5 continued on next page. . .

(b) $\sqrt{7n^2 + 2n - 8} = \theta(n)$

Proof. $\sqrt{7n^2 + 2n - 8} = 0(n)$ Let c = 3 and $n_0 = 1$, then for any $n \geq n_0$.
Since $\sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n^2}$.

$$\begin{aligned} \sqrt{7n^2 + 2n - 8} &\leq \sqrt{7n^2 + 2n^2} \\ &= \sqrt{9n^2} \\ c &= 3n \end{aligned}$$

∎

Proof. $\sqrt{7n^2 + 2n - 8} = \Omega(n)$ Let c = 2 and $n_0 = 1$, then for any $n \geq n_0$.
Since $\sqrt{7n^2} \leq \sqrt{7n^2 + 2n}$.

$$\begin{aligned} \sqrt{7n^2} &\leq \sqrt{7n^2 + 2n} \\ \sqrt{7} * \sqrt{n^2} &= \\ \lfloor 2.6 \rfloor * n &= \\ 2n &= \end{aligned}$$

Since $\sqrt{7n^2 + 2n - 8} = 0(n)$ and $\sqrt{7n^2 + 2n - 8} = \Omega(n)$ we can conclude $\sqrt{7n^2 + 2n - 8} = \theta(n)$. ∎