NYU CS Bridge to Tandon Course: Homework #7

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Problem 3

a. Exercise 8.2.2

Give complete proof for the growth rates of the polynomials below. You should provide specific value for C and n_0 and prove algebraically that the functions satisfy the definition for O and Ω

(b) $f(n) = n^3 + 3n^2 + 4$ Prove that $f = \theta(n^3)$

$0f(n)$ = we will show that $n^3 + 3n^2 + 4 \;\; f = O(n^3)$

Proof. $n^3 + 3n^2 + 4 = 0(n^3)$
Let c = 8 and n_0 = 1, then for any $n \geq n_0$.
Since $n^3 + 3n^2 + 4 \leq n^3 + 3n^3 + 4n^3$.

$$\begin{aligned} n^3 + 3n^2 + 4 \leq &= n^3 + 3n^3 + 4n^3 \\ &= 8n^3 \\ c &= 8 \end{aligned}$$

■

Proof. $n^3 + 3n^2 + 4 = 0(n^3) = \Omega(n^3)$ Let c = 1 and n_0 = 1, then for any $n \geq n_0$.
Since $n^3 \leq n^3 + 3n^2 + 4$.

$$\begin{aligned} n^3 \leq &= n^3 + 3n^2 + 4 \\ n^3 &= \\ 1 &= \end{aligned}$$

Since $n^3 + 3n^2 + 4 = 0(n^3)$ and $n^3 + 3n^2 + 4 = \Omega(n^3)$ we can conclude $5n^3 + 2n^2 + 3n = \theta(n^3)$. ■

b. Exercise 8.3.5

Worse Case time complexity - mystery AlgorithmDescribe in English how the sequence of numbers is changed by the algorithm. (Hint: try the algorithm out on a small list of positive and negative numbers with p = 0)?

Solution: *i* starts from the beginning of the sequence while j starts from the end of the sequence. they move towards the middle one by one until they meet. The first while loop find the element greater than or equal to *p* and the second while loop finds the Right most elements less than p. if two element of first inner loop p and second inner loop p exist they swap these elements. lastly outer while loop execute and return left sequence all less than *p* and right sequence all greater or equal to *p*

What is the total number of times that the lines "i := i + 1" or "j := j - 1" are executed on a sequence of length n? Does your answer depend on the actual values of the numbers in the sequence or just the length of the sequence? If so, describe the inputs that maximize and minimize the number of times the two lines are executed.

Solution: $n = n - 1$.

What is the total number of times that the swap operation is executed? Does your answer depend on the actual values of the numbers in the sequence or just the length of the sequence? If so, describe the inputs that maximize and minimize the number of times the swap is executed.)?

Solution: $\lfloor \frac{n}{2} \rfloor$: The sequence of length of n the swap operation execute $\lfloor \frac{n}{2} \rfloor$ depending on the actual value. It is value on left half of the length are greater than or equal to p and the right is less than p. when execution is minimized the when there is no value that you cannot swap example: left sequence < p and right sequence is >= p then it will execute 0 times.

Give an asymptotic lower bound for the time complexity of the algorithm. Is it important to consider the worst-case input in determining an asymptotic lower bound (using) on the time complexity of the algorithm? (Hint: argue that the number of swaps is at most the number of times that i is incremented or j is decremented)?

Solution: Two while loop will run $n - 1$ times and the swap operation will run $\lfloor \frac{n}{2} \rfloor$ times depending on the sequence. Thus the asymptotic lower bound is $\Omega(n)$

Give a matching upper bound (using O-notation) for the time complexity of the algorithm?

Solution: $n - 1 + \lfloor \frac{n}{2} \rfloor$: $O(n)$

Problem 4

a. Exercise 5.1.2

$$Digits = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$Letters = \{a, b, c, d, ...z\}$$
$$SpecialCharacters = \{*, \&, \$, \#\}$$

(b) String of Length 7,8,9 Character can be special characters, digit, or letter

Solution: $40^7 + 40^8 + 40^9$

(c) String of Length 7,8,9 Character can be special characters, digit, or letter. first character cannot be a letter

Solution: $14 * (40^6 + 40^7 + 40^8)$

b. Exercise 5.3.2

(a) How many strings are there over the set {a,b,c} that have length 10 in which no two consecutive character are the same?

Solution: $3 * 2^9 = 1536$.

c. Exercise 5.3.3 *License plate number in a certain state consist of seven Character. the first character is a digit(0 through 9). The next four characters are capital letters(A through Z) and the last tow character are digits. Therefore a license plate number in this state can be string of the form : DIGIT - LETTER - LETTER - LETTER - LETTER - DIGIT - DIGIT*

(b) How many license plate number are possible if no digit appear more than once?

Solution: $10 * 26^4 * 9 * 8$.

(c) No digit or letter appear more than once?

Solution: $10 * 9 * 8 * 26 * 25 * 24 * 23.$

a. Exercise 5.2.3 *Let B = {0, 1}. B^n is the set of binary strings with n bits. Define the set E_n to be the set of binary strings with n bits that have an even number of 1's. Note that zero is an even number, so a string with zero 1's (i.e., a string that is all 0's) has an even number of 1's.*

(a) Show a bijection between B^9 *and* E_{10} Explain why your function is bijection

Solution: $f : |B^9| \rightarrow |E_{10}|$ when $x \exists B^9$ x is counted by collecting number of 1's in the string. if number of 1's in B^9 is odd then we add 1 to the last bit of E_{10}. $B^9(000000001) \rightarrow E_{10}(0000000011)$ if 1's in B^9 is even add 0

to last bit of E_{10}. $B^9(000000011)$ to $E_{10}(0000000110)$. .

$$|E_{10}| = \frac{|B^9|}{2} * 2 = |B^9|$$
$$|B^9| = 2^9$$
$$|E_{10}| = \frac{2^9}{2} * 2 = 2^9$$

(b) What is $|E^{10}|$?

Solution: B^9 to E^{10} is a bijection, thus $|E^{10}| = |B^9| = 2^9$.

Problem 5

a. Exercise 5.4.2 *At a certain University in the U.S all phone number are 7 -digit long and start with either 825 or 825*

(a) How many different phone numbers are possible?

Solution: $2 * 10^4 = 20000$.

(b) How many different phone numbers are there in which the last four digits are all different?

Solution: $2 * P(10, 4) = 10,080.$

b. Exercise 5.5.3 *How many 10 - bit strings are there subject to each of the following restrictions?*

(a) No restrictions

Solution: $2^{10} = 1024$

(b) String starts with 001

Solution: $2^7 = 128$

(c) String starts with 001 or 10

Solution: $2^7 + 2^8 = 384$

(d) The first two bits are the same as the last two bits.

Solution: $2^2 * 2^6 = 2^8 = 256$

(e) String has exactly six 0's

Solution: $\binom{10}{4} = 210$

(f) String has exactly six 0's and first bit is 1

Solution: $\binom{9}{3} = 84$

(g) There is exactly one 1 in the first half and exactly three 1's in the second half

Solution: $\binom{5}{1}\ \binom{5}{3} = 50$

c. Exercise 5.5.5 *There are 30 boys and 35 girls that try out for a chorus. The choir director will select 10 girls and 10 boys from the children trying out. How many are there for choir director to make his selection?*

(a) 30 boys 10 selection. 35 girls 10 selection

Solution: $\binom{30}{10}\ \binom{35}{10} = 30045015 + 3247943160 = 3277988175.$

d. Exercise 5.5.8

(c) How many five -card hands are made entirely of heart and diamonds?

Solution: $\binom{26}{5}$ = (65780) 13 ranks in hearts and 13 ranks in diamond = 26 we then choose 5 cards from there .

(d) How many five - card hands have four cards of the same rank?

Solution: There are 13 cards in a rank so we can choose $\binom{13}{4}$ There are total of 52 card deck we can subtract 4 cards that we choose. and remaining cards will be = 48 and choose one card from there. = $\binom{13}{4} * \binom{48}{1}$ = 624.

(e) A "Full house" is a five card hand that has two cards of the same rank and three cards of the same rank. How many five - card hands contain full house?

Solution: $\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$ = 3744.

(f) How many Five - card hand do not have any two card of the same rank?

Solution: $\binom{13}{5} * 4^5$.

e. Exercise 5.6.6 *A country has two political parties, the Demonstrators and the Repudiators. Suppose that the national senate consists of 100 members, 44 of which are Demonstrators and 56 of which are Repudiators.*

(a) How many ways are there to select a committee of 10 senate members with the same number of Demonstrators and Repudiators?

Solution: $\binom{44}{5}$ $\binom{56}{5}$ = 1086008 + 3819816 = 4905824.

(b) Suppose that each party must select a speaker and a vice speaker. How many ways are there for the two speakers and two vice speakers to be selected?

Solution: $P(44, 2) * P(56, 2)$ = 44 * 43 * 56* 55 = 5827360.

Problem 6

a. Exercise 5.7.2
A 5 - card hand is drawn from a deck of standard playing card

(a) How many 5-card hands have at least one club?

Solution: $\binom{52}{5} - \binom{39}{5}$ we first count all the possible way from 52 cards. then count by complement there are $\binom{39}{5}$ possible card without club. then subtract all possible ways with possible way without club to get the answer .

(b) How many 5-card hands have at least two cards with the same rank?

Solution: $\binom{52}{5} - \binom{13}{5} * 4^5$

b. Exercise 5.8.4 *20 different comic books will be distributed to five kids.*

(a) How many ways are there to distribute the comic books if there are no restrictions on how many go to each kid (other than the fact that all 20 will be given out)?

Solution: 5^{20} = 9.5367432e+13.

(b) How many ways are there to distribute the comic books if they are divided evenly so that 4 go to each kid?

Solution: $\frac{20!}{4!4!4!4!4!}$.

Problem 7

How many one to one function are there from a set with five elements to sets with the following number of elements?

(a) 4

Solution: 0 Since target's cardinality is less than the domain it can not be one to one function.

(b) 5

Solution: $5! = 5 * 4 * 3 * 2 * 1 = 120$.

(c) 6

Solution: $P(6,5) = 6 * 5 * 4 * 3 * 2 = 720$.

(d) 7

Solution: $P(7,5) = 7 * 6 * 5 * 4 * 3 = 2520$.