NYU CS Bridge to Tandon Course: Homework #2

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Problem 5

1.12.2 *Solve the following questions from the Discrete Math zyBook.*

(b)

$$\begin{array}{l} p \to (q \land r) \\ \underline{\neg q \qquad\quad} \\ \therefore \neg q \end{array}$$

1	$\neg q$	Hypothesis
2	$\neg q \lor \neg r$	Addition 1
3	$\neg(q \land r)$	De Morgan's Law 2
4	$p \to (q \land r)$	Hypothesis
5	$\neg p$	Modus tollen 3,4

(e)

$$\begin{array}{l} p \lor q \\ \neg p \lor r \\ \underline{\neg q \qquad} \\ \therefore r \end{array}$$

1	$p \lor q$	Hypothesis
2	$\neg p \lor r$	Hypothesis
3	$q \lor r$	Resolution 1,2
4	$\neg q$	Hypothesis
5	r	Disjunctive Syllogism 3,4

Exercise 1.12.3

(c)

$$\begin{array}{l} p \lor q \\ \underline{\neg p \quad} \\ \therefore q \end{array}$$

1	$p \lor q$	Hypothesis
2	$\neg(\neg p) \lor q$	Double negation 1
3	$\neg p \to q$	Conditional Identity 2
4	$\neg p$	Hypothesis
5	q	Modus ponens 3,4

Problem 5 continued on next page. . .

Exercise 1.12.5

(c)

$$\begin{array}{l} \text{I will buy a new car and a new house only if I get a job} \\ \text{I am not going to get a job.} \\ \hline \therefore \text{ I will not buy a new car.} \end{array}$$

Solution:

- p: I will get a job
- q: I will buy a new car
- r: I will buy a new house

$$\begin{array}{rl} & (q \wedge r) \to p \\ & \neg p \\ \hline \therefore & \neg q \end{array}$$

The argument is **Invalid**. When $p = F$ and $q, r = T$, both of the hypotheses are True and the conclusion is False.

(d)

$$\begin{array}{rl} & \text{I will buy a new car and a new house only if I get a job} \\ & \text{I am not going to get a job.} \\ & \text{I will buy a new house} \\ \hline \therefore & \text{I will not buy a new car.} \end{array}$$

Solution:

- p: I will get a job
- q: I will buy a new car
- r: I will buy a new house

$$\begin{array}{rl} & (q \wedge r) \to p \\ & \neg p \\ & r \\ \hline \therefore & \neg q \end{array}$$

1	$(q \wedge r) \to p$	Hypothesis
2	$\neg p$	Hypothesis
3	$\neg(q \wedge r)$	Modules tollen 1,2
4	$\neg q \vee \neg r$	De Morgan's Law 3
5	$\neg r \vee \neg q$	Commutative law 4
6	r	Hypothesis
7	$\neg(\neg r)$	Double negation law 6
8	$\neg q$	Disjunctive syllogism 5,7

Thus, the argument is **Valid**.

Exercise 1.13.3

(b)

$$\begin{array}{rl} & \exists x(P(x) \vee Q(x)) \\ & \underline{\exists x \neg Q(x) \qquad\quad} \\ \therefore & \exists x P(x) \end{array}$$

Solution:

	P	Q
a	F	T
b	F	F

The hypothesis $\exists x(P(x) \vee Q(x))$ is True when $x = a$, and the hypotheses $(\exists x \neg Q(x))$ is True when $x = b$. But, the conclusion $\exists x P(x)$ is False.

Since both hypotheses are True but the conclusion is False, the argument is **Invalid**.

Exercise 1.13.5

(d)

Every Student who missed class got a detention
Penelope is a student in the class
Penelope did not miss class
∴ Penelope did not get a detention.

Solution:

- P(x): x missed a class
- Q(x): x got a detention

$$\begin{array}{rl} & \forall x(P(x) \to Q(x) \\ & \text{Penelope is a student in the class} \\ & \underline{\neg P(Penelope \qquad\qquad\qquad\quad} \\ \therefore & \neg Q(Penelope) \end{array}$$

The argument is **Invalid**. When $P(Penelope) = F$ and $Q(Penelope) = T$, both of the hypotheses are True but the conclusion is False.

(e)

Every Student who missed class or got a detention did not get an A
Penelope is a student in the class
Penelope got an a

$\therefore$ Penelope did not get a detention.

Solution:

- P(x): x missed a class
- Q(x): x got a detention
- R(x): x got an A

$$\forall x(P(x) \lor Q(x)) \to \neg R(x)$$
Penelope, student in the class
$$R(Penelope)$$

$$\therefore \neg Q(Penelope)$$

The argument is Valid.

1	$\forall x(P(x) \lor Q(x)) \to \neg R(x)$	Hypothesis
2	Penelope, student in the class	Hypothesis
3	$(P(Penelope) \lor Q(Penelope)) \to \neg R(Penelope)$	Universal Instantiation 1,2
4	$R(Penelope)$	Hypothesis
5	$\neg(\neg R(Penelope))$	Double Negation Law 4
6	$\neg(P(Penelope) \lor Q(Penelope))$	Modus tollens 3,5
7	$\neg P(Penelope) \land \neg Q(Penelope)$	De Morgan's Law 6
8	$\neg Q(Penelope)$	Simplification 7

Problem 6

Exercise 2.4.1 . *Prove each of the following statements using a direct proof.*

(d) *The product of two odd integer is an odd integer.*

Proof. Let x and y be two odd integers. We shall prove that xy is an odd integer.

Since x is odd, there is an integer m such that $x = 2m+1$. Since y is odd, there is an integer n such that $y = 2n+1$.

$$\begin{aligned} xy &= (2m+1)(2n+1) \\ &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1 \end{aligned}$$

Since m and n are integers, $2mn + m + n$ is also an integer.
As $xy = 2k + 1$, where $k = 2mn + m + n$ is an integer, xy is odd ■

Exercise 2.4.3 *Proving algebraic statements with direct proofs*

(b) *If x is a real number and* $x \leq 3$*, then* $12 - 7x + x^2 \geq 0$.

Proof. Let x be a real number and we shall prove $12 - 7x + x^2 \geq 0$.
$12 - 7x + x^2$ can be broken down into $(x-3)(x-4)$. This gives

$$(x-3)(x-4) \geq 0$$

Since $x \leq 3$, we can subtract 3 from both sides of the inequality to get

$$(x-3) \leq 0$$

Subtracting 1 from both sides of the inequality gives

$$(x-4) \leq -1$$

Since $(x-3)$ and $(x-4)$ are both negative real numbers, product of two negative numbers is at least greater than zero.

Therefore, $12 - 7x + x^2 \geq 0$ ■

Problem 7

Exercise 2.5.1 .

(d) For every integer n if $n^2 - 2n + 7$ is even then n is odd.

Proof. Assume n is an even integer and we will show $n^2 - 2n + 7$ is odd. So $n = 2k$ for some integer k

$$\begin{aligned} n^2 - 2n + 7 &= (2k)^2 - 2(2k) + 7 \\ &= 4k^2 - 4k + 7 \\ &= 2(2k^2 - 2k + 3) + 1 \end{aligned}$$

Since k is a integer, $2k^2 - 2k + 3$ is also an integer. Therefore, $n^2 - 2n + 7 = 2m + 1$, where $m = 2k^2 - 2k + 3$ is an integer. We can conclude that $n^2 - 2n + 7$ is an odd integer ∎

Exercise 2.5.4 .

(a) For every pair of real number x and y, if $x^3 + xy^2 \leq x^2y + y^3$ then $x \leq y$

Proof. Assume for every pair of real number x and y, and $x > y$ we will show that $x^3 + xy^2 > x^2y + y^3$

Since x and y are real numbers and $x > y$, either $xory \neq 0$. Thus, $x^2 + y^2 \neq 0$ and $x^2 + y^2 > 0$.

Multiply both side of the equation $x > y$ by $x^2 + y^2$ gives

$$\begin{aligned} &x * (x^2 + y^2) > y * (x^2 + y^2) \\ &x^3 + xy^2 > x^2y + y^3 \\ &\boxed{x^3 + xy^2 > x^2y + y^3} \end{aligned}$$

Since $x^2 + y^2$ must be positive, the inequality holds. ∎

(b) For every pair of real number x and y, if $x + y > 20$ then $x > 10$ or $y > 10$

Proof. Assume for every pair of real number x, y and $x \leq 10$, and $y \leq 10$. We will show that $x + y \leq 20$

We assign x and y to their maximum value and we get

$$\begin{aligned} x &= 10 \\ y &= 10 \end{aligned}$$

we can rewrite $x + y \leq 20$:

$$\begin{aligned} &x + y \leq 20 \\ &10 + 10 \leq 20 \\ &20 \leq 20 \end{aligned}$$

■

Exercise 2.5.5 .

(c) For every non -zero number x, if x is irrational, then $\frac{1}{x}$ is also irrational

Proof. Let x be non-zero real number.Assume $\frac{1}{x}$ is not an irrational number. we will show x is not irrational

Since every real number is either rational or irrational $\frac{1}{x}$ is a real number and not irrational then $x \neq 0$ so $\frac{1}{x}$ is rational

Thus $\frac{1}{x} = \frac{a}{b}$ where a and b is an integer and $b \neq 0$

Switching reciprocal on both side $x = \frac{b}{a}$ is also rational. ■

Problem 8

Exercise 2.6.6 .

(c) The average of three real number is greater than or equal to at least on of the numbers.

Proof. Let x, y, z be real number. Assume average of these three number is less than all of the three numbers

$$
\begin{aligned}
&x\frac{x+y+z}{3} < y\frac{x+y+z}{3} < z\frac{x+y+z}{3} < x+y+z \\
&\frac{3x+3y+3z}{3} < x+y+z \\
&Divide\ by\ 3\ =\ x+y+z < x+y+z \\
&= x+y+z < x+y+z
\end{aligned}
$$

Since sum of three number is not less then the sum. this contradict the assumption. The average of three real number is greater than or equal to at least one of the number. ■

(d) There is no smallest integer

Proof. Assume there is smallest integer x

when we subtract x by $1 = x - 1 < x$ This contradict assume that x is the smallest integer ■

Problem 9

Exercise 2.7.2 .

(b) If integers x and y have the same parity, then $x + y$ is even. The parity of a number tells whether the number is odd or even. If x and y have the same parity they are either both even or both odd

Proof. Proof by case
Case 1: x and y are both even. Thus $x = 2m$ and $y = 2n$ for some integers of m and n

We have : $x + y = 2m + 2n = 2(m + n)$

Since m and n are both integer, $m + n$ is also an integer. And $x + y$ can be expressed as 2 times an integer, which makes $x + y$ even

Case 2: x and y are both odd. Thus $x = 2m + 1$ and $y = 2n + 1$ for some integers of m and n

We have : $x + y = (2m + 1) + (2n + 1) = 2(m + n + 1)$

Since m and n are both integers, $m + n + 1$ is also an integer. And $x + y$ can be expressed as 2 times an integer, which makes $x + y$ even ■