NYU CS Bridge to Tandon Course: Homework #3

Winter 2022

Jae Butler **Jiyoon Jung**

Problem 7

a. Exercise 3.1.1 *Use the definitions for the sets given below to determine whether each statement is true of false.*

$$A = \{x \in \mathbb{Z} : \text{x is an integer multiple of 3}\}$$
$$B = \{x \in \mathbb{Z} : \text{x is a perfect square}\}$$
$$C = \{4, 5, 9, 10\}$$
$$D = \{2, 4, 11, 14\}$$
$$E = \{3, 6, 9\}$$
$$F = \{4, 6, 16\}$$

(a) $27 \in A$

True: 27 is a multiple of 3, $9 * 3 = 27$.

(b) $27 \in B$

False: There is no integer y such $y^2 = 27$.

(c) $100 \in B$

True: 10^2 is equal to 100.

(d) $E \subseteq C$ or $C \subseteq E$

False: $3 \in E$ but $3 \notin C$, and $10 \in C$ but $10 \notin E$

(e) $E \subseteq A$

True: 3, 6, 9 is Multiple of 3.

(f) $A \subset E$

False: $12 \notin E$ $15 \notin E$.... .

(g) $E \in A$

False: E {3 ,6, 9} is not an element of A

Problem 7 continued on next page. . .

b. Exercise 3.1.2 *Use the definitions for the sets given below to determine whether each statement is true of false.*

$$A = \{x \in \mathbb{Z} : \text{x is an integer multiple of 3 }\}$$
$$B = \{x \in \mathbb{Z} : \text{x is a perfect square}\}$$
$$C = \{4, 5, 9, 10\}$$
$$D = \{2, 4, 11, 14\}$$
$$E = \{3, 6, 9\}$$
$$F = \{4, 6, 16\}$$

(a) $15 \subset A$

False: 15 is an element not a set.

(b) $\{15\} \subset A$

True: 15 is an integer multiple of 3. $3 \in A$ but $3 \notin \{15\}$

(c) $\varnothing \subset A$

True

(d) $A \subseteq A$

True Since they share same sets

(e) $\varnothing \in B$

False: $\varnothing$ is a set while set B contains integer elements.

c. Exercise 3.1.5 *Express each set using set builder notation. then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite*

(b) {3 ,6 ,9 ,12 ,.....}

Let A = {3 ,6 ,9 ,12 ,.....}

A = $\{x \in \mathbb{Z} :$ x is an integer multiple of 3 and $x \geq 3\}$ The set is infinite.

(d) {0 ,10 ,20, 30....., 1000 }

Let B = {0 ,10 ,20, 30....., 1000 }

B = $\{x \in \mathbb{N} :$ x is an integer multiple of 10 and $0 \leq x \leq 1000$ } The set is finite, hence |B| = 101

Problem 7 continued on next page. . .

d. Exercise 3.2.1 *Sets of Sets - True or False.*

Let X = $\{1, \{1\}, \{1, 2\}, 2, \{3\}, 4\}$

(a) $2 \in X$

$\boxed{\textbf{True}}$

(b) $\{2\} \subseteq X$

$\boxed{\textbf{True}}$

(c) $\{2\} \in X$

$\boxed{\textbf{False}}$

(d) $3 \in X$

$\boxed{\textbf{False}}$

(e) $\{1, 2\} \in X$

$\boxed{\textbf{True}}$

(f) $\{1, 2\} \subseteq X$

$\boxed{\textbf{True}}$

(g) $\{2, 4\} \subseteq X$

$\boxed{\textbf{True}}$

(h) $\{2, 4\} \in X$

$\boxed{\textbf{False}}$

(i) $\{2, 3\} \subseteq X$

$\boxed{\textbf{False}}$

(j) $\{2, 3\} \in X$

$\boxed{\textbf{False}}$

(k) $|X| = 7$

$\boxed{\textbf{False}\text{, Cardinality is 6}}$

Problem 8

Exercise 3.2.4 . *A subset of a power Set.*

(b) Let A = $\{1, 2, 3\}$ What is $\{X \in P(A) : 2 \in X\}$

size 0	$\emptyset$		
size 1	$\{1\}$	$\{2\}$	$\{3\}$
size 2	$\{1, 2\}$	$\{1, 3\}$	$\{2, 3\}$
size 3	$\{1, 2, 3\}$		

Let B = $\{X \in P(A) : 2 \in X\}$
$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$
$\boxed{B = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}}$

Problem 9

a. Exercise 3.3.1 *Define Sets A,B,C and D as follow.*

$$A = \{-3, 0, 1, 4, 17\}$$
$$B = \{-12, -5, 1, 4, 6\}$$
$$C = \{x \in \mathbb{Z} : \text{x is odd}\}$$
$$D = \{x \in \mathbb{Z} : \text{x is positive}\}$$

(c) $A \cap C$

Answer: {-3, 1, 17}

(d) $A \cup (B \cap C)$

Answer: {-5, -3, 0, 1, 4, 17}

(e) $A \cap B \cap C$

Answer: {1}

b. Exercise 3.3.3 *Unions and intersections of sequences of sets, part 2*

$$A_i = \{i^0, i^1, i^2\}$$
$$B_i = \{x \in \mathbb{R} : -i \leq x1/i\}$$
$$C_i = \{x \in \mathbb{R} : -1/i \leq x \leq 1/i\}$$

(a) $\bigcap_{i=2}^{5} A_i$

$$\begin{aligned}\bigcap_{i=2}^{5} A_i &= A_2 \cap A_3 \cap A_4 \cap A_5 \\ &= \{2^0, 2^1, 2^2\} \cap \{3^0, 3^1, 3^2\} \cap \{4^0, 4^1, 4^2\} \cap \{5^0, 5^1, 5^2\} \\ &= \{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\} \\ &= \boxed{\mathbf{\{1\}}}\end{aligned}$$

(b) $\bigcup_{i=2}^{5} A_i$

$$\begin{aligned}\bigcup_{i=2}^{5} A_i &= A_2 \cup A_3 \cup A_4 \cup A_5 \\ &= \{2^0, 2^1, 2^2\} \cup \{3^0, 3^1, 3^2\} \cup \{4^0, 4^1, 4^2\} \cup \{5^0, 5^1, 5^2\} \\ &= \{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\} \\ &= \boxed{\mathbf{\{1, 2, 3, 4, 5, 9, 16, 25\}}}\end{aligned}$$

(e) $\bigcap_{i=1}^{100} C_i$

$$\begin{aligned}\bigcap_{i=1}^{100} C_i &= \{-1 \le x \le 1\} \cap \{-\frac{1}{2} \le x \le \frac{1}{2}\} \cdots \cap \{-\frac{1}{100} \le x \le \frac{1}{100}\} \\ &= \boxed{\{\mathbf{x} \in \mathbb{R} : -\mathbf{\tfrac{1}{100}} \le \mathbf{x} \le \mathbf{\tfrac{1}{100}}\}} \\ &\because -\frac{1}{100} \le x \le \frac{1}{100} \text{ is the range of x that is part of all } C_1, C_2, C_3 \ldots C_{100}\end{aligned}$$

(f) $\bigcup_{i=1}^{100} C_i$

$$\begin{aligned}\bigcup_{i=1}^{100} C_i &= \{-1 \le x \le 1\} \cup \{-\frac{1}{2} \le x \le \frac{1}{2}\} \cdots \cup \{-\frac{1}{100} \le x \le \frac{1}{100}\} \\ &= \boxed{\{\mathbf{x} \in \mathbb{R} : \mathbf{-1} \le \mathbf{x} \le \mathbf{1}\}} \\ &\because -1 \le x \le 1 \text{ is the range of x that entails all } C_1, C_2, C_3 \ldots C_{100}\end{aligned}$$

c. Exercise 3.3.4 *Power sets and set operations..*

- $A = \{a, b\}$
- $B = \{b, c\}$

(b) $P(A \cup B)$

$$\begin{aligned}A \cup B &= \{a, b\} \cup \{b, c\} \\ &= \{a, b, c\} \\ P(A \cup B) &= \boxed{\{\varnothing, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{c}\}, \{\mathbf{a}, \mathbf{b}\}, \{\mathbf{a}, \mathbf{c}\}, \{\mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}\}}\end{aligned}$$

(d) $P(A) \cup P(B)$

$$\begin{aligned}P(A) &= \{\varnothing, \{a\}, \{b\}, \{a, b\}\} \\ P(B) &= \{\varnothing, \{b\}, \{c\}, \{b, c\}\} \\ P(A) \cup P(B) &= \boxed{\{\varnothing, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{c}\}, \{\mathbf{a}, \mathbf{b}\}, \{\mathbf{b}, \mathbf{c}\}\}}\end{aligned}$$

Problem 10

a. Exercise 3.5.1 *Cartesian product of three small sets*

- A = { tall, grande, venti}
- B = { foam, no-foam}
- C = { non-fat, whole}

(b) Write an Element from the set B x A x C

B x A x C = {foam, tall, non-fat}

(c) Write the set B x C using roster notation.

B x C ={foam, non-fat}, {foam, whole}, {no-foam, non-fat},{no-foam, whole}

b. Exercise 3.5.3 *Indicate which of the following statements are true.*

(b) $\mathbb{Z}^2 \subseteq \mathbb{R}^2$

True: if (x,y) $\in \mathbb{Z}^2$ are both element of $\mathbb{Z}$, since $\mathbb{Z} \subseteq \mathbb{R}$ then x and y are both element of $\mathbb{R}$

(c) $\mathbb{Z}^2 \cap \mathbb{Z}^3 = \varnothing$

False: The elements in $\mathbb{Z}^2$ are pairs but elements in $\mathbb{Z}^3$ are triplet. Thus the two set have no element in common

(e) For any three sets, A, B, and C if $A \subseteq B$ then $A \times C \subseteq B \times C$

True: if $x \in A \times C$ then $x \in A$ and $y \in C$. Since $A \subseteq B$, x also exists in B

c. Exercise 3.5.6 *Express the following sets using the roster method. Express the element as strings, not n -tuples.*

(d) $\{xy : where\ x \in \{0\} \cup \{0\}^2\ and\ y \in \{1\} \cup \{1\}^2\}$

$$LetA = \{xy : where\ x \in \{0\} \cup \{0\}^2\ and\ y \in \{1\} \cup \{1\}^2\}$$
$$x \in \{0, 00\}$$
$$y \in \{1, 11\}$$
$$A = \boxed{\{\mathbf{01}, \mathbf{011}, \mathbf{001}, \mathbf{0011}\}}$$

(e) $\{xy : x \in \{aa, ab\}\ and\ y \in \{a\} \cup \{a\}^2\}$

$$LetA = \{xy : where\ x \in \{aa, ab\}\ and\ y \in \{a\} \cup \{a\}^2\}$$
$$y \in \{a, aa\}$$
$$A = \boxed{\textbf{\{aaa, aaaa, aba, abaa\}}}$$

d. Exercise 3.5.7 *Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as String.*

- $A = \{a\}$
- $B = \{b, c\}$
- $C = \{a, b, d\}$

(c) $(A \times B) \cup (A \times C)$

$$\begin{aligned}(A \times B) &= \{ab, ac\} \\ (A \times C) &= \{aa, ab, ad\} \\ (A \times B) \cup (A \times C) &= \{ab, ac\} \cup \{aa, ab, ad\} \\ &= \boxed{\{\mathbf{ab, ac, aa, ad}\}}\end{aligned}$$

(f) $P(A \times B)$

$$\begin{aligned}(A \times B) &= \{ab, ac\} \\ P(A \times B) &= \boxed{\{\emptyset, \{\mathbf{ab}\}, \{\mathbf{ac}\}, \{\mathbf{ab}, \mathbf{ac}\}\}}\end{aligned}$$

(g) $P(A) \times P(B)$ use ordered pair notation for elements of the Cartesian product

$$\begin{aligned}P(A) &= \{\emptyset, \{a\}\} \\ P(B) &= \{\emptyset, \{b\}, \{c\}, \{b, c\}\} \\ P(A) \times P(B) &= \boxed{(\emptyset, \emptyset), (\emptyset, \{\mathbf{b}\}), (\emptyset, \{\mathbf{c}\}), (\emptyset, \{\mathbf{b}, \mathbf{c}\}), (\{\mathbf{a}\}, \emptyset), (\{\mathbf{a}\}, \{\mathbf{b}\}), (\{\mathbf{a}\}, \{\mathbf{c}\}), (\{\mathbf{a}, \mathbf{b}, \mathbf{c}\})\}}\end{aligned}$$

Problem 11

Exercise 3.6.2 .*Proving set identities.*

(b) $(B \cup A) \cap (\overline{B} \cup A) = \mathrm{A}$

1	$(B \cup A) \cap (\overline{B} \cup A) = \mathrm{A}$	
2	$(B \cap \overline{B}) \cup A$	Distributive Law
3	$\varnothing \cup A$	Complement law
4	A	Identity law

(c) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

1	$\overline{A \cap \overline{B}} = \overline{A} \cup B$	
2	$\overline{A} \cup \overline{\overline{B}}$	De Morgan's law
3	$\overline{A} \cup B$	Double Negation Law

b. Exercise 3.6.3 *Showing set equation that are not identities.*

(b) $A - (B \cap A) = A$

Solution: A = {1,2,3} and B = {2,3} then$A - (B \cap A) = A$= {1}

(d) $(B - A) \cup A = A$

Solution: B = {1,2} and A = {2,3} then $(B - A) \cup A$ = {1,2,3}

c.Exercise 3.6.4 .*Proving set identities with the set difference operation.*

(b) $A \cap (B - A) = \varnothing$

1	$A \cap (B - A) = \varnothing$	
2	$A \cap (B \cap \overline{A})$	Sub Subtraction Law
3	$A \cap (\overline{A} \cap B)$	Commutative laws
4	$(A \cap \overline{A}) \cap B$	Associative law
5	$(\varnothing \cap B)$	Complement laws
6	$(\varnothing)$	Domination Laws

(c) $A \cup (B - A) =$ A $\cup B$

1	$A \cup (B - A) =$ A $\cup B$	
2	$A \cup (B \cap \overline{A})$	Sub Subtraction Law
3	$(A \cup B) \cap (A \cup \overline{A})$	Distributive law
4	$(A \cup B) \cap U$	Complement Law
5	$(A \cup B)$	Identity Law