introduction to

PROBABILITY

Bur Oak Math Club - Oct 23 2024 - Grade 9/10

About Probability

Probability is a branch of mathematics that studies **how likely an event is to occur.** This is calculated through the following formula:

$$P(A) = \frac{n}{N} = \frac{\#\,outcomes\ in\ A}{\#\,outcomes\ in\ Sample\ Space}$$

where a **probability of 0** shows the event is **impossible** and a **probability of 1** shows the event is **certain.**

P(A) is used to represent the probability of **A** happening.

Important Terms

Independent events refer to events where one action **does not affect** the chances of another action. (eg. If I roll a die twice, the outcome of the first roll does not affect the outcome of the second)

Dependent events occur when the outcome of one event **affects** the probability of another event. (eg. If I draw two cards without replacement, the result of the first will affect the result of the second).

Mutually exclusive events are events that cannot occur at the same time. (eg. I cannot roll a 5 and an even number on one die at the same time).

Formulas:

$P(A \cap B)$:

Probability of A **AND** B occurring

$P(A \cup B)$:

Probability of A **OR** B occurring

$$P(A) = \frac{\text{number of favourable events}}{\text{number of total events}}$$

$$P(A) = \frac{n(A)}{n}$$

$$P(B) = \frac{n(B)}{n}$$

$$P(A \cap B) = P(A)\,P(B)$$

<u>for Mutually Exclusive Events</u>

$$P(A \cup B) = P(A) + P(B)$$

<u>for non-Mutual Events</u>

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Formulas:

Permutation

- Permutation is the arrangement of items in which **order matters**
- Number of ways of **selection and arrangement of items** in which Order Matters

$$\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$$

Combination

- Combination is the selection of items in which **order does not matters** .
- Number of ways of **selection of items** in which Order does not Matters

$$\boxed{{}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}}$$

Past UWaterloo

Problems of the Week

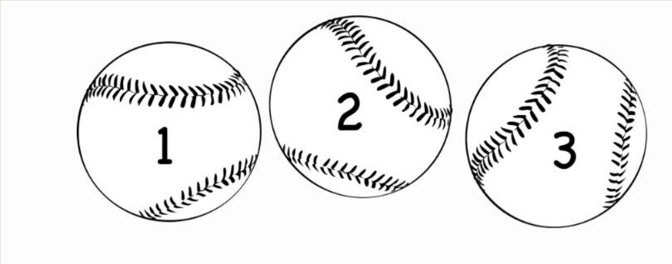
Data Management

Bur Oak Math Club - **Oct 23 2024** - **Grade 9/10**

Week One Problem D

Ivy has created a game for her school's math fair. She put three baseballs, numbered 1, 2, and 3, into a bag. Without looking, a player will randomly draw a baseball from the bag, record its number, and then put the baseball back into the bag. They will do this two more times and then calculate the sum of the three numbers recorded. If the sum is less than 8, the player will win a prize.

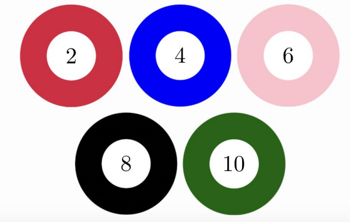
What is the probability that a player will win a prize when they play this game once?



Week Twenty Problem D

Five balls are placed in a bag. Each ball is labelled with a 2, 4, 6, 8, or 10, with no ball having the same label as any other. Adeleke randomly chooses a ball, records the integer on the ball, and replaces the ball into the bag. Then Bo randomly chooses a ball, records the integer on the ball, and replaces the ball into the bag. Finally, Carlos randomly chooses a ball, records the integer on the ball, and replaces the ball into the bag.

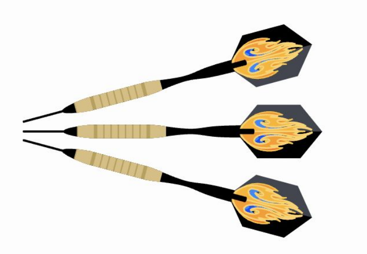
Determine the probability that the product of the three recorded integers is not a power of 2.



Week Seven Problem D

Kurtis is creating a game for a math fair. They attach n circles, each with radius 1 metre, onto a square wall with side length n metres, where n is a positive integer, so that none of the circles overlap. Participants will throw a dart at the wall and if the dart lands on a circle, they win a prize. Kurtis wants the probability of winning the game to be at least $\frac{1}{2}$.

If they assume that each dart hits the wall at a single random point, then what is the largest possible value of n?

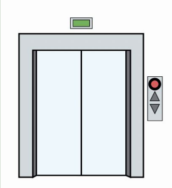


Week Twenty-Six Problem D

There are six people in an elevator. The sum of all six of their ages is 190 and the median age is 22. From youngest to oldest, the names of the people in the elevator are Ashish, Brook, Calista, Dipak, Enid, and Freyja.

The elevator stops and Ashish and Enid get off. The mean (average) age of the remaining four people in the elevator is then 30. The elevator then stops again and Brook and Calista get off. The mean age of the remaining two people in the elevator is then 40.

If Ashish is 18 years old, and each person's age is a different positive integer, how old is Freyja?



Past UWaterloo

CIMC Problems

Probability

Bur Oak Math Club - **Oct 23 2024** - **Grade 9/10**

CIMC 2021 PART A Q6

6. Dragomir has 6 pairs of socks in a drawer. Each pair of socks is different from every other pair in the drawer. He randomly removes 4 individual socks from the drawer, one at a time. What is the probability that there is exactly 1 matching pair of socks among these 4 socks?

CIMC 2018 PART B Q2

2. A bag contains n balls numbered from 1 to n, with $n \geq 2$. There are $n \times (n-1)$ ways in which Julio can remove one ball from the bag and then remove a second ball. This is because there are n possible choices for the first ball and then $n-1$ possible choices for the second ball. For example, when a bag contains 6 balls numbered from 1 to 6, 4 of which are black and 2 of which are gold, there are $6 \times 5 = 30$ ways in which he can remove two balls in this way, and $4 \times 3 = 12$ ways in which both balls are black.

   a. A bag contains 11 balls numbered from 1 to 11, 7 of which are black and 4 of which are gold. Julio removes two balls as described above. What is the probability that both balls are black?

   b. For some integer $g \geq 2$, a second bag contains 6 black balls and g gold balls; the balls are numbered from 1 to $g+6$. Julio removes two balls as described above. The probability that both balls are black is $\frac{1}{8}$. Determine the value of g.

   c. For some integer $x \geq 2$, a third bag contains $2x$ black balls and x gold balls; the balls are numbered from 1 to $3x$. Julio removes two balls as described above. The probability that both balls are black is $\frac{7}{16}$. Determine the value of x.

   d. For some integer $r \geq 3$, a fourth bag contains 10 black balls, 18 gold balls, and r red balls; the balls are numbered from 1 to $r+28$. This time, Julio draws three balls one after another. The probability that two of these three balls are black and one of these three balls is gold is at least $\frac{1}{3000}$. What is the largest possible value of r?

CIMC 2022 PART B Q3

3. A straight path is 2 km in length. Beatrice walks from the beginning of the path to the end of the path at a constant speed of 5 km/h. Hieu cycles from the beginning of the path to the end of the path at a constant speed of 15 km/h.

b. Suppose that Beatrice starts at a time that is exactly b minutes after 9:00 a.m. and that Hieu starts at a time that is exactly h minutes after 9:00 a.m., where b and h are integers from 0 to 59, inclusive, that are each randomly and independently chosen.

i. Determine the probability that there is a time at which Beatrice and Hieu are at the same place on the path. (Beatrice and Hieu are at the same place at a given time if they start at the same time or finish at the same time, or are at the same place at some point between the beginning and end of the path at the same time.)

ii. One day, Beatrice uses a scooter and travels from the beginning of the path to the end of the path at a constant speed of x km/h, where $x > 5$ and $x < 15$. Hieu still cycles from the beginning of the path to the end of the path at a constant speed of 15 km/h. If the probability that there is a time at which Beatrice and Hieu are at the same place on the path is $\frac{13}{200}$, determine the range of possible values of x.