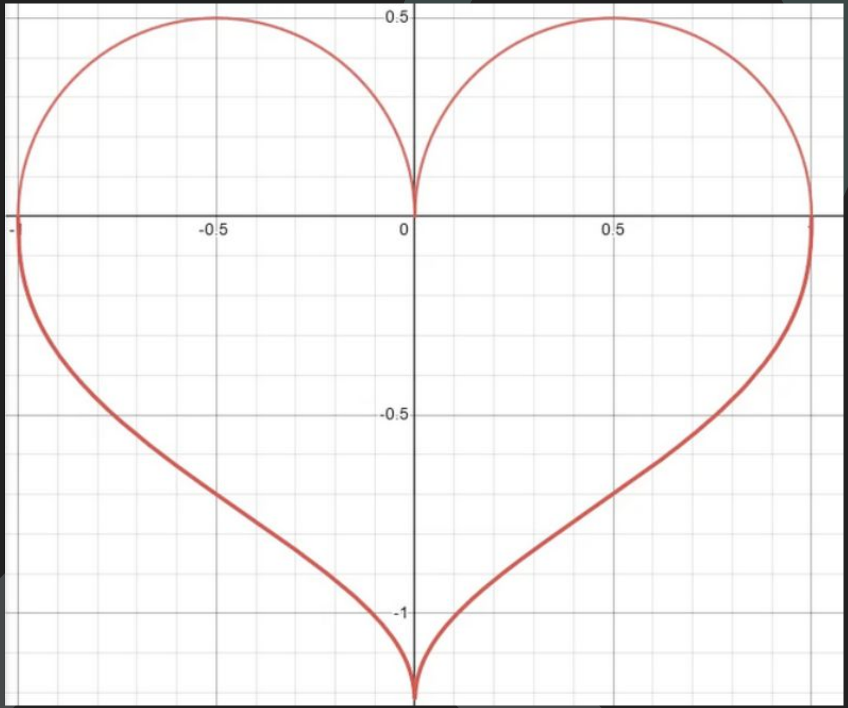
Analytic Geometry

by Mr. Muzsi’s two favourite students :)

Introduction

What is Analytical Geometry

- A type of geometry that involves coordinates!
- Supposed to learn it in Grade 9 and 10, but it got taken out
- We gotchu! B)

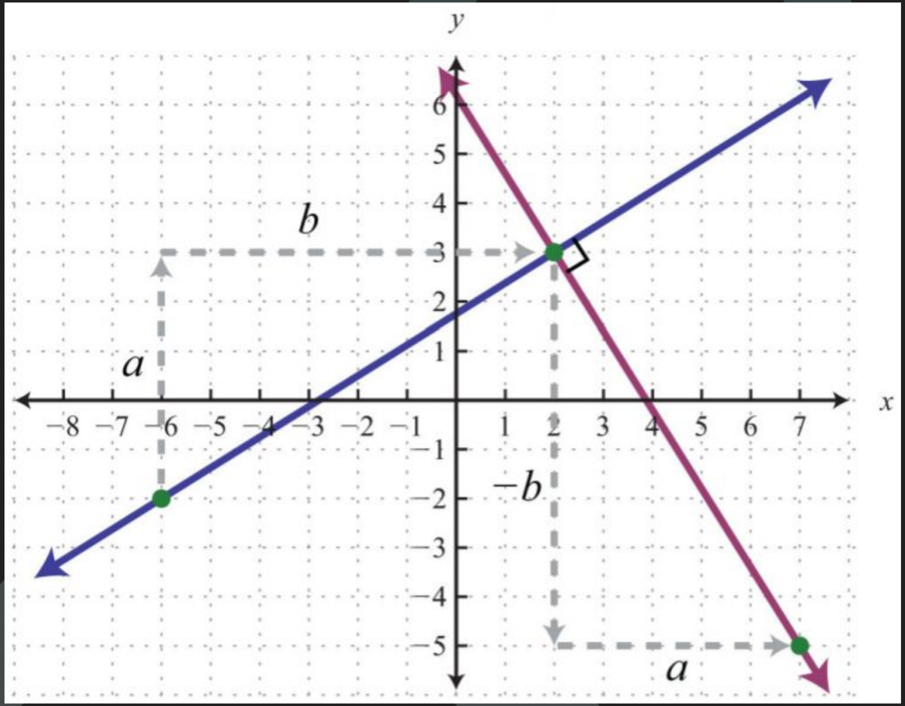


Description	Formula
Slope of a line through points (x_1, y_1) and (x_2, y_2)	$\dfrac{y_2 - y_1}{x_2 - x_1}$
Two perpendicular lines with slopes m_1 and m_2	$m_2 = -\dfrac{1}{m_1}$
Standard form of an equation of a line with slope $-\dfrac{A}{B}$, x-intercept $-\dfrac{C}{A}$, and y-intercept $-\dfrac{C}{B}$	$Ax + By + C = 0$
Equation of a line with slope m through the point (x_0,y_0)	$y - y_0 = m(x - x_0)$
Equation of a line with intercepts at $(a, 0)$ and $(0, b)$	$\dfrac{x}{a} + \dfrac{y}{b} = 1$
Formula for the midpoint of $A(x_1,y_1)$ and $B(x_2,y_2)$	$\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
Distance D between points $A(x_1,y_1)$ and $B(x_2,y_2)$	$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
(Minimum) distance D between the line $Ax + By + C = 0$ and the point (x_0,y_0)	$D = \dfrac{\lvert Ax_0 + By_0 + C\rvert}{\sqrt{A^2 + B^2}}$
Area of a triangle with vertices $A(x_1,y_1)$, $B(x_2,y_2)$, and $C(x_3,y_3)$	$\dfrac{1}{2}\lvert x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3\rvert$
Equation of a circle centred at (h,k) with radius r	$(x - h)^2 + (y - k)^2 = r^2$

Slope of a Perpendicular Line

- Given a line with slope m, a line perpendicular to it will have a slope of -1/m

- $m_2 = -1/m_1$ where m_1 and m_2 are slopes of perpendicular lines

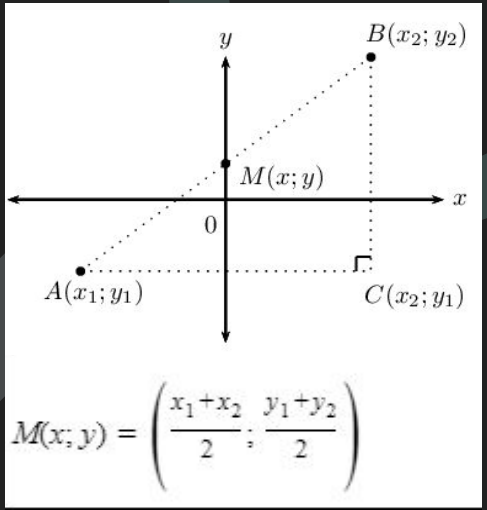


Midpoint Formula

- $M(x,y) = ((x_1+x_2)/2 , (y_1+y_2)/2)$

- This formula finds the midpoint of a line segment

- Idea is to find the midpoint with respect to the x-axis and midpoint with respect to the y-axis

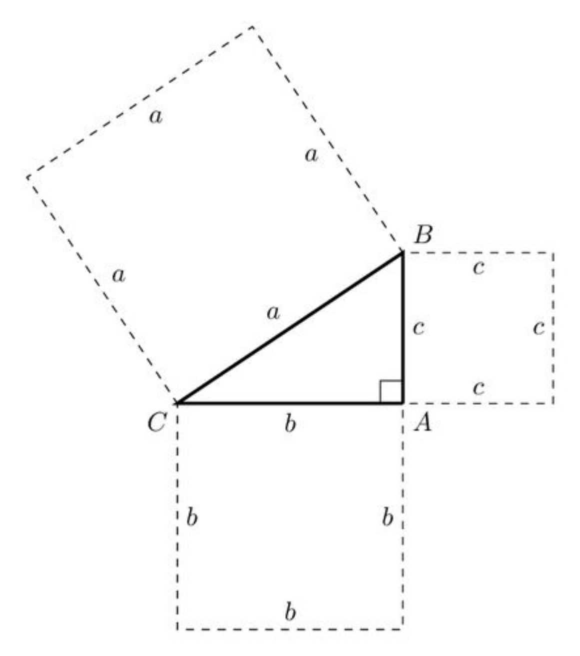


Pythagorean Theorem

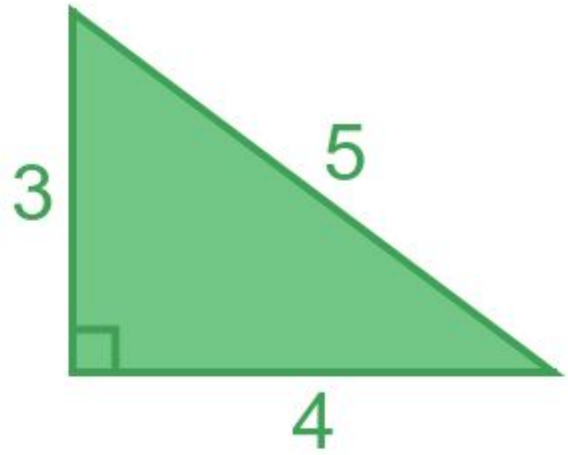
- A popular analytical geometry formula

- $\mathbf{A^2 + B^2 = C^2}$

- Applies to all right triangles



Example

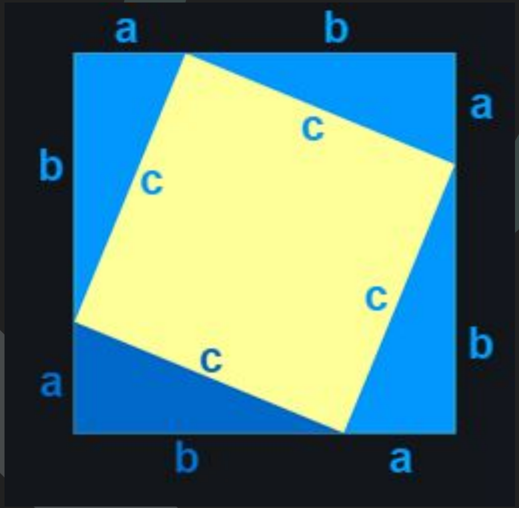


$$5^2 = 3^2 + 4^2$$

$$25 = 9 + 16$$

Proof of Pythagorean Theorem

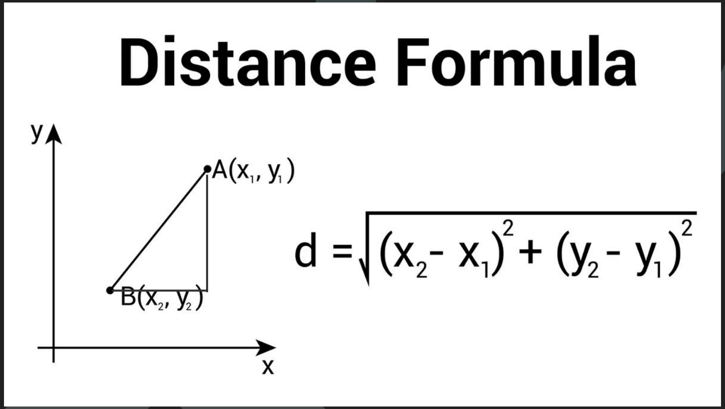
- A smaller square with side length c is inscribed in a larger square with side length a + b so that it forms 4 right angle triangles
- Total $\text{Area}_{\text{Four Triangles}} = 4(½\ ab) = 2ab$
- $\text{Area}_{\text{Small Square}} = c^2$
- $\text{Area}_{\text{Large Square}} = (a+b)(a+b) = 2ab + c^2$
- $a^2 + 2ab + b^2 = 2ab + c^2$
- $a^2 + b^2 = c^2$ (proof)



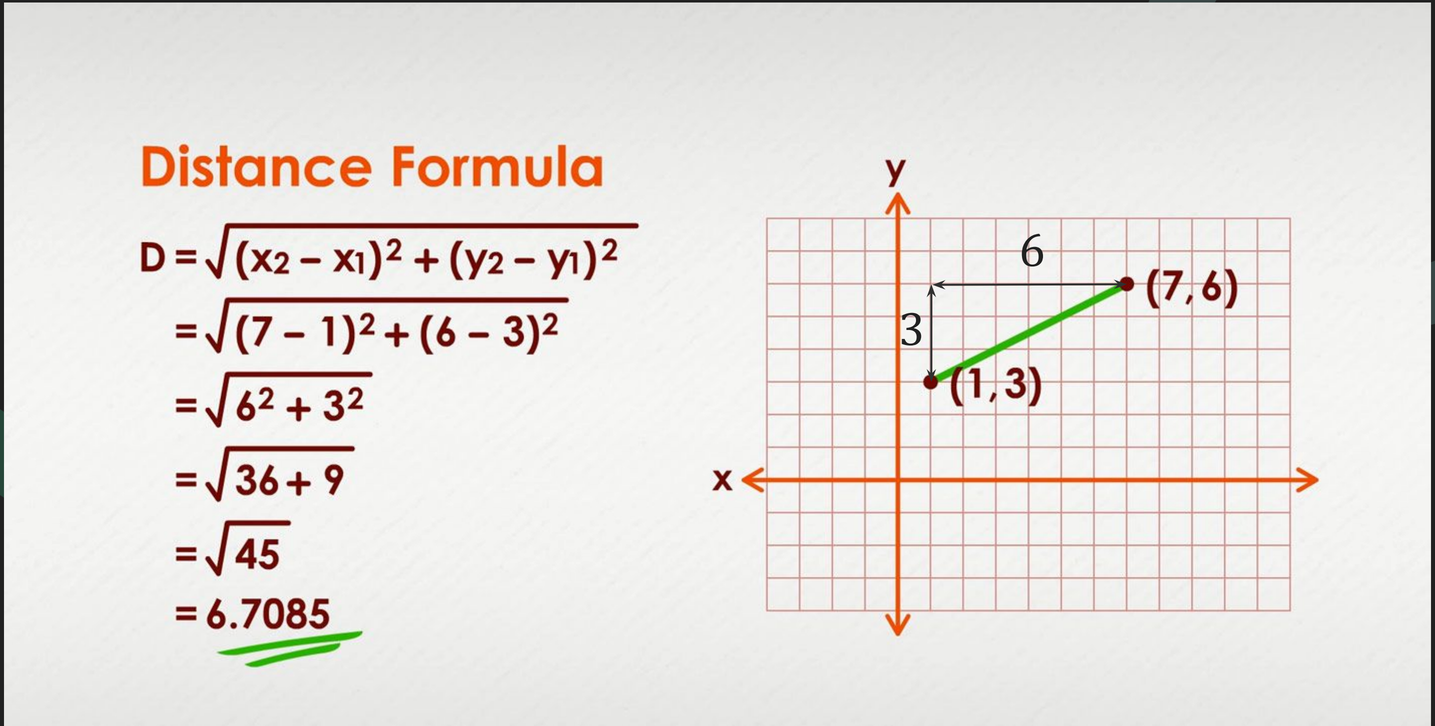
Distance Formula

- $\Delta d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

- Idea is to build a right triangle and use pythagorean theorem to find our hypotenuse (our distance)



Example


Distance Formula
$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(7 - 1)^2 + (6 - 3)^2}$
$= \sqrt{6^2 + 3^2}$
$= \sqrt{36 + 9}$
$= \sqrt{45}$
$= 6.7085$
y
x
6
3
(7, 6)
(1, 3)

Equation of a circle

- The equation of a circle is as follows:
  $(x - h)^2 + (y - k)^2 = r^2$
  - Horizontally shifted h units, vertically shifted k units → centre at (h,k)
  - Circle has radius r

Sample Problems

Example 1

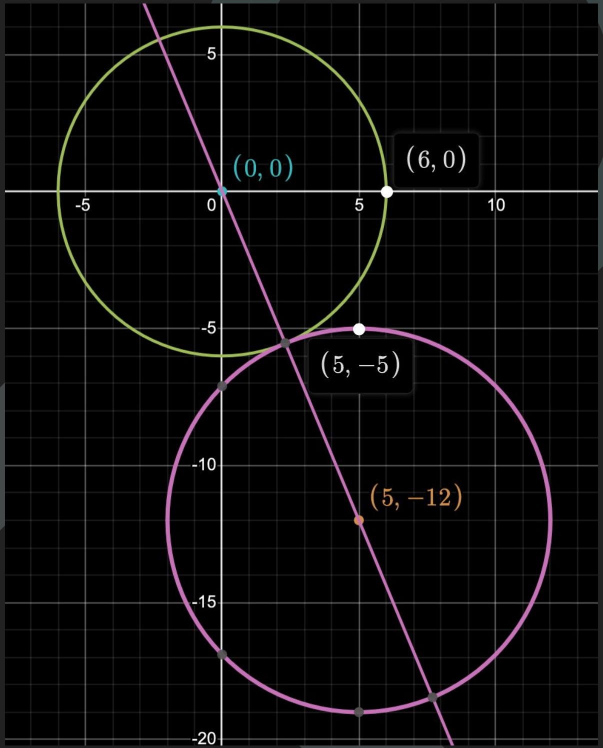
Find all value(s) of k so that the circle with equation $x^2 + y^2 = k^2$ will intersect the circle with equation $(x - 5)^2 + (y + 12)^2 = 49$ in exactly one point.

Solution

Second circle is shifted 5 units right and 12 units down → √(5^2+12^2) = 13 → circle is 13 units away from origin

Second circle has radius 7

First circle has centre at origin → 13-7=6 → k = 6



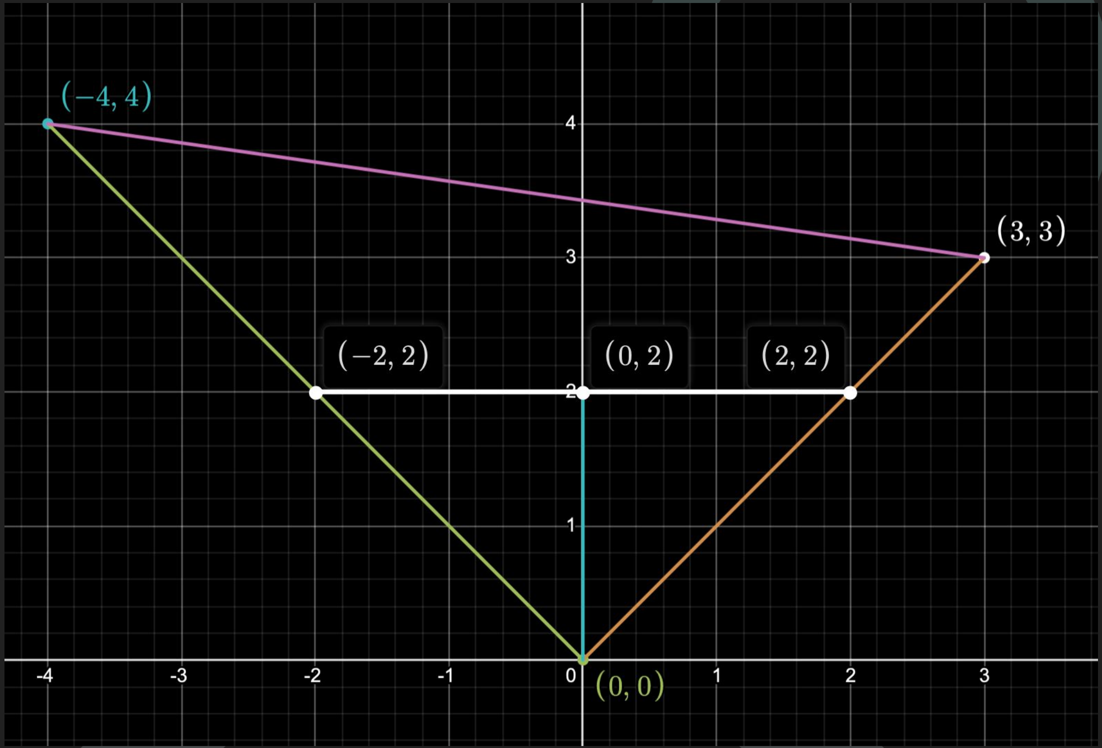
Example 2

If triangle ABC has vertices $A(0,0)$, $B(3,3)$, and $C(-4,4)$, determine the equation of the bisector of $\angle CAB$.

Solution

$y = 2 \rightarrow m = 0$

Perpendicular line is $x = 0$



Example 3

Find the equation of the set of points equidistant from C(0,3) and D(6,0)

Solution

$y = m_1x + b_1$

$m_1 = (0\text{-}3)/(6\text{-}0)$

$b_1 = 3$

$y = \text{-}0.5x+3$

$y = m_2x + b_2$

$m_2=\text{-}1/m_1=2$

$y=2x + b_2$

$(6\text{-}0)/2=2(3\text{-}0)/2+b_2$

$y=2x - 4.5$