

Objective 3 - Construct Linear Model

Construct a model equation for the real-life situation.

Scenarios we normally use linear functions to model are:

Finance Cost to produce, Utility Bill, Depreciation of Value;

Motion Relative distance of two objects, Using round-trip times to calculate distance,

Chemistry Mixing two different concentrations of solutions;

Statistics Line of best fit.

After you complete each of the questions below, see if you can put together a “general form” for the linear model you built.

Question 1 A company sells doughnuts. They incur a fixed cost of \$11000 for rent, insurance, and other expenses. It costs \$0.1 to produce each doughnut. The company sells each doughnut for \$0.2.

Part A. Construct a linear model that describes their total costs, C , as a function of the number of doughnuts, x , they produce.

$$C(x) = \boxed{0.1x + 11000}$$

Part B. Construct a linear model that describes their total profits, P , as a function of the number of doughnuts, x , they produce.

$$P(x) = \boxed{0.2x}$$

Part C. Construct a linear model that describes their total revenue, R , as a function of the number of doughnuts, x , they produce.

$$R(x) = \boxed{0.1x - 11000}$$

Question 2 Aubrey is a college student going into her first year at UF. She will receive Bright Futures, which covers her tuition plus a \$300 educational expense each Fall and Spring semester. Before college, Aubrey saved up \$9000. She knows she will need to pay \$900 in rent a month, \$40 for food a week, and \$32 in other weekly expenses.

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Part A. Construct a linear model that describes her total costs, C as a function of the number of months, x , she is at UF **during Fall semester**.

$$C(x) = \boxed{1188x}$$

Part B. Construct a linear model that describes her total income, I , as a function of the number of months, x , she is at UF **during Fall semester**.

$$I(x) = \boxed{9300}$$

Part C. Construct a linear model that describes their total budget, B , as a function of the number of months, x , she is at UF **during Fall semester**.

$$B(x) = \boxed{-1188x + 9300}$$

Question 3 Two UFPD are patrolling the campus on foot. To cover more ground, they split up and begin walking in different directions. Office A is walking at 2 mph while Office B is walking at 2 mph.

Part A. Construct a linear model that describes Officer A's distance from their starting point, D , as a function of minutes, m , that have passed.

$$D(m) = \boxed{\frac{1}{30}m}$$

Part B. Construct a linear model that describes their total distance from each other, T_1 , as a function of minutes, m , that have passed if they were walking in exactly opposite directions (e.g., North/South).

$$T_1(m) = \boxed{\frac{1}{15}m}$$

Part C. Construct a linear model that describes their total distance from each other, T_2 , as a function of minutes, m , that have passed if they were walking in exactly 90 degrees from each other (e.g., North/East).

$$T_2(m) = \boxed{\frac{1}{15}\sqrt{\frac{1}{2}}m}$$

Hint: For Part C, draw a picture and think about how the Pythagorean Theorem could be used. Remember: these are all linear models! So if your model has a non-linear **variable**, is there a reason we would ignore part of the domain?

Question 4 A bicyclist is training for a race on a hilly path. Their bike keeps track of their speed at any time, but not the distance traveled. Their speed traveling up a hill is 2mph, 7mph when traveling down a hill, and 4mph when traveling along a flat portion.

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Figure 1: Training path.

Hint: Distance is equal to rate times time.

Part A. Construct linear models that describe their distance, D in miles, on a particular portion of the path in terms of the time, t in hours, spent on that part of the path.

$$D_{\text{up}}(t) = \boxed{2t}$$

$$D_{\text{down}}(t) = \boxed{7t}$$

$$D_{\text{flat}}(t) = \boxed{4t}$$

Part B. Construct linear models that describe their time, t in hours, on a particular portion of the path in terms of the length, D in miles, of that part of the path.

$$t_{\text{up}}(D) = \boxed{\frac{1}{2}D}$$

$$t_{\text{down}}(D) = \boxed{\frac{1}{7}D}$$

$$t_{\text{flat}}(D) = \boxed{\frac{1}{4}D}$$

Part C. Construct a linear model that describes the total distance of the path, D , in terms of the time spent on a particular path if we knew that the time spent on each path was equal.

$$D(t) = \boxed{13t}$$

Part D. Construct a linear model that describes the total time T spent on the path in terms of the distance of a particular part of the path if we knew that all parts of the path are equal length.

$$T(D) = \boxed{\frac{25}{28}D}$$

Question 5 Kappa Delta is hosting an all-you-can-eat pancake fundraiser to support the prevention of child abuse. Adult (18+) tickets are \$9 and teen

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(10-17) tickets are \$4. Children under 10 are let in without a ticket. The ticket-sellers only kept track of the total number of tickets sold, 98, and total revenue, \$732.

Part A. Construct a linear model that describes the total number of adult tickets, y , sold in terms of the number of teen tickets, x , sold.

$$y = \boxed{-x + 98}$$

Part B. Construct a linear model that describes the revenue made from selling many adult tickets, R_a , in terms of the number of teen tickets, x , sold.

$$R_a = \boxed{-9x + 882}$$

Part C. Construct a linear model that describes the total revenue made, R_T , in terms of the number of teen tickets, x , sold.

$$R_T = \boxed{-5x + 882}$$

Hint: Part A: Is there way to build an equation that relates adult tickets, teen tickets, and total tickets? Solving this equation for adult tickets would give you the linear model.

Part B: It may be easier to first build this model in terms of y , then use your answer from part A.

Part C: Think about how to model total revenue, then use your answer in Part B to make part of the model.

Question 6 Chemists commonly create a solution by mixing two products of differing concentrations together. For example, a chemist could have large amounts of a 10% acid solution and a 30% acid solution, but need a 8 liter 15% solution.

Part A. Construct a linear model that describes the volume of the 30% acid solution, v_{30} , in terms of the volume of the 10% acid solution, v .

$$v_{30} = \boxed{-v + 8}$$

Part B. Construct a linear model that describes the amount of acid in a 10% acid solution, A_{10} , in terms of the volume of the 10% acid solution, v .

$$A_{10} = \boxed{0.100000000000000000v}$$

Part C. Construct a linear model that describes the amount of acid in a 30% acid solution, A_{30} , in terms of the volume of the 10% acid solution, v .

$$A_{30} = \boxed{-0.300000000000000000v + 2.4000000000000000}$$

Part D. Construct a linear model that describes the amount of acid in a 15% acid solution, A_{15} , in terms of the volume of the 10% acid solution, v .

$$A_{15} = \boxed{-0.200000000000000000v + 2.4000000000000000}$$

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Hint: Parts A-D: Think about what you did in the last problem. How can we use that same structure in this new setting?
