Objective 3 - Lowest-Degree Polynomial

Construct a lowest-degree polynomial given its zeros.

Link to section in online textbook

First, watch this video to learn how to construct a polynomial, given its zeros. Now practice constructing polynomials from zeros with the questions below.

Question 1 *Construct the lowest-degree polynomial given the zeros below.*

$$-4, -5, -5$$

$f(x) = \boxed{1}x^3 + \boxed{14}x^2 + \boxed{65}x + \boxed{100}$

Question 2 *Construct the lowest-degree polynomial given the zeros below.*

$$-4, -3, 3$$

$f(x) = \boxed{1}x^3 + \boxed{4}x^2 + \boxed{-9}x + \boxed{-36}$

Question 3 *Construct the lowest-degree polynomial given the zeros below.*

$$\frac{5}{3}, \frac{5}{3}, -\frac{3}{2}$$

$f(x) = \boxed{18}x^3 + \boxed{-33}x^2 + \boxed{-40}x + \boxed{75}$

Hint: *Remember back to what it meant to be in Standard Form for linear functions: we did not have any fractions as coefficients. How would we rewrite a factor that has a fraction in it, like* $\left(x - \frac{3}{4}\right)$*?*

Question 4 *Construct the lowest-degree polynomial given the zeros below.*

Learning outcomes:
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$$-3, \frac{4}{3}, -\frac{4}{3}$$

$f(x) = \boxed{9}x^3 + \boxed{27}x^2 + \boxed{-16}x + \boxed{-48}$

Hint: *Remember back to what it meant to be in Standard Form for linear functions: we did not have any fractions as coefficients. How would we rewrite a factor that has a fraction in it, like* $\left(x - \frac{3}{4}\right)$*?*

We focused on building polynomials with integer and rational zeros. What would we do if we had other types of zeros, like irrational or complex?

Theorem 1. *Complex and Irrational roots for polynomials come in "_________" pairs.*

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

tells us something about the types of zeros a quadratic function may have:

- *2 different, rational zeros*
  - *e.g.,* $\frac{1}{4}$ *and* -3 *for the polynomial* $4x^2 + 11x - 3$
- *2 copies of a rational zero*
  - *e.g.,* $\frac{1}{3}$ *and* $\frac{1}{3}$ *for the polynomial* $9x^2 - 2x + 1$
- *2 different, irrational zeros*
  - *e.g.,* $\frac{1}{2} - \sqrt{2}$ *and* $\frac{1}{2} + \sqrt{2}$ *for the polynomial* $4x^2 - 4x - 7$
- *2 different, complex zeros*
  - *e.g.,* $\frac{1}{4} - 3i$ *and* $\frac{1}{4} + 3i$ *for the polynomial* $16x^2 - 8x + 145$

Let's focus on the irrational and complex zeros. These occur when the number under the square root is either (1) not a perfect square or (2) negative. Let's look closer at the form these zeros take by looking at the subgroups the numbers belong to.

Case 1: $b^2 - 4ac$ *is positive and is **not** a perfect square.*

$$x = \frac{integer}{integer} \pm \frac{irrational}{integer}$$

$$x = rational \pm irrational$$

Case 2: $b^2 - 4ac$ *is negative.*

$$x = \frac{integer}{integer} \pm \frac{complex}{integer}$$

$$x = rational \pm complex$$

Question 5 *What word describes the relationship between the zeros* $x = rational - complex$ *and* $x = rational + complex$*?*

They are $\boxed{conjugate}$ *pairs!*

Hint: *What are* $3 + 4i$ *and* $3 - 4i$ *to each other?*

We use this theorem to construct polynomials with irrational and/or complex roots.

Question 6 *Construct the lowest-degree polynomial given the zeros below.*

$$\sqrt{7}, -\frac{2}{3}$$

$f(x) = \boxed{3}x^3 + \boxed{2}x^2 + \boxed{-21}x + \boxed{-14}$

Hint: *If* $\sqrt{7}$ *is a zero to the polynomial, then* $-\sqrt{7}$ *is also! Multiply* $(x-\sqrt{7})(x+\sqrt{7})$ *first, then use the third zero to finish building the polynomial.*

Question 7 *Construct the lowest-degree polynomial given the zeros below.*

$$4 + \sqrt{2}, -\frac{2}{7}$$

$f(x) = \boxed{7}x^3 + \boxed{-54}x^2 + \boxed{82}x + \boxed{28}$

Hint: *Be careful with how you set up this problem. Again, multiply the conjugate factors together first. If you did this right, there should be no radicals left!*

Question 8 *Construct the lowest-degree polynomial given the zeros below.*

$$2i, -\frac{5}{3}$$

$f(x) = \boxed{3}x^3 + \boxed{5}x^2 + \boxed{12}x + \boxed{20}$

Question 9 *Construct the lowest-degree polynomial given the zeros below.*

$$5+5i, -\frac{1}{2}$$

$f(x) = \boxed{2}x^3 + \boxed{-19}x^2 + \boxed{90}x + \boxed{50}$