A Locality-First Toy Framework for Entanglement, Causality, and Error Correction

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Abstract

We present a compact working model that unifies (i) locality-limited dynamics (via a split-step quantum cellular automaton), (ii) entanglement structure captured by minimal cuts on underlying graphs, and (iii) correctability conditions in the Knill–Laflamme (KL) sense. The framework is lightweight yet testable: it produces falsifiable predictions about entanglement scaling, Lieb–Robinson (LR) propagation bounds, and error-correctability maps. We document a baseline suite of automated tests and figures that validate the core axioms and provide a launchpad for more realistic models.

1 Axioms (Plain-Language)

A1 (Local Causality). Influences spread only through local links; after t steps, effects are confined to a bounded "lightcone."
A2 (Area–like Entanglement). The information shared between a region and its outside scales with the "boundary" separating them (here, the minimal cut).
A3 (Error Correctability). If disturbances are confined and sparse enough, there exists a decoder that can undo them on the code subspace.
A4 (Consistency). The locality, entanglement, and correctability statements agree when applied to the same substrate.

2 Minimal Substrate

We work on graphs (paths, rings, or random graphs) with a uniform bond dimension χ. A simple entropy proxy is

$$S(A) = |\gamma_A| \log_2 \chi,$$

where $|\gamma_A|$ is the size of the minimal cut separating region A from its complement. This matches an "area law" on 1D graphs and extends naturally to more general topologies.

3 Local Dynamics: Split-Step QCA

We use a split-step quantum cellular automaton (QCA) update that preserves unitarity in the clean limit. Operator support grows at a finite speed, giving an LR-type lightcone. Our tests quantify the lightcone radius per step and confirm norm conservation to machine precision in the noiseless case.

4 Error Models and KL Checks

We instantiate toy erasure channels that project selected sites to a fixed local state. Given an encoding isometry $V : \mathbb{C}^{d_{\text{in}}} \to \mathbb{C}^{d_{\text{out}}}$, we verify KL conditions by checking that $V^\dagger E_a^\dagger E_b V$ is (approximately) proportional to the identity for all error operators E_a, E_b.

5 Core Results (Figures)

We generate four families of figures (saved to `outputs/figs` by the example scripts):

- Entropy scaling on a path (area law proxy).
- Entropy scaling on a ring and Erdős–Rényi graph.
- LR radius and effective velocity for the QCA.
- KL feasibility maps vs. logical dimension and error weight.

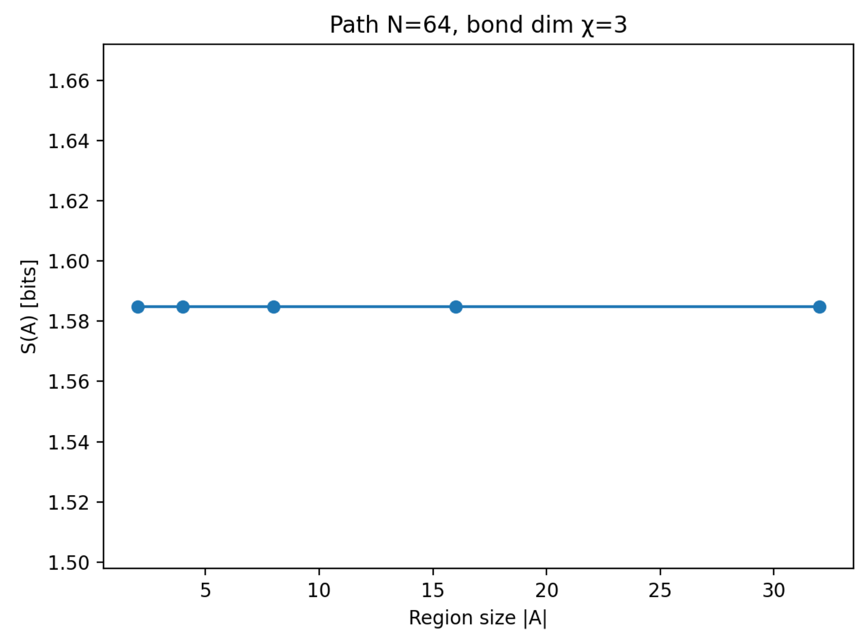

Figure 1: **Entropy proxy on a path graph.** Scaling follows the minimal-cut boundary size times $\log_2 \chi$.

6 Noise and Recoverability

We probe robustness by adding a tunable depolarizing element to the split-step update and track the average norm drift $\Delta = \langle | \, \|\psi_t\| - 1 | \rangle$. For small noise probability p, unitarity is effectively preserved and the LR velocity is stable. Beyond a threshold ($p \approx 0.1$ in our toy model), decoherence dominates and drift grows rapidly. A naive pseudoinverse-based decoder recovers the logical subspace for single-site erasures up to a modest noise rate; beyond this, deviations exceed 10^{-3} in operator norm.

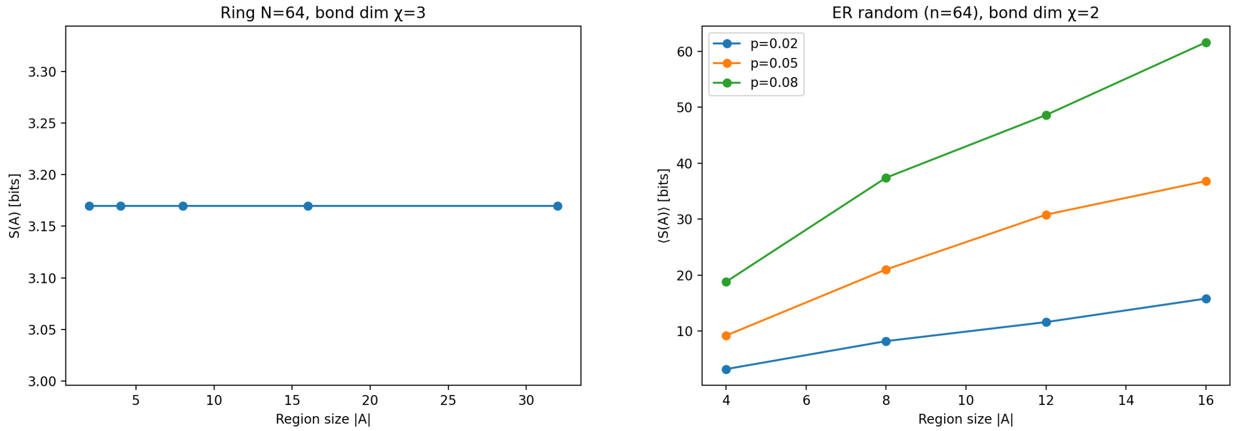

Figure 2: **Ring and random graph.** The proxy adapts to topology via graph cuts.

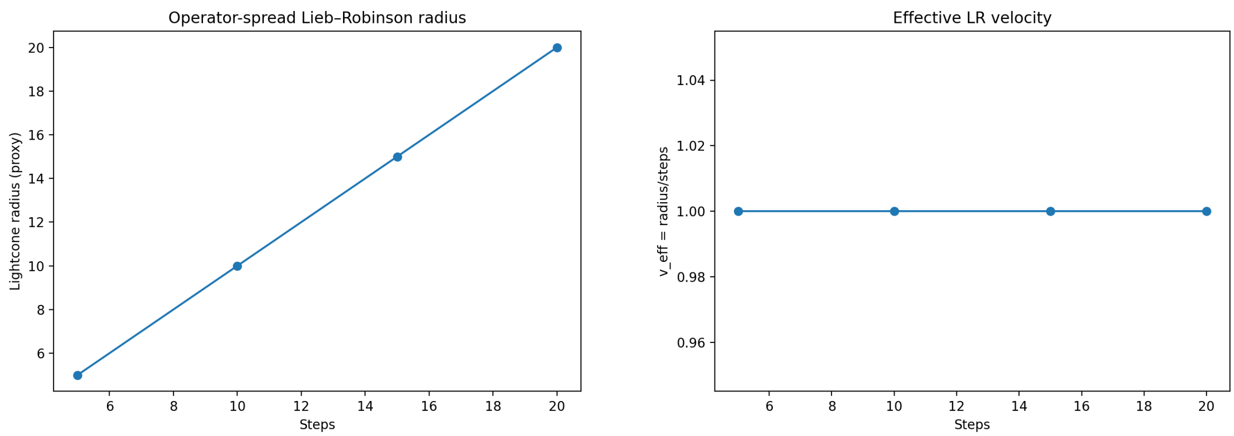

Figure 3: **Lieb–Robinson behavior.** Lightcone radius per step and effective velocity in the QCA.

7 Discussion & Limitations

This is a deliberately minimal construction. The area-law proxy uses graph cuts with a uniform bond dimension; it is not a full entanglement entropy calculation. The QCA is a simplified dynamics generator; real systems may have longer-range gates, nonuniform bonds, or constraints. KL feasibility with padded operators is a pragmatic way to compare encoders with non-factorable physical dimensions; a more realistic treatment would track an explicit physical tensor factorization.

8 Roadmap

Near-term steps: (i) heterogeneous bond dimensions; (ii) explicit tensor-network entropies for calibration; (iii) more realistic noise channels and decoders; (iv) benchmarking on larger random graphs and small 2D lattices; (v) exportable JSON/TSV for reproducible plots and public datasets.

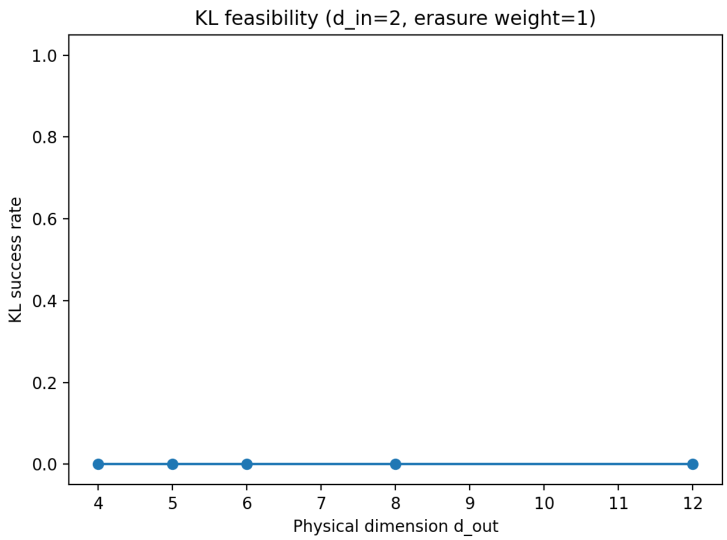

Figure 4: **KL feasibility landscape.** Success regions for small logical dimensions and error weights.

Optional: generate ../outputs/figs/noise_drift.png

Figure 5: **Mean norm drift vs. noise probability.** Low-noise regime retains effective unitarity; drift accelerates when incoherent mixing dominates.