

center speaker, the absence of a hole in the middle becomes a rare "special case" for perhaps just one observing location and perhaps not even one location.

This is true regardless of the stereo recording technique or number of microphones. No bridging of microphones can fill the hole in the center if there is no speaker there to reproduce it except sometimes for one unique observing location. The center speaker is capable of affording the "solid sound curtain."

VIII. TOE-IN

Slightly later than the famed Symposium, Snow showed the desirability to angle or "toe-in" the outside speakers.^{8,12} The effect is to reduce the shift of the virtual sound sources for different observing locations, thus supplementing the function of the center speaker.

Even prior to the revival of the bridged center speaker, this writer advocated corner speaker spacing and toe-in for stereo.¹³ The effect was to enable listeners at the flanks to hear stereo, but listeners at all points experienced the hole in the middle. The virtual sound sources remained at one side or the other. But without the toe-in, the listener on the flank heard only the

nearest speaker. Thus, toe-in accomplishes its function even with only two speakers.

The principle has to do with projecting more sound energy toward the opposite side of the listening area. A natural "floodlight" spatial radiation pattern is desirable—the "spotlight" pattern would be fatal—and the toe-in for the vast majority of suitable listening rooms becomes 45°.

The center speaker is still necessary. Combined with toe-in of flanking units, the error figures¹¹ derived for a central listening position were of the order of 0.12 and increased to 0.16 for observers at the flanks of the listening area. With the center speaker off and the toe-in eliminated, the error was 0.62. For the spacings used, a "live" error figure of 0.04 obtained. Error figures for three independent channels and for the two channels with bridged center speaker differed by insignificant amounts.

IX. CONCLUSION

The requirements of good audio and good stereo may be summed up in the eight cardinal points.

Large books could be written on these points. The foregoing is an attempt to touch the high spots, while affording some basic bibliography for one who would delve further.

¹³ P. W. Klipsch, "Experiences in stereophony," *Audio*, vol. 39, pp. 16–17, 41–42; July, 1955.

"Colorless" Artificial Reverberation*

M. R. SCHROEDER† AND B. F. LOGAN†, MEMBER, IRE

Summary—Electronic devices are widely used to introduce in sound signals an artificial reverberation subjectively similar to that caused by multiple reflections in a room. Attention is focused on those devices employing delay loops. Usually, these devices have a comb-like frequency response which adds an undesired "color" to the sound quality. Also, for a given reverberation time, the density of echoes is far below that encountered in a room, giving rise to a noticeable flutter effect in transient sounds. A class of all-pass filters is described which may be employed in cascade to obtain "colorless" reverberation with high echo density.

INTRODUCTION

ELECTRONIC devices are widely used today to add reverberation to sound. Ideally, such artificial reverberators should act on sound signals exactly like real, three-dimensional rooms. This is not

simple to achieve, unless one uses a reverberation chamber or the electrical equivalent of a three-dimensional space. Reverberation chambers (and plates¹) are preferred by broadcast stations and record manufacturers because of their high quality and lack of undesirable side effect, but they are not truly *artificial* reverberators.

In this paper, we shall focus our attention on electronic reverberators consisting of delay-lines, disk or tape-delay, and amplifiers. Electronic reverberators are both cheaper than real rooms and have wider applicability, notably in the home (unless one wants to convert the basement into a reverberation chamber). They can also be employed to increase the reverberation time of auditoriums, thereby adapting them to concert hall use, without changing the architecture.

* Received by the PGA, December 8, 1960; revised manuscript received, February 27, 1961. Reprinted with permission from the *J. Audio Engrg. Soc.*, vol. 9, pp. 192–197; July, 1961.

† Bell Telephone Labs., Inc., Murray Hill, N. J.

¹ W. Kuhl, "The acoustical and technological properties of the reverberation plate," *European Broadcasting Union Rev.*, Pt. A—Technical, No. 49; May, 1958.

Before attempting the difficult task of reproducing room characteristics by delay-lines, it is wise to recall some of the important properties of large rooms.

THE FREQUENCY RESPONSE OF LARGE ROOMS

A room can be characterized by its normal modes of vibration. It has been shown² that the density of modes is nearly independent of room shape and is proportional to the square of the frequency:

$$\text{number of modes per cps} = \left(\frac{4\pi V}{c^3} \right) f^2.$$

Here V = the volume, c = the velocity of sound, and f the frequency.

Above a certain critical frequency,³ given by

$$f_c = 2000\sqrt{T/V} \text{ (reverberation time } T \text{ in seconds, } V \text{ in m}^3\text{),}$$

the density of modes becomes so high that many modes overlap. In this frequency range, which is of prime interest for large rooms, the concept of individual normal modes loses its practical (though not its theoretical) significance. The behavior of the room is governed by the collective action of many simultaneously excited and interfering modes resulting in a very irregular amplitude-frequency response.^{4,5} However, the fluctuations are so rapid (on the frequency scale) that the ear, in listening to a non-steady sound, does not perceive these irregularities.⁶ (The response fluctuations can be heard by exciting the rooms with a sinewave of slowly varying frequency and listening with one ear.) When the room response is measured using, instead of a sinewave, a psychoacoustically more appropriate test signal, such as narrow bands of noise, the response would indeed be much smoother.

It is this apparent smoothness of a room's frequency response which people have found particularly difficult to imitate with artificial reverberators. In this paper we shall describe electronic reverberators which have perfectly flat amplitude-frequency responses. Thus, they not only overcome this long-standing difficulty but are actually superior to rooms *in this one respect*.

However, a flat frequency response is not the only requirement for a high-quality reverberator. Before we can hope to successfully design one, we must also know something about the *transient* behavior of rooms.

² P. M. Morse and R. H. Bolt, "Sound waves in rooms," *Rev. Mod. Phys.*, vol. 16, pp. 69–150; April, 1944.

³ M. R. Schroeder, "Die statistischen Parameter der Frequenzkurven von grossen Räumen," *Acustica*, vol. 4, Beiheft 2, pp. 594–600; 1954.

⁴ E. C. Wente, "Characteristics of sound transmission in rooms," *J. Acoust. Soc. Am.*, vol. 7, pp. 123–126; October, 1935.

⁵ H. Kuttruff und R. Thiele, "Über die Frequenzabhängigkeit des Schalldrucks im Räumen," *Acustica*, vol. 4, Beiheft 2, pp. 614–617; 1954.

⁶ A. F. Nickson and R. W. Muncey, "Frequency irregularity in rooms," *Acustica*, vol. 5, pp. 44–48; 1955.

THE TRANSIENT BEHAVIOR OF ROOMS

How does a room respond to excitation with a short impulse? If we record the sound pressure at some location in the room as a function of time, we first observe an impulse corresponding to the direct sound which has traveled from the sound source to the pick-up point without reflection at the walls. After that we see a number of discrete low-order echos which correspond to one or a few reflections at the walls and the ceiling. Gradually, the echo density increases to a statistical "clutter." In fact, it can be shown⁷ that the echo density is proportional to the square of the elapsed time:

$$\text{number of echos per second} = \frac{4\pi c^3}{V} t^2.$$

The time after which the echo response becomes a statistical clutter depends on the width of the exciting impulse. For a pulse of width Δt , the critical time after which individual echos start overlapping is about

$$t_c = 5 \cdot 10^{-5} \sqrt{V/\Delta t} \text{ (} V \text{ in m}^3\text{)}.$$

Thus, for transients of 1-msec duration and a volume of 10,000 m³ (350,000 ft³), the response is statistical for times greater than 150 msec. In this region, the concept of the individual echo loses its practical significance. The echo response is determined by the collective behavior and interference of many overlapping echos.⁸

Another important characteristic of large "diffuse" rooms is that all modes have the same or nearly the same reverberation time and thus decay at equal rates as evidenced by a straight-line decay when plotting the sound level in decibels vs elapsed time.

Still another property of acoustically good rooms is the absence of "flutter" echos, *i.e.*, periodic echos resulting from sound waves bouncing back and forth between parallel hard walls. Such periodicities in the echo response are closely associated with one-dimensional modes of sound propagation which can be avoided by splaying the walls and placing "diffusors" in the sound path.

THE CONDITIONS TO BE MET BY ARTIFICIAL REVERBERATORS

After this brief review, we are in a position to formulate conditions to be met by artificial reverberators.

1) The frequency response must be flat when measured with narrow bands of noise, the bandwidth corresponding to that of the transients in the sound to be reverberated. This condition is, of course, fulfilled by re-

⁷ L. Cremer, "Die wissenschaftlichen Grundlagen der Raumakustik," Band 1 ("Geometrische Raumakustik"), S. Hirzel Verlag, Stuttgart, Germany, vol. 1, p. 27; 1948.

⁸ R. C. Jones, "Theory of fluctuations in the decay of sound," *J. Acoust. Soc. Am.*, vol. 11, pp. 324–332; January, 1940.

verberators which have a flat response even for sinusoidal excitation.

2) The normal modes of the reverberator must overlap and cover the entire audio frequency range.

3) The reverberation times of the individual modes must be equal or nearly equal so that different frequency components of the sound decay with equal rates.

4) The echo density a short interval after shock excitation must be high enough so that individual echos are not resolved by the ear.

5) The echo response must be free from periodicities (flutter echos).

In addition to these five conditions, a sixth one must be met which is not apparent from the above review of room behavior but easily violated by electronic reverberators:

6) The amplitude-frequency response must not exhibit any apparent periodicities. Periodic or comb-like frequency responses produce an unpleasant hollow, reedy, or metallic sound quality and give the impression that the sound is transmitted through a hollow tube or barrel.

This condition is a particularly important one because long reverberation times are achieved by circulating the sound by means of delay in feedback loops. The responses of such loops, which are the equivalent of one-dimensional sound transmission, are inherently periodic and special precautions are required to make these periodicities inaudible.

In the following, a basic reverberator is described which fulfills conditions 1), 3) and 6) ideally. By connecting several of these reverberating elements in series, conditions 2), 4) and 5) can also be satisfied without violating the others.

TWO SIMPLE REVERBERATORS

The simplest reverberator consists of a delay-line, disk, or tape-delay which gives a single echo after a delay time τ . Its impulse response is

$$h(t) = \delta(t - \tau), \quad (1)$$

where $\delta(t)$ is the Dirac delta-function (an ideal impulse). The spectrum of the delayed impulse is

$$H(\omega) = e^{-i\omega\tau}, \quad (2)$$

where ω is the radian frequency. The absolute value of $H(\omega)$ is one. This means that all frequencies are passed equally well and without gain or loss.

In order to produce multiple echos without using more (expensive) delay, one inserts the delay line into a feedback loop, as shown in Fig. 1, with gain g of magnitude less than one (so that the loop will be stable). The impulse response, illustrated in Fig. 1(b), is now an exponentially decaying repeated echo:

$$h(t) = \delta(t - \tau) + g\delta(t - 2\tau) + g^2\delta(t - 3\tau) + \dots \quad (3)$$

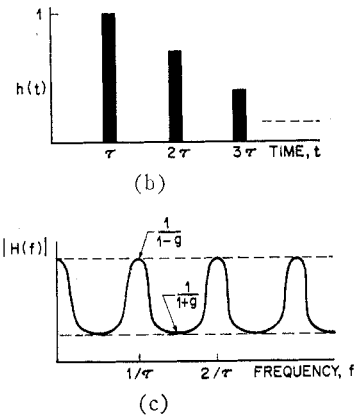
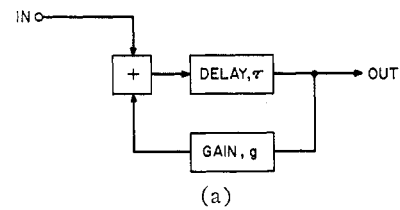


Fig. 1—(a) Delay in feedback loop. (b) Impulse response. (c) Frequency response. Simple reverberators with exponentially decaying echo response. Frequency response resembles comb.

The corresponding complex frequency response is

$$H(\omega) = e^{-i\omega\tau} + ge^{-2i\omega\tau} + g^2e^{-3i\omega\tau} + \dots, \quad (4)$$

or, using the formula for summing geometric series,

$$H(\omega) = \frac{e^{-i\omega\tau}}{1 - ge^{-i\omega\tau}}. \quad (5)$$

By taking the absolute square of $H(\omega)$, one obtains the squared amplitude-frequency response:

$$|H(\omega)|^2 = \frac{1}{1 + g^2 - 2g \cos \omega\tau}. \quad (6)$$

As can be seen, $|H(\omega)|$ is no longer independent of frequency. In fact, for $\omega = 2n\pi/\tau$ ($n=0, 1, 2, 3, \dots$), the response has maxima (for positive g) given by

$$H_{\max} = \frac{1}{1 - g}, \quad (7)$$

and, for $\omega = (2n+1)\pi/\tau$, minima given by

$$H_{\min} = \frac{1}{1 + g}. \quad (8)$$

The ratio of the response maxima to minima is

$$H_{\max}/H_{\min} = \frac{1 + g}{1 - g}. \quad (9)$$

For a loop gain of $g=0.7$ (-3 db), this ratio is $1.7/0.3 = 5.7$ or 15 db!

The amplitude-frequency response of a delay in a feedback loop has the appearance of a comb with periodic maxima and minima, as shown in Fig. 1(c). Each

response maximum corresponds to one normal mode. The natural frequencies are thus spaced $1/\tau$ cps apart.

The 3-db-bandwidth of each peak is approximately

$$\Delta f = \frac{-\ln g}{\pi\tau}, \quad (10)$$

where "ln" denotes the logarithm to the base $e=2.718 \dots$. Converting to logarithms to the base 10 (log), one obtains

$$\Delta f = \frac{1}{20\pi \log e} \frac{-\gamma}{\tau} = 0.0367 \frac{-\gamma}{\tau}, \quad (11)$$

where γ is the loop gain in decibels: $\gamma = 20 \log g$. For $\gamma = -3$ db, the bandwidth is about $0.11/\tau$ or only one-ninth of the spacing of the natural frequencies. The subjective effect of this resonant response is the hollow or reedy sound quality mentioned above.

ALL-PASS REVERBERATORS

In our search for better reverberators, we discovered that a certain mixture of the output of the multiply delayed and the undelayed sound resulted in an equal response of the reverberator for all frequencies. The mixing ratio that accomplishes this and results in unity gain for all frequencies is $(-g)$ for the undelayed sound and $(1-g^2)$ for the multiply delayed sound. The corresponding circuit is shown in Fig. 2. Its impulse response is given by

$$h(t) = -g\delta(t) + (1-g^2) \cdot [\delta(t-\tau) + g\delta(t-2\tau) + \dots]. \quad (12)$$

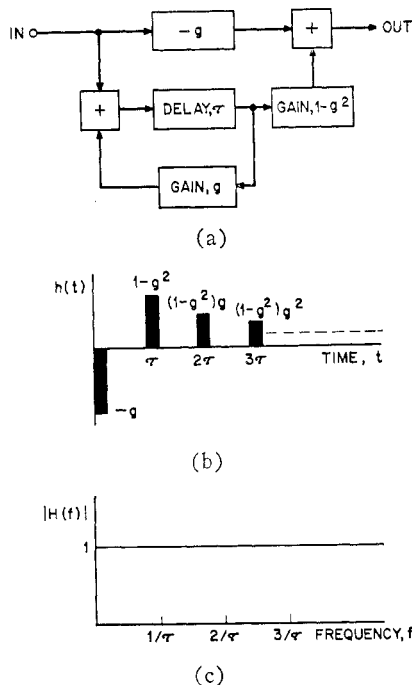


Fig. 2—(a) All-pass filter. (b) Impulse response. (c) Frequency response. Modification of simple reverberator. By adding proper amount of undelayed signal, frequency response of reverberator becomes flat (all-pass reverberator).

The corresponding frequency response is

$$H(\omega) = -g + (1-g^2) \frac{e^{-i\omega\tau}}{1 - ge^{-i\omega\tau}}, \quad (13)$$

or

$$H(\omega) = \frac{e^{-i\omega\tau} - g}{1 - ge^{-i\omega\tau}}, \quad (14)$$

or

$$H(\omega) = e^{-i\omega\tau} \frac{1 - ge^{i\omega\tau}}{1 - ge^{-i\omega\tau}}. \quad (15)$$

What is the absolute value of this $H(\omega)$? The first factor on the right has, of course, absolute value one. The second factor is the quotient of two conjugate complex vectors, *i.e.*, its absolute value is also one. Thus,

$$|H(\omega)| = 1. \quad (16)$$

In other words, the addition of a suitably proportioned undelayed path has converted the comb filter (6) into an all-pass filter (16). This is not a mere academic result. The conversion of a comb filter into an all-pass filter is accompanied by a marked improvement of the sound quality from the hollow sound of the former to the perfectly "colorless" quality of the latter.

Now we are in possession of a basic reverberating element which passes all frequencies with equal gain and thus fulfills conditions 1) and 6) above. The spacings and decay rates of the normal modes (though no longer "visible" as resonant peaks of the amplitude-frequency response) are the same as those for the previously discussed comb filter. Thus, condition 3), requiring equal decay rates for the normal modes, is also fulfilled.

Whether the normal modes overlap (condition 2) can no longer be judged on the basis of the amplitude-frequency response because it is constant. However, the phase-frequency response still reflects the distribution of normal modes and thus must conform to condition 2). The phase-lag of $H(\omega)$ as a function of frequency is, with (15),

$$\phi(\omega) = \omega\tau + 2 \arctan \frac{g \sin \omega\tau}{1 - g \cos \omega\tau}. \quad (17)$$

A more convenient quantity to consider is the rate of change of phase-lag with respect to radian frequency:

$$\frac{d\phi}{d\omega} = \frac{1 - g^2}{1 + g^2 - 2g \cos \omega\tau} \tau, \quad (18)$$

which has exactly the same dependence on ω as the square amplitude-frequency response $|H(\omega)|^2$ of the corresponding comb filter [see (6)]. The physical significance of $d\phi/d\omega$ is that of the envelope or "group" delay of a narrow band of frequencies around ω . According to

(18), for a loop gain of $g=0.7$ this envelope delay fluctuates as much as 32:1 for different frequency bands, with the long delays occurring, of course, for frequencies near the natural frequencies, $2\pi n/\tau$ ($n=0, 1, 2, \dots$), of the filter. The half-width of the envelope delay peaks is the same as that for squared amplitude [see (10)]. Thus, for a loop gain of -3 db, only one-ninth of all frequency components suffer a large envelope delay, while the remaining frequencies are much less delayed. This constitutes a very unequal treatment of different frequency components and violates condition 2).

The remaining two conditions, 4) and 5), are also violated as we shall see immediately. The relationship between reverberation time T (defined by a 60-db decay) and the two parameters of the reverberator, the delay τ and the loop gain γ in decibels, is as follows. For every trip around the feedback loop the sound is attenuated γ db. Thus, the 60-db decay time is

$$T = \frac{60}{-\gamma} \tau. \quad (19)$$

For $\gamma = -3$ db,⁹ we have $T = 20 \cdot \tau$. Thus, in order to achieve, for example, 2 seconds of artificial reverberation, the loop delay must be 0.1 sec. With this loop delay the basic reverberating element shown in Fig. 2 produces one echo every one-tenth of a second. This constitutes a most undesirable periodic flutter echo. Also, the echo density (ten echos per second) is much too low to give a continuous reverberation. Thus, conditions 4) and 5) are violated.

How can one obtain a less periodic time response and a greater echo density without giving up the all-pass characteristic?¹⁰ If several all-pass feedback loops with incommensurate loop delays are connected in *series*, as illustrated in Fig. 3, the combined frequency response remains flat, while the echo response becomes aperiodic and the echo density increases.

In addition, a better coverage of the frequency axis with normal modes is achieved. In fact, the envelope delay response of the series connection is a sum of terms like (18) with different τ 's. Since each of these terms "covers" only one-ninth of the frequency axis, at least five all-pass feedback loops in series are required. On the other hand, one can also show that *too many* all-pass feedback loops in tandem are bad because they lead to a very unnatural, nonexponential reverberation which builds up to its maximum intensity rather slowly before it starts decaying. We shall spare the reader the mathe-

matical details of this peculiar reverberation because he has suffered already too much, we are afraid.

Fig. 4 shows the impulse response of five all-pass filters connected in series with loop delays of 100, 68, 60, 19.7, 5.85 msec. The loop gains are $+0.7, -0.7, +0.7, +0.7, +0.7$, respectively. This combination of delays and gains was arrived at after considerable experimentation observing the response to a variety of sounds, both on the oscilloscope and by listening, and using a smooth envelope of the decay as a criterion. The appearance of the echo response is quite random and not unlike that of real rooms.^{11,12} An increase in pulse density with increasing time can also be noticed.

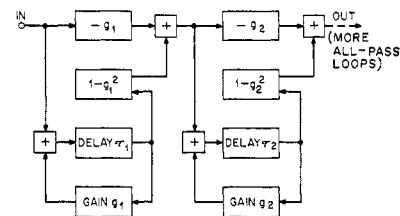


Fig. 3—Series connection of several all-pass reverberators with incommensurate delays to make echo response aperiodic and increase echo density.

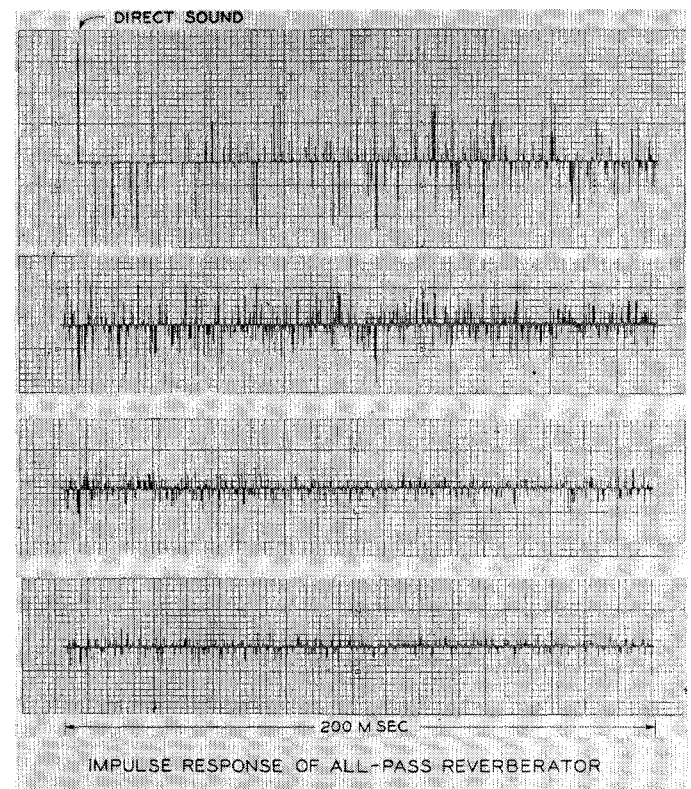


Fig. 4—Echo response of all-pass reverberator consisting of five simple reverberators connected in series, as shown in Fig. 3.

⁹ We do not consider open-loop gains greater than 0.7 (-3 db), because in practice it is difficult to maintain the desired closed loop characteristics with gains too close to unity.

¹⁰ N. V. Franssen, "Neuentwicklung in der räumlichen Schallwiedergabe," Bericht über die 4. Tonmeistertagung, Detmold, Germany; October, 1957, p. 10; Franssen has suggested the use of multiple feedback, proportioning the gains to give a flat open-loop frequency response. This allows larger loop gains without incurring instability. However, a feedback loop around an all-pass filter results in a nonflat frequency response, unless supplemented by a direct path of suitable gain as described above.

¹¹ E. Meyer and R. Thiele, "Raumakustische Untersuchungen in zahlreichen Konzertsälen und Rundfunkstudios unter Anwendung neuerer Messverfahren," *Acustica*, vol. 6, Beiheft 2, pp. 425-444; 1956.

¹² G. R. Schodder, "Über die Verteilung der energiereicheren Schallrückwürfe in Sälen," *Acustica*, vol. 6, Beiheft 2, pp. 445-465; 1956.

DIGITAL COMPUTER SIMULATION

The impulse response shown in Fig. 4 was obtained from a large digital computer (IBM 7090) in conjunction with special digital-to-analog conversion and plotting apparatus. In addition to using the computer as a "draftsman," many of the actual reverberation experiments were performed with the help of the digital computer. In this research method (sometimes called "digital simulation"¹³), ordinary ("analog") tape recordings of the sound to be reverberated are prepared and converted into digital tapes by means of special conversion equipment. The computer then reads the digital tape and acts upon it exactly like any desired real equipment would act on the sound signal. To facilitate the programming of the computer, our engineers make use of a special translation program, developed at Bell Telephone Laboratories, which translates their block diagrams into the computer language. Thus, the computer first "compiles" its own program on the basis of the block diagram information and then acts on the sound as the block diagram would do. It then prepares one (or several) digital output tapes which are converted back into analog tape recordings and evaluated by listening. Needless to say, this is a very powerful research tool, especially when complex equipment is to be evaluated! In this manner we have studied the subjective quality of a great variety of reverberators with both flat and nonflat frequency responses.

APPLICATION TO QUASI-STEREOPHONY

The late Holger Lauridsen¹⁴ of the Danish State Radio has discovered a method of splitting a single audio signal into two "quasi-stereophonic" signals which give the listener all the "ambience" of multichannel stereophony but permits, of course, no correct localization of individual sound sources. In order to achieve this, Lauridsen has used delay networks connected to form a pair of interleaved comb filters.¹⁵ However, these comb filters give rise to unpleasant sound qualities—not quite as pronounced as in artificial reverberators,

¹³ E. E. David, Jr., "Digital simulation in research on human communication," *PROC. IRE*, vol. 49, pp. 319-329; January, 1961.

¹⁴ H. Lauridsen, "Nogle forsøg med forskellige former rum akustik gengivelse," *Ingeniøren*, no. 47, p. 906; 1954.

¹⁵ M. R. Schroeder, "An artificial stereophonic effect obtained from a single audio signal," *J. Audio Engrg. Soc.*, vol. 6, pp. 74-79; April, 1958.

but nevertheless easily perceptible. We have overcome this disadvantage by using a pair of all-pass filters, like the one shown in Fig. 2, to split the single-channel audio signal. This idea is described in greater detail in a forthcoming publication.¹⁶

SUMMARY

Several reverberators of the all-pass type were successfully simulated on a digital computer (IBM 7090). Others were instrumented with delay-lines and tape-delay. No coloration of the reverberation sound was detected in any of these electronic reverberators. Our listening experience with all-pass reverberators indicates that the problem of unequal response to different frequencies has been solved and sound "coloration" completely eliminated. Audible flutter echos have been avoided by the use of several all-pass feedback loops with "incommensurate" delays in series.

The application of all-pass reverberators to the problem of increasing the reverberation time of auditoriums and concert halls by purely electroacoustic means¹⁷ remains to be studied. Here the flat frequency response is particularly important for two reasons: 1) It minimizes acoustic feedback problems. The "ringing" and instability due to the unavoidably irregular frequency response of the room can be reduced by shifting all frequency components of the reverberated sound by a small constant amount.¹⁸ 2) A flat response of the reverberator contributes to the high sound quality required in concert hall applications. Ultimate acceptance of electroacoustic techniques in concert halls and opera houses is assured only if the artificial effects are not recognized as such by the music-loving public.

ACKNOWLEDGMENT

We thank Miss N. Nordahl for assistance in the computer simulation experiments and Messrs. A. J. Prestigiacoma, F. K. Harvey and J. G. Cisek for conducting a variety of auxiliary experiments.

¹⁶ M. R. Schroeder, "Improved quasi-stereophony and colorless artificial reverberation," *J. Acoust. Soc. Am.*, vol. 33, pp. 1061-1064; August, 1961.

¹⁷ R. Vermeulen, "Stereo reverberation," *J. Audio Engrg. Soc.*, vol. 6, pp. 124-130; April, 1958.

¹⁸ M. R. Schroeder, "Stop feedback in public address systems," *Radio Electronics*, vol. 31, pp. 40-42; February, 1960. See also "Improvement of acoustic feedback stability in public address systems," *Proc. 3rd Internat. Congr. Acoustics*, Elsevier Publishing Co., Amsterdam, The Netherlands, vol. 2, pp. 771-775; 1961.