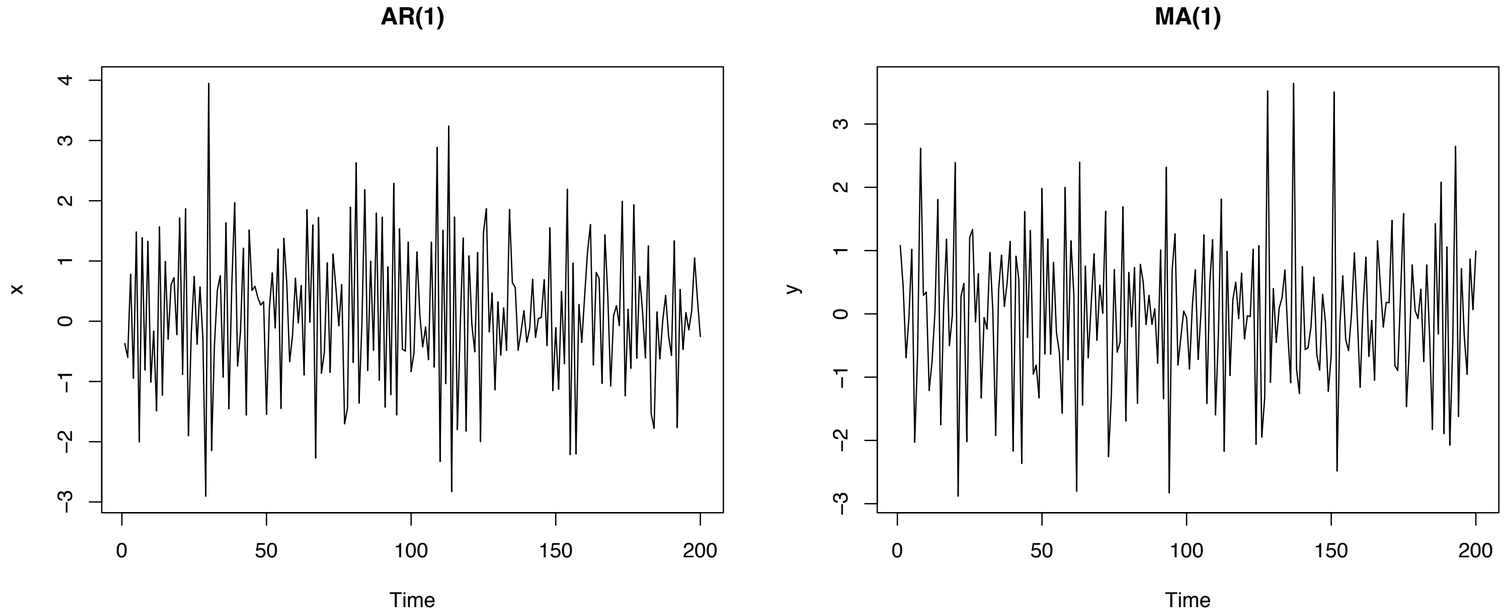
ARIMA MODELING WITH R

AR and MA Models

AR and MA Models

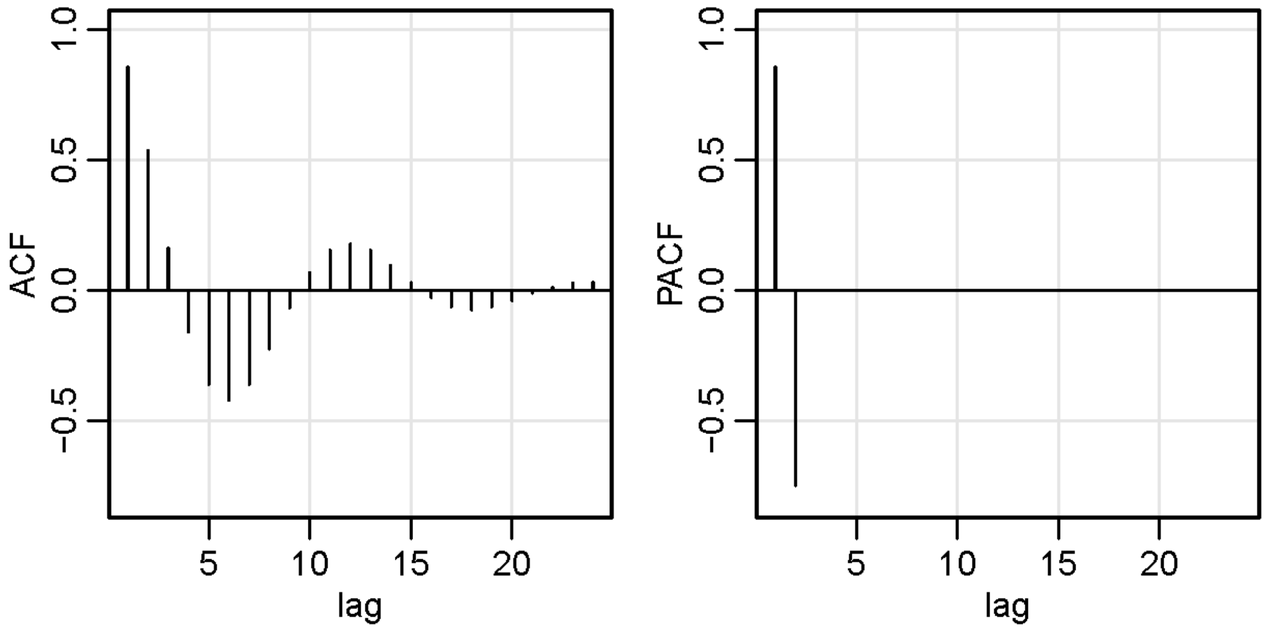
```
> x <- arima.sim(list(order = c(1, 0, 0), ar = -.7), n = 200)
> y <- arima.sim(list(order = c(0, 0, 1), ma = -.7), n = 200)

> par(mfrow = c(1, 2))
> plot(x, main = "AR(1)")
> plot(y, main = "MA(1)")
```



ACF and PACF

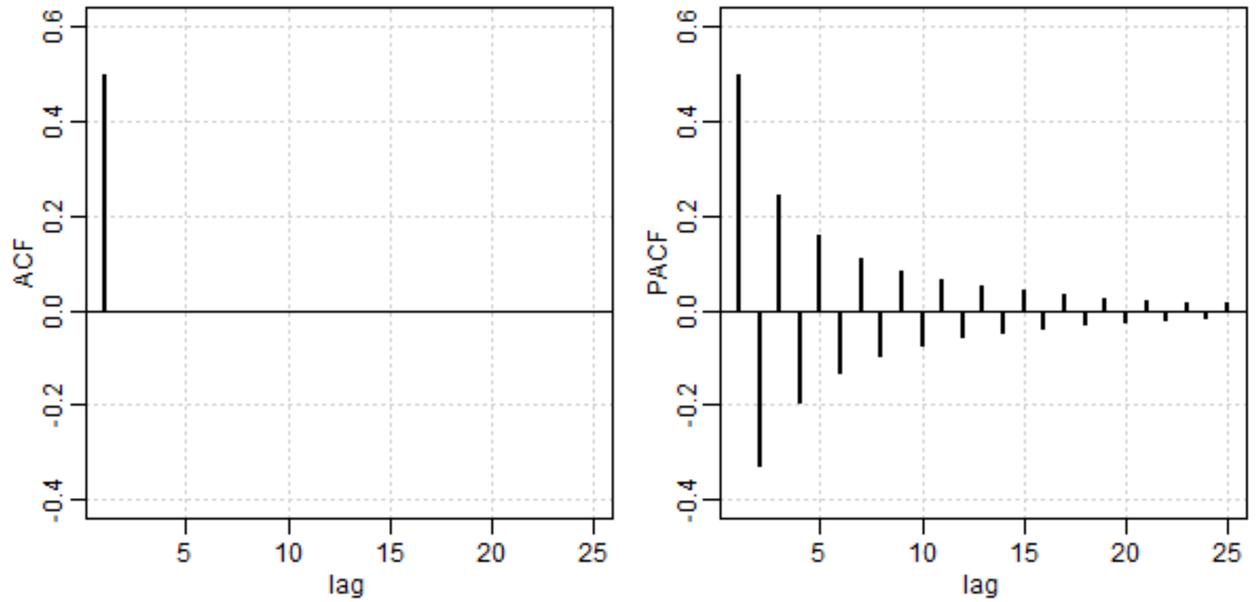
	AR(p)	MA(q)	ARMA(p, q)
ACF	Tails off	Cuts off lag q	Tails off
PACF	Cuts off lag p	Tails off	Tails off



AR(2)

ACF and PACF

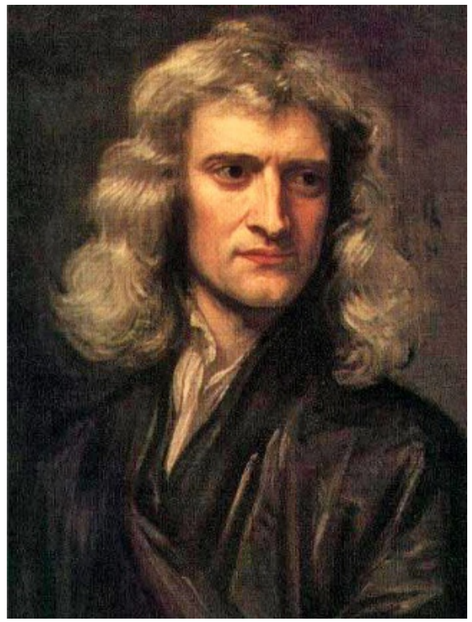
	AR(p)	MA(q)	ARMA(p, q)
ACF	Tails off	Cuts off lag q	Tails off
PACF	Cuts off lag p	Tails off	Tails off



MA(1)

Estimation

- Estimation for time series is similar to using least squares for regression
- Estimates are obtained numerically using ideas of Gauss and Newton



Estimation with astsa

- AR(2) with mean 50:

$$X_t = 50 + 1.5(X_{t-1} - 50) - .75(X_{t-2} - 50) + W_t$$

```
> x <- arima.sim(list(order = c(2, 0, 0),
         ar = c(1.5, -.75)),
         n = 200) + 50
> x_fit <- sarima(x, p = 2, d = 0, q = 0)
> x_fit$ttable

      Estimate     SE  t.value p.value
ar1     1.5429 0.0435  35.4417       0
ar2    -0.7752 0.0434 -17.8650       0
xmean  49.6984 0.3057 162.5788       0
```

Estimation with astsa

- MA(1) with mean 0:

$$X_t = W_t - .7W_{t-1}$$

```
> y <- arima.sim(list(order = c(0, 0, 1), ma = -.7), n = 200)
> y_fit <- sarima(y, p = 0, d = 0, q = 1)
> y_fit$ttable

      Estimate      SE   t.value  p.value
ma1    -0.7459  0.0513  -14.5470   0.0000
xmean   0.0324  0.0191    1.6946   0.0917
```

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Let’s practice!

ARIMA MODELING WITH R

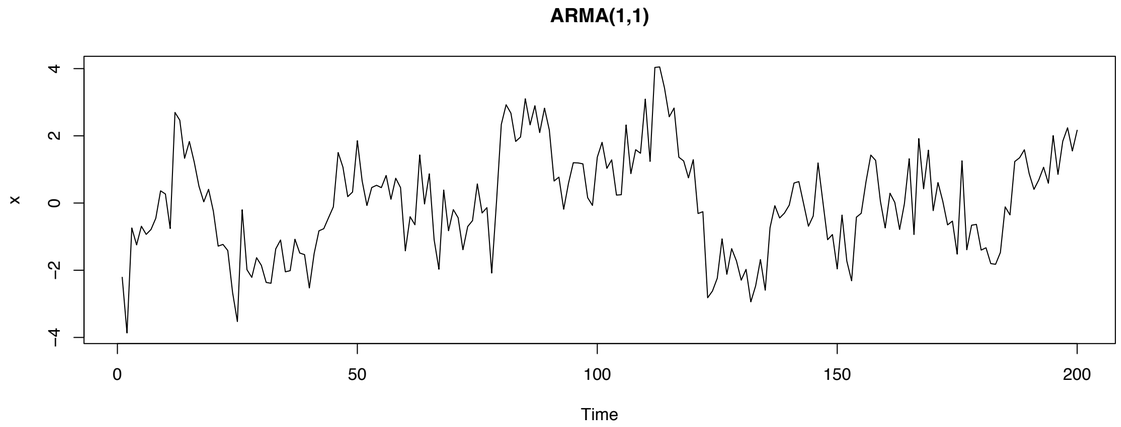
AR and MA Together

AR and MA Together: ARMA

$$X_t = \phi X_{t-1} + W_t + \theta W_{t-1}$$

auto-regression with correlated errors

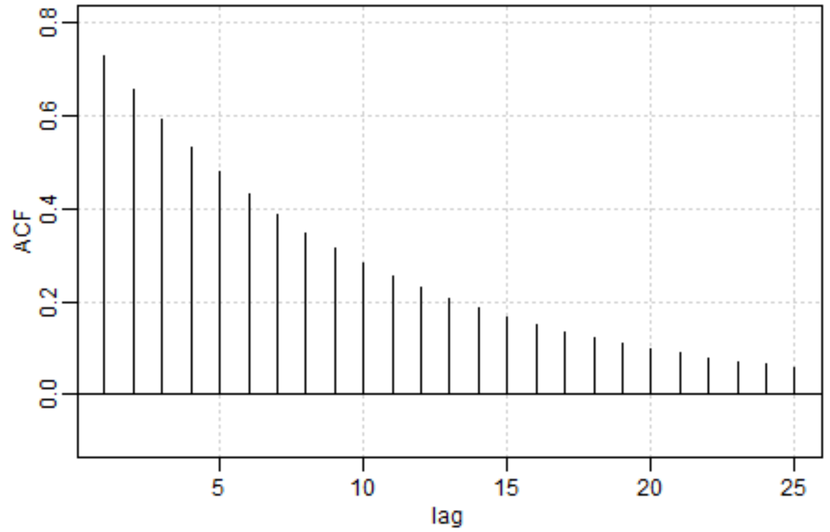
```
> x <- arima.sim(list(order = c(1, 0, 1),
                      ar = .9,
                      ma = -.4),
                      n = 200)
> plot(x,  main = "ARMA(1, 1)")
```

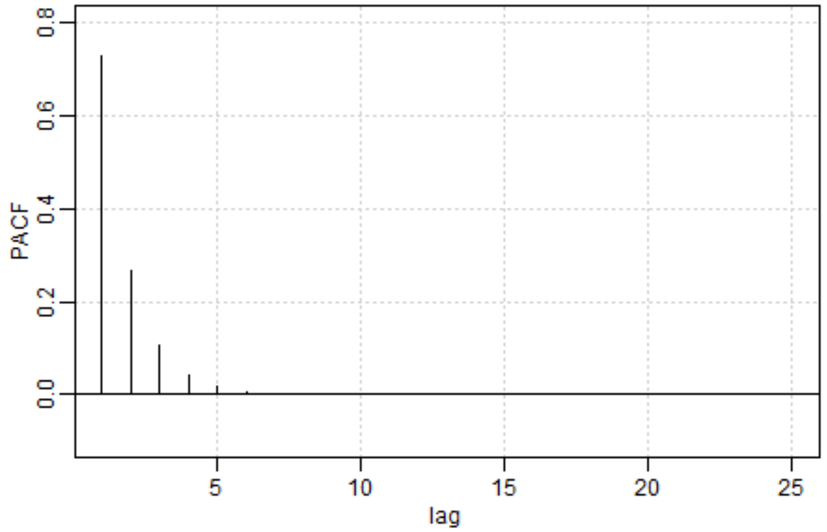


ACF and PACF of ARMA Models

	AR(p)	MA(q)	ARMA(p, q)
ACF	Tails off	Cuts off lag q	Tails off
PACF	Cuts off lag p	Tails off	Tails off

$$X_t = .9X_{t-1} + W_t - .4W_{t-1}$$





Estimation

$$X_t = .9X_{t-1} + W_t - .4W_{t-1}$$

```
> x <- arima.sim(list(order = c(1, 0, 1),
          ar = .9,
          ma = -.4),
          n = 200) + 50
> x_fit <- sarima(x, p = 1, d = 0, q = 1)
> x_fit$ttable

      Estimate     SE  t.value p.value
ar1     0.9083 0.0424  21.4036       0
ma1    -0.4458 0.0879  -5.0716       0
xmean  49.5647 0.4079 121.5026       0
```

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Let’s practice!

ARIMA MODELING WITH R

Model Choice and Residual Analysis

AIC and BIC

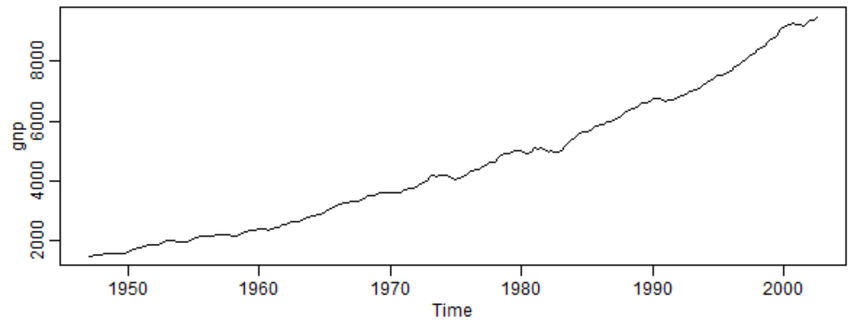
Error **Number of Parameters**

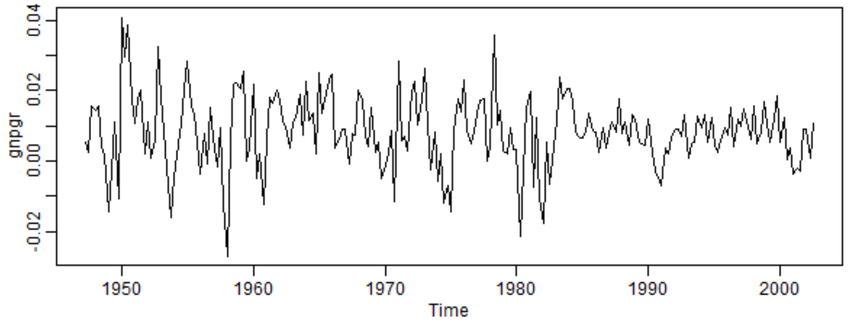
$$average(observed - predicted)^2 \quad + \quad k(p+q)$$

- AIC and BIC measure the error and penalize (differently) for adding parameters
- For example, AIC has $k = 2$ and BIC has $k = log(n)$
- Goal: find the model with the smallest AIC or BIC

Model Choice: AR(1) vs. MA(2)

```
> gnpgr <- diff(log(gnp))

> sarima(gnpgr, p = 1, d = 0, q = 0)

$AIC                              $BIC
[1] -8.294403              [1]  -9.263748

> sarima(gnpgr, p = 0, d = 0, q = 2)

$AIC                              $BIC
[1] -8.297695              [1] -9.251712
```

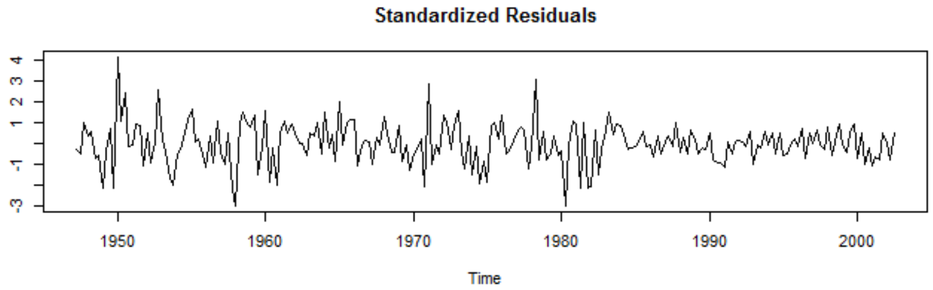


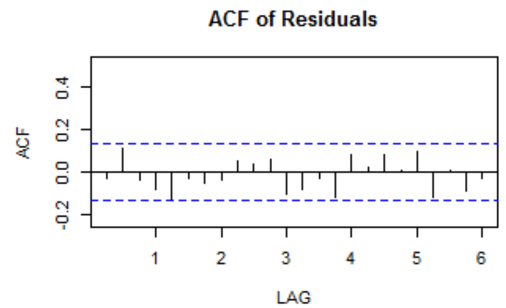


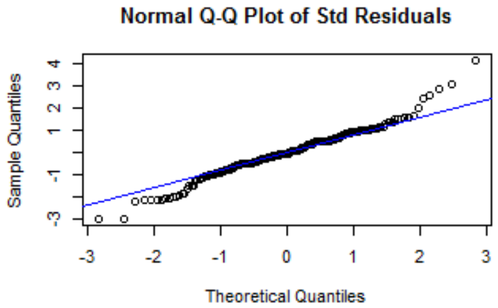
Residual Analysis

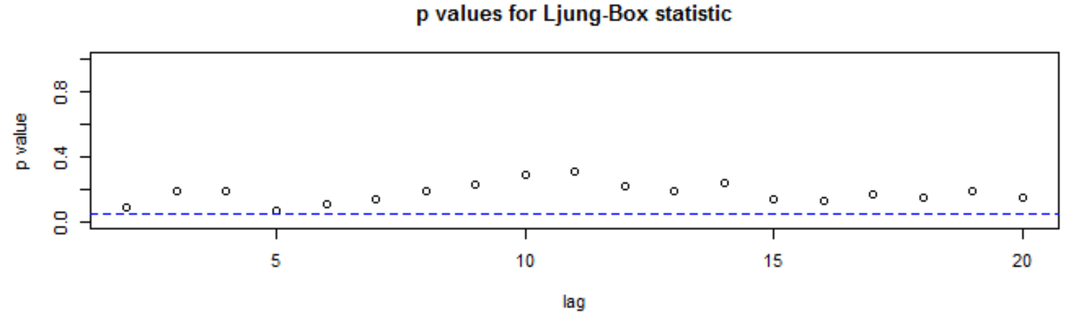
sarima() includes residual analysis graphic showing:

1. Standardized residuals
2. Sample ACF of residuals
3. Normal Q-Q plot
4. Q-statistic p-values



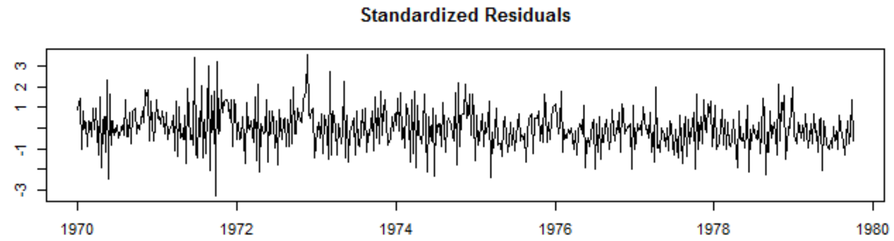


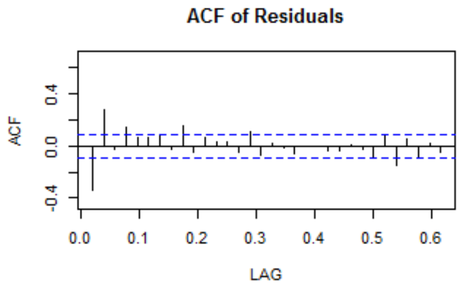


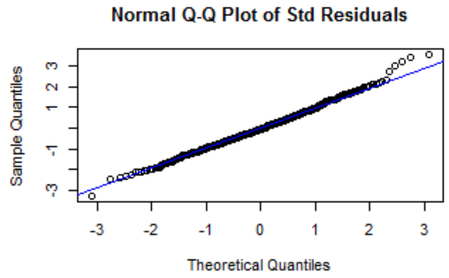


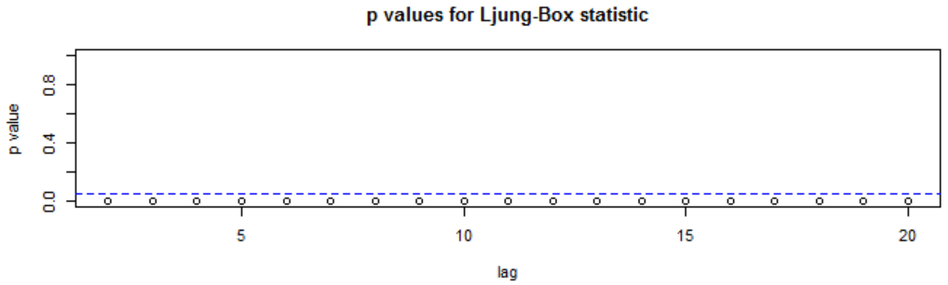
Bad Residuals

- Pattern in the residuals
- ACF has large values
- Q-Q plot suggests normality
- Q-statistic - all points below line









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