2019 FRM Part I

百题巅峰班

定量分析

2019 年 3 月

2. Quantitative Analysis

2.1. Key Point: Conditional Probability

2.1.1. 重要知识点

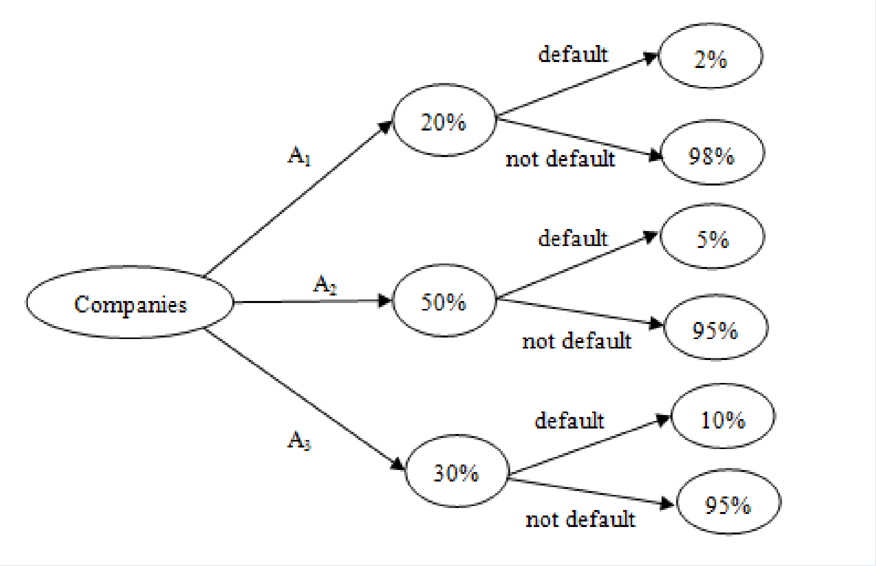
2.1.1.1. Conditional Probability:

$$P(A|B)=\frac{P(AB)}{P(B)} \quad ; \quad P(B)>0$$

$$P(B|A)=\frac{P(AB)}{P(A)} \quad ; \quad P(A)>0$$

$$P(B)=P(A_1)P(B|A_1)+P(A_2)P(B|A_2)+\ldots+P(A_n)P(B|A_n)$$

$$P(A_1|B)=\frac{P(B|A_1)}{P(B)}\times P(A_1)$$



2.1.2. 基础题

Q-1. Suppose there are two events A and B. The probability of A occurrence equals that of B. P(AB) = 4%, If event A occurred, the probability of B occurs is 80%. What is the probability of neither occurs?

A. 86%

B. 90%

C. 94%

D. 96%

Q-2. An analyst develops the following probability distribution about the state of the economy and the market.

Initial Probability P(A)	Conditional Probability P(B |A)
Good economy 60%	Bull market 50%

	Normal market 30%
	Bear market 20%
Poor economy 40%	Bull market 20%
	Normal market 30%
	Bear market 50%

Which of the following statements about this probability distribution is least likely accurate?

A. The probability of a normal market is 0.30.

B. The probability of having a good economy and a bear market is 0.12.

C. Given that the economy is good, the chance of a poor economy and a bull market is 0.15.

D. Given that the economy is poor, the combined probability of a normal or a bull market is 0.50.

Q-3. In country X, the probability that a letter sent through the postal system reaches its destination is 2/3. Assume that each postal delivery is independent of every other postal delivery, and assume that if a wife receives a letter from her husband, she will certainly mail a response to her husband. Suppose a man in country X mails a letter to his wife (also in country X) through the postal system. If the man does not receive a response letter from his wife, what is the probability that his wife received his letter?

A. 1/3

B. 3/5

C. 2/3

D. 2/5

Q-4. An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year, and 60% of policyholders who have only a homeowner policy will renew next year. The company estimates that 80% of policyholders who have both an auto and a homeowner policy will renew at least one of those policies next year. Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowner policy, and 15% of policyholders have both an auto and a homeowner policy. Using the company's estimates, what is the percentage of policyholders that will renew at least one policy next year?

A. 20%

B. 29%

C. 41%

D. 53%

2.2. Variance and Covariance

2.2.1. 重要知识点

2.2.1.1. Variance and Covariance

$$\mathrm{var}(X) = \sigma^2 = E(X-\mu)^2$$

$$\mathrm{var}(X) = E(X^2) - [E(X)]^2$$

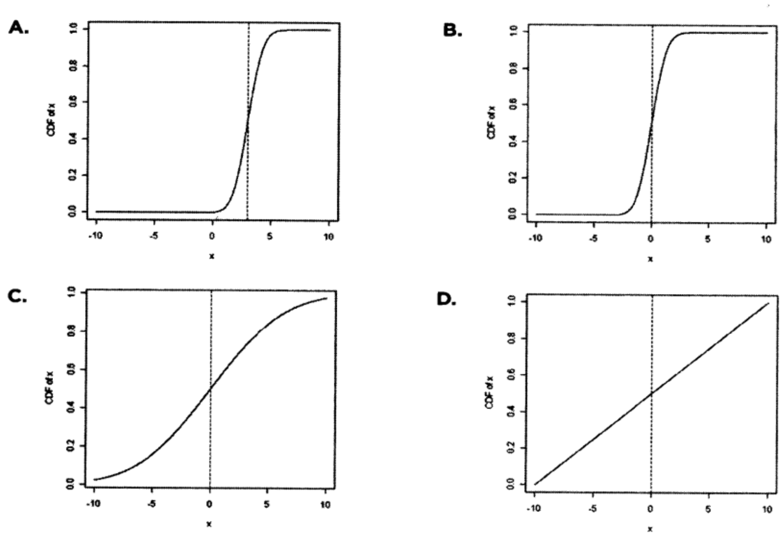
$$\mathrm{var}(X \pm Y) = \mathrm{var}(X) + \mathrm{var}(Y) \pm 2\,\mathrm{cov}(X, Y)$$

$$\mathrm{Cov}(X,Y) = E[(X\text{-}E(X))(Y\text{-}E(Y))] = E(XY)\text{-}E(X)E(Y)$$

$$\mathrm{Cov}(ax+by,\ cx+dy) = ac\sigma_x^2 + bd\sigma_y^2 + (ad+bc)\mathrm{Cov}(x, y)$$

2.2.2. 基础题

Q-5. The following graphs show the cumulative distribution function (CDF) of four different random variables. The dotted vertical line indicates the mean of the distribution. Assuming each random variable can only be values between -10 and 10, which distribution has the highest variance?



Q-6. Roy Thomson, a global investment risk manager of FBN Bank, is assessing Markets A and B using a two-factor model:

$$R_i = \alpha_i + \beta_{i,1}F_1 + \beta_{i,2}F_2 + \varepsilon_i$$

where Ri is the return for asset i; β is the factor sensitivity; And F is the factor. The random error , has a mean of zero and is uncorrelated with the factors and with the random error of the other asset returns. In order to determine the covariance between Markets A and B, Thomson developed the following factor covariance matrix for global assets:

Factor Covariance Matrix for Global Assets		
	Global Equity Factor	Global Bond Factor
Global Equity Factor	0.3424	0.0122
Global Bond Factor	0.0122	0.0079

Suppose the factor sensitivities to the global equity factor are 0.70 for market A and 0.85 for Market B, and the factor sensitivities to the global bond factors are 0.30 for market A and 0.55 for Market B. The covariance between Market A and Market B is closest to:

A. 0.213

B. 0.461

C. 0.205

D. 0.453

Q-7. Let X and Y be two random variables representing the annual returns of two different portfolios. If E[X] = 3, E[Y] = 4 and E[XY] = 11, then what is Cov[X, Y]?

A. 0.213

B. 0.461

C. 0.205

D. 0.453

Use the following data to answer Questions 8 and 9.

Probability Matrix			
Returns	R_B = 50%	R_B = 20%	R_B = -30%
R_A = -10%	40%	0%	0%
R_A = 10%	0%	30%	0%
R_A = 30%	0%	0%	30%

Q-8. Given the probability matrix above, the standard deviation of Stock B is closest to?

A. 0.11

B. 0.22

C. 0.33

D. 0.15

Q-9. Given the probability matrix above, the covariance between Stock A and B is closest to:Let X and Y be two random variables representing the annual returns of two different portfolios. If E[X] = 3, E[Y] = 4 and E[XY] = 11, then what is Cov[X, Y]?

A. -0.160

B. -0.055

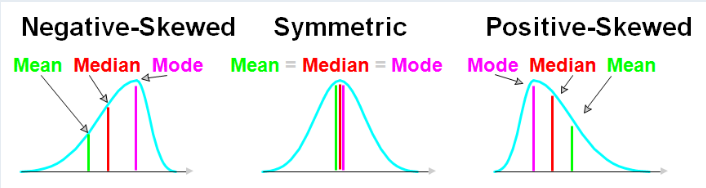
C. 0.004

D. 0.020

2.3. Skewness & Kurtosis

2.3.1. 重要知识点

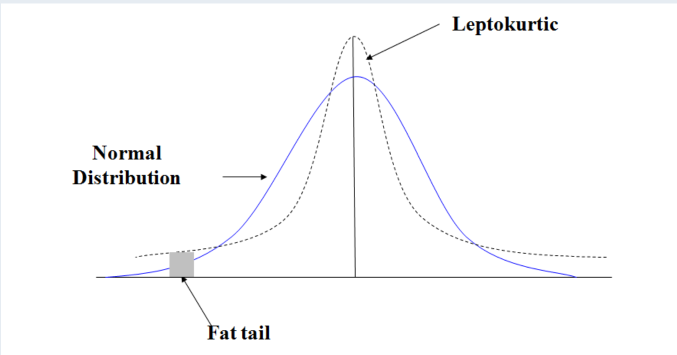
2.3.1.1. Skewness: $S=\frac{E(X-\mu)^3}{\sigma^2}$



2.3.1.2. Kurtosis: $K=\frac{E(X-\mu)^4}{\sigma^4}$

	leptokurtic	Mesokurtic (normal distribution)	platykurtic
Sample kurtosis	>3	=3	<3
Excess kurtosis	>0	=0	<0

Excess Kurtosis = Kurtosis – 3



2.3.2. 基础题

Q-10. An analyst gathered the following information about the return distributions for two portfolios during the same time period:

Portfolio	Skewness	Kurtosis
A	-1.6	1.9
B	0.8	3.2

The analyst states that the distribution for Portfolio A is more peaked than a normal distribution and that the distribution for Portfolio B has a long tail on the left side of the distribution. Which of the following is correct?

A. The analyst's assessment is correct.

B. The analyst's assessment is correct for Portfolio A and incorrect for portfolio B.

C. The analyst's assessment is incorrect for Portfolio A but is correct for portfolio B.

D. The analyst is incorrect in his assessment for both portfolios.

Q-11. Which of the following exhibit positively skewed distributions?

I. Normal Distribution

II. Lognormal Distribution

III. The Returns of Being Short a Put Option

IV. The Returns of Being Long a Call Option

A. II only

B. III only

C. II and IV only

D. I, III, and IV only

Q-12. Which type of distribution produces the lowest probability for a variable to exceed a specified extreme value X which is greater than the mean assuming the distributions all have the same mean and variance?

A. A leptokurtic distribution with a kurtosis of 4

B. A leptokurtic distribution with a kurtosis of 8

C. A normal distribution

D. A platykurtic distribution

Q-13. An analyst is concerned with the symmetry and peakedness of a distribution of returns over a period of time for a company she is examining. She does some calculations and finds that the median return is 4.2%, the mean return is 3.7%, and the mode return is 4.8%. She also finds that the measure of kurtosis is 2. Based on this information, the correct characterization of the distribution of returns over time is:

	Skewness	Kurtosis
A.	Positive	Leptokurtic
B.	Positive	Platykurtic
C.	Negative	Platykurtic
D.	Negative	Leptokurtic

Q-14. In looking at the frequency distribution of weekly crude oil price changes between 1984 and 2008, an analyst notices that the frequency distribution has a surprisingly large number of observations for extremely large positive price changes and a smaller number, but still a surprising one of observations for extremely large negative price changes. The analyst provides you with the following statistical measures. Which measures would help you identify these characteristics of the frequency distribution?

I. Serial correlation of weekly price chances

II. Variance of weekly price changes

III. Skewness of weekly price changes

IV. Kurtosis of weekly price changes

A. I, II, III and IV

B. II only

C. III and IV only

D. I, III and IV only

2.4. Chebyshev's Inequality

2.4.1. 重要知识点

2.4.1.1. HPY, r_{MM}, r_{BD}, EAY, BEY 的计算及转化

- 对任何一组观测值，个体落于均值周围 k 个标准差之内的概率不小于 1 - $1/k^2$ ，对任意 k>1。

$$P(|X-\mu| \le k\sigma) \ge 1-\frac{1}{k^2}, \quad k>1$$

At least	$1-\frac{1}{2^2}=1-\frac{1}{4}=\frac{3}{4}=75\%$	Lie within	2	Standard deviations of the mean
	$1-\frac{1}{3^2}=1-\frac{1}{9}=\frac{8}{9}=89\%$		3	
	$1-\frac{1}{4^2}=1-\frac{1}{16}=\frac{15}{16}=94\%$		4	

2.4.2. 基础题

Q-15. Using Chebyshev's inequality, what is the proportion of observations from a population of 250 that must lie within three standard deviations of the mean, regardless of the shape of the distribution?

A. 75%

B. 99%

C. 89%

D. 54%

2.5. Distribution

2.5.1. 重要知识点

2.5.1.1. Discrete Distribution;

2.5.1.1.1. Bernoulli Distribution

$$p(x)=\begin{cases} p, & x=1 \\ 1-p, & x=0 \end{cases}$$

2.5.1.1.2. Binomial distribution

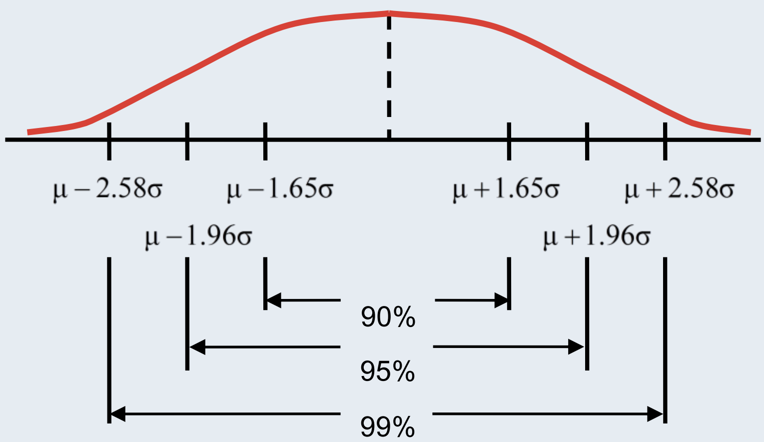
$$p(x)=P(X=x)=C_n^x p^x (1-p)^{n-x}=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

	Expectation	Variance
Bernoulli random variable (Y)	p	p(1-p)
Binomial random variable (X)	np	np(1-p)

2.5.1.2. Continuous Distribution.

2.5.1.2.1. Normal Distribution

$$\overline{X} \sim N(u,\ \sigma^2) \rightarrow Z=\frac{(\overline{X}-\mu)}{\sigma/\sqrt{n}} \sim N(0,\ 1)$$



2.5.1.2.2. Lognormal Distribution

$$\ln X \sim N(\mu,\sigma^2)$$

➢ If lnX is normal, then X is lognormal; if a variable is lognormal, its natural log is normal.

2.5.1.2.3. Poisson Distribution

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

2.5.1.2.4. *t*- Distribution

$$t = \frac{\bar{X} - \mu}{S / \sqrt{n}} \sim t_{(n-1)}$$

Fatter tail than normal distribution.

2.5.1.2.5. Chi-Square Distribution

$$(n\text{-}1)\left(\frac{S^2}{\sigma^2}\right) \sim \chi^2_{(n\text{-}1)}$$

2.5.1.2.6. F-Distribution

$$F = \frac{S_1^2}{S_2^2} \sim F_{(n_1-1),\ (n_2-1)},\ (S_1 > S_2)$$

2.5.2. 基础题

Q-16. A portfolio manager holds five bonds in a portfolio and each bond has a 1-year default probability of 17%. The event of default for each of the bonds is independent.What is the probability of exactly two bonds defaulting over the next year?

A. 1.9%

B. 5.7%

C. 16.5%

D. 32.5%

Q-17. A fixed income portfolio manager currently holds a portfolio of bonds of various companies. Assuming all these bonds have the same annualized probability of default and that the defaults are independent, the number of defaults in this portfolio over the next year follows which type of distribution?

A. Bernoulli

B. Normal

C. Binomial

D. Exponential

Q-18. A multiple choice exam has ten questions, with five choices per question. If you need at least three correct answers to pass the exam, what is the probability that you will pass simply by guessing?

A. 0.8%

B. 20.1%

C. 67.8%

D. 32.2%

Q-19. A call center receives an average of two phone calls per hour. The probability that they will receive 20 calls in an 8-hour day is closest to:

A. 5.59%

B. 16.56%

C. 3.66%

D. 6.40%

Q-20. Which of the following statements are TRUE?

I. The sum of two random normal variables is also a random normal variable.

II. The product of two random normal variables is also a random normal variable.

III. The sum of two random lognormal variables is also a random lognormal variable.

IV. The product of two random lognormal variables is also a random lognormal variable.

A. I and II only

B. II and III only

C. III and IV only

D. I and IV only

Q-21. Suppose that a quiz consists of 20 true-false questions. A student has not studied for the exam and just randomly guesses the answers. How would you find the probability that the student will get 8 or fewer answers correct?

A. Find the probability that X = 8 in a binomial distribution with n = 20 and p = 0.5.

B. Find the area between 0 and 8 in a uniform distribution that goes from 0 to 20.

C. Find the probability that X = 8 for a normal distribution with mean of 10 and standard deviation of 5.

D. Find the cumulative probability for 8 in a binomial distribution with n = 20 and p = 0.5.

Q-22. A portfolio manager holds three bonds in one of his portfolios and each has a 1-year default probability of 15%. The event of default for each of the bonds is independent. What is the mean and variance of the number of bonds defaulting over the next year?

A. Mean = 0.15, variance = 0.32

B. Mean = 0.45, variance = 0.38

C. Mean = 0.45, variance = 0.32

D. Mean = 0.15, variance = 0.38

Q-23. Assume that a random variable follows a normal distribution with a mean of 80 and a standard deviation of 24. What percentage of this distribution is between 32 and 116?

A. 4.56%

B. 8.96%

C. 13.36%

D. 91.04%

Q-24. The recent performance of Prudent Fund, with USD 50 million in assets, has been weak and the institutional sales group is recommending that it be merged with Aggressive Fund, a USD 200 million fund. The returns on Prudent Fund are normally distributed with a mean of 3% and a standard deviation of 7% and the returns on Aggressive Fund are normally distributed with a mean of 7% and a standard deviation of 15%. Senior management has asked you to estimate the likelihood that returns on the combined portfolio will exceed 26%. Assuming the returns on the two funds are independent, your estimate for the probability that the returns on the combined fund will exceed 26% is closest to:

A. 1.0%

B. 2.5%

C. 5.0%

D. 10.0%

2.6. The Central Limit Theorem

2.6.1. 重要知识点

2.6.1.1. Nominal, ordinal, interval, ratio scales

- If $x_1, x_2 \cdots x_n$ represent n independent identically distributed random variables with mean μ and a finite variance σ^2, regardless of the distribution of these n variables, as $n \to \infty$, the distribution of the sample mean $\overline{X} = \sum X_i / n$ is close to the normal distribution with mean μ and variance σ^2 / n .

2.6.2. 基础题

Q-25. If the mean P/E of 30 stocks in a certain industrial sector is 18 and the sample standard deviation is 3.5, standard error of the mean is CLOSEST to:

A. 0.12

B. 0.34

C. 0.64

D. 1.56

2.7. Confidence Intervals

2.7.1. 重要知识点

2.7.1.1. In general, we have:

$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ when the variance is known

$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ when the variance is unknown

2.7.2. 基础题

Q-26. For a sample of the past 30 monthly stock returns for McCreary, Inc., the mean return is 4% and the sample standard deviation is 20%. Since the population variance is unknown, the standard error of the sample is estimated to be:

$$S_X = \frac{20\%}{\sqrt{30}} = 3.65\%$$

The related t-table values are ($t_{i,j}$ denotes the (100-j)th percentile of t-distribution value with i degrees of freedom):

$t_{29,2.5\%}$	2.045
$t_{29,5.0\%}$	1.699
$t_{30,2.5\%}$	2.042
$t_{29,5.0\%}$	1.697

What is the 95% confidence interval for the mean monthly return?

A. [-3.453%, 11.453%]

B. [-2.201%, 10.201%]

C. [-2.194%, 10.194%]

D. [-3.464%, 11.464%]

Q-27. Using the prior 12 monthly returns, an analyst estimates the mean monthly return of stock XYZ to be -0.75% with a standard error of 2.70%.

ONE-TAILED T-DISTRIBUTION TABLE			
Degrees of Freedom	**α**		
	0.10	**0.05**	**0.025**
8	1.397	1.860	2.306
9	1.383	1.833	2.262

10	1.372	1.812	2.228
11	1.363	1.796	2.201
12	1.356	1.782	2.179

Using the t-table above, the 95% confidence interval for the mean return is between:

A. -6.69% and 5.19%

B. -6.63% and 5.15%

C. -5.60% and 4.10%

D. -5.56% and 4.06%

Q-28. A risk manager is examining a Hong Kong trader's profit and loss record for the last week, as shown in the table below:

Trading Day	Profit/Loss (HKD million)
Monday	10
Tuesday	80
Wednesday	90
Thursday	-60
Friday	30

The profits and losses are normally distributed with a mean of 4.5 million HKD and assume that transaction costs can be ignored. Part of the t-table is provided below:

Percentage Point of the t-Distribution

P(T > t) = α

	α		
Degrees of Freedom	0.3	0.2	0.15
4	0.569	0.941	1.19
5	0.559	0.92	1.16

According to the information provided above, what is the probability that this trader will record a profit of at least HKD 30 million on the first trading day of next week?

A. About 15%

B. About 20%

C. About 80%

D. About 85%

2.8. Hypothesis Testing

2.8.1. 重要知识点

2.8.1.1. The null hypothesis (H0) and alternative hypothesis (Ha)

- One-tailed test vs. Two-tailed test

- One-tailed test: H0: μ≥0 Ha: μ < 0 H0: μ≤0 Ha: μ > 0
- Two-tailed test: H0: μ = 0 Ha: μ≠0

2.8.1.2. Critical Value

- The distribution of test statistic (z, t, χ^2 , F)
- Significance level (α)
- One-tailed or two-tailed test

2.8.1.3. Summary of hypothesis

Test type	Assumptions	H_0	Test-statistic	distribution
Mean hypothesis testing	Normally distributed population, known population variance	$\mu = 0$	$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	N(0,1)
	Normally distributed population, unknown population variance	$\mu = 0$	$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$	t(n-1)
Variance hypothesis testing	Normally distributed population	$\sigma^2 = \sigma_0^2$	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2(n-1)$
	Two independent normally distributed populations	$\sigma_1^2 = \sigma_2^2$	$F = s_1^2 / s_2^2$	$F(n_1 - 1, n_2 - 1)$

2.8.2. 基础题

Q-29. Hedge Fund has been in existence for two years. Its average monthly return has been 6% with a standard deviation of 5%. Hedge Fund has a stated objective of controlling volatility as measured by the standard deviation of monthly returns. You are asked to test the null hypothesis that the volatility of Hedge Fund’s monthly returns is equal to 4% versus the alternative hypothesis that the volatility is greater than 4%. Assuming that all monthly returns are independently and identically normally distributed, and using the tables below, what is the correct test to be used and what is the correct conclusion at the 5% level of significance?

t Table: Inverse of the one-tailed probability of the Student’s t-distribution

Df	One-tailed Probability = 5.0%	One-tailed Probability 2.5%
22	1.717	2.074
23	1.714	2.069
24	1.711	2.064

Chi-Square Table: Inverse of the one-tailed probability of the Chi-Squared distribution

Df	One-tailed Probability = 5.0%	One-tailed Probability = 2.5%
22	33.9244	36.7807
23	35.1725	38.0757
24	36.4151	39.3641

A. t-test; reject the null hypothesis

B. Chi-square test; reject the null hypothesis

C. t-test; do not reject the null hypothesis

D. Chi-square test; do not reject the null hypothesis

Q-30. Based on 21 daily returns of an asset, a risk manager estimates the standard deviation of the asset's daily returns to be 2%. Assuming that returns are normally distributed and that there are 260 trading days in a year, what is the appropriate Chi-square test statistic if the risk manager wants to test the null hypothesis that the true annual volatility is 25% at a 5% significance level?

A. 25.80

B. 33.28

C. 34.94

D. 54.74

Q-31. Using a sample size of 61 observations, an analyst determines that the standard deviation of the returns from a stock is 21%. Using a 0.05 significance level, the analyst:

A. Can conclude that the standard deviation of returns is higher than 14%.

B. Cannot conclude that the standard deviation of returns is higher than 14%.

C. Can conclude that the standard deviation of returns is not higher than 14%.

D. None of the above.

Q-32. Bob tests the null hypothesis that the population mean is less than or equal to 45. From a population size of 3,000,000 people, 81 observations are randomly sampled. The corresponding sample mean is 46.3 and sample standard deviation is 4.5. What is the value of the most appropriate test statistic for the test of the population mean, and what is the correct decision at the 1 percent significance level?

A. z = 0.29, and fail to reject the null hypothesis.

B. z = 2.60, and reject the null hypothesis.

C. t = 0.29, and accept the null hypothesis.

D. t = 2.60, and neither reject nor fail to reject the null hypothesis.

Q-33. An analyst wants to test whether the standard deviation of return from pharmaceutical stocks is lower than 0.2. For this purpose, he obtains the following data from a sample of 30 pharmaceutical stocks. Mean return from pharmaceutical stocks = 8%. Standard deviation of return from pharmaceutical stocks = 12%. Mean return from the market = 12%. Standard deviation of return from the market = 16%. What is the appropriate test statistic for this test?

A. t-statistic

B. z-statistic

C. F-statistic

D. χ^2 statistic

2.9. Best Linear Unbiased Estimator

2.9.1. 重要知识点

2.9.1.1. Unbiasedness、Efficiency、Consistency、Linearity

2.9.1.2. The OLS estimator is BLUE.

2.9.2. 基础题

Q-34. If the variance of the sampling distribution of an estimator is smaller than all other unbiased estimators of the parameter of interest, the estimator is:

A. Reliable

B. Efficient

C. Unbiased

D. Consistent

Q-35. Analyst Rob has identified an estimator, denoted T(.), which qualifies as the best linear unbiased estimator (BLUE). If T(.) is BLUE, which of the following must also necessarily be TRUE?

A. T(.) must have the minimum variance among all possible estimators.

B. T(.) must be the most efficient (the "best") among all possible estimators.

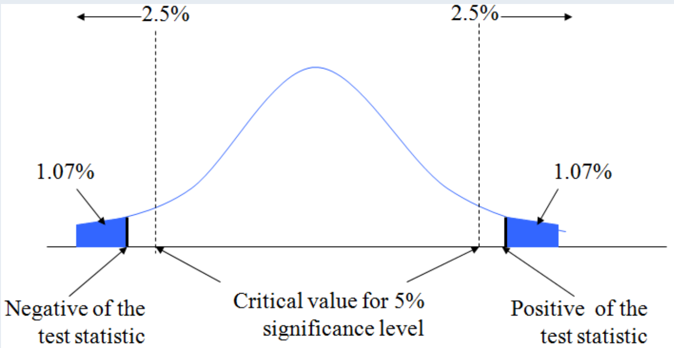
C. It is possible that T(.) is the maximum likelihood (MLE) estimator of variance; i.e., SUM([X - average (X)]^2)/(n-1).

D. Among the class of unbiased estimators that are linear, T(.) has the smallest variance.

2.10. P-value Testing

2.10.1. 重要知识点

2.10.1.1. P-value Testing:



Alpha=5%
P-value=2.14%

- **Decision rule:**
  - If P-value<alpha, we will reject the null hypothesis
  - If P-value>=alpha, we will not reject the null hypothesis

2.10.2. 基础题

Q-36. Which of the following statements regarding hypothesis testing is correct?

A. Type II error refers to the failure to reject the H_1 when it is actually false.

B. Hypothesis testing is used to make inferences about the parameters of a given population on the basis of statistics computed for a sample that is drawn from another population.

C. All else being equal, the decrease in the chance of making a Type I error comes at the cost of increasing the probability of making a Type II error.

D. If the p-value is greater than the significance level, then the statistics falls into the reject intervals.

2.11. Type I & Type II errors

2.11.1. 重要知识点

2.11.1.1. The P (Type I error) equals to the significance level α.

	True Condition	
Decision	H_0 is true	H_0 is false
Do not reject H_0	Correct decision	Incorrect decision **Type II error**
Reject H_0	Incorrect decision **Type I error** Significance level, α, =P(Type I error)	Correct decision Power of the test =1- P(Type II error)

- Given the sample size, Type I and II errors cannot be reduced simultaneously.

2.11.2. 基础题

Q-37. When testing a hypothesis, which of the following statements is correct when the level of significance of the test is decreased?

A. The likelihood of rejecting the null hypothesis when it is true decreases.

B. The likelihood of making a Type I error increases.

C. The null hypothesis is rejected more frequently, even when it is actually false.

D. The likelihood of making a Type II error decreases.

Q-38. An oil industry analyst with a large international bank has constructed a sample of 1,000 individual firms on which she plans to perform statistical analyses. She considers either decreasing the level of significance used to test hypotheses from 5% to 1%, or removing 500 state-run firms from her sample. What impact will these changes have on the probability of making Type I and Type II errors?

	Level of significance decrease	Reduction in sample size
A.	P(Type I error) increases	P(Type I error) increases
B.	P(Type I error) decreases	P(Type II error) increases
C.	P(Type II error) increases	P(Type I error) decreases
D.	P(Type II error) decreases	P(Type II error) decreases

Q-39. According to the Basel back-testing framework guidelines, penalties start to apply if there are five or more exceptions during the previous year. The Type I error rate of this test is 11 percent. If the true coverage is 97 percent of exceptions instead of the required 99 percent, the power of the test is 87 percent. This implies that there is a (an):

A. 89% probability regulators will reject the correct model.

B. 11% probability regulators will reject the incorrect model.

C. 87% probability regulators will not reject the correct model.

D. 13% probability regulators will not reject the incorrect model.

2.12. Regression & Variance Analysis

2.12.1. 重要知识点

2.12.1.1. Simple Linear Regression:

$$Y_i = B_0 + B_1 \times X_i + \varepsilon_i$$

2.12.1.2. Ordinary least squares (OLS):

$$\text{minimize} \sum e_i^2 = \sum [Y_i - (b_0 + b_1 \times X_i)]^2$$

$$\text{minimize} \sum e_i^2 = \sum [Y_i - (b_0 + \sum_{i=1}^{k} b_i \times X_i)]^2$$

2.12.1.3. The Assumptions of Classical Linear Regression Model:

- A linear relationship exists between X and Y;
- X is uncorrelated with the error term;
- The expected value of the error term is zero;
- The variance of the error term is constant (i.e., the error terms are homoskedastic);
- The error term is uncorrelated across observations;
- The error term is normally distributed.

2.12.1.4. Regression Assumption Violations:

- Heteroskedasticity occurs when the variance of the residuals is not rho same across all observations in the sample.
- Multicollinearity refers to the condition when two or more of the independent variables, or linear combinations of the independent variables, in a multiple regression are highly correlated with each other.
- Omitted variable bias occurs when the omitted variable is correlated with the included regressor and is a determinant of the dependent variable.
- Serial correlation refers to the situation in which the residual terms are correlated with one another.

2.12.1.5. Analysis of Variance (ANOVA) Table:

	df	SS	MSS
Regression	k	ESS	ESS/k
Residual	n-k-1	RSS	RSS/(n-k-1)
Total	n-1	TSS	-

Total sum of squares = explained sum of squares + sum of squared residuals

$$\sum (Y_i - \overline{Y})^2 = \sum (\hat{Y} - \overline{Y})^2 + \sum (Y_i - \hat{Y})^2$$

$$\text{TSS} = \text{ESS} + \text{SSR}$$

$$R^2 = \frac{\text{ESS}}{\text{TSS}} = 1 - \frac{\text{SSR}}{\text{TSS}} \quad ; \quad r = \pm\sqrt{R^2}$$

2.12.1.6. Adjusted R-Squared:

$$\text{Adjusted } R^2 = 1 - (1 - R^2) \times \frac{n-1}{n-k-1}$$

2.12.1.7. Multiple Linear Regressions:

$$Y_i = B_0 + B_1 \times X_{1,i} + B_2 \times X_{2,i} + \varepsilon_i$$

2.12.2. 基础题

Q-40. Samantha Xiao is trying to get some insight into the relationship between the return on stock LMD ($\mathbf{R_{LMD,t}}$) and the return on the S&P 500 index ($\mathbf{R_{S\&P,t}}$). Using historical data she estimates the following:

Annual mean return for LMD	11%
Annual mean return for S&P 500 index	7%
Annual volatility for S&P 500 index	18%
Covariance between the returns of LMD and S&P 500 index	6%

Assuming she uses the same data to estimate the regression model given by:

$$R_{LMD,t} = \alpha + \beta R_{S\&P,t} + \varepsilon_t$$

Using the ordinary least squares technique, which of the following models will she obtain?

A. $R_{LMD,t} = -0.02 + 0.54 R_{S\&P,t} + \varepsilon_t$

B. $R_{LMD,t} = -0.02 + 1.85 R_{S\&P,t} + \varepsilon_t$

C. $R_{LMD,t} = 0.04 + 0.54 R_{S\&P,t} + \varepsilon_t$

D. $R_{LMD,t} = 0.04 + 1.85 R_{S\&P,t} + \varepsilon_t$

Q-41. For a sample of 400 firms, the relationship between corporate revenue (Yi) and the average years of experience per employee (Xi) is modeled as follows:

$$Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i,\ i = 1, 2\ldots, 400$$

You wish to test the joint null hypothesis that $\beta_1 = 0$ and $\beta_2 = 0$ at the 95% confidence level. The p-value for the t-statistic for β_1 is 0.07, and the p-value for the t-statistic for β_2 is 0 06. The p-value for the F-statistic for the regression is 0.045. Which of the following statements is correct?

A. You can reject the null hypothesis because each β is different from 0 at the 95% confidence level.

B. You cannot reject the null hypothesis because neitherβis different from 0 at the 95% confidence level.

C. You can reject the null hypothesis because the F-statistic is significant at the 95% confidence level.

D. You cannot reject the null hypothesis because the F-statistic is not significant at the 95%

confidence level.

Q-42. An analyst is testing a hypothesis that the beta, β, of stock CDM is 1. The analyst runs an ordinary least squares regression of the monthly returns of CDM, RCDM, on the monthly returns of the S&P 500 index, Rm, and obtains the following relation:

$$R_{CDM} = 0.86\ R_m - 0.32$$

The analyst also observes that the standard error of the coefficient of R_m is 0.80. In order to test the hypothesis H_0: $\beta = 1$ against H_1: $\beta \neq 1$, what is the correct statistic to calculate?

A. t-statistic

B. Chi-square test statistic

C. F test statistic

D. Sum of squared residuals

Q-43. The proper selection of factors to include in an ordinary least squares estimation is critical to the accuracy of the result. When does omitted variable bias occur?

A. Omitted variable bias occurs when the omitted variable is correlated with the included regressor and is a determinant of the dependent variable.

B. Omitted variable bias occurs when the omitted variable is correlated with the included regressor but is not a determinant of the dependent variable.

C. Omitted variable bias occurs when the omitted variable is independent of the included regressor and is a determinant of the dependent variable.

D. Omitted variable bias occurs when the omitted variable is independent of the included regressor but is not a determinant of the dependent variable.

Q-44. Which of the following is assumed in the multiple least squares regression model?

A. The dependent variable is stationary.

B. The independent variables are not perfectly multicollinear.

C. The error terms are heteroskedastic.

D. The independent variables are homoskedastic.

Q-45. Which of the following statements about the ordinary least squares regression model (or simple regression model) with one independent variable are correct?

I. In the ordinary least squares (OLS) model, the random error term is assumed to have zero mean and constant variance.

II. In the OLS model, the variance of the independent variable is assumed to be positively correlated with the variance of the error term.

III. In the OLS model, it is assumed that the correlation between the dependent variable and the random error term is zero.

IV. In the OLS model, the variance of the residuals is assumed to be constant.

A. I, II, III and IV

B. II and IV only

C. I and IV only

D. I, II, and III only

Use the following information to answer the following question .

Regression Statistics

R squared	0.8537
R sq. adj.	0.8120
Std. error	10.3892
Num obs.	10

ANOVA

	df	**SS**	**MS**	**F**	**P-value**
Explained	2	4410.4500	2205.2250	20.4309	0.0012
Residual	7	755.5500	107.9357		
Total	9	5166.0000			

	Coefficients	**Std. Error**	**t-Stat**	**P-value**
Intercept	35.5875	6.1737	5.7644	0.0007
X_1	1.8563	1.6681	1.1128	0.3026
X_2	7.4250	1.1615	6.3923	0.0004

Q-46. Based on the results and a 5% level of significance, which of the following hypotheses can be rejected?

I. H_0: $B_0 = 0$

II. H_0: $B_1 = 0$

III. H_0: $B_2 = 0$

IV. H_0: $B_1 = B_2 = 0$

A. I, II, and III

B. I and IV

C. III and IV

D. I, III, and IV

Q-47. Paul Graham, FRM® is analyzing the sales growth of a baby product launched three years ago by a regional company. He assesses that three factors contribute heavily towards the growth and comes up with the following results:

$Y = b + 1.5 X_1 + 1.2X_2 + 3X_3$

Sum of Squared Regression [SSR] = 869.76

Sum of Squared Errors [SEE] = 22.12

Determine what proportion of sales growth is explained by the regression results.

A. 0.36

B. 0.98

C. 0.64

D. 0.55

Q-48. Many statistical problems arise when estimating relationships using regression analysis. Some of these problems are due to the assumptions behind the regression model. Which one of the following is NOT one of these problems?

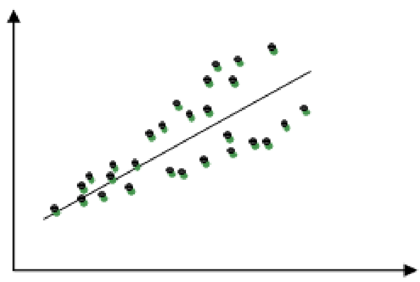
A. Stratification

B. Multicollinearity

C. Heteroscedasticity

D. Autocorrelation

Q-49. An analyst is performing a regression. The dependent variable is portfolio return while the independent variable is the years of experience of the portfolio manager. In his analysis, the resulting scatter plot is as follow:



The analyst can conclude that the portfolio returns exhibit:

A. Heteroskedasticity

B. Homoskedasticity

C. Perfect multicollinearity

D. Non-perfect multicollinearity

Q-50. A regression of a stock's return (in percent) on an industry index's return (in percent) provides the following results:

	Coefficient	Standard Error
Intercept	2.1	2.01
Industry index	1.9	0.31

	Degrees of Freedom	SS
Explained	1	92.648
Residual	3	24.512
Total	4	117.160

Which of the following statements regarding the regression is correct?

I. The correlation coefficient between the X and Y variables is 0.889.

II. The industry index coefficient is significant at the 99% confidence interval.

III. If the return on the industry index is 4%, the stock's expected return is 10.3%.

IV. The variability of industry returns explains 21% of the variation of company returns.

A. III only

B. I and II only

C. II and IV only

D. I, II, and IV

Q-51. An analyst is given the data in the following table for a regression of the annual sales for Company XYZ, a maker of paper products, on paper product industry sales.

Parameters	Coefficient	Standard Error of the Coefficient
Intercept	-94.88	32.97
Slope (industry sales)	0.2796	0.0363

The correlation between company and industry sales is 0.9757. Which of the following is closest to the value and reports the most likely interpretation of the R^2?

A. 0.048, indicating that the variability of industry sales explains about 4.8% of the variability of company sales.

B. 0.048, indicating that the variability of company sales explains about 4.8% of the variability of industry sales.

C. 0.952, indicating that the variability of industry sales explains about 95.2% of the variability of company sales.

D. 0.952, indicating that the variability of company sales explains about 95.2% of the variability of industry sales.

Q-52. A risk manager performs an ordinary least squares (OLS) regression to estimate the sensitivity of a stock's return to the return on the S&P 500. This OLS procedure is designed to:

A. Minimize the square of the sum of differences between the actual and estimated S&P 500 returns.

B. Minimize the square of the sum of differences between the actual and estimated stock returns.

C. Minimize the sum of differences between the actual and estimated squared S&P 500 returns.

D. Minimize the sum of squared differences between the actual and estimated stock returns.

Q-53. Using data from a pool of mortgage borrowers, a credit risk analyst performed an ordinary least squares regression of annual savings (in GBP) against annual household income (in GBP) and obtained the following relationship:

Annual Savings = 0.24 * Household Income - 25.66, $R^2 = 0.50$

Assuming that all coefficients are statistically significant, which interpretation of this result is correct?

A. For this sample data, the average error term is GBP -25.66.

B. For a household with no income, annual savings is GBP 0.

C. For an increase of GBP 1,000 in income, expected annual savings will increase by GBP 240.

D. For a decrease of GBP 2,000 in income, expected annual savings will increase by GBP 480.

Q-54. A risk manager has estimated a regression of a firm's monthly portfolio returns against the returns of three U.S. domestic equity indexes: the Russell 1000 index, the Russell 2000 index, and the Russell 3000 index. The results are shown below.

Regression Statistics	
Multiple R	0.9
R Square	0.9
Adjusted R Square	0.9
Standard Error	0.0
Observations	192

Regression Output	Coefficients	Standard Error	t-Stat	P-value
Intercept	0.0023	0.0006	3.530	0.0005
Russell 1000	0.1093	1.5895	0.068	0.9452

Russell 2000	0.1055	0.1384	0.762	0.4470
Russell 3000	0.3533	1.7274	0.204	0.8382

Correlation Matrix	Portfolio Returns	Russell 1000	Russell 2000	Russell 3000
Portfolio	1.000			
Russell 1000	0.937	1.000		
Russell 2000	0.856	0.813	1.000	
Russell 3000	0.945	0.998	0.845	1.000

Based on the regression results, which statement is correct?

A. The estimated coefficient of 0.3533 indicates that the returns of the Russell 3000 index are more statistically significant in determining the portfolio returns than the other two indexes.

B. The high adjusted R^2 indicates that the estimated coefficients on the Russell 1000, Russell 2000, and Russell 3000 indexes are statistically significant.

C. The high p-value of 0.9452 indicates that the regression coefficient of the returns of Russell 1000 is more statistically significant than the other two indexes.

D. The high correlations between each pair of index returns indicate that multicollinearity exists between the variables in this regression.

2.13. White Noise

2.13.1. 重要知识点

2.13.1.1. White Noise 掌握概念：

- If we want to forecast a series, we'd like its mean and covariance structure to be stable over time, in which case the series is covariance stationary.
- Many economic, business, financial, and government series are not covariance stationary. For example, many series that are clearly non-stationary in levels appear covariance stationary in growth rates.
- The autocorrelations are just the "simple" or "regular" correlations between y(t) and y(t-T) The partial autocorrelations measure the association between y(t) and y(t-τ) after controlling for the effects of y(t-1),..., y(t-T + 1); they measure the partial correlation between y(t) and y(t-τ).
- We use y to denote the observed series.

$$y_t = \varepsilon_t$$
$$\varepsilon_t \sim N\left(0, \sigma^2\right)$$

- Where the "shock" is uncorrelated over time. We say y(t) is serially uncorrelated. Such a process, with zero mean, constant variance, and no serial correlation, is called zero-mean white noise, or simply white noise.

2.13.2. 基础题

Q-55. In regard to white noise, each of the following statements is true except?

A. If a process is zero-mean white noise, then is must be Gaussian white noise.

B. If a process is Gaussian (aka, normal) white noise, then it must be (zero-mean) white noise.

C. If a process is Gaussian (aka, normal) white noise, then it must be independent white noise.

D. If a process is stationary, has zero mean, has constant variance and it serially uncorrelated, then the process is white noise.

2.14. Box-Pierce Q-statistic & Ljung-Box Q-statistic

2.14.1. 重要知识点

2.14.1.1. Box-Pierce Q-statistic & Ljung-Box Q-statistic

- H_0: All the correlation observed in the series are independent of each other and hence Autocorrelation of the series is zero.
- For Box-Pierce Q-statistic, the formula used is:

$$Q_{BP} = T\sum_{\tau=1}^{m}\hat{\rho}^2(\tau)$$

- Whereas in case of Ljung-Box Q-statistic, the test statistic is derived as:

$$Q_{LB} = T(T+2)\sum_{\tau=1}^{m}\hat{\rho}^2(\tau)\left(\frac{1}{T-\tau}\right)$$

- Where:
- T = Sample size
- $\hat{\rho}(\tau) =$ Sample autocorrelation function for τ lags
- m = number of lags under observation

2.14.2. 基础题

Q-56. For a certain time series, you have produced a correlogram with an autocorrelation function that includes twenty four monthly observations; m = degrees of freedom = 24. Your calculated Box-Pierce Q-statistic is 19.50 and your calculated Ljung-Box Q-statistic is 27.90. You want to determine if the series is white noise. Which is your best conclusion (given CHISQ.INV(0.95, 24) = 36.41)?

A. With 95% confidence, you accept the series as white noise (more accurately, you fail to reject the null).

B. With 95% confidence, you accept the series as partial white noise (due to Box-Pierce) but reject the null (due to Ljung-Box).

C. With 95% confidence, you reject both null hypotheses and conclude the series is not white noise.

D. With 95% confidence, you reject both null hypotheses but conclude the series is white noise because the sum of the statistics is greater than the critical value.

2.15. Modeling and Forecasting Trend

2.15.1. 重要知识点

2.15.1.1. Modeling and Forecasting Trend

- One of the ways of selecting best fit model is by estimating the Mean Squared Error (MSE) of the model. The model with least MSE would be chosen for fitting the data series. MSE is computed as:

$$\mathrm{MSE} = \frac{\sum_{t=1}^{T} e_t^2}{T}$$

- As degree of freedom represents the choice of freely selecting the variables during the model fitting exercise therefore, to reduce the MSE bias, the degree of freedom must be deducted from the sample size to arrive at adjusted MSE, commonly referred as S2.

$$S^2 = \frac{\sum_{t=1}^{T} e_t^2}{T-k}$$

- Apart from S2, the other model selection criteria also resort to the technique of degree of freedom penalty for out of sample prediction. Those criterions are Akaike Information Criteria (AIC) & Schwarz Information Criterion (SIC), which are given as follows:

$$\mathrm{AIC} = e^{\left(\frac{2k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$$

$$\mathrm{SIC} = T^{\left(\frac{k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$$

- The penalty factors for s^2, Akaike information criterion (AIC), and Schwarz information criterion (SIC) are (T/T – k), $e^{(2k/T)}$, and $T^{(k/T)}$, respectively. **SIC has the largest penalty factor.**

2.15.1.2. Choice between the four indicators:

- We evaluate model selection criteria in terms of a key property called consistency.
  - As the sample size gets large, the chosen measure will choose the true model correctly or with the biggest probability.
  - SIC is consistency while others are not.

If the no model discussed has the property of consistency. We're then led to a different optimality property, called asymptotic efficiency.

- As the sample size gets large, an asymptotically efficient model selection criterion chooses the model has the fastest speed to approach to the true error variance.
- The AIC, although inconsistent, is asymptotically efficient, whereas the SIC is not.

2.15.2. 基础题

Q-57. Richard Frank, FRM, is running a regression model to forecast in-sample data. He is concerned about data mining and over-fitting the data. Which of the following criteria provides the highest penalty factor based on degrees of freedom?

A. Mean squared error (MSE)

B. Unbiased mean squared error (s^2)

C. Akaike information criterion (AIC)

D. Schwarz information criterion (SIC)

Q-58. Which of the following criteria is consistency ?

A. Mean squared error (MSE)

B. Unbiased mean squared error (s^2)

C. Akaike information criterion (AIC)

D. Schwarz information criterion (SIC)

Q-59. Which of the following criteria is symptotically efficient ?

A. Mean squared error (MSE)

B. Unbiased mean squared error (s^2)

C. Akaike information criterion (AIC)

D. Schwarz information criterion (SIC)

2.16. Modeling Cycles: MA, AR, and ARMA Models

2.16.1. 重要知识点

2.16.1.1. AR(1) model

$$y_t = \varphi y_{t-1} + \varepsilon_t;\ \varepsilon_t \sim WN(0,\sigma^2)$$
$$(1-\varphi L)y_t = \varepsilon_t$$

- The AR(1) model is capable of capturing much more persistent dynamics than is the MA(1). is the condition for covariance stationary in the AR(1).

- If ϕ is positive, the autocorrelation decay is one-sided. If ϕ is negative, the decay involves back-and-forth oscillations.

2.16.1.2. AR(p) model

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \ldots + \varphi_p y_{t-p} + \varepsilon_t; \varepsilon_t \sim WN(0,\sigma^2)$$
$$\Phi(L)y_t = (1-\varphi_1 L - \varphi_2 L^2 + \ldots - \varphi_p L^p)y_t = \varepsilon_t$$

2.16.1.3. MA(1) model

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1} = (1+\theta L)\varepsilon_t; \varepsilon_t \sim WN(0,\sigma^2)$$

- The current value of the observed series is expressed as a function of current and lagged unobservable shocks.
- MA(1) process with parameter θ = 0.95 varies a bit more than the process with a parameter of θ = 0.4.

2.16.1.4. MA(q) model

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} = \Theta(L)\varepsilon_t; \varepsilon_t \sim WN(0,\sigma^2)$$
$$\Theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$$

2.16.1.5. ARMA(p,q) model

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \ldots + \varphi_p y_{t-p} + \theta\varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t; \varepsilon_t \sim WN(0,\sigma^2)$$

- ARMA models are often both highly accurate and highly parsimonious.

2.16.2. 基础题

Q-60. Which of the following statements is correct regarding the usefulness of an autoregressive (AR) process and an autoregressive moving average (ARMA) process when modeling seasonal data?

I. They both include lagged terms and, therefore, can better capture a relationship in motion.

II. They both specialize in capturing only the random movements in time series data.

A. I only.

B. II only.

C. Both I and II.

D. Neither I nor II.

2.17. Modeling and Forecasting Seasonality

2.17.1. 重要知识点

2.17.1.1. The pure seasonal dummy model is:

$$y_t = \sum_{i=1}^{s} \gamma_i D_{it} + \varepsilon_t$$

2.17.1.2. Trend may be included as well, in which case the model is

$$y_t = \beta_1 TIME_t + \sum_{i=1}^{s} \gamma_i D_{it} + \varepsilon_t$$

2.17.2. 基础题

Q-61. Winnie is an analyst in the retail industry. She is modeling a company's sales over time and has noticed a quarterly seasonal pattern. If Winnie includes an intercept term in her model, how many dummy variables should she use to model the seasonality component?

A. 2.

B. 3.

C. 4.

D. 5.

Q-62. Consider the following regression equation utilizing dummy variables for explaining quarterly SALES in terms of the quarter of their occurrence:

$SALES_t = \beta_0+\beta_2 D_{2,t}+\beta_3 D_{3,t}+\beta_4 D_{4,t} + e_t$

where:

SALES=a quarterly observation of EPS

$D_{2,t}$= 1if period t is the second quarter, $D_{2,t}$= 0 otherwise

$D_{3,t}$=1 if period t is the third quarter, $D_{3,t}$= 0 otherwise

$D_{4,t}$ = 1 if period t is the fourth quarter, $D_{4,t}$ = 0 otherwise

The intercept term β_0 represents the average value of sales for the:

A. First quarter

B. Second quarter

C. Third quarter.

D. Fourth quarter.

Q-63. Which of the following scenarios would produce a forecasting model that exhibits perfect multicollinearity? A model that includes:

I Only one seasonal dummy that is equal to 1.

II A holiday variation variable that accounts for an “Easter dummy variable.”

III A trading-day variation variable for modeling trading volume throughout the year.

IV A dummy variable for each season, plus an intercept.

A. II only.

B. I and III.

C. IV only.

D. All not.

2.18. Monte Carlo Simulation

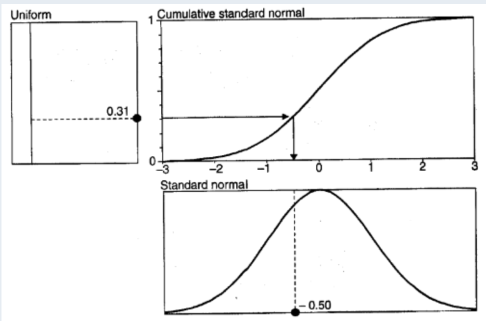
2.18.1. 重要知识点

2.18.1.1. GBM: $\Delta S_t = S_t(\mu\Delta t + \sigma\varepsilon\sqrt{\Delta t})$

Monte Carlo Simulation	
Basic Steps	☐ Specify the data generating process. ☐ Estimate an unknown variable. ☐ Save the estimate from step 2. ☐ Go back to step 1 and repeat this process N times.
Reducing Standard Error $\frac{s}{\sqrt{N}}$	☐ The standard error estimate of a Monte Carlo simulation can be reduced by a factor of 10 by increasing N by a factor of 100. ☐ Variance reduction technique • Antithetic Variates • Control Variates • Random Number Re-Usage across Experiments

Variance Reduction Technique	
Antithetic Variates	☐ Reduces sampling error by rerunning the simulation using a complement set of the original set of random variables.
Control Variates	☐ Replaces a variable x that has unknown properties in a Monte Carlo simulation with a similar variable y that has known properties. The new x* variable estimate will have a smaller sampling error than the original x variable if the control statistic and statistic of interest are highly correlated.
Random Number Re-Usage	☐ Reusing sets of random number draws across Monte Carlo experiments reduces the estimate variability across experiments.

2.18.1.2. Inverse Transform Method:



2.18.2. 基础题

Q-64. A portfolio manager has asked each of four analysts to use Monte Carlo simulation to price a path-dependent derivative contract on a stock. The derivative expires in nine months and the risk-free rate is 4% per year compounded continuously. The analysts generate a total of 20,000 paths using a geometric Brownian motion model, record the payoff for each path, and present the results in the table shown below.

Analyst	Number of Paths	Average Derivative Payoff per Path (USD)
1	2,000	43
2	4,000	44
3	10,000	46
4	4,000	45

What is the estimated price of the derivative?

A. USD 43.33

B. USD 43.77

C. USD 44.21

D. USD 45.10

Q-65. Which of the following statements about Monte Carlo simulation is incorrect?

A. Correlations among variables can be incorporated into a Monte Carlo simulation.

B. Monte Carlo simulations can handle time-varying volatility.

C. Monte Carlo methods can be used to estimate value-at-risk (VaR) but cannot be used to price options.

D. For estimating VaR, Monte Carlo methods generally require more computing power than historical simulations.

Q-66. Consider a stock that pays no dividends, has a vol. of 30% per annum, and provide an expected return of 15% per annum with continuous compounding. The stock price movements follow GBM. Consider a time interval of 1 week and the initial stock price is 100, then the stock price increase has a normal distribution with:

A. Mean = 0.268%, standard deviation = 4.03%

B. Mean = 0.278%, standard deviation = 4.13%

C. Mean = 0.288%, standard deviation = 4.16%

D. Mean = 0.288%, standard deviation = 4.27%

Q-67. Consider a stock that pays no dividends, has a volatility of 25% per annum and an expected return of 13% per annum. Suppose that the current share price of the stock, S0, is

USD 30. You decide to model the stock price behavior using a discrete-time version of geometric Brownian motion and to simulate paths of the stock price using Monte Carlo simulation. Let Δt denote the time interval used and let St denote the stock price at time interval t. So, according to your model, $S_{t+1} = S_t \times (1 + 0.13 \times \Delta t + 0.25 \times \sqrt{\Delta t} \times \varepsilon)$, where ε is a standard normal variable.

To implement this simulation, you generate a path of the stock price by starting at t = 0, generating a sample for ε updating the stock price according to the model, incrementing t by 1, and repeating this process until the end of the horizon is reached. Which of the following strategies for generating a sample for Δ will implement this simulation properly?

A. Generate a sample for ε by using the inverse of the standard normal cumulative distribution of a sample value drawn from a uniform distribution between 0 and 1.

B. Generate a sample for ε by sampling from a normal distribution with mean 0.13 and standard deviation 0.25.

C. Generate a sample for ε by using the inverse of the standard normal cumulative distribution of a sample value drawn from a uniform distribution between 0 and 1. Use Cholesky decomposition to correlate this sample with the sample from the previous time interval.

D. Generate a sample for ε by sampling from a normal distribution with mean 0.13 and standard deviation 0.25. Use Cholesky decomposition to correlate this sample with the sample from the previous time interval.

Q-68. A risk manager has been requested to provide some indication of accuracy of a Monte Carlo simulation. Using 1,000 replications of a normally distributed variable S, the relative error in the one-day 99% VaR is 5%. Under these conditions:

A. Using 1,000 replications of a long option position on S should create a larger relative error.

B. Using 10,000 replications should create a larger relative error.

C. Using another set of 1,000 replications will create an exact measure of 5.0% for relative error.

D. Using 1,000 replications of a short option position on S should create a larger relative error.

2.19. The Bootstrap

2.19.1. 重要知识点

2.19.1.1. An alternative to generating random numbers from a hypothetical distribution is to Sample from historical data. $S_{t+1} = S_t\left[1 + R_{m(1)}\right]$

- **Advantage of bootstrap:**
  - Can include fat tails, jumps, or any departure from the normal distribution.
  - Account for correlations across series because one draw consists of the simultaneous returns for N series, such as stock, bonds, and currency prices.
- **Limitation of bootstrap:**
  - For small sample sizes, it may be a poor approximation of the actual one.
  - Relies heavily on the assumption that returns are independent.

2.19.2. 基础题

Q-69. Which of the following statements about simulation is invalid?

A. The historical simulation approach is a nonparametric method that makes no specific assumption about the distribution of asset returns.

B. When simulating asset returns using Monte Carlo simulation, a sufficient number of trials must be used to ensure simulated returns are risk neutral.

C. Bootstrapping is an effective simulation approach that naturally incorporates correlations between asset returns and non-normality of asset returns, but does not generally capture autocorrelation of asset returns.

D. Monte Carlo simulation can be a valuable method for pricing derivatives and examining asset return scenarios.

2.20. GARCH(1,1) & EWMA

2.20.1. 重要知识点

2.20.1.1. EWMA Model:

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$

2.20.1.2. GARCH (1, 1) Model:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

2.20.1.3. Correlation Estimation:

$$\hat{\rho}_{XY} = \frac{\text{cov}_n}{\sigma_{x,n}\sigma_{y,n}}$$

$$\text{cov}_n = \lambda\,\text{cov}_{n-1} + (1-\lambda)x_{n-1}y_{n-1}$$
$$\text{cov}_n = \omega + \alpha x_{n-1}y_{n-1} + \beta\,\text{cov}_{n-1}$$

2.20.2. 基础题

Q-70. The GARCH model is useful for simulating asset returns. Which of the following statements about this model is FALSE?

A. The Exponentially Weighted Moving Average (EWMA) approach of RiskMetrics is a particular case of a GARCH process.

B. The GARCH allows for time-varying volatility.

C. The GARCH can produce fat tails in the return distribution.

D. The GARCH imposes a positive conditional mean return.

Q-71. Suppose that the current daily volatilities of asset X and asset Y are 1.0% and 1.2%, respectively. The prices of the assets at close of trading yesterday were $30 and $50 and the estimate of the coefficient of correlation between the returns on the two assets made at this time was 0.50. Correlations and volatilities are updated using a GARCH (1, 1) model. The estimates of the model's parameters are α = 0.04 and β = 0.94. For the correlation ω = 0.000001, and for the volatilities ω = 0.000003. If the prices of the two assets at close of trading today are $31 and $51, how is the correlation estimate updated?

A. 0.539

B. 0.549

C. 0.559

D. 0.569

Q-72. Which of the following GARCH models will take the shortest time to revert to its mean?

A. $h_t = 0.05 + 0.03r_{t-1}^2 + 0.96h_{t-1}$

B. $h_t = 0.03 + 0.02r_{t-1}^2 + 0.95h_{t-1}$

C. $h_t = 0.02 + 0.01r_{t-1}^2 + 0.97h_{t-1}$

D. $h_t = 0.01 + 0.01r_{t-1}^2 + 0.98h_{t-1}$

Q-73. The current estimate of daily volatility is 1.5%. The closing price of an asset yesterday was $30.00. The closing price of the asset today is $30.50. Using the EWMA (Exponentially Weighted Moving Average) model (with λ = 0.94), the updated estimate of volatility is:

A. 1.5096%

B. 1.5085%

C. 1.5092%

D. 1.5083%

Q-74. Given λ of 0.94, under an infinite series, what is the weight assigned to the seventh prior daily squared return?

A. 4.68%

B. 4.40%

C. 4.14%

D. 3.89%

Q-75. A risk analyst is estimating the variance of stock returns on day n, given by σ_n^2, using the equation: $\sigma_n^2 = \gamma V_L + \alpha\mu_{n-1}^2 + \beta\sigma_{n-1}^2$

Where μ_{n-1} and σ_{n-1} represent the return and volatility on day n-1, respectively. If the values of α and β are as indicated below, which combination of values indicates that the variance follows a stable GARCH (1,1) process?

A. α = 0.084427 and β = 0.909073

B. α = 0.084427 and β = 0.925573

C. α = 0.084427 and β = 0.925573

D. α = 0.090927 and β = 0.925573

Q-76. Which of the following four statements on models for estimating volatility is INCORRECT?

A. In the exponentially weighted moving average (EWMA) model, some positive weight is assigned to the long-run average variance.

B. In the EWMA model, the weights assigned to observations decrease exponentially as the observations become older.

C. In the GARCH (1,1) model, a positive weight is estimated for the long-run average variance.

D. In the GARCH (1,1) model, the weights estimated for observations decrease exponentially as the observations become older.

Q-77. Which of the following statements about the exponentially weighted moving average (EWMA) model and the generalized autoregressive conditional heteroscedasticity (GARCH(1,1)) model is correct?

A. The EWMA model is a special case of the GARCH(1,1) model with the additional assumption that the long-run volatility is zero.

B. A variance estimate from the EWMA model is always between the prior day's estimated variance and the prior day's squared return.

C. The GARCH(1,1) model always assigns less weight to the prior day's estimated variance than the EWMA model.

D. A variance estimate from the GARCH(1,1) model is always between the prior day's estimated variance and the prior day's squared return.

2.21. Copula

2.21.1. 重要知识点

2.21.1.1. Copula

- Interdependence of returns of two or more assets is usually calculated using the correlation coefficient, which only works well with normal distributions, whereas in practice, distributions in financial markets are mostly skewed.
- Copulas are used to describe the dependence between random variables. A copula is a multivariate probability distribution for which the marginal probability distribution of each variable is uniform.
- Gaussian Copula: To use Gaussian copula, variables having marginal distributions are mapped into new variables that have standard normal distributions in order to arrive at a bivariate normal joint distribution.
- Student-t copula: This is very similar to Gaussian copula, with the exception that variables are assumed to have a bivariate Student-t distribution rather than a bivariate normal distribution.
- Tail dependence is the tendency for extreme values for two variables to occur together. Tail dependence of a pair of random variables describes their co-movements in the tails of the distributions. The choice of the copula affects tail dependence. Tail dependence is higher in a bivariate Student t-distribution than in a bivariate normal distribution.

2.21.2. 基础题

Q-78. Consider the following three statements about tail dependence:

I. Tail dependence is the tendency for extreme values for two or more variables to occur together.

II. The choice of the copula affects tail dependence.

III. The tail dependence is higher in a bivariate Student t-distribution than in a bivariate normal distribution

Which of the above is (are) true?

A. None are true.

B. Only I is true.

C. Only III is true.

D. All are true.

Q-79. Suppose a risk manager wishes to create a correlation copula to estimate the risk of loan defaults during a financial crisis. Which type of copula will most accurately measure tail risk?

A. Gaussian copula

B. Student’s t-copula

C. Gaussian one-factor copula

D. Standard normal copula