Mathematics behind GAN

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2018 c 1 œ 26 F

Contents

Math behind GAN

Definition

GAN is composed of two networks: Descrimitive Network, and Generative Network.

GAN abstract

1. GAN is a framework for estimating generative models via an *adversarial process*

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GAN abstract

1. GAN is a framework for estimating generative models via an *adversarial process*
2. simultaneously train two models: A *generative* model G and A *discriminative* model D.
3. This framework corresponds to a minimax two-player game.

Previous Generative Models

1. deep Boltzmann machine

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Mathematics for Adversarial nets

Generator

1. data x
2. input noise variables $p_z(z)$
3. mapping to data space as $G(z;\theta_g)$,where G is a differentiable funciton represented by a multilayer perceptron with parameter θ_g.

Mathematics for Adversarial nets 2

Discriminator

1. $D(x;\theta_d)$ which is a multilayer perceptron that outputs a single scalar.
2. $D(x)$ represents the probability that x came from data rather than p_g

Mathematics for Adversarial nets 3

minimax playgame

$$\min_G \max_D V(D,G) =$$
$$E_{x\sim p_{data}}[\log D(x)] + E_{z\sim p_z}[\log(1 - D(G(z))]$$

Mathematics for Adversarial nets 4

Optimium D

$$\max_D V(D,G) = E_{x\sim p_{data}}[\log D(x)] + E_{z\sim p_z}[\log(1-D(G(z))]$$
$$= E_{x\sim p_{data}}[\log D(x)] + E_{x\sim p_g}[\log(1-D(x)]$$
$$= \int_x p_{data}(x)[\log D(x)]dx + \int_x p_g(x)\log(1-D(x))dx$$

Mathematics for Adversarial nets 4

Optimium D

1. $\max_D V(D,G) = \int_x p_{data}(x)[\log D(x)]dx + \int_x p_g(x)\log(1-D(x))dx$
2. for given x, $p_{data}(x)$ is constant,marked as a
3. for given x, $p_g(x)$ is constant,marked as b
4. $f(D) = a\log D + b\log(1-D)$
5. To find max of $f(D)$, $\frac{\partial f(D)}{\partial D} = 0$
6. We get $D = \frac{a}{a+b}$
7. That is,for given G, $D^\star = \frac{p_{data}(x)}{p_g(x)+p_{data}(x)}$

KL divergence

Definition

$$KL(p||q) = \sum_{k=1}^{N} p_k \log \frac{p_k}{q_k}$$

What's the mean of KL divergence

the divergence (distance) of two distributions.