Mathematics behind GAN

Li Jun

2018 c 1 œ 26 F

Contents

Math behind GAN

Definition

GAN is composed of two networks: Descrimitive Network, and Generative Network.

GAN abstract

1. GAN is a framework for estimating generative models via an *adversarial process*

GAN abstract

1. GAN is a framework for estimating generative models via an *adversarial process*
2. simultaneously train two models: A *generative* model G and A *discriminative* model D.

GAN abstract

1. GAN is a framework for estimating generative models via an *adversarial process*
2. simultaneously train two models: A *generative* model G and A *discriminative* model D.
3. This framework corresponds to a minimax two-player game.

Previous Generative Models

1. deep Boltzmann machine

Previous Generative Models

1. deep Boltzmann machine
2. Generative stochastic networks

Previous Generative Models

1. deep Boltzmann machine
2. Generative stochastic networks
3. variational autoencoders(VAEs)

Previous Generative Models

1. deep Boltzmann machine
2. Generative stochastic networks
3. variational autoencoders(VAEs)
4. ...

Mathematics for Adversarial nets

Generator

1. data x
2. input noise variables $p_z(z)$
3. mapping to data space as $G(z;\theta_g)$,where G is a differentiable funciton represented by a multilayer perceptron with parameter θ_g.

Mathematics for Adversarial nets 2

Discriminator

1. $D(x; \theta_d)$ which is a multilayer perceptron that outputs a single scalar.
2. $D(x)$ represents the probability that x came from data rather than p_g

Mathematics for Adversarial nets 3

minimax playgame

$$\min_{G} \max_{D} V(D, G) = E_{x \sim p_{data}}[\log D(x)] + E_{z \sim p_z}[\log(1 - D(G(z)))]$$

Mathematics for Adversarial nets 4

Optimium D

$$\max_D V(D,G) = E_{x\sim p_{data}}[\log D(x)] + E_{z\sim p_z}[\log(1-D(G(z))]$$
$$= E_{x\sim p_{data}}[\log D(x)] + E_{x\sim p_g}[\log(1-D(x)]$$
$$= \int_x p_{data}(x)[\log D(x)]dx + \int_x p_g(x)\log(1-D(x))dx$$

Mathematics for Adversarial nets 5

Optimium D

1. $\max_D V(D,G) = \int_x p_{data}(x)[\log D(x)]dx + \int_x p_g(x)\log(1-D(x))dx$
2. for given x, $p_{data}(x)$ is constant,marked as a
3. for given x, $p_g(x)$ is constant,marked as b
4. $f(D) = a\log D + b\log(1-D)$
5. To find max of $f(D)$, $\frac{\partial f(D)}{\partial D} = 0$
6. We get $D = \frac{a}{a+b}$
7. That is,for given G, $D^\star = \frac{p_{data}(x)}{p_g(x)+p_{data}(x)}$

Mathematics for Adversarial nets 6

Optimium D

$$forgivenG, D^\star = \frac{p_{data}(x)}{p_g(x)+p_{data}(x)} \max_D V(D^\star, G) =$$
$$\int_x p_{data}(x)[\log D^*(x)]dx + \int_x p_g(x)\log(1 - D^*(x))dx$$
$$=$$
$$\int_x p_{data}(x)[\log \frac{p_{data}(x)}{p_{data}(x)+p_g(x)}]dx + \int_x p_g(x)[\log \frac{p_g(x)}{p_{data}(x)+p_g(x)}]dx$$
$$= \int_x p_{data}(x)[\log \frac{\frac{1}{2}p_{data}(x)}{\frac{1}{2}(p_{data}(x)+p_g(x))}]dx +$$
$$\int_x p_g(x)[\log \frac{\frac{1}{2}p_g(x)}{\frac{1}{2}(p_{data}(x)+p_g(x))}]dx$$
$$= -\log 4 + KL(p_{data}||\tfrac{1}{2}(p_{data}+p_g)) + KL(p_g||\tfrac{1}{2}(p_{data}+p_g))$$

KL divergence

Definition

$KL(p||q) = \sum_{k=1}^{N} p_k \log \frac{p_k}{q_k}$

What's the mean of KL divergence

the divergence (distance) of two distributions.