NavPy Documentation

Release 0.1

NavPy Team

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This will be NavPy's documentation site. (under construction). Visit later!

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Module Documenation

This is the documentation for using the individual module functions.

1.1 Coordinate Transformations

1.1.1 Direction Cosine Matrices

```
navpy.angle2dcm(rotAngle1, rotAngle2, rotAngle3, input_unit='rad', rotation_sequence='ZYX', out-
put_type='ndarray')
```

This function converts Euler Angle into Direction Cosine Matrix (DCM). The DCM is described by three sucessive rotation rotAngle1, rotAngle2, and rotAngle3 about the axis described by the rotation_sequence.

The default rotation_sequence='ZYX' is the aerospace sequence and rotAngle1 is the yaw angle, rotAngle2 is the pitch angle, and rotAngle3 is the roll angle. In this case DCM transforms a vector from the locally level coordinate frame (i.e. the NED frame) to the body frame.

This function can batch process a series of rotations (e.g., time series of Euler angles).

```
Parameters rotAngle1, rotAngle2, rotAngle3: angles \{(N,), (N,1), \text{ or } (1,N)\}
```

They are a sequence of angles about successive axes described by rotation_sequence.

```
input_unit : {'rad', 'deg'}, optional
    Rotation angles. Default is 'rad'.

rotation_sequence : {'ZYX'}, optional
    Rotation sequences. Default is 'ZYX'.

output_type : {'ndarray','matrix'}, optional
    Output type. Default is 'ndarray'.

Returns C : {3x3} Direction Cosine Matrix
```

Notes

Programmer: Adhika Lie Created: May 03, 2011 Last Modified: January 12, 2016

```
navpy.dcm2angle(C, output_unit='rad', rotation_sequence='ZYX')
```

This function converts a Direction Cosine Matrix (DCM) into the three rotation angles. The DCM is described by three sucessive rotation rotAngle1, rotAngle2, and rotAngle3 about the axis described by the rotation_sequence.

The default rotation_sequence='ZYX' is the aerospace sequence and rotAngle1 is the yaw angle, rotAngle2 is the pitch angle, and rotAngle3 is the roll angle. In this case DCM transforms a vector from the locally level coordinate frame (i.e. the NED frame) to the body frame.

This function can batch process a series of rotations (e.g., time series of direction cosine matrices).

```
Parameters C : \{(3,3), (N,3,3), \text{ or } (3,3,N)\}
```

direction consine matrix that rotates the vector from the first frame to the second frame according to the specified rotation sequence.

```
output_unit : {'rad', 'deg'}, optional
    Rotation angles. Default is 'rad'.
rotation_sequence : {'ZYX'}, optional
```

Returns rotAngle1, rotAngle2, rotAngle3: angles

Rotation sequences. Default is 'ZYX'.

They are a sequence of angles about successive axes described by rotation_sequence.

Notes

The returned rotAngle1 and 3 will be between +/- 180 deg (+/- pi rad). In contrast, rotAngle2 will be in the interval +/- 90 deg (+/- pi/2 rad).

In the 'ZYX' or '321' aerospace sequence, that means the pitch angle returned will always be inside the closed interval +/- 90 deg (+/- pi/2 rad). Applications where pitch angles near or larger than 90 degrees in magnitude are expected should used alternate attitude parameterizations like quaternions.

1.1.2 ECEF and NED

navpy.ecef2ned (ecef, lat_ref, lon_ref, alt_ref, latlon_unit='deg', alt_unit='m', model='wgs84')

Transform a vector resolved in ECEF coordinate to its resolution in the NED coordinate. The center of the NED

coordiante is given by lat_ref, lon_ref, and alt_ref.

Parameters ecef: $\{(N,3)\}$ input vector expressed in the ECEF frame

lat_ref: Reference latitude, unit specified by latlon_unit, default in deg

lon_ref: Reference longitude, unit specified by latlon_unit, default in deg

alt: Reference altitude, unit specified by alt_unit, default in m

Returns ned: $\{(N,3)\}$ array like ecef position, unit is the same as alt_unit

Examples

```
>>> import numpy as np
>>> from navpy import ecef2ned
>>> lat
```

navpy.ned2ecef (ned, lat_ref, lon_ref, alt_ref, latlon_unit='deg', alt_unit='m', model='wgs84')

Transform a vector resolved in NED (origin given by lat_ref, lon_ref, and alt_ref) coordinates to its ECEF representation.

Parameters ned: $\{(N,3)\}$ input array, units of meters

lat_ref: Reference latitude, unit specified by latlon_unit, default in deg

lon_ref: Reference longitude, unit specified by latlon_unit, default in deg

alt_ref: Reference altitude, unit specified by alt_unit, default in m

Returns ecef: {(N,3)} array like ned vector, in the ECEF frame, units of meters

Notes

The NED vector is treated as a relative vector, and hence the ECEF representation returned is NOT converted into an absolute coordinate. This means that the magnitude of *ned* and *ecef* will be the same (bar numerical differences).

Examples

```
>>> import navpy
>>> ned = [0, 0, 1]
>>> lat_ref, lon_ref, alt_ref = 45.0, -93.0, 250.0 # deg, meters
>>> ecef = navpy.ned2ecef(ned, lat_ref, lon_ref, alt_ref)
>>> print("NED:", ned)
>>> print("ECEF:", ecef)
>>> print("Notice that 'down' is not same as 'ecef-z' coordinate.")
```

1.1.3 ECEF and LLA

```
navpy.ecef2lla(ecef, latlon_unit='deg')
```

Calculate the Latitude, Longitude and Altitude of a point located on earth given the ECEF Coordinates.

Parameters ecef: {(N,3)} array like input of ECEF coordinate in X, Y, and Z column, unit is meters

latlon_unit: {('deg','rad')} specifies the output latitude and longitude unit

Returns lat: {(N,)} array like latitude in unit specified by latlon_unit

lon: {(N,)} array like longitude in unit specified by latlon_unit

 $alt : \{(N,)\}$ array like altitude in meters

References

[R1]

```
navpy.lla2ecef (lat, lon, alt, latlon_unit='deg', alt_unit='m', model='wgs84')
Convert Latitude, Longitude, Altitude, to ECEF position
```

Parameters lat: {(N,)} array like latitude, unit specified by latlon_unit, default in deg

lon: {(N,)} array like longitude, unit specified by latlon_unit, default in deg

alt: {(N,)} array like altitude, unit specified by alt_unit, default in m

Returns ecef: {(N,3)} array like ecef position, unit is the same as alt_unit

navpy.lla2ned(lat, lon, alt, lat_ref, lon_ref, alt_ref, latlon_unit='deg', alt_unit='m', model='wgs84')

Convert Latitude, Longitude, Altitude to its resolution in the NED coordinate. The center of the NED coordinate is given by lat ref, lon ref, and alt ref.

For example, this can be used to convert GPS data to a local NED frame.

Parameters lat: {(N,)} array like latitude, unit specified by latlon_unit, default in deg

lon: {(N,)} array like longitude, unit specified by latlon_unit, default in deg

alt: {(N,)} array like altitude, unit specified by alt_unit, default in m

lat_ref: Reference latitude, unit specified by latlon_unit, default in deg

lon_ref: Reference longitude, unit specified by latlon_unit, default in deg

alt: Reference altitude, unit specified by alt_unit, default in m

Returns ned: $\{(N,3)\}$ array like ecef position, unit is the same as alt_unit

1.1.4 NED and LLA

navpy.ned211a (ned, lat_ref, lon_ref, alt_ref, latlon_unit='deg', alt_unit='m', model='wgs84')

Calculate the Latitude, Longitude and Altitude of points given by NED coordinates where NED origin given by lat_ref, lon_ref, and alt_ref.

Parameters ned: $\{(N,3)\}$ array like input of NED coordinate in N, E, and D column, unit is meters

lat ref: Reference latitude, unit specified by latlon unit, default in deg

lon_ref: Reference longitude, unit specified by latlon_unit, default in deg

alt_ref: Reference altitude, unit specified by alt_unit, default in m

latlon_unit : {('deg','rad')} specifies the output latitude and longitude unit

Returns lat: {(N,)} array like latitude in unit specified by latlon_unit

lon: {(N,)} array like longitude in unit specified by latlon_unit

 $alt: \{(N,)\}$ array like altitude in meters

navpy.**lla2ned** (*lat*, *lon*, *alt*, *lat_ref*, *lon_ref*, *alt_ref*, *latlon_unit='deg'*, *alt_unit='m'*, *model='wgs84'*)

Convert Latitude, Longitude, Altitude to its resolution in the NED coordinate. The center of the NED coordinate is given by lat ref, lon ref, and alt ref.

For example, this can be used to convert GPS data to a local NED frame.

Parameters lat: {(N,)} array like latitude, unit specified by latlon_unit, default in deg

lon: {(N,)} array like longitude, unit specified by latlon unit, default in deg

alt: {(N,)} array like altitude, unit specified by alt_unit, default in m

lat_ref: Reference latitude, unit specified by latlon_unit, default in deg

lon_ref: Reference longitude, unit specified by latlon_unit, default in deg

alt: Reference altitude, unit specified by alt_unit, default in m

Returns ned: $\{(N,3)\}$ array like ecef position, unit is the same as alt_unit

1.2 Attitude Transformations

navpy.angle2quat (rotAngle1, rotAngle2, rotAngle3, input_unit='rad', rotation_sequence='ZYX')

Convert a sequence of rotation angles to an equivalent unit quaternion

This function can take inputs in either degree or radians, and can also batch process a series of rotations (e.g., time series of Euler angles). By default this function assumes aerospace rotation sequence but can be changed using the rotation_sequence keyword argument.

```
Parameters rotAngle1, rotAngle2, rotAngle3: \{(N,), (N,1), or (1,N)\}
```

They are a sequence of angles about successive axes described by rotation_sequence.

```
input_unit : { 'rad', 'deg' }, optional
   Rotation angles. Default is 'rad'.

rotation_sequence : { 'ZYX' }, optional
   Rotation sequences. Default is 'ZYX'.
```

Returns $q0: \{(N,)\}$ array like scalar component of the quaternion

qvec: {(N,3)} array like vector component of the quaternion

Notes

Convert rotation angles to unit quaternion that transfroms a vector in F1 to F2 according to

$$v_q^{F2} = q^{-1} \otimes v_q^{F1} \otimes q$$

where \otimes indicates the quaternion multiplication and v_q^F is a pure quaternion representation of the vector v_q^F . The scalar component of v_q^F is zero. For aerospace sequence ('ZYX'): rotAngle1 = psi, rotAngle2 = the, and rotAngle3 = phi

```
>>> import numpy as np
>>> from navpy import angle2quat
>>> psi = 0
>>> theta = np.pi/4.0
>>> phi = np.pi/3.0
>>> q0, qvec = angle2quat(psi,theta,phi)
>>> q0
0.80010314519126557
>>> qvec
array([ 0.46193977,  0.33141357, -0.19134172])
```

```
navpy.dcm2quat (C, rotation_sequence='ZYX')
Convert a DCM to a unit quaternion
```

Parameters C: direction consine matrix that rotates the vector from the first frame

to the second frame according to the specified rotation_sequence. rotation_sequence: {'ZYX'}, optional. Rotation sequences. Default is 'ZYX'.

```
Returns q0: \{(N,)\} array_like
```

Scalar component of the quaternion

```
qvec: {(N,3)} array_like
```

Vector component of the quaternion

Examples

```
navpy.qmult(p0, pvec, q0, qvec)
```

Quaternion Multiplications r = p x q

Parameters $p0, q0 : \{(N,)\}$ array_like

Scalar component of the quaternion

```
pvec, qvec: {(N,3)} array_like
```

Vector component of the quaternion

Returns r0: $\{(N,)\}$ array like scalar component of the quaternion

rvec: {(N,3)} array like vector component of the quaternion

```
>>> import numpy as np
>>> from navpy import qmult
>>> p0, pvec = 0.701057, np.array([-0.69034553, 0.15304592, 0.09229596])
>>> q0, qvec = 0.987228, np.array([ 0.12613659, 0.09199968, 0.03171637])
>>> qmult(q0,qvec,p0,pvec)
(0.76217346258977192, array([-0.58946236, 0.18205109, 0.1961684 ]))
>>> s0, svec = 0.99879, np.array([ 0.02270747, 0.03430854, -0.02691584])
>>> t0, tvec = 0.84285, np.array([ 0.19424161, -0.18023625, -0.46837843])
>>> qmult(s0,svec,t0,tvec)
(0.83099625967941704, array([ 0.19222498, -0.1456937 , -0.50125456]))
>>> qmult([p0, s0],[pvec, svec],[q0, t0], [qvec, tvec])
(array([ 0.76217346, 0.83099626]), array([[-0.59673664, 0.24912539, 0.03053588], [ 0.19222498
```

navpy.quat2angle(q0, qvec, output_unit='rad', rotation_sequence='ZYX')

Convert a unit quaternion to the equivalent sequence of angles of rotation about the rotation_sequence axes.

This function can take inputs in either degree or radians, and can also batch process a series of rotations (e.g., time series of quaternions). By default this function assumes aerospace rotation sequence but can be changed using the rotation_sequence keyword argument.

Parameters q0: {(N,), (N,1), or (1,N)} array_like

Scalar component of the quaternion

qvec: $\{(N,3),(3,N)\}$ array_like

Vector component of the quaternion

rotation_sequence: {'ZYX'}, optional

Rotation sequences. Default is 'ZYX'.

Returns rotAngle1, rotAngle2, rotAngle3: {(N,), (N,1), or (1,N)} array_like

They are a sequence of angles about successive axes described by rotation_sequence.

output_unit : {'rad', 'deg'}, optional

Rotation angles. Default is 'rad'.

Notes

Convert rotation angles to unit quaternion that transfroms a vector in F1 to F2 according to

$$v_q^{F2} = q^{-1} \otimes v_q^{F1} \otimes q$$

where \otimes indicates the quaternion multiplication and v_q^F is a pure quaternion representation of the vector v_q^F . The scalar component of v_q^F is zero. For aerospace sequence ('ZYX'): rotAngle1 = psi, rotAngle2 = the, and rotAngle3 = phi

```
>>> import numpy as np
>>> from navpy import quat2angle
>>> q0 = 0.800103145191266
>>> qvec = np.array([0.4619398,0.3314136,-0.1913417])
>>> psi, theta, phi = quat2angle(q0,qvec)
>>> psi
1.0217702360987295e-07
>>> theta
0.7853982192745731
>>> phi
1.0471976051067484
```

```
>>> psi, theta, phi = quat2angle(q0,qvec,output_unit='deg')
>>> psi
5.8543122160542875e-06
>>> theta
45.00000320152342
>>> phi
60.000003088824108
```

```
>>> q0 = [ 0.96225019,  0.92712639,  0.88162808]
>>> qvec = np.array([[-0.02255757,  0.25783416,  0.08418598],
>>> psi, theta, phi = quat2angle(q0,qvec,output_unit='deg')
>>> psi
array([ 9.99999941,  19.99999997,  29.9999993 ])
>>> theta
array([ 30.00000008,  39.99999971,  50.00000025])
>>> phi
array([ -6.06200867e-07,  5.00000036e+00,  1.00000001e+01])
```

navpy.quat2dcm(q0, qvec, rotation_sequence='ZYX', output_type='ndarray')

Convert a single unit quaternion to one DCM

Parameters q0: {(N,), (N,1), or (1,N)} array_like

Scalar component of the quaternion

qvec: $\{(N,3),(3,N)\}$ array_like

Vector component of the quaternion

rotation_sequence : {'ZYX'}, optional

Rotation sequences. Default is 'ZYX'.

output_type : {'ndarray','matrix'}, optional

Output is either numpy array (default) or numpy matrix.

Returns C_N2B: direction consine matrix that rotates the vector from the first frame

to the second frame according to the specified rotation_sequence.

Examples

```
>>> q0 = 0.9811

>>> qvec = np.array([-0.0151, 0.0858, 0.1730])

>>> C = quat2dcm(q0,qvec,output_type='matrix')

>>> C

matrix([[ 9.25570440e-01, 3.36869440e-01, -1.73581360e-01],

[ -3.42051760e-01, 9.39837700e-01, 5.75800000e-05],

[ 1.63132160e-01, 5.93160200e-02, 9.84972420e-01]])
```

1.2.1 Utilities

```
navpy.wrapToPi(e)
Wraping angle to [-pi,pi] interval
```

[-0.018968]

```
navpy.omega2rates(pitch, roll, input_unit='rad', euler_angles_order='roll_pitch_yaw', out-put_type='ndarray')
```

This function is used to create the transformation matrix to go from: [p, q, r] -> [roll_rate, pitch_rate, yaw_rate]

where pqr are xyz body rotation-rate measurements expressed in body frame. Yaw, pitch, and roll are the Euler angles. We assume the Euler angles are 3-2-1 (i.e Yaw -> Pitch -> Roll) transformations that go from navigation-frame to body-frame.

```
Parameters pitch: pitch angle, units of input_unit.
```

```
roll: roll angle, units of input_unit.
```

input_unit : units for input angles { 'rad', 'deg' }, optional

euler_angles_order : {'roll_pitch_yaw', 'yaw_pitch_roll'}, optional

Assumed order of Euler Angles attitude state vector (see Notes).

output_type : {'ndarray' or 'matrix'}, optional

Numpy array (default) or matrix

Returns R: transformation matrix, from xyz body-rate to Euler angle-rates

numpy 'output_type' 3x3 (Note: default return variable is an ARRAY, not a matrix)

Notes

Since the returned transformation matrix is used to transform one vector to another, the assumed attitude variables order matters. The <code>euler_angles_order</code> parameter can be used to specify the assumed order.

The difference is demonstrated by example:

```
By default euler_angles_order='roll_pitch_yaw' R = omega2rates(pitch, roll) [ roll_rate] [omega_x] [pitch_rate] = dot(R,[omega_y]) [ yaw_rate] [omega_z]
```

Now assume our attitude state is [yaw, pitch, roll].T R = omega2rates(pitch, roll, euler_angles_order='yaw_pitch_roll') [yaw_rate] [omega_x] [pitch_rate] = dot(R,[omega_y]) [roll_rate] [omega_z]

References

[1] Equation 2.74, Aided Navigation: GPS with High Rate Sensors, Jay A. Farrel 2008

[2] omega2rates.m function at: http://www.gnssapplications.org/downloads/chapter7/Chapter7_GNSS_INS_Functions.tar.gz

1.3 Earth Modeling

```
\verb"navpy.llarate" (\textit{VN}, \textit{VE}, \textit{VD}, \textit{lat}, \textit{alt}, \textit{lat\_unit} = \textit{'deg'}, \textit{alt\_unit} = \textit{'m'})
```

Calculate Latitude, Longitude, Altitude Rate given locally tangent velocity

Parameters VN: $\{(N,)\}$ array like earth relative velocity in the North direction, m/s

 $VE : \{(N,)\}$ array like earth relative velocity in the East direction, m/s

VD: {(N,)} array like earth relative velocity in the Down direction, m/s

lat: {(N,)} array like latitudes, unit specified in lat_unit, default deg

1.3. Earth Modeling 11

alt: {(N,)} array like altitudes, unit specified in alt_unit, default m

Returns lla_dot: {(N,3)} np.array of latitude rate, longitude rate, altitude rate.

The unit of latitude and longitude rate will be the same as the unit specified by lat_unit and the unit of altitude rate will be the same as alt_unit

See also:

earthrad called by this method

Examples

navpy.llarate(VN, VE, VD, lat, alt, lat_unit='deg', alt_unit='m')

Calculate Latitude, Longitude, Altitude Rate given locally tangent velocity

Parameters $VN : \{(N,)\}$ array like earth relative velocity in the North direction, m/s

 $VE : \{(N,)\}$ array like earth relative velocity in the East direction, m/s

 \mathbf{VD} : $\{(N,)\}$ array like earth relative velocity in the Down direction, m/s

lat: {(N,)} array like latitudes, unit specified in lat_unit, default deg

alt: {(N,)} array like altitudes, unit specified in alt_unit, default m

Returns lla_dot: $\{(N,3)\}$ np.array of latitude rate, longitude rate, altitude rate.

The unit of latitude and longitude rate will be the same as the unit specified by lat_unit and the unit of altitude rate will be the same as alt_unit

See also:

earthrad called by this method

13

navpy.earthrad(lat, lat_unit='deg', model='wgs84')

Calculate radius of curvature in the prime vertical (East-West) and meridian (North-South) at a given latitude.

Parameters lat: {(N,)} array like latitude, unit specified by lat_unit, default in deg

Returns $\mathbf{R}_{\mathbf{N}}$: {(N,)} array like, radius of curvature in the prime vertical (East-West)

R_M: {(N,)} array like, radius of curvature in the meridian (North-South)

Examples

navpy.earthrad(lat, lat_unit='deg', model='wgs84')

Calculate radius of curvature in the prime vertical (East-West) and meridian (North-South) at a given latitude.

Parameters lat: {(N,)} array like latitude, unit specified by lat_unit, default in deg

Returns $\mathbf{R}_{\mathbf{N}}$: {(N,)} array like, radius of curvature in the prime vertical (East-West)

 $\mathbf{R}_{\mathbf{M}}$: {(N,)} array like, radius of curvature in the meridian (North-South)

Examples

1.3. Earth Modeling

```
navpy.earthrate(lat, lat_unit='deg', model='wgs84')
```

Calculate the earth rotation rate resolved on NED axis given VN, VE, VD, lat, and alt.

```
Paul Groves's Notation: \omega_{IE}^N, Eq. (2.75), Ch. 2.3, pp. 44
```

Parameters lat: {(N,)} array like latitudes, unit specified in lat_unit, default deg

Returns $e: \{(N,3)\}$ np.array of the earth's rotation rate

The unit is in rad/seconds.

References

[1] P. Groves, GNSS, Inertial, and Integrated Navigation Systems, Artech House, 2008

1.4 Miscellaneous Utilities

```
navpy.skew (w, output_type='ndarray')
Make a skew symmetric 2-D array
```

Parameters $\mathbf{w} : \{(3,)\}$ array_like

Returns $C : \{(3,3)\}$ skew symmetric representation of w

CHAPTER	2
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User Guide

This is the user guide demonstrating the utility of NavPy.

CHAPTER 3

Indices and tables

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ekeli, C.,"Inertial Navigation Systems With Geodetic Applications", Walter de Gruyter, New York, 20	001, pp.

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