Shear strength of soil

One of the most important and the most controversial engineering properties of soil is its shear strength or ability to resist sliding along internal surfaces within a mass. The stability of a cut, the slope of an earth dam, the foundations of structures, the natural slopes of hillsides and other structures built on soil depend upon the shearing resistance offered by the soil along the probable surfaces of slippage.

Basic concept of shirring resistance and shearing strength

The basic concept of shearing resistance and shearing strength can be made clear by studying first the basic principles of friction between solid bodies. Consider a prismatic block **B** resting on a plane surface **MN** as shown in **Fig. 1**. Block **B** is subjected to the force $\mathbf{P_n}$ which acts at right angles to the surface **MN**, and the force $\mathbf{F_a}$ that acts tangentially to the plane. The normal force $\mathbf{P_n}$ remains constant whereas $\mathbf{F_a}$ gradually increases from zero to a value which will produce sliding. If the tangential force $\mathbf{F_a}$ is relatively small, block **B** will remain at rest, and the applied horizontal force will be balanced by an equal and opposite force $\mathbf{F_r}$ on the plane of contact. This resisting force is developed as a result of roughness characteristics of the bottom of block **B** and plane surface **MN**. The angle $\boldsymbol{\delta}$ formed by the resultant **R** of the two forces $\mathbf{F_r}$ and $\mathbf{P_n}$ with the normal to the plane **MN** is known as the angle of obliquity.

If the applied horizontal force $\mathbf{F_a}$ is gradually increased, the resisting force $\mathbf{F_r}$ will likewise increase, always being equal in magnitude and opposite in direction to the applied force. Block **B** will start sliding along the plane when the force $\mathbf{F_a}$ reaches a value which will increase the angle of obliquity to a certain maximum value $\boldsymbol{\delta}$. If block **B** and plane surface **MN** are made of the same material, the angle $\boldsymbol{\delta_m}$ is equal to $\boldsymbol{\phi}$ which is termed the angle of friction, and the value **tan** $\boldsymbol{\phi}$ is termed the **coefficient of friction**. If block **B** and plane surface **MN** are made of dissimilar materials, the angle $\boldsymbol{\delta}$ is termed the **angle of wall friction**. The applied horizontal force $\mathbf{F_a}$ on block **B** is a shearing force and the developed force is friction or shearing resistance. The maximum shearing resistance which the materials are capable of developing is called the shearing strength.

If another experiment is conducted on the same block with a higher normal load $\mathbf{P_n}$ the shearing force $\mathbf{F_a}$ will correspondingly be greater. A series of such experiments would show that the shearing force $\mathbf{F_a}$ is proportional to the normal load $\mathbf{P_n}$, that is

$$F_a = P_n \tan\phi$$

If **A** is the overall contact area of block **B** on plane surface **MN**, the relationship may be written as in termed of shear strength

$$\tau = \frac{F_a}{A} = \frac{P_n \tan\phi}{A}$$

$$\tau = \sigma \tan\phi$$

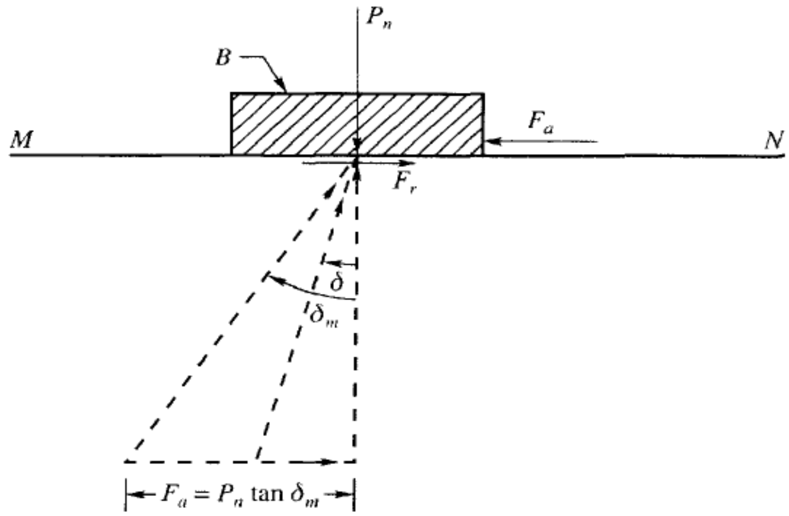


Fig. 1: Basic concept of shearing resistance and strength

The Coulomb equation

The basic concept of friction as explained in **section above** applies to soils which are purely granular in character. Soil which are not purely granular exhibit an additional strength which is due to the cohesion between the particles. It is, therefore, still customary to separate the shearing strength s of such soils into two components, one due to the **cohesion** between the soil particles and the other due to the **friction** between them. The fundamental shear strength equation proposed by the French Engineer **Coulomb** (1776) is

$$\tau_f = c + \sigma \tan \phi \qquad (1)$$

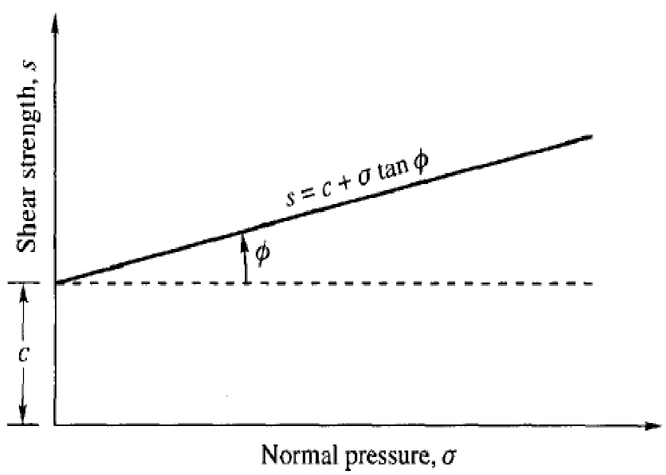


Fig. 2: Coulomb's law

This equation expresses the assumption that the cohesion C is independent of the normal pressure **σ** acting on the plane of failure. At zero normal pressure, the shear strength of the soil is expressed as

$$\tau_f = c$$

According to above equation, the cohesion of a soil is defined as the shearing strength at zero normal pressure on the plane of rupture.

In Coulomb's equation **c** and **ϕ** are empirical parameters, the values of which for any soil depend upon several factors; the most important of these are:

1. The past history of the soil.
2. The initial state of the soil, i.e., whether it is saturated or unsaturated.
3. The permeability characteristics of the soil.
4. The conditions of drainage allowed to take place during the test.

Since **c** and **ϕ** in Coulomb's **Eq. 1** depend upon many factors, **c** is termed as apparent cohesion and **ϕ** the angle of shearing resistance. For cohesion less soil **c** equal to **0**, then Coulomb's equation becomes

$\tau_f = \sigma \tan \phi$

Methods of determining shear strength parameters

The shear strength parameters **c** and **ϕ** of soils either in the undisturbed or remolded states may be determined by any of the following methods:

1. Laboratory methods
   - Direct or box shear test
   - Triaxial compression test
2. Field method: Vane shear test or by any other indirect methods

Direct shear test

The original form of apparatus for the direct application of shear force is the shear box. The box shear test, though simple in principle. The apparatus consists of a square brass box split horizontally at the level of the center of the soil sample, which is held between metal grilles and porous stones as shown in **Fig. 3**. Vertical load is applied to the sample as shown in the figure and is held constant during a test. A gradually increasing horizontal load is applied to the lower part of the box until the sample fails in shear. The shear load at failure is divided by the cross-sectional area of the sample to give the ultimate shearing strength. The vertical load divided by the area of the sample gives the applied vertical stress **σ**. The test may be repeated with a few more samples having the same initial conditions as the first sample. Each sample is tested with a different vertical load.

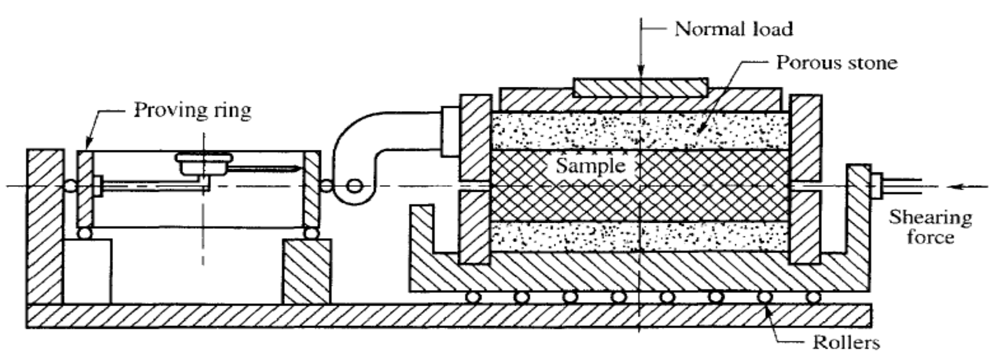


Fig. 3: Constant rate of strain shear box

Procedure for determining shearing strength of soil

In the direct shear test, a sample of soil is placed into the shear box. The size of the box normally used for clays and sands is **6** x **6 cm** and the sample is **2 cm** thick. A large box of size **30** x **30 cm** with sample thickness of **15 cm** is sometimes used for gravelly soils.

The soils used for the test are either undisturbed samples or remolded. If undisturbed, the specimen has to be carefully trimmed and fitted into the box. If remolded samples are required, the soil is placed into the box in layers at the required initial water content and tamped to the required dry density.

After the specimen is placed in the box, and all the other necessary adjustments are made, a known normal load is applied. Then a shearing force is applied. The normal load is held constant throughout the test but the shearing force is applied at a constant rate of strain. The shearing displacement is recorded by a dial gauge.

The results of tests are plotted on a graph, with normal stress along the horizontal axes and the shear stress that produced failure of the specimen along the vertical axis **Fig. 4**. A straight line drawn connecting the plotted points is extended to intersect the ordinate. The angle between this straight line and a horizontal line is the angle of internal friction **ϕ**, and the shear stress where the straight line intersects the ordinate is the cohesion of soil **c**.

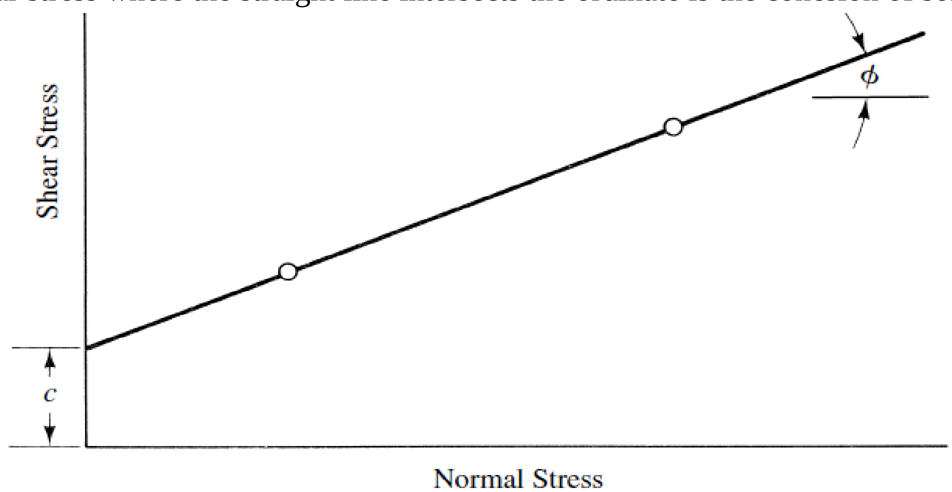


Fig. 4: Shear diagram for direct shear test

Triaxial Compression Test

A diagrammatic layout of a triaxial test apparatus is shown in **Fig. 5**. In the triaxial compression test, three or more identical samples of soil are subjected to uniformly distributed fluid pressure around the cylindrical surface. The sample is sealed in a watertight rubber membrane. Then axial load is applied to the soil sample until it fails. Although only compressive load is applied to the soil sample, it fails by shear on internal faces. It is possible to determine the shear strength of the soil from the applied loads at failure.

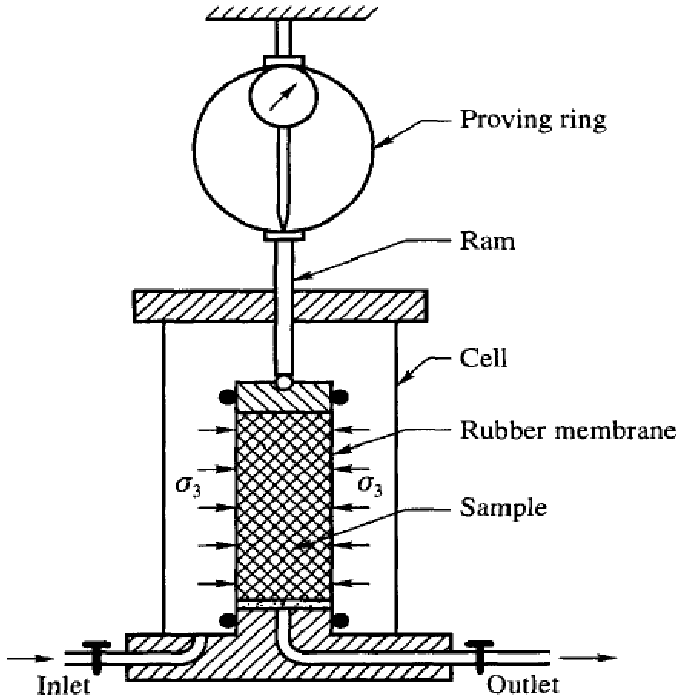


Fig. 5: Triaxial test apparatus

Advantages and disadvantages of direct and triaxial shear tests

Direct shear tests are generally suitable for cohesion less soils except fine sand and silt whereas the triaxial test is suitable for all types of soils and tests. Undrained and consolidated undrained tests on clay samples can be made with the box-shear apparatus. The advantages of the triaxial over the direct shear test are:

1. The stress distribution across the soil sample is more uniform in a triaxial test as compared to a direct shear test.
2. The measurement of volume changes is more accurate in the triaxial test.
3. The complete state of stress is known at all stages during the triaxial test, whereas only the stresses at failure are known in the direct shear test.
4. In the case of triaxial shear, the sample fails along a plane on which the combination of normal stress and the shear stress gives the maximum angle of obliquity of the resultant with the normal, whereas in the case of direct shear, the sample is sheared only on one plane which is the horizontal plane which need not be the plane of actual failure.
5. Pore water pressures can be measured in the case of triaxial shear tests whereas it is not possible in direct shear tests.

Advantages of direct shear tests are:

1. The direct shear machine is simple and fast to operate.
2. A thinner soil sample is used in the direct shear test thus facilitating drainage of the pore water quickly from a saturated specimen.
3. Direct shear requirement is much less expensive as compared to triaxial equipment.

Mohr – Coulomb failure theory

The failure of a soil mass is more nearly in accordance of the Mohr theory of failure than other theory and the interpretation of the triaxial compression test depends to a large extent on this fact. The Mohr theory is based on the postulate that a material will fail when the shearing stress on the plane along which the failure is presumed to occur is a unique function of the normal stress acting on that plane. The material fails along the plane only when the angle between the resultant of the shearing stress and the normal stress is a maximum, that is, where the combination of normal and shearing stresses produces the maximum obliquity angle **δ**.

According to **Coulomb's Law**, the condition of failure is that the shear stress is

$$\tau_f \leq c + \sigma \tan\phi \qquad (2)$$

Mohr diagram for triaxial compression test at failure

Consider a cylindrical specimen of soil possessing both cohesion and friction is subjected to a conventional triaxial compression test. In the conventional test the lateral pressure $\boldsymbol{\sigma_3}$ is held constant and the vertical pressure $\boldsymbol{\sigma_1}$ is increased at a constant rate of stress or strain until the sample fails. If $\boldsymbol{\sigma_1}$ is the peak value of the vertical pressure at which the sample fails, the two principal stresses that are to be used for plotting the Mohr circle of rupture are $\boldsymbol{\sigma_3}$ and $\boldsymbol{\sigma_1}$. In **Fig. 6** the values of $\boldsymbol{\sigma_1}$ and $\boldsymbol{\sigma_3}$ are plotted on the **σ**-axis and a circle is drawn with ($\boldsymbol{\sigma_1 - \sigma_3}$) as diameter, the soil fails along a plane which makes an angle ($\boldsymbol{\alpha = 45° + \phi/2}$) with the major principal plane. In **Fig. 4** the two lines $\boldsymbol{P_oP_l}$ and $\boldsymbol{P_oP_2}$ are the conjugate rupture planes. The two lines $\boldsymbol{M_oN}$ and $\boldsymbol{M_oN_1}$ drawn tangential to the rupture circle at points $\boldsymbol{P_l}$ and $\boldsymbol{P_2}$ are called Mohr envelopes.

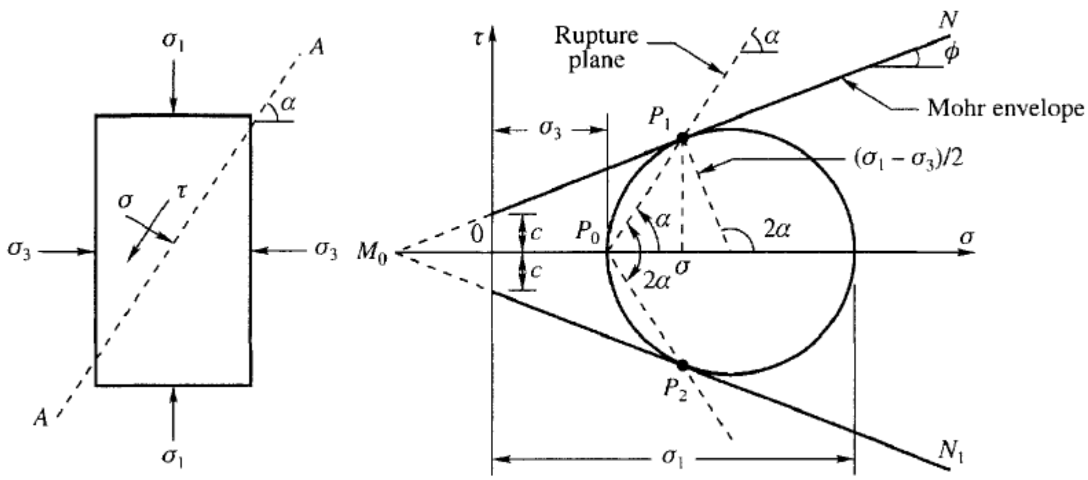


Fig. 6: Mohr diagram for triaxial test at failure for **c-ϕ** soil

The various information that can be obtained from the figure includes

1. The angle of shearing resistance **ϕ** equal the slope of the Mohr envelope.
2. The apparent cohesion **c** is the intercept of the Mohr envelope on the **τ**-axis.
3. The inclination of the rupture plane is **α**.
4. The angle between the conjugate planes is **2α**.

If the soil is cohesion less with (**c** = **0**) the Mohr envelopes pass through the origin, and if the soil is purely cohesive with (**ϕ** = **0**) the Mohr envelope is parallel to the abscissa. The Mohr envelopes for these two types of soils are shown in **Fig. 7**.

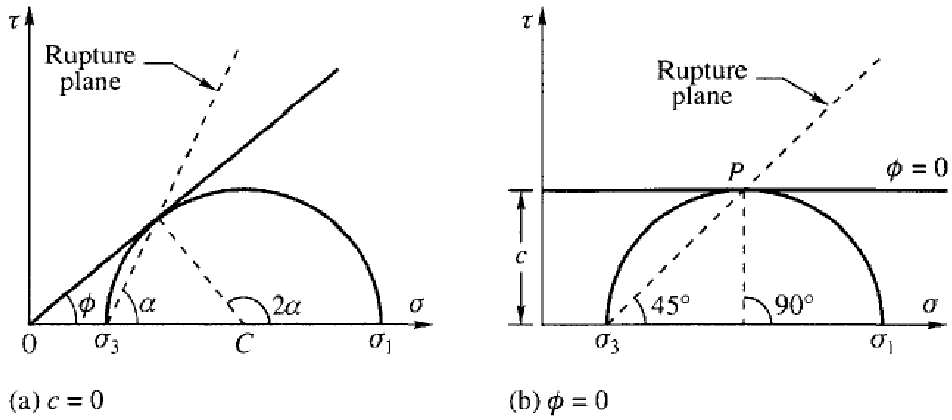


Fig. 7: Mohr diagram for soil

Effective stresses

The voids may completely be filled with water or partly with water and air. Shear stresses are to be carried only by the skeleton of solid particles. However, the total normal stresses on any plane are, in general, the sum of two components, stress carried by solid particles and pressure in the fluid in the void space.

This visualization of the distribution of stresses between solid and fluid has two important consequences:

1. When a specimen of soil is subjected to external pressure, the volume changes of the specimen is not due to the total normal stress but due to the difference between the total normal stress **σ** and the pressure of the fluid in the void space. The pressure in the fluid is the pore pressure **u**.
   $\sigma' = \sigma - u$
2. The shear strength of soils, as of all granular materials, is largely determined by the frictional forces arising during slip at the contacts between the soil particles. For practical purposes the shear strength equation of Coulomb is given by
   $$\tau_f = c' + (\sigma - u)\tan\phi = c' + \sigma' \tan\phi \qquad (3)$$

Where:

c' is apparent cohesion in terms of effective stresses

ϕ' is angle of shearing resistance in terms of effective stresses

σ is total normal pressure to the plane considered

u is pore pressure.

The effective stress parameters **c'** and **ϕ'** of a given sample of soil may be determined provided the pore pressure **u** developed during the shear test is measured. The pore pressure **u** is developed when the testing of the soil is done under **undrained** conditions. However, if free drainage takes place during testing, there will not be any development of pore pressure. In such cases, the total stresses themselves are effective stresses.

Types of laboratory tests

The laboratory tests on soils may be on

1. Undisturbed samples, or
2. Remolded samples.

Further, the tests may be conducted on soil that are:

1. Fully saturated, or
2. Partially saturated.

The type of test to be adopted depends upon how best we can simulate the field conditions. Generally speaking, the various shear tests for soils may be classified as follows:

1. Unconsolidated-Undrained Tests (UU)
   The samples are subjected to an applied pressure under conditions in which drainage is prevented, and then sheared under conditions of no drainage.
2. Consolidated-Undrained or Quick Tests (CD)
   The samples are allowed to consolidate under an applied pressure and then sheared under conditions of no drainage.
3. Consolidated-Drained or Slow Tests (CD)
   The samples are consolidated as in the previous test, but the shearing is carried out slowly under conditions of no excess pressure in the pore space.

Example. 1

A shear box test was carried out on sandy clay yielding the following results

Normal load (N)	108	202	295	390	484	576
Shear load at failure (N)	172	227	266	323	374	425

If the rea of shear plane is 60 x 60 mm. Determine the apparent cohesion and angle of shear resistance for the soil.

Solution

$\mathrm{A} = 60 * 60 * 10^{-6} = 3.6 * 10^{-3}\mathrm{m}^2$

$$\sigma_\mathrm{n} = \frac{108 * 10^{-3}}{3.6 * 10^{-3}} = 30\,\mathrm{kN/m^2}$$

$$\tau_\mathrm{f} = \frac{172 * 10^{-3}}{3.6 * 10^{-3}} = 47.8\,\mathrm{kN/m^2}$$

Normal stress σ_n (kN/m^2)	30.0	56.1	81.9	108.3	134.4	160.0
Shear stress at failure τ_f (kN/m^2)	47.8	63.1	73.9	89.7	103.9	118.1

Apparent cohesion

$\mathrm{c} = 33\,\mathrm{kN/m^2}$

Angle of shear resistance for the soil

$\phi = 28^\mathrm{o}$

Example. 2

A drained triaxial compression test was carried out on samples of the same soil. The results were as follow. Determine the shear strength parameters of soil, assuming the pore water pressure at failure to be zero.

Test number	1	2	3
Cell pressure (kN/m^2)	100	200	300
Deviator stress at failure (kN/m^2)	210	438	644

Solution

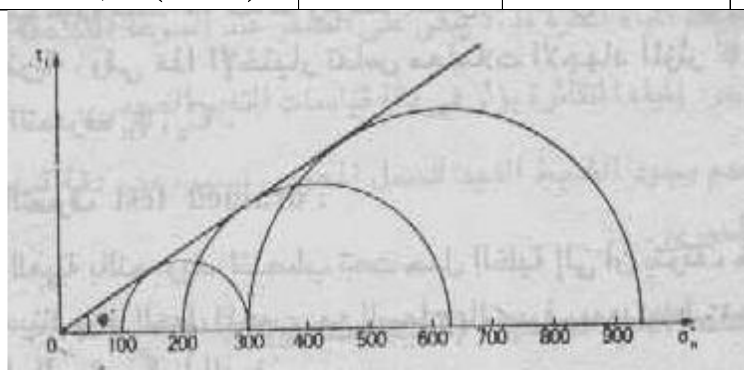
Minor principal stress,

$\sigma_3 = \text{cell pressure}$

Major principal stress,

$\sigma_1 = \text{cell pressure} + \text{deviator stress} = \sigma_3 + \text{deviator stress}$

Test number	1	2	3
Minor principal stress, σ_3 (kN/m^2)	100	200	300
Deviator stress at failure (kN/m^2)	210	438	644
Major principal stress, σ_1 (kN/m^2)	310	638	944



Apparent cohesion

$c = 0$

Angle of shear resistance for the soil

$\phi = 31^o$

Example. 3

The following results were obtained from a consolidated-undrained test on a normally consolidated clay soil.

Test number	1	2	3
Cell pressure (kN/m^2)	100	200	300
Deviator stress at failure (kN/m^2)	137	210	283
Pore pressure at failure (kN/m^2)	22	86	147

Determine the values of apparent cohesion and angle of shearing resistance in terms of effective stress and total stress for the soil.

Solution

Apparent cohesion and angle of shearing resistance in terms of effective stress

The effective stresses at failure are,

$\sigma_1' = \sigma_1 - u$

$\sigma_3' = \sigma_3 - u$

$\sigma_1' - \sigma_3' = \sigma_1 - \sigma_3$

Test number	1	2	3
σ_3 (kN/m^2)	100	200	300
u (kN/m^2)	22	86	147
σ'_3 (kN/m^2)	78	114	153
(σ'_1- σ'_3) (kN/m^2)	137	210	283
σ'_1 (kN/m^2)	215	324	436

$c' = 0$

$\phi' = 29^o$

Apparent cohesion and angle of shearing resistance in terms of total stress

$c = 24\,\mathrm{kN/m^2}$

$\phi = 16^o$

Problem. 1

In a shear box test on a clay soil the shear load was applied immediately after the normal load. The following results were obtained. Determine the apparent cohesion and angle of shearing resistance for the soil.

Test number	1	2	3	4
Normal stress σ_n (kN/m^2)	120	230	340	450
Shear stress at failure τ_f (kN/m^2)	133	150	168	186

Problem. 2

A series of direct shear tests was performed on a soil sample. Each test was carried out until the specimen sheared (failed). The laboratory data for the tests are tabulated as follows. Determine the soil's cohesion and angle of internal friction.

Test number	1	2	3	4
Normal stress σ_n (kN/m^2)	10	19	28	48
Shear stress at failure τ_f (kN/m^2)	22	25	28	35

Problem. 3

In a consolidated-undrained triaxial test on a normally consolidated clay at a cell pressure of 150kN/m^2, the deviator stress at failure was 260kN/m^2 and the pore water pressure was 50kN/m^2. Draw the appropriate shear strength envelope and determine the other corresponding parameter when angle of friction is zero and cohesion of soil is zero.

Problem. 4

Triaxial compression tests on three specimens of a soil sample were performed. Each test was carried out until the specimen experienced shear failure. The test data are tabulated as follows. Find the soil's cohesion and angle of internal friction.

Test number	1	2	3
Cell pressure (kN/m^2)	70	140	210
Deviator stress at failure (kN/m^2)	280	330	360

Problem. 5

The following results were obtained from a consolidated undrained test on an over consolidated clay soil. Determine the apparent cohesion and angle of shearing resistance for the soil.

Test number	1	2	3
Cell pressure (kN/m^2)	100	250	400
Deviator stress at failure (kN/m^2)	340	410	474
Pore pressure at failure (kN/m^2)	- 42	64	177