**Faculty of Engineering**
**Mechanical Engineering Department**

# Cash Flow and Compound Interest Factors

## Terminology and Symbols

- The equations and procedures of engineering economy utilize the following terms and symbols:
  - ***P*** = value or amount of money at a time designated as the present or time 0. Also, ***P*** is referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC);
  - ***F*** = value or amount of money at some future time. Also, ***F*** is called future worth (FW) and future value (FV);
  - ***A*** = series of consecutive, equal, end-of-period amounts of money. Also, ***A*** is called the annual worth (AW) and equivalent uniform annual worth (EUAW);
  - ***n*** = number of interest periods;
  - ***i*** = interest rate or rate of return per time period.

# Example 1

- A new college graduate plans to borrow \$10,000 now to help in buying a car. She has arranged to repay the entire principal plus 8% per year interest after 5 years. Identify the engineering economy symbols involved and their values for the total owed after 5 years.

- **Solution**

- In this case, $P$ and $F$ are involved, since all amounts are single payments, as well as $n$ and $i$. Time is expressed in years.

$P$ = \$10,000 $i$ = 8% per year $n$ = 5 years $F$ = ?

- The future amount $F$ is unknown.

# Example 2

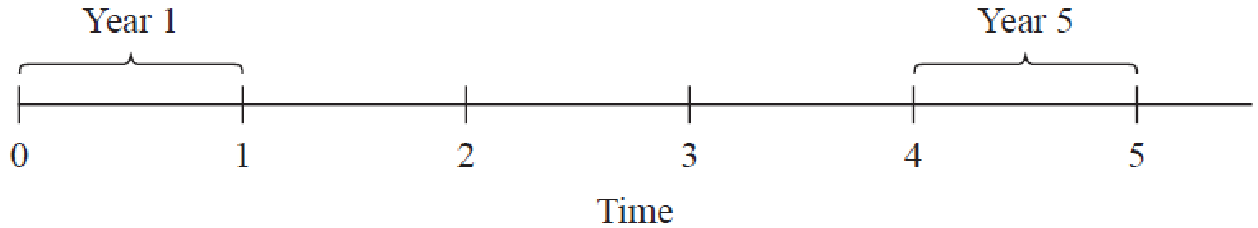
- On July 1, 2018, your new employer deposits $5000 into your money market account, as part of your employment bonus. The account pays interest at 5% per year. You expect to withdraw an equal annual amount each year for the following 10 years. Identify the symbols and their values.

- **Solution**

  $P$ = $5000
  $A$ = ? per year
  $i$ = 5% per year
  $n$ = 10 years.

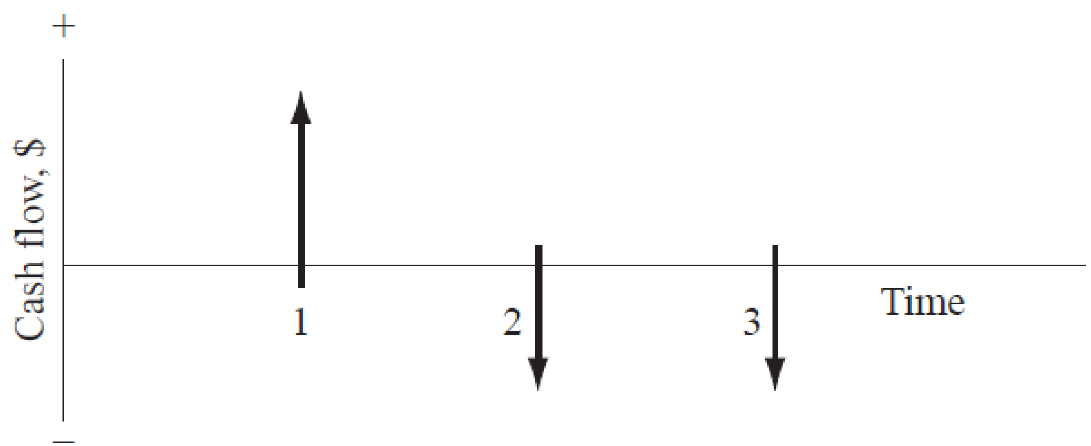
# Cash Flow Diagram

- The cash flow diagram is a very important tool in an economic analysis, especially when the cash flow series is complex.
- It is a graphical representation of cash flows drawn on a time scale. The diagram includes what is known, what is estimated and what is needed.



# Cash Flow Diagram

- The direction of the arrows on the cash flow diagram is important.
  - A vertical arrow pointing up indicates a positive cash flow.
  - An arrow pointing down indicates a negative cash flow.

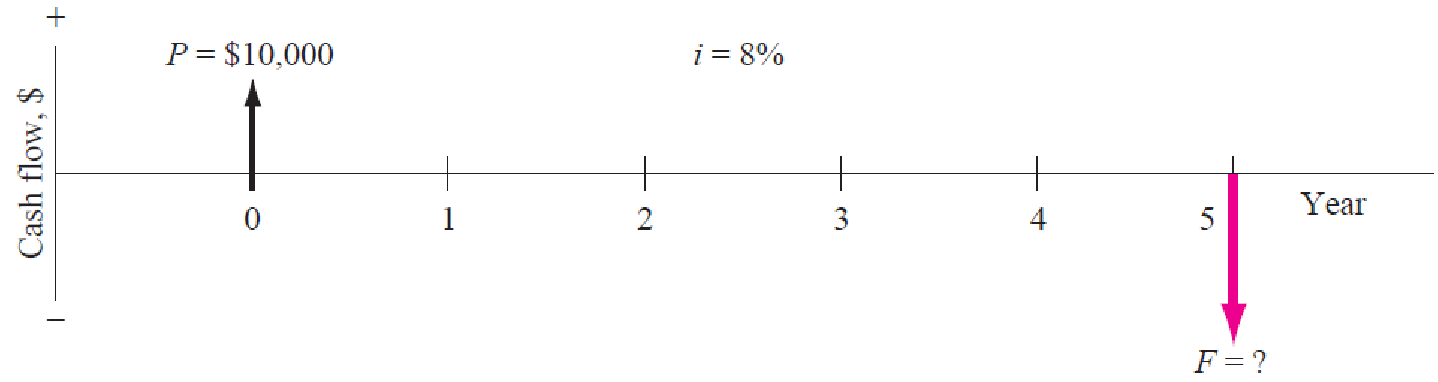


# Example 3

- Reread Example 1, where *P* = $10,000 is borrowed at 8% per year and *F* is sought after 5 years. Construct the cash flow diagram.

- **Solution**

- The present sum *P* is a cash inflow of the loan principal at year 0, and the future sum *F* is the cash outflow of the repayment at the end of year 5. The interest rate should be indicated on the diagram.



# Single-Payment Formulas

- The most fundamental equation in engineering economy is the one that determines the amount of money *F* accumulated after *n* years (or periods) from a ***single*** present worth *P*, with interest compounded one time per year.

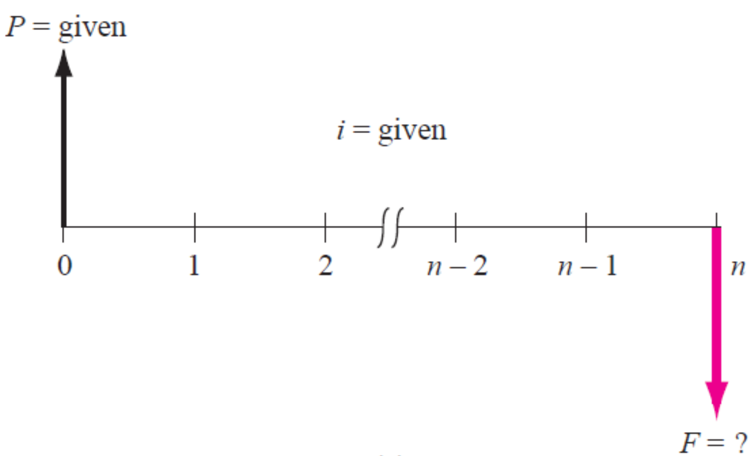
$$F = P(1 + i)^n$$

- The term $(1+i)^n$ is called a factor and is known as the ***single-payment compound amount factor***, but it is usually referred to as the *F/P factor*.

# Single-Payment Formulas

- ***Single-payment present worth factor***, or the *P/F factor*.

$$P = F\left[\frac{1}{(1+i)^n}\right]$$

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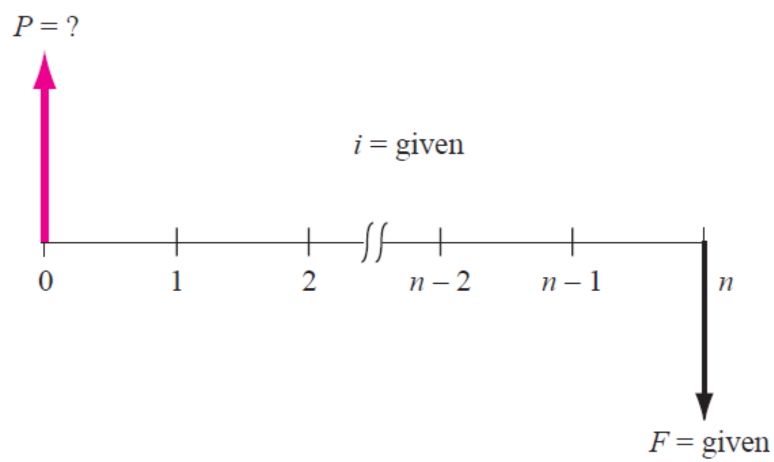



(b)

# Single-Payment Formulas

- A standard notation has been adopted for all factors. It is always in the general form (*X*/*Y*,*i*,*n*). The letter *X* represents what is sought, while the letter *Y* represents what is given.
- For example, ***F*/*P*** means **find *F* when given *P***. The *i* is the interest rate in percent, and *n* represents the number of periods involved.
- Thus, (*F*/*P*,6%,20) represents the factor that is used to calculate the future amount *F* accumulated in 20 periods if the interest rate is 6% per period. The *P* is given.

# Example 4

- An engineer received a bonus of $12,000 that he will invest now. He wants to calculate the equivalent value after 24 years. Assume a rate of return of 8% per year for each of the 24 years. Find the amount he can pay down.

- **Solution**

- The symbols and their values are:

  *P* = $12,000   *F* = ?   *i* = 8% per year   *n* = 24 years

$$
\begin{aligned}
F = P(1+i)^n &= 12{,}000(1+0.08)^{24} \\
&= 12{,}000(6.341181) \\
&= \$76{,}094.17
\end{aligned}
$$

# Uniform Series Formulas

- There are four uniform series formulas that involve *A*, where *A* means the cash flow occurs in *consecutive interest periods*, and in *same amount* in each period.

- The formulas relate a present worth *P* or a future worth *F* to a uniform series amount *A*. The two equations that relate *P* and *A* are as follows:

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

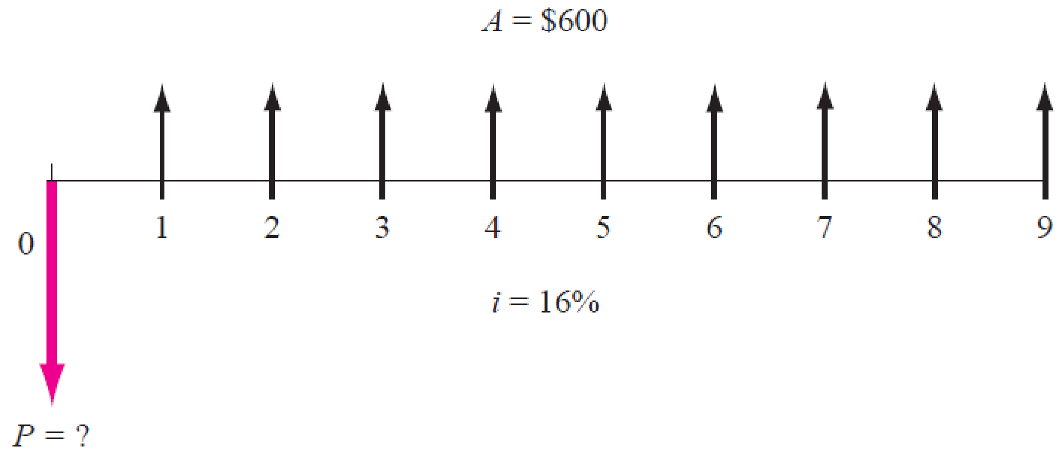
# Example 5

- How much money should you be willing to pay now for a guaranteed \$600 per year for 9 years starting next year, at a rate of return of 16% per year?

- **Solution**

- The present worth is:

$P = 600(P/A,16\%,9)$
$\quad = 600(4.6065) = \$2763.90$

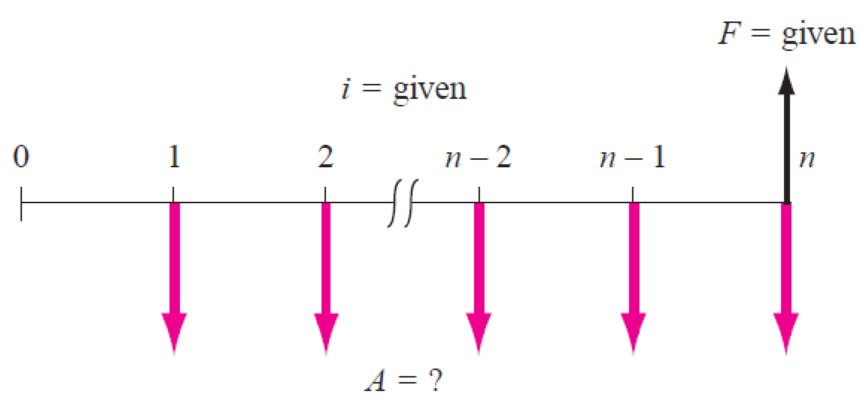


# Uniform Series Formulas

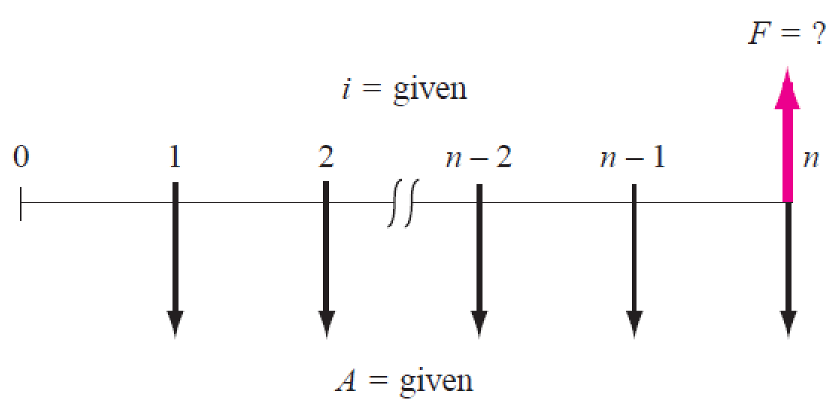
- The uniform series formulas that relate *A* and *F* follow.

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$



(a)



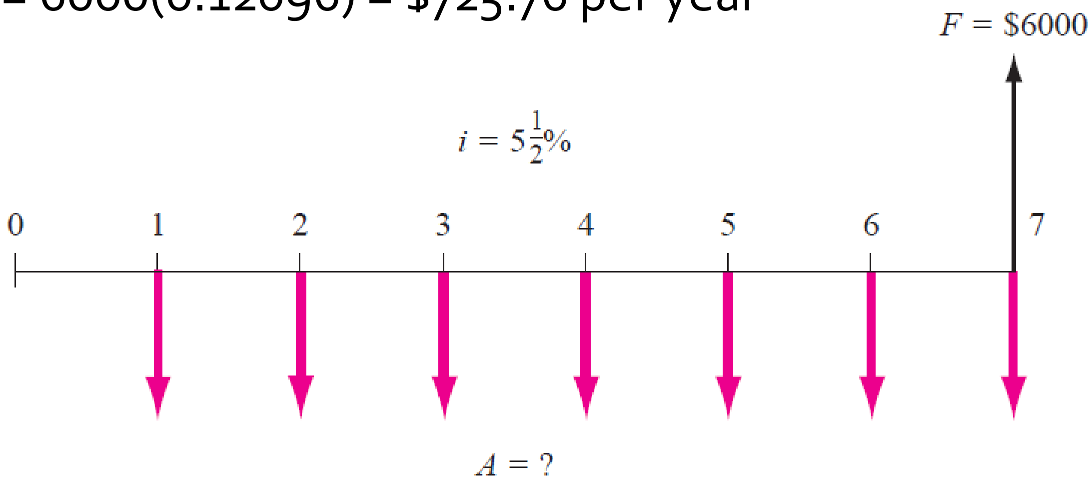
(b)

# Example 6

- How much money must an electrical contractor deposit every year in his savings account starting 1 year from now at 5.5% per year in order to accumulate $6000 seven years from now?

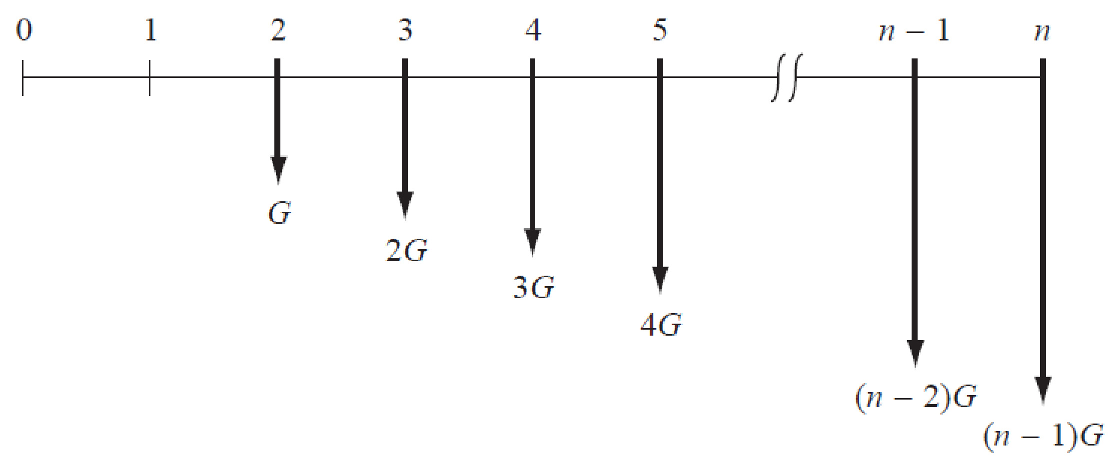
- **Solution**

$A$ = $6000($A/F$,5.5%,7) = 6000(0.12096) = $725.76 per year



# Gradient Formulas

- Sometimes the cash flows that occur in consecutive interest periods are not the same amount (not an *A* value), but they do change in a predictable way. These cash flows are known as *gradients*, and there are two general types: arithmetic and geometric.
- An *arithmetic* gradient is one wherein the cash flow changes (increases or decreases) by the same amount in each period.



## Gradient Formulas

$$P = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

- This equation finds the present worth of the *gradient only*. It does not include the base amount of money that the gradient was built upon.
- The general equation to find *P* of an arithmetic gradient is:

$$\begin{aligned} \boldsymbol{P} &= \textbf{Present worth of base amount} + \textbf{present worth of gradient amount} \\ &= \boldsymbol{A(P/A, i\%, n) + G(P/G, i\%, n)} \end{aligned}$$

where $A$ = amount in *period* 1
$G$ = amount of *change* in cash flow between periods 1 *and* 2
$n$ = number of periods from 1 through $n$ of gradient cash flow
$i$ = interest rate per period

# Example 7

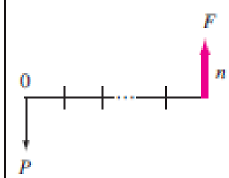
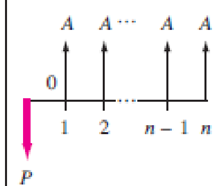
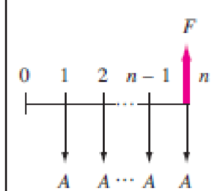
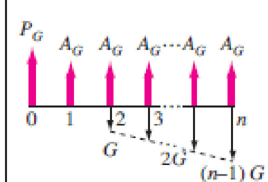
- The Highway Department expects the cost of maintenance for a piece of heavy construction equipment to be \$5000 in year 1, to be \$5500 in year 2, and to increase annually by \$500 through year 10. At an interest rate of 10% per year, determine the present worth of 10 years of maintenance costs.

- **Solution**

- The cash flow includes an increasing gradient with $G$ = \$500 and a base amount of \$5000 starting in year 1.

$$
\begin{aligned}
P &= 5000(P/A,10\%,10) + 500(P/G,10\%,10) \\
&= 5000(6.1446) + 500(22.8913) \\
&= \$42,169
\end{aligned}
$$

# Relations for Cash Flows with End-of-Period Compounding

| Type | Find/Given | Factor Notation and Formula | Relation | Sample Cash Flow Diagram |
|---|---|---|---|---|
| **Single Amount** | $F/P$ Compound amount | $(F/P,i,n) = (1+i)^n$ | $F = P(F/P,i,n)$ |   |
| | $P/F$ Present worth | $(P/F,i,n) = \dfrac{1}{(1+i)^n}$ | $P = F(P/F,i,n)$ | |
| **Uniform Series** | $P/A$ Present worth | $(P/A,i,n) = \dfrac{(1+i)^n - 1}{i(1+i)^n}$ | $P = A(P/A,i,n)$ |   |
| | $A/P$ Capital recovery | $(A/P,i,n) = \dfrac{i(1+i)^n}{(1+i)^n - 1}$ | $A = P(A/P,i,n)$ | |
| | $F/A$ Compound amount | $(F/A,i,n) = \dfrac{(1+i)^n - 1}{i}$ | $F = A(F/A,i,n)$ |   |
| | $A/F$ Sinking fund | $(A/F,i,n) = \dfrac{i}{(1+i)^n - 1}$ | $A = F(A/F,i,n)$ | |
| **Arithmetic Gradient** | $P_G/G$ Present worth | $(P/G,i,n) = \dfrac{(1+i)^n - in - 1}{i^2(1+i)^n}$ | $P_G = G(P/G,i,n)$ |   |
| | $A_G/G$ Uniform series | $(A/G,i,n) = \dfrac{1}{i} - \dfrac{n}{(1+i)^n - 1}$ (Gradient only) | $A_G = G(A/G,i,n)$ | |

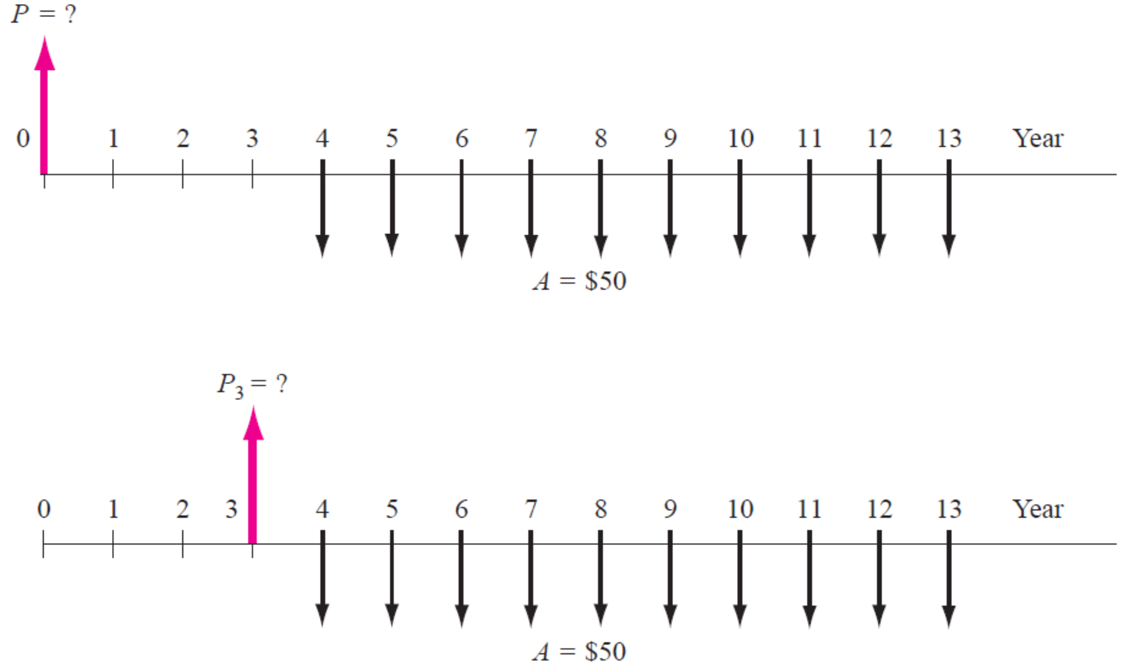
# Interest Factor Tables

5% TABLE 10 Discrete Cash Flow: Compound Interest Factors 5%

| | Single Payments | | Uniform Series Payments | | | | Arithmetic Gradients | |
|---|---|---|---|---|---|---|---|---|
| n | F/P Compound Amount | P/F Present Worth | A/F Sinking Fund | F/A Compound Amount | A/P Capital Recovery | P/A Present Worth | P/G Gradient Present Worth | A/G Gradient Uniform Series |
| 1 | 1.0500 | 0.9524 | 1.00000 | 1.0000 | 1.05000 | 0.9524 | | |
| 2 | 1.1025 | 0.9070 | 0.48780 | 2.0500 | 0.53780 | 1.8594 | 0.9070 | 0.4878 |
| 3 | 1.1576 | 0.8638 | 0.31721 | 3.1525 | 0.36721 | 2.7232 | 2.6347 | 0.9675 |
| 4 | 1.2155 | 0.8227 | 0.23201 | 4.3101 | 0.28201 | 3.5460 | 5.1028 | 1.4391 |
| 5 | 1.2763 | 0.7835 | 0.18097 | 5.5256 | 0.23097 | 4.3295 | 8.2369 | 1.9025 |
| 6 | 1.3401 | 0.7462 | 0.14702 | 6.8019 | 0.19702 | 5.0757 | 11.9680 | 2.3579 |
| 7 | 1.4071 | 0.7107 | 0.12282 | 8.1420 | 0.17282 | 5.7864 | 16.2321 | 2.8052 |
| 8 | 1.4775 | 0.6768 | 0.10472 | 9.5491 | 0.15472 | 6.4632 | 20.9700 | 3.2445 |
| 9 | 1.5513 | 0.6446 | 0.09069 | 11.0266 | 0.14069 | 7.1078 | 26.1268 | 3.6758 |
| 10 | 1.6289 | 0.6139 | 0.07950 | 12.5779 | 0.12950 | 7.7217 | 31.6520 | 4.0991 |
| 11 | 1.7103 | 0.5847 | 0.07039 | 14.2068 | 0.12039 | 8.3064 | 37.4988 | 4.5144 |
| 12 | 1.7959 | 0.5568 | 0.06283 | 15.9171 | 0.11283 | 8.8633 | 43.6241 | 4.9219 |
| 13 | 1.8856 | 0.5303 | 0.05646 | 17.7130 | 0.10646 | 9.3936 | 49.9879 | 5.3215 |
| 14 | 1.9799 | 0.5051 | 0.05102 | 19.5986 | 0.10102 | 9.8986 | 56.5538 | 5.7133 |
| 15 | 2.0789 | 0.4810 | 0.04634 | 21.5786 | 0.09634 | 10.3797 | 63.2880 | 6.0973 |
| 16 | 2.1829 | 0.4581 | 0.04227 | 23.6575 | 0.09227 | 10.8378 | 70.1597 | 6.4736 |
| 17 | 2.2920 | 0.4363 | 0.03870 | 25.8404 | 0.08870 | 11.2741 | 77.1405 | 6.8423 |
| 18 | 2.4066 | 0.4155 | 0.03555 | 28.1324 | 0.08555 | 11.6896 | 84.2043 | 7.2034 |
| 19 | 2.5270 | 0.3957 | 0.03275 | 30.5390 | 0.08275 | 12.0853 | 91.3275 | 7.5569 |
| 20 | 2.6533 | 0.3769 | 0.03024 | 33.0660 | 0.08024 | 12.4622 | 98.4884 | 7.9030 |
| 21 | 2.7860 | 0.3589 | 0.02800 | 35.7193 | 0.07800 | 12.8212 | 105.6673 | 8.2416 |
| 22 | 2.9253 | 0.3418 | 0.02597 | 38.5052 | 0.07597 | 13.1630 | 112.8461 | 8.5730 |
| 23 | 3.0715 | 0.3256 | 0.02414 | 41.4305 | 0.07414 | 13.4886 | 120.0087 | 8.8971 |
| 24 | 3.2251 | 0.3101 | 0.02247 | 44.5020 | 0.07247 | 13.7986 | 127.1402 | 9.2140 |
| 25 | 3.3864 | 0.2953 | 0.02095 | 47.7271 | 0.07095 | 14.0939 | 134.2275 | 9.5238 |
| 26 | 3.5557 | 0.2812 | 0.01956 | 51.1135 | 0.06956 | 14.3752 | 141.2585 | 9.8266 |
| 27 | 3.7335 | 0.2678 | 0.01829 | 54.6691 | 0.06829 | 14.6430 | 148.2226 | 10.1224 |
| 28 | 3.9201 | 0.2551 | 0.01712 | 58.4026 | 0.06712 | 14.8981 | 155.1101 | 10.4114 |
| 29 | 4.1161 | 0.2429 | 0.01605 | 62.3227 | 0.06605 | 15.1411 | 161.9126 | 10.6936 |
| 30 | 4.3219 | 0.2314 | 0.01505 | 66.4388 | 0.06505 | 15.3725 | 168.6226 | 10.9691 |
| 31 | 4.5380 | 0.2204 | 0.01413 | 70.7608 | 0.06413 | 15.5928 | 175.2333 | 11.2381 |
| 32 | 4.7649 | 0.2099 | 0.01328 | 75.2988 | 0.06328 | 15.8027 | 181.7392 | 11.5005 |
| 33 | 5.0032 | 0.1999 | 0.01249 | 80.0638 | 0.06249 | 16.0025 | 188.1351 | 11.7566 |
| 34 | 5.2533 | 0.1904 | 0.01176 | 85.0670 | 0.06176 | 16.1929 | 194.4168 | 12.0063 |
| 35 | 5.5160 | 0.1813 | 0.01107 | 90.3203 | 0.06107 | 16.3742 | 200.5807 | 12.2498 |
| 40 | 7.0400 | 0.1420 | 0.00828 | 120.7998 | 0.05828 | 17.1591 | 229.5452 | 13.3775 |
| 45 | 8.9850 | 0.1113 | 0.00626 | 159.7002 | 0.05626 | 17.7741 | 255.3145 | 14.3644 |
| 50 | 11.4674 | 0.0872 | 0.00478 | 209.3480 | 0.05478 | 18.2559 | 277.9148 | 15.2233 |
| 55 | 14.6356 | 0.0683 | 0.00367 | 272.7126 | 0.05367 | 18.6335 | 297.5104 | 15.9664 |
| 60 | 18.6792 | 0.0535 | 0.00283 | 353.5837 | 0.05283 | 18.9293 | 314.3432 | 16.6062 |
| 65 | 23.8399 | 0.0419 | 0.00219 | 456.7980 | 0.05219 | 19.1611 | 328.6910 | 17.1541 |
| 70 | 30.4264 | 0.0329 | 0.00170 | 588.5285 | 0.05170 | 19.3427 | 340.8409 | 17.6212 |
| 75 | 38.8327 | 0.0258 | 0.00132 | 756.6537 | 0.05132 | 19.4850 | 351.0721 | 18.0176 |
| 80 | 49.5614 | 0.0202 | 0.00103 | 971.2288 | 0.05103 | 19.5965 | 359.6460 | 18.3526 |
| 85 | 63.2544 | 0.0158 | 0.00080 | 1245.09 | 0.05080 | 19.6838 | 366.8007 | 18.6346 |
| 90 | 80.7304 | 0.0124 | 0.00063 | 1594.61 | 0.05063 | 19.7523 | 372.7488 | 18.8712 |
| 95 | 103.0347 | 0.0097 | 0.00049 | 2040.69 | 0.05049 | 19.8059 | 377.6774 | 19.0689 |
| 96 | 108.1864 | 0.0092 | 0.00047 | 2143.73 | 0.05047 | 19.8151 | 378.5555 | 19.1044 |
| 98 | 119.2755 | 0.0084 | 0.00042 | 2365.51 | 0.05042 | 19.8323 | 380.2139 | 19.1714 |
| 100 | 131.5013 | 0.0076 | 0.00038 | 2610.03 | 0.05038 | 19.8479 | 381.7492 | 19.2337 |

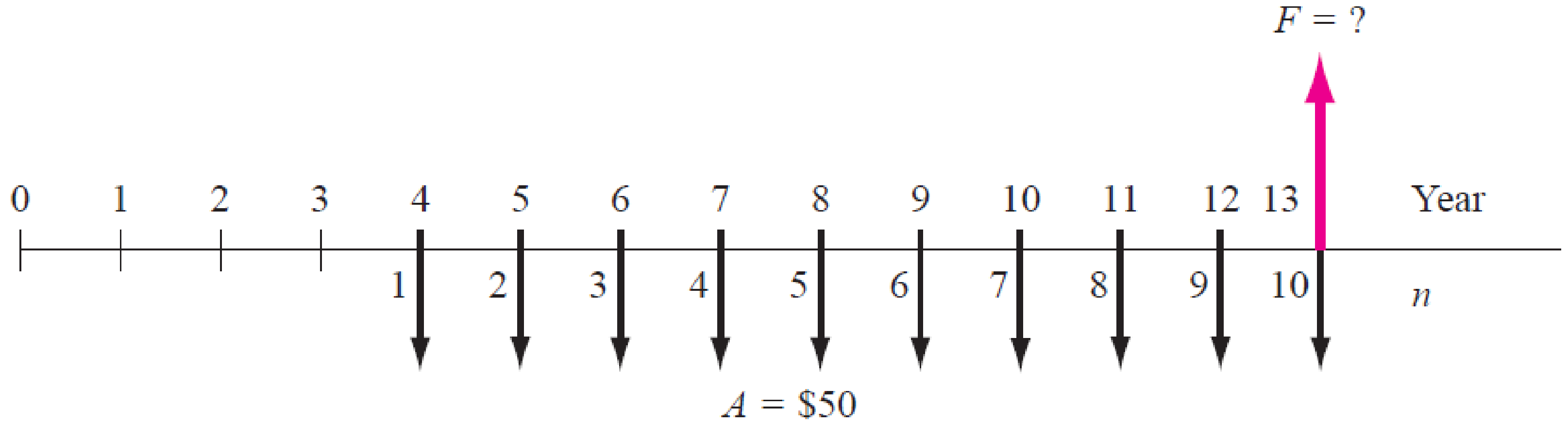
# Shifted Series

- When a uniform series begins at a time other than at the end of period 1, it is called a *shifted series*. In this case use the *P*/*A* factor to compute the "present worth", and then find the present worth in year 0 by using the (*P*/*F*,*i*,*n*) factor.



# Shifted Series

- To determine a future worth or *F* value, recall that the *F/A* factor has the F located in the same period as the last uniform-series amount.
- ***Renumbering*** the cash flow diagram to avoid errors in counting.

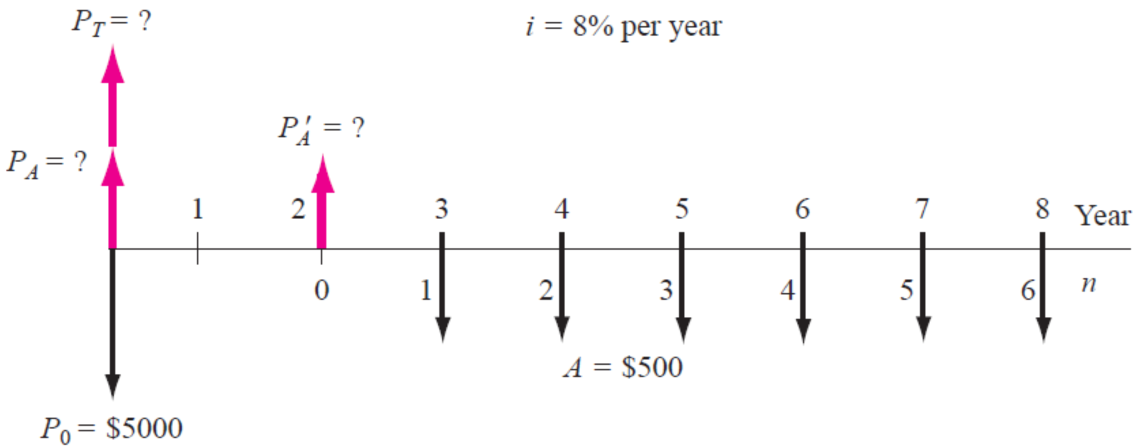


# Example 8

- An engineering technology group just purchased new CAD software for \$5000 now and annual payments of \$500 per year for 6 years starting 3 years from now for annual upgrades. What is the present worth of the payments if the interest rate is 8% per year?

- **Solution**

- First, find the value of the shifted series $P_A' = \$500(P/A,8\%,6)$



# Example 8

- **Solution**

- Since is located in year 2, now find in year 0.

$P_A = P_A'(P/F,8\%,2)$

$P_T = P_o + P_A$
$= 5000 + 500(P/A,8\%,6)(P/F,8\%,2)$
$= 5000 + 500(4.6229)(0.8573)$
$= \$6981.60$