Faculty of Engineering
Mechanical Engineering Department

Comparison of Alternatives for Decision Making

Analysis Methods that Compare Equivalent Values

- **Present Worth Analysis**: Find the equivalent value of cash flows at time 0.
- **Annual Worth Analysis**: Find the equivalent annual worth of all cash flows.
- **Rate of Return Analysis:** Compare the interest rate (ROR) of each alternative's cash flows to a minimum value you will accept.
- **Benefit/Cost Ratio**: Use equivalent values of cash flows to form ratios that can be easily analyzed.

Present Worth Analysis of Equal-Life Alternatives

- In present worth analysis, the *P* value, now called *PW*, is calculated at the MARR for each alternative.
- This converts all future cash flows into present dollar equivalents.
- The PW comparison of alternatives with equal lives is straightforward.
- If both alternatives are used in identical capacities for the same time period, they are termed *equal-service* alternatives.

Present Worth Analysis of Equal-Life Alternatives

- For mutually exclusive alternatives the following guidelines are applied:
  - **One alternative:**

Calculate PW at the MARR. If PW ≥ 0, the alternative is financially viable.

  - **Two or more alternatives:**

Calculate the PW of each alternative at the MARR. Select the alternative with the PW value that is numerically largest, that is, less negative or more positive.

Example 1

- Perform a present worth analysis of equal-service machines with the costs shown below, if the MARR is 10% per year. Revenues for all three alternatives are expected to be the same.

	Electric-Powered	Gas-Powered	Solar-Powered
First cost, $	−2500	−3500	−6000
Annual operating cost (AOC), $/year	−900	−700	−50
Salvage value, $	200	350	100
Life, years	5	5	5

Example 1

- **Solution**

- The PW of each machine is calculated at i = 10% for n = 5 years.

$$PW_E = -2500 - 900(P/A,10\%,5) + 200(P/F,10\%,5) = \$-5788$$
$$PW_G = -3500 - 700(P/A,10\%,5) + 350(P/F,10\%,5) = \$-5936$$
$$PW_S = -6000 - 50(P/A,10\%,5) + 100(P/F,10\%,5) = \$-6127$$

- The electric-powered machine is selected since the PW of its costs is the lowest.

Annual Worth Analysis

- The annual worth (AW) method is commonly used for comparing alternatives.
- All cash flows are converted to an equivalent uniform annual amount over one life cycle of the alternative.
- The annual worth method is typically the easiest of the evaluation techniques to perform, when the MARR is specified.
- The selection guidelines for the AW method are the same as for the PW method.
  - **One alternative:** AW ≥ 0, the alternative is financially viable.
  - **Two or more alternatives:** Choose the numerically largest AW value (lowest cost or highest income).

Example 2

- A company wishes to evaluate two similar pieces of equipment by which it can meet new state environmental requirements for dust emissions. The MARR is 12% per year. Determine which alternative is economically better using (a) the AW method, and (b) AW method with a 3-year study period.

Equipment	X	Y
First cost, $	40,000	75,000
AOC, $ per year	25,000	15,000
Life, years	4	6
Salvage value, $	10,000	7,000
Estimated value after 3 years, $	14,000	20,000

Example 2

- **Solution**

a. Calculating AW values over the respective lives indicates that Y is the better alternative.
$AW_X = -40{,}000(A/P{,}12\%{,}4) - 25{,}000 + 10{,}000(A/F{,}12\%{,}4) = \$-36{,}077$
$AW_Y = -75{,}000(A/P{,}12\%{,}6) - 15{,}000 + 7{,}000(A/F{,}12\%{,}6) = \$-32{,}380$

b. All *n* values are 3 years and the "salvage values" become the estimated market values after 3 years. Now X is economically better.
$AW_X = -40{,}000(A/P{,}12\%{,}3) - 25{,}000 + 14{,}000(A/F{,}12\%{,}3) = \$-37{,}505$
$AW_Y = -75{,}000(A/P{,}12\%{,}3) - 15{,}000 + 20{,}000(A/F{,}12\%{,}3) = \$-40{,}299$

Rate of Return Analysis

- The basis for calculating an unknown rate of return is an equivalence relation in PW, AW, or FW terms.
- The objective is to find the interest rate, represented as i^*, at which the cash flows are equivalent.
- The calculations are the reverse of those made previously, where the interest rate was known.

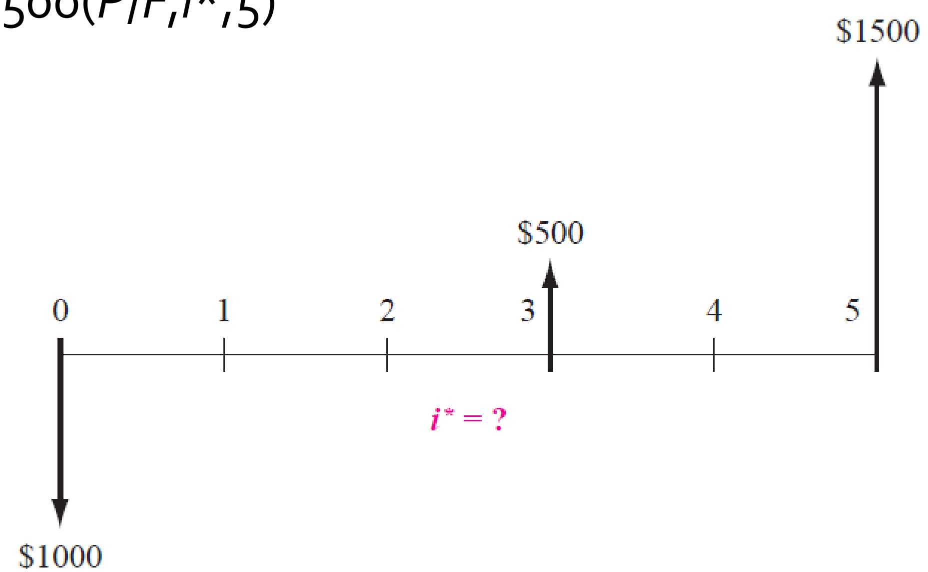
Rate of Return Analysis

- For example, if you invest $1000 now and are promised payments of $500 three years from now and $1500 five years from now, the rate of return relation using PW factors is.

$$1000 = 500(P/F,i^*,3) + 1500(P/F,i^*,5)$$

$$0 = -1000 + 500(P/F,i^*,3) + 1500(P/F,i^*,5)$$

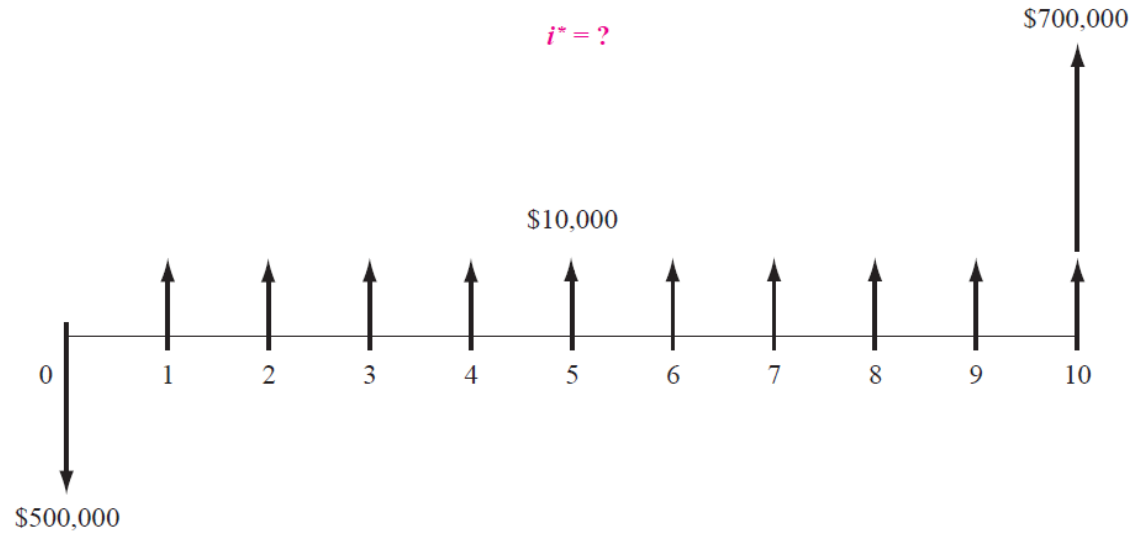
$$i^* = 16.9\%$$



Example 3

- The HVAC engineer for a company requested that $500,000 be spent on software and hardware to improve the efficiency of the environmental control systems. This is expected to save $10,000 per year for 10 years in energy costs and $700,000 at the end of 10 years in equipment refurbishment costs. Find the rate of return.

- **Solution**



Example 3

$0 = -500{,}000 + 10{,}000(P/A,i^*,10) + 700{,}000(P/F,i^*,10)$

- Try $i = 5\%$.

$0 = -500{,}000 + 10{,}000(P/A,5\%,10) + 700{,}000(P/F,5\%,10)$
$= \$6946$

- The result is positive, indicating that the return is more than 5%.
- Try $i = 6\%$.

$0 = -500{,}000 + 10{,}000(P/A,6\%,10) + 700{,}000(P/F,6\%,10)$
$= \$-35{,}519$

- Since 6% is too high, linearly interpolate between 5% and 6%.

$$i^* = 5.00 + \frac{6946 - 0}{6946 - (-35{,}519)}(1.0)$$
$$= 5.00 + 0.16 = 5.16\%$$

Benefit/Cost Analysis of a Single Project

- The benefit/cost ratio, a fundamental analysis method for public sector projects, was developed to introduce more objectivity into public sector economics.
- All cost and benefit estimates must be converted to a common equivalent monetary unit (PW, AW, or FW) at the discount rate (interest rate).
- The decision guideline for a single project is simple:
  - **If B/C ≥ 1.0**, accept the project as economically acceptable for the estimates and discount rate applied.
  - **If B/C < 1.0**, the project is not economically acceptable.

Benefit/Cost Analysis of a Single Project

- The *conventional B/C ratio* is the most widely used. It subtracts disbenefits from benefits.

$$\text{B/C} = \frac{\text{benefits} - \text{disbenefits}}{\text{costs}} = \frac{B - D}{C}$$

- The *modified B/C ratio* places benefits (including income and savings), disbenefits, and maintenance and operation (M&O) costs in the numerator.

$$\text{Modified B/C} = \frac{\text{benefits} - \text{disbenefits} - \text{M\&O costs}}{\text{initial investment}}$$

Example 4

- The Ford Foundation expects to award $15 million in grants to public high schools to develop new ways to teach the fundamentals of engineering that prepare students for university-level material. The grants will extend over a 10-year period and will create an estimated savings of $1.5 million per year in faculty salaries and student-related expenses. The Foundation uses a discount rate of 6% per year. This grants program will share Foundation funding with ongoing activities, so an estimated $200,000 per year will be removed from other program funding.

To make this program successful, a $500,000 per year operating cost will be incurred from the regular M&O budget. Use the B/C method to determine if the grants program is economically justified.

Example 4

- **Solution**

- Use annual worth as the common monetary equivalent.
  AW of investment cost. 15,000,000(*A*/*P*,6%,10) = $2,038,050 per year
  AW of M&O cost. $500,000 per year
  AW of benefit. $1,500,000 per year
  AW of disbenefit. $200,000 per year

$$\text{B/C} = \frac{1{,}500{,}000 - 200{,}000}{2{,}038{,}050 + 500{,}000} = \frac{1{,}300{,}000}{2{,}538{,}050} = 0.51$$

$$\text{Modified B/C} = \frac{1{,}500{,}000 - 200{,}000 - 500{,}000}{2{,}038{,}050} = 0.39$$

- The project is not justified, since B/C < 1.0.