Soil mechanics 2

Lateral earth pressure

Lateral earth pressure is a function of several factors, such as (a) the type and amount of wall movement, (b) the shear strength parameters of the soil, (c) the unit weight of the soil, and (d) the drainage conditions in the backfill.

Figure 1 shows a retaining wall of height H. For similar types of backfill,

a. The wall may be restrained from moving (Figure 1.a). The lateral earth pressure on the wall at any depth is called the at-rest earth pressure.
b. The wall may tilt away from the soil that is retained (Figure 1.b). With sufficient wall tilt, a triangular soil wedge behind the wall will fail. The lateral pressure for this condition is referred to as active earth pressure.
c. The wall may be pushed into the soil that is retained (Figure 1.c). With sufficient wall movement, a soil wedge will fail. The lateral pressure for this condition is referred to as passive earth pressure.

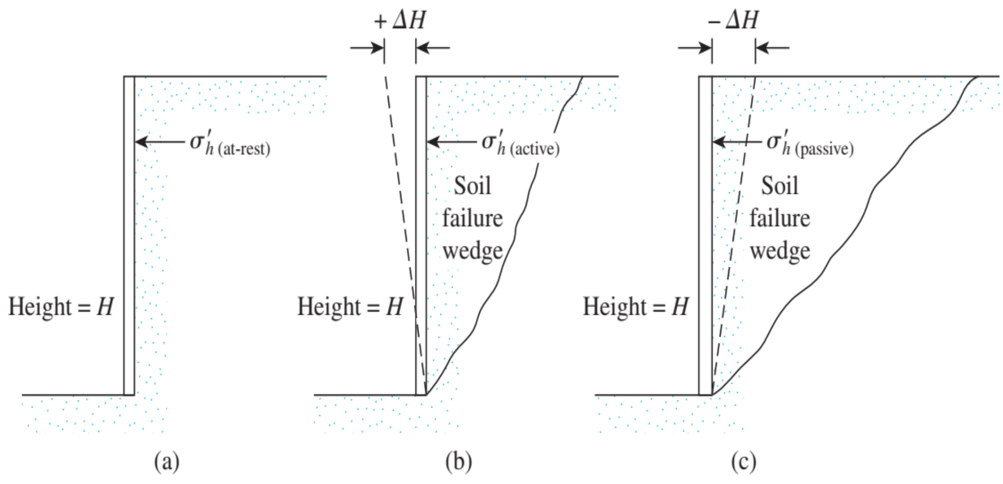

Figure 1: Nature of lateral earth pressure on a retaining wall

7.2 Lateral Earth Pressure at Rest

Consider a vertical wall of height H, as shown in Figure 7.3, retaining a soil having a unit weight of γ. A uniformly distributed load, q/unit area, is also applied at the ground surface. The shear strength of the soil is

$$s = c' + \sigma' \tan \phi'$$

where

c' = cohesion
ϕ' = effective angle of friction
σ' = effective normal stress

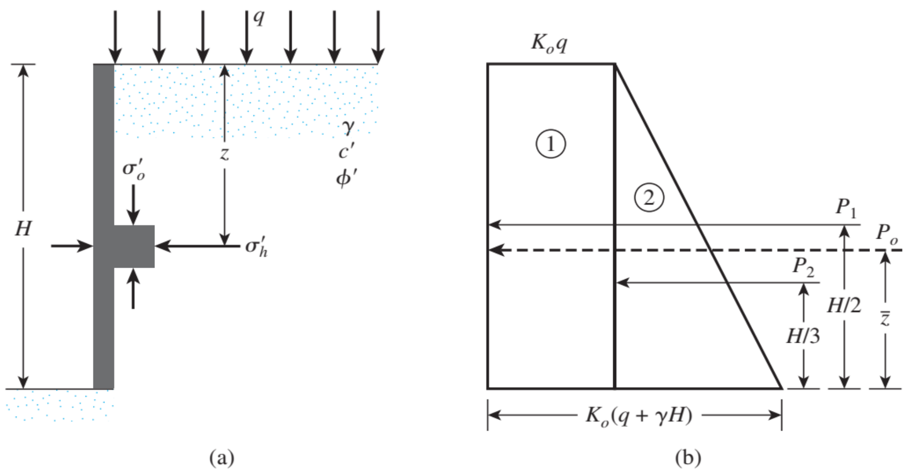

Figure 7.3 At-rest earth pressure

At any depth z below the ground surface, the vertical subsurface stress is

$$\sigma'_o = q + \gamma z \tag{7.1}$$

If the *wall is at rest and is not allowed to move at all,* either away from the soil mass or into the soil mass (i.e., there is zero horizontal strain), the lateral pressure at a depth z is

$$\sigma_h = K_o \sigma'_o + u \tag{7.2}$$

where

u = pore water pressure
K_o = coefficient of at-rest earth pressure

For normally consolidated soil, the relation for K_o (Jaky, 1944) is

$$\boxed{K_o \approx 1 - \sin \phi'} \tag{7.3}$$

Equation (7.3) is an empirical approximation.

For overconsolidated soil, the at-rest earth pressure coefficient may be expressed as (Mayne and Kulhawy, 1982)

$$K_o = (1 - \sin\phi')\mathrm{OCR}^{\sin\phi'} \tag{7.4}$$

where OCR = overconsolidation ratio.

With a properly selected value of the at-rest earth pressure coefficient, Eq. (7.2) can be used to determine the variation of lateral earth pressure with depth z. Figure 7.3b shows the variation of σ'_h with depth for the wall depicted in Figure 7.3a. Note that if the surcharge $q = 0$ and the pore water pressure $u = 0$, the pressure diagram will be a triangle. The total force, P_o, *per unit length* of the wall given in Figure 7.3a can now be obtained from the area of the pressure diagram given in Figure 7.3b and is

$$P_o = P_1 + P_2 = qK_oH + \tfrac{1}{2}\gamma H^2 K_o \tag{7.5}$$

where

P_1 = area of rectangle 1
P_2 = area of triangle 2

The location of the line of action of the resultant force, P_o, can be obtained by taking the moment about the bottom of the wall. Thus,

$$\bar{z} = \frac{P_1\left(\dfrac{H}{2}\right) + P_2\left(\dfrac{H}{3}\right)}{P_o} \tag{7.6}$$

If the water table is located at a depth $z < H$, the at-rest pressure diagram shown in Figure 7.3b will have to be somewhat modified, as shown in Figure 7.4. If the effective unit weight of soil below the water table equals γ' (i.e., $\gamma_{\mathrm{sat}} - \gamma_w$), then

at $z = 0$, $\quad \sigma'_h = K_o\sigma'_o = K_oq$
at $z = H_1$, $\quad \sigma'_h = K_o\sigma'_o = K_o(q + \gamma H_1)$

and

at $z = H_2$, $\quad \sigma'_h = K_o\sigma'_o = K_o(q + \gamma H_1 + \gamma' H_2)$

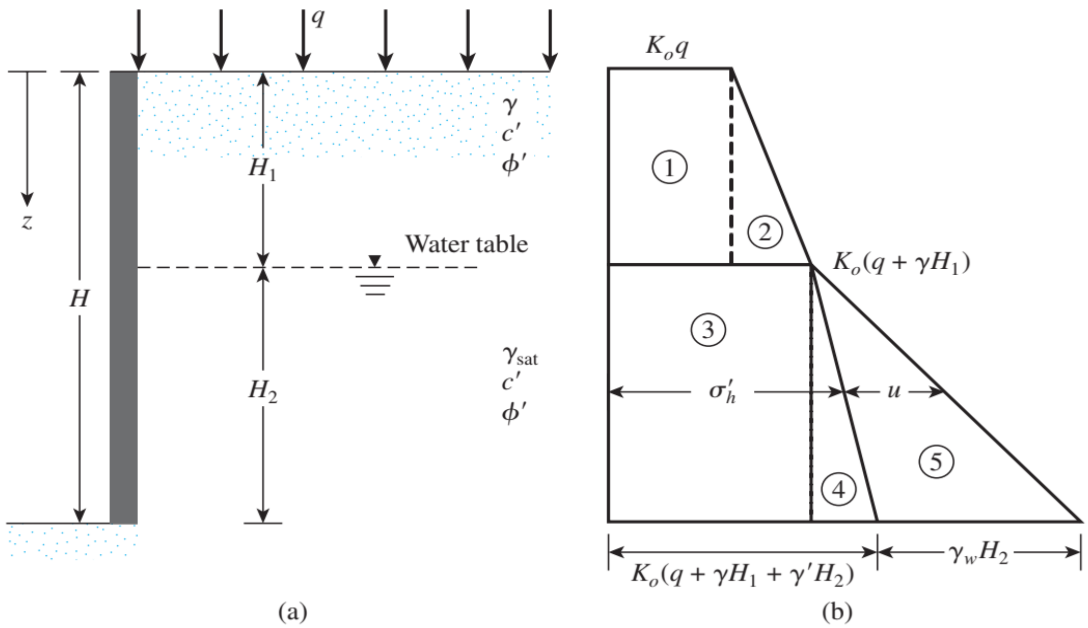

Figure 7.4 At-rest earth pressure with water table located at a depth $z < H$

Note that in the preceding equations, σ'_o and σ'_h are effective vertical and horizontal pressures, respectively. Determining the total pressure distribution on the wall requires adding the hydrostatic pressure u, which is zero from $z = 0$ to $z = H_1$ and is $H_2\gamma_w$ at $z = H_2$. The variation of σ'_h and u with depth is shown in Figure 7.4b. Hence, the total force per unit length of the wall can be determined from the area of the pressure diagram. Specifically,

$$P_o = A_1 + A_2 + A_3 + A_4 + A_5$$

where A = area of the pressure diagram.

So,

$$P_o = K_o q H_1 + \tfrac{1}{2}K_o\gamma H_1^2 + K_o(q + \gamma H_1)H_2 + \tfrac{1}{2}K_o\gamma' H_2^2 + \tfrac{1}{2}\gamma_w H_2^2 \tag{7.7}$$

Example 7.1

For the retaining wall shown in Figure 7.5(a), determine the lateral earth force at rest per unit length of the wall. Also determine the location of the resultant force. Assume OCR = 1.

Solution

$$K_o = 1 - \sin\phi' = 1 - \sin 30° = 0.5$$

At $z = 0$, $\sigma'_o = 0$; $\sigma'_h = 0$

At $z = 2.5$ m, $\sigma'_o = (16.5)(2.5) = 41.25\ \text{kN/m}^2$;

$$\sigma'_h = K_o\sigma'_o = (0.5)(41.25) = 20.63\ \text{kN/m}^2$$

At $z = 5$ m, $\sigma'_o = (16.5)(2.5) + (19.3 - 9.81)2.5 = 64.98\ \text{kN/m}^2$;

$$\sigma'_h = K_o\sigma'_o = (0.5)(64.98) = 32.49\ \text{kN/m}^2$$

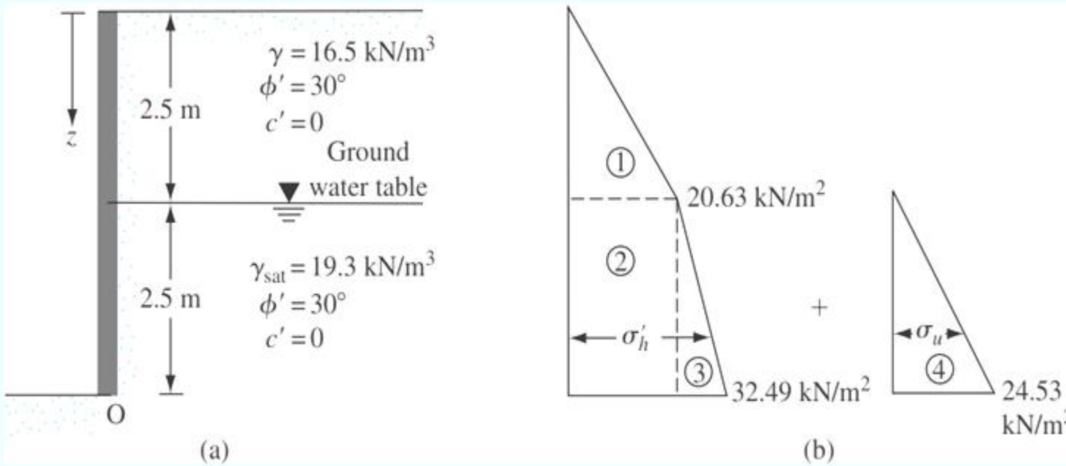

Figure 7.5

The hydrostatic pressure distribution is as follows:

From $z = 0$ to $z = 2.5$ m, $u = 0$. At $z = 5$ m, $u = \gamma_w(2.5) = (9.81)(2.5) = 24.53$ kN/m^2. The pressure distribution for the wall is shown in Figure 7.5b.

The total force per unit length of the wall can be determined from the area of the pressure diagram, or

$$
\begin{aligned}
P_o &= \text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4} \\
&= \tfrac{1}{2}(2.5)(20.63) + (2.5)(20.63) + \tfrac{1}{2}(2.5)(32.49 - 20.63) \\
&\quad + \tfrac{1}{2}(2.5)(24.53) = \mathbf{122.85\ kN/m}
\end{aligned}
$$

The location of the center of pressure measured from the bottom of the wall (point O) =

$$
\begin{aligned}
\bar{z} &= \frac{(\text{Area 1})\left(2.5+\dfrac{2.5}{3}\right)+(\text{Area 2})\left(\dfrac{2.5}{2}\right)+(\text{Area 3}+\text{Area 4})\left(\dfrac{2.5}{3}\right)}{P_o} \\
&= \frac{(25.788)(3.33) + (51.575)(1.25) + (14.825 + 30.663)(0.833)}{122.85} \\
&= \frac{85.87 + 64.47 + 37.89}{122.85} = \mathbf{1.53\ m}
\end{aligned}
$$

Active Pressure

7.3 Rankine Active Earth Pressure

The lateral earth pressure described in Section 7.2 involves walls that do not yield at all. However, if a wall tends to move away from the soil a distance Δx, as shown in Figure 7.6a, the soil pressure on the wall at any depth will decrease. For a wall that is *frictionless,* the horizontal stress, σ'_h, at depth z will equal $K_o\sigma'_o(=K_o\gamma z)$ when Δx is zero. However, with $\Delta x > 0, \sigma'_h$ will be less than $K_o\sigma'_o$.

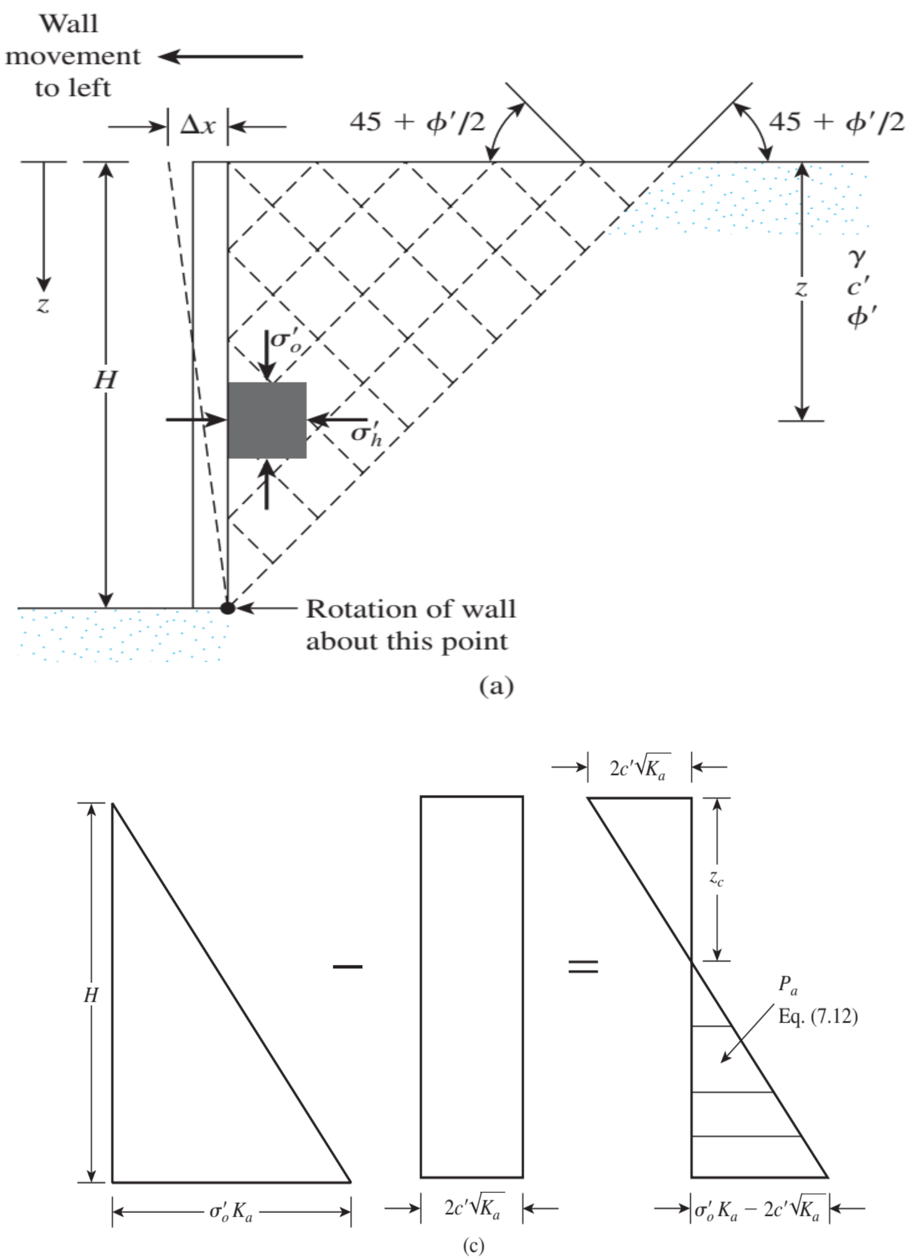

Figure 7.6 Rankine active pressure

The Mohr's circles corresponding to wall displacements of $\Delta x = 0$ and $\Delta x > 0$ are shown as circles *a* and *b*, respectively, in Figure 7.6b. If the displacement of the wall, Δx, continues to increase, the corresponding Mohr's circle eventually will just touch the Mohr–Coulomb failure envelope defined by the equation

$$s = c' + \sigma' \tan \phi'$$

This circle, marked *c* in the figure, represents the failure condition in the soil mass; the horizontal stress then equals σ'_a, referred to as the *Rankine active pressure*. The *slip lines* (failure planes) in the soil mass will then make angles of $\pm(45 + \phi'/2)$ with the horizontal, as shown in Figure 7.6a.

Equation (1.87) relates the principal stresses for a Mohr's circle that touches the Mohr–Coulomb failure envelope:

$$\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

For the Mohr's circle *c* in Figure 7.6b,

Major principal stress, $\sigma'_1 = \sigma'_o$

and

Minor principal stress, $\sigma'_3 = \sigma'_a$

Thus,

$$\sigma'_o = \sigma'_a \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

$$\sigma'_a = \frac{\sigma'_o}{\tan^2\left(45 + \frac{\phi'}{2}\right)} - \frac{2c'}{\tan\left(45 + \frac{\phi'}{2}\right)}$$

or

$$\begin{aligned} \sigma'_a &= \sigma'_o \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \tan\left(45 - \frac{\phi'}{2}\right) \\ &= \sigma'_o K_a - 2c'\sqrt{K_a} \end{aligned} \tag{7.8}$$

where $K_a = \tan^2(45 - \phi'/2)$ = Rankine active-pressure coefficient.

The variation of the active pressure with depth for the wall shown in Figure 7.6a is given in Figure 7.6c. Note that $\sigma'_o = 0$ at $z = 0$ and $\sigma'_o = \gamma H$ at $z = H$. The pressure distribution shows that at $z = 0$ the active pressure equals $-2c'\sqrt{K_a}$, indicating a tensile stress that decreases with depth and becomes zero at a depth $z = z_c$, or

$$\gamma z_c K_a - 2c'\sqrt{K_a} = 0$$

and

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} \tag{7.9}$$

The depth z_c is usually referred to as the *depth of tensile crack,* because the tensile stress in the soil will eventually cause a crack along the soil–wall interface. Thus, the total Rankine active force per unit length of the wall before the tensile crack occurs is

$$P_a = \int_0^H \sigma'_a dz = \int_0^H \gamma z K_a dz - \int_0^H 2c'\sqrt{K_a} dz$$
$$= \tfrac{1}{2}\gamma H^2 K_a - 2c' H\sqrt{K_a} \tag{7.10}$$

After the tensile crack appears, the force per unit length on the wall will be caused only by the pressure distribution between depths $z = z_c$ and $z = H$, as shown by the hatched area in Figure 7.6c. This force may be expressed as

$$P_a = \tfrac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) \tag{7.11}$$

or

$$P_a = \frac{1}{2}\left(H - \frac{2c'}{\gamma\sqrt{K_a}}\right)\left(\gamma H K_a - 2c'\sqrt{K_a}\right) \tag{7.12}$$

However, it is important to realize that the active earth pressure condition will be reached only if the wall is allowed to "yield" sufficiently. The necessary amount of outward displacement of the wall is about $0.001H$ to $0.004H$ for granular soil backfills and about $0.01H$ to $0.04H$ for cohesive soil backfills.

Note further that if the *total stress* shear strength parameters (c, ϕ) were used, an equation similar to Eq. (7.8) could have been derived, namely,

$$\sigma_a = \sigma_o \tan^2\left(45 - \frac{\phi}{2}\right) - 2c \tan\left(45 - \frac{\phi}{2}\right)$$

Example 7.2

A 6-m-high retaining wall is to support a soil with unit weight $\gamma = 17.4\ \text{kN/m}^3$, soil friction angle $\phi' = 26°$, and cohesion $c' = 14.36\ \text{kN/m}^2$. Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs, and determine the line of action of the resultant in both cases.

Solution

For $\phi' = 26°$,

$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2(45 - 13) = 0.39$$

$$\sqrt{K_a} = 0.625$$

$$\sigma'_a = \gamma H K_a - 2c'\sqrt{K_a}$$

From Figure 7.6c, at $z = 0$,

$$\sigma'_a = -2c'\sqrt{K_a} = -2(14.36)(0.625) = -17.95 \text{ kN/m}^2$$

and at $z = 6$ m,

$$\sigma'_a = (17.4)(6)(0.39) - 2(14.36)(0.625)$$
$$= 40.72 - 17.95 = 22.77 \text{ kN/m}^2$$

Active Force before the Tensile Crack Appeared: Eq. (7.10)

$$P_a = \tfrac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a}$$
$$= \tfrac{1}{2}(6)(40.72) - (6)(17.95) = 122.16 - 107.7 = \mathbf{14.46 \text{ kN/m}}$$

The line of action of the resultant can be determined by taking the moment of the area of the pressure diagrams about the bottom of the wall, or

$$P_a\bar{z} = (122.16)\left(\frac{6}{3}\right) - (107.7)\left(\frac{6}{2}\right)$$

Thus,

$$\bar{z} = \frac{244.32 - 323.1}{14.46} = -\mathbf{5.45 \text{ m}}.$$

Active Force after the Tensile Crack Appeared: Eq. (7.9)

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} = \frac{2(14.36)}{(17.4)(0.625)} = 2.64 \text{ m}$$

Using Eq. (7.11) gives

$$P_a = \tfrac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) = \tfrac{1}{2}(6 - 2.64)(22.77) = \mathbf{38.25 \text{ kN/m}}$$

Figure 7.6c indicates that the force $P_a = 38.25$ kN/m is the area of the hatched triangle. Hence, the line of action of the resultant will be located at a height $\bar{z} = (H - z_c)/3$ above the bottom of the wall, or

$$\bar{z} = \frac{6 - 2.64}{3} = \mathbf{1.12 \text{ m}}$$

■

Passive Pressure

7.10 Rankine Passive Earth Pressure

Figure 7.23a shows a vertical frictionless retaining wall with a horizontal backfill. At depth z, the effective vertical pressure on a soil element is $\sigma'_o = \gamma z$. Initially, if the wall does not yield at all, the lateral stress at that depth will be $\sigma_h' = K_o \sigma_o'$. This state of stress is illustrated by the Mohr's circle a in Figure 7.23b. Now, if the wall is pushed into the soil mass by an amount Δx, as shown in Figure 7.23a, the vertical stress at depth z will stay the same; however, the horizontal stress will increase. Thus, σ'_h will be greater than $K_o \sigma'_o$. The state of stress can now be represented by the Mohr's circle b in Figure 7.23b. If the wall moves farther inward (i.e., Δx is increased still more), the stresses at depth z will ultimately reach the state represented by Mohr's circle c. Note that this Mohr's circle touches the Mohr–Coulomb failure envelope, which implies that the soil behind the wall will fail by being pushed upward. The horizontal stress, σ'_h, at this point is referred to as the *Rankine passive pressure,* or $\sigma'_h = \sigma'_p$.

For Mohr's circle c in Figure 7.23b, the major principal stress is σ'_p, and the minor principal stress is σ'_o. Substituting these quantities into Eq. (1.87) yields

$$\sigma'_p = \sigma'_o \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right) \tag{7.60}$$

Now, let

$$\begin{aligned} K_p &= \text{Rankine passive earth pressure coefficient} \\ &= \tan^2\left(45 + \frac{\phi'}{2}\right) \end{aligned} \tag{7.61}$$

Then, from Eq. (7.60), we have

$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p} \tag{7.62}$$

Equation (7.62) produces (Figure 7.23c), the passive pressure diagram for the wall shown in Figure 7.23a. Note that at $z = 0$,

$$\sigma'_o = 0 \quad \text{and} \quad \sigma'_p = 2c'\sqrt{K_p}$$

and at $z = H$,

$$\sigma'_o = \gamma H \quad \text{and} \quad \sigma'_p = \gamma H K_p + 2c'\sqrt{K_p}$$

The passive force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_p = \tfrac{1}{2}\gamma H^2 K_p + 2c' H\sqrt{K_p} \tag{7.63}$$

The approximate magnitudes of the wall movements, Δx, required to develop failure under passive conditions are as follows:

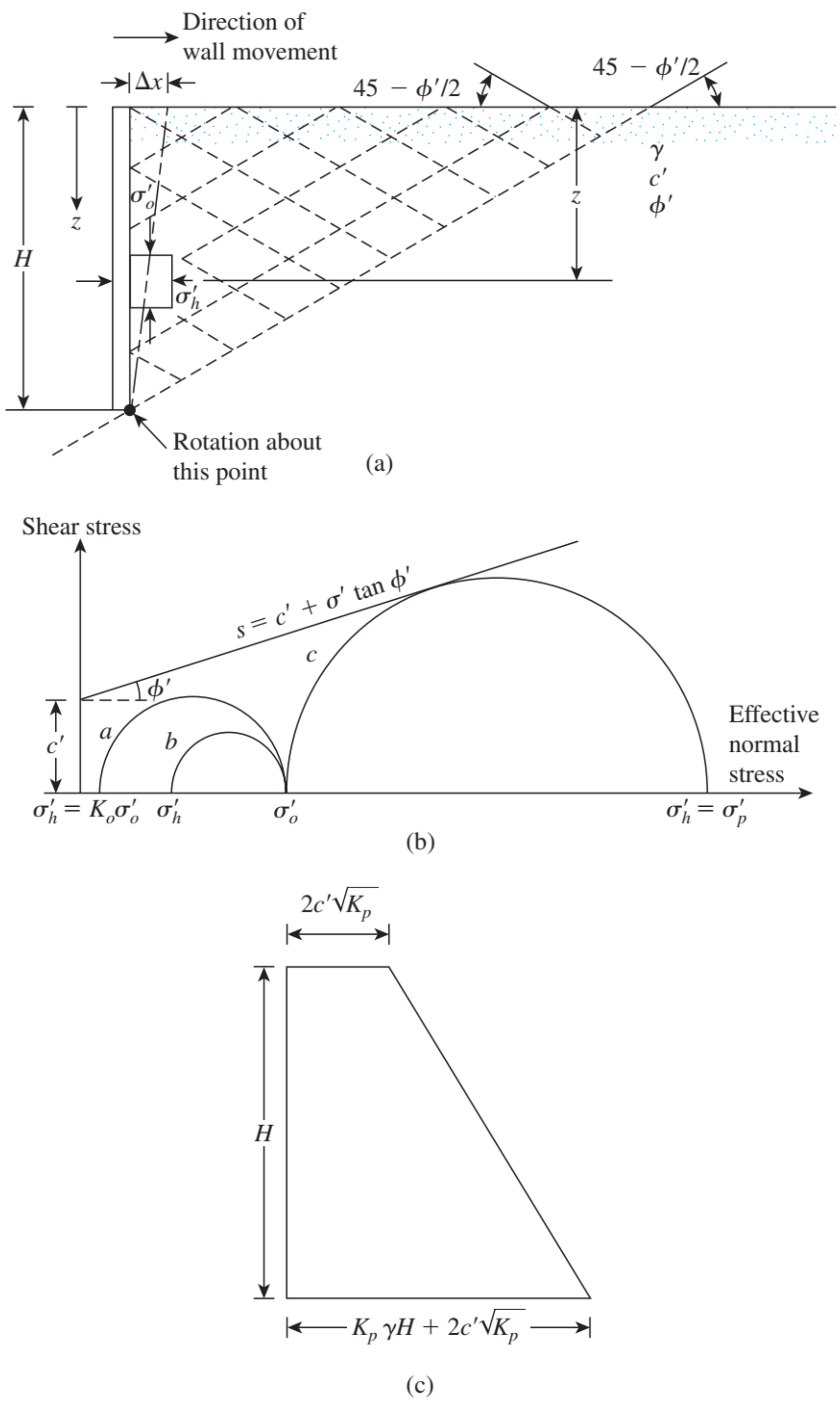

Figure 7.23 Rankine passive pressure

If the backfill behind the wall is a granular soil (i.e., $c' = 0$), then, from Eq. (7.63), the passive force per unit length of the wall will be

$$P_p = \frac{1}{2}\gamma H^2 K_p \tag{7.64}$$

Example 7.12

A 3-m high wall is shown in Figure 7.24a. Determine the Rankine passive force per unit length of the wall.

Solution

For the top layer

$$K_{p(1)} = \tan^2\left(45 + \frac{\phi'_1}{2}\right) = \tan^2(45 + 15) = 3$$

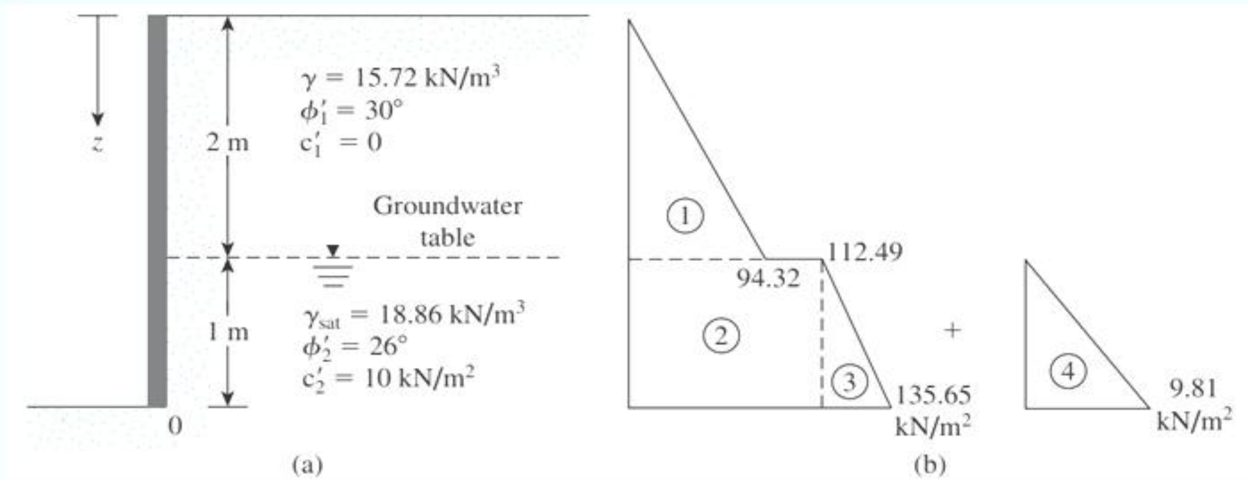

Figure 7.24

From the bottom soil layer

$$K_{p(2)} = \tan^2\left(45 + \frac{\phi'_2}{2}\right) = \tan^2(45 + 13) = 2.56$$

$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p}$$

where

σ'_o = effective vertical stress

at $z = 0, \sigma'_o = 0, c'_1 = 0, \sigma'_p = 0$

at $z = 2$ m, $\sigma'_o = (15.72)(2) = 31.44$ kN/m², $c'_1 = 0$

So, for the top soil layer

$$\sigma'_p = 31.44K_{p(1)} + 2(0)\sqrt{K_{p(1)}} = 31.44(3) = 94.32 \text{ kN/m}^2$$

At this depth, that is $z = 2$ m, for the bottom soil layer

$$\sigma'_p = \sigma'_o K_{p(2)} + 2c'_2\sqrt{K_{p(2)}} = 31.44(2.56) + 2(10)\sqrt{2.56}$$
$$= 80.49 + 32 = 112.49 \text{ kN/m}^2$$

Again, at $z = 3$ m,

$$\sigma'_o = (15.72)(2) + (\gamma_{sat} - \gamma_w)(1)$$
$$= 31.44 + (18.86 - 9.81)(1) = 40.49 \text{ kN/m}^2$$

Hence,

$$\sigma'_p = \sigma'_o K_{p(2)} + 2c'_2\sqrt{K_{p(2)}} = 40.49(2.56) + (2)(10)(1.6)$$
$$= \mathbf{135.65 \text{ kN/m}^2}$$

Note that, because a water table is present, the hydrostatic stress, u, also has to be taken into consideration. For $z = 0$ to 2 m, $u = 0$; $z = 3$ m, $u = (1)(\gamma_w) = 9.81 \text{ kN/m}^2$.

The passive pressure diagram is plotted in Figure 6.24b. The passive force per unit length of the wall can be determined from the area of the pressure diagram as follows:

Area no.	Area	
1	$(\frac{1}{2})(2)(94.32)$	$= 94.32$
2	$(112.49)(1)$	$= 112.49$
3	$(\frac{1}{2})(1)(135.65 - 112.49)$	$= 11.58$
4	$(\frac{1}{2})(9.81)(1)$	$= 4.905$
		$P_P \approx 223.3$ **kN/m**

Problem

7.2 Use Eq. (7.3), Figure P7.2, and the following values to determine the at-rest lateral earth force per unit length of the wall. Also find the location of the resultant.
$H = 5$ m, $H_1 = 2$ m, $H_2 = 3$ m, $\gamma = 15.5\ \mathrm{kN/m^3}$, $\gamma_{sat} = 18.5\ \mathrm{kN/m^3}$, $\phi' = 34°$, $c' = 0$, $q = 20\ \mathrm{kN/m^2}$, and OCR = 1.

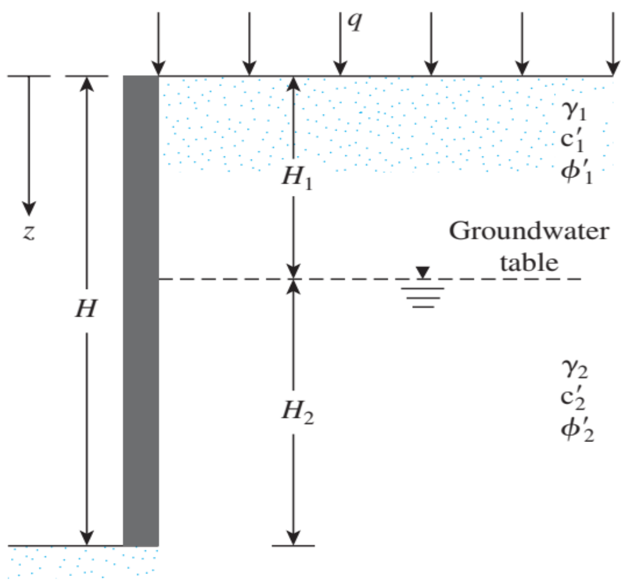

Figure P7.2

Also determine active and passive lateral earth force per unit length of the wall.