INTERNATIONAL UNIVERSITY OF AFRICA

CIVIL ENGINEERING DEPARTMENT

ANALYSIS AND DESIGN OF STEEL WORKS II

GRADE 4

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Lecture No 7

MEMBERS SUBJECT TO AXIAL LOAD AND BENDING

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MEMBERS SUBJECT TO AXIAL LOAD AND BENDING

4.1 Introduction

Situations will often arise in which the loading on a member cannot reasonably be represented as a single dominant effect. Such problems require an understanding of the way in which the various structural actions interact with one another. In the simplest of cases, this may account to nothing more than a direct summation of 'unity' factors (e.g. applied axial load/axial resistance plus applied moment/moment capacity). Alternatively, for more complex problems, careful consideration of the complicated interaction between individual load components and the resulting deformations is necessary. انظر الترجمه

The design of members subject to axial load and bending is influenced by the method of frame analysis, the shape of the cross-section used and the type of restraint provided.

In order to perform satisfactorily, the combined effects of axial load and bending must not cause the member to fail due to:

- *Local buckling*
- *Inadequate cross-section capacity*
- *Overall member buckling.*

Therefore, a member subject to axial load and bending must be checked for *each of these failure modes.*

The additional complexity of buckling associated with compressive loads means that it is more convenient to discuss the cases of *tension plus bending* and *compression plus bending separately*.

4.2 Section classification

In order to ensure that a member does not fail due to local buckling the cross-section should be ***classified*** and the design carried out according to theclass of cross-section.

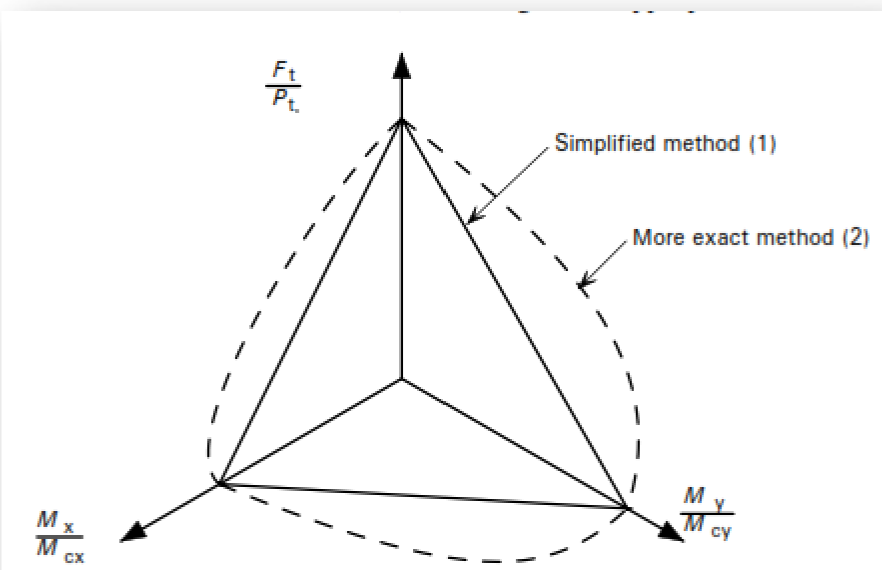
Classification of sections is discussed before.

4.3 Tension members with moments

4.3.1 Cross-section capacity

Figure 4.1 illustrates the type of three-dimensional interaction failure envelope that controls the ultimate strength of steel members under combined biaxial bending and tension. Each axis represents a single load component, ***F***, ***Mx*** or ***My***

Two failure envelopes are illustrated, one a simple plane surface (the 'simplified method') and one a doubly curved surface that generally lies further from the origin (the 'more exact method'). Both envelopes intersect the individual axes at points that represent the member's capacity under that form of load acting ***singly***.



4.1 Simple and more exact failure envelopes for strength under combined loading

Simplified method

The simplified method of BS 5950-1:2000 is expressed as:

$$\frac{F_t}{P_t} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \leq 1$$

F_t	is the axial tension at the critical location
P_t	is the tension capacity of the section
M_x	is the moment about the major axis at the critical location
M_{cx}	is the moment capacity about the major axis of the section
M_y	is the moment about the minor axis at the critical location
M_{cy}	is the moment capacity about the minor axis of the section.

The expression should be evaluated at the critical location within the member, which is usually where the moments and/or the forces are largest.

The interaction expression above is based on the simple assumption that the stresses due to axial load and moments are additive, as shown in Figure 4.2.
Thus, for an elastic distribution, the cross-section capacity is reached when the total stress at an extreme fibre reaches the yield stress of the member.

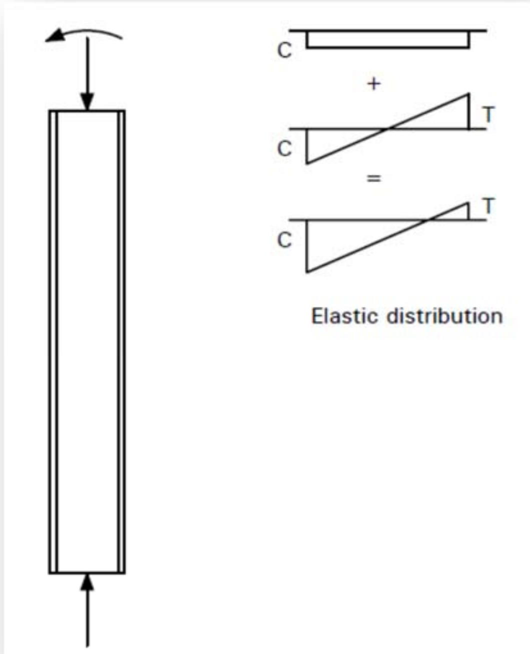


Figure 4.2 Stress due to axial load and bending

More exact method

Alternatively, there is a more exact method that may be used for tension members with moments.

The advantage of using the more exact method is that the interaction is non-linear and therefore the calculations *give a larger cross-section capacity*. The difference between the simple and the more exact methods is shown graphically in Figure 4.1.

we will skip the exact method since it depend on the plastic analysis and it is out of the scope at this level and the simplified method is quite satisfactory

4.4 Compression members with moments

4.4.1 General

Compression members with moments must be checked for cross-section capacity and member buckling resistance.

4.4.2 Cross-section capacity

Simplified method

The simplified method for compression members with moments is very similar to the simplified method for tension members with moments, the one difference being that for Class 4 slender sections the capacities (axial and bending) are based on the effective section area.

For compression members the following expressions should be satisfied. For non slender members (i.e. Class 1, 2 or 3)

$$\frac{F_c}{A_g p_y} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \leq 1$$

where:

p_y is the member design strength

A_g is the gross area of the cross-section

A_{eff} is the effective area of the cross-section

These expressions should be satisfied for the critical locations of the member i.e. *where the axial loads and bending moments are greatest.*

4.4.3 Member buckling resistance

The code again gives two methods for checking the member buckling resistance; *a simplified method* and a *more exact method.*

Simplified Method

Compression members with moments can be checked for member buckling resistance by using two interaction formulae. The first expression deals with buckling generally and the second with buckling about the minor axis.

$$\frac{F_c}{P_c} + \frac{m_x M_x}{p_y Z_x} + \frac{m_y M_y}{p_y Z_y} \leq 1$$

1-General equation

$$\frac{F_c}{P_{cy}} + \frac{m_{LT} M_{LT}}{M_b} + \frac{m_y M_y}{p_y Z_y} \leq 1$$

2-Buckling About The Minor Axis

where:

F_c	is the applied axial compression
P_c	is the minimum of P_{cx} and P_{cy}
P_{cx}	is the compression resistance considering buckling about the major axis only
P_{cy}	is the compression resistance considering buckling about the minor axis only
M_x	is the maximum major axis moment within the segment length L_x governing P_{cx}
M_y	is the maximum minor axis moment within the segment length L_y governing P_{cy}
M_{LT}	is the maximum major axis moment within the segment length L governing M_b
Z_x	is the elastic section modulus about the major axis, $Z_{x,eff}$ should be used for Class 4 slender sections

Z_y is the elastic section modulus about the minor axis, $Z_{y,eff}$ should be used for Class 4 slender sections

m_x, m_y and m_{LT} are uniform moment factors, which take account of the shape of the bending moment diagram between restraints (see later).

The first of the two interaction expressions (1) is a combination of the following expressions, (3) and (4), relating to buckling about the major axis due to axial compression and major axis bending (3) and to buckling about the minor axis due to axial compression and minor axis bending (4).

$$\frac{F_c}{P_{cx}} + \frac{m_x M_x}{p_y Z_x} \le 1 \quad \text{for major axis bending} \longrightarrow \boxed{3}$$

$$\frac{F_c}{P_{cy}} + \frac{m_y M_y}{p_y Z_y} \le 1 \quad \text{for minor axis bending} \longrightarrow \boxed{4}$$

The second expression (2) in this Section checks buckling about the minor axis due to axial compression, major axis bending and minor axis bending.

This form of buckling is analogous to lateral torsional buckling in beams. The column buckles in a mode involving twisting and minor axis bending.

The twisting mode distinguishes it from minor axis buckling and reduces the buckling load. It is significant for I and H sections that buckle at low axial loads. It is generally not relevant for tubular sections, apart from rectangular hollow sections with a large depth to width ratio.

In this case the value of P_{cy} is specifically used as we are considering buckling about the minor axis. Figure 4.3 shows the differences between in-plane and out-of-plane buckling

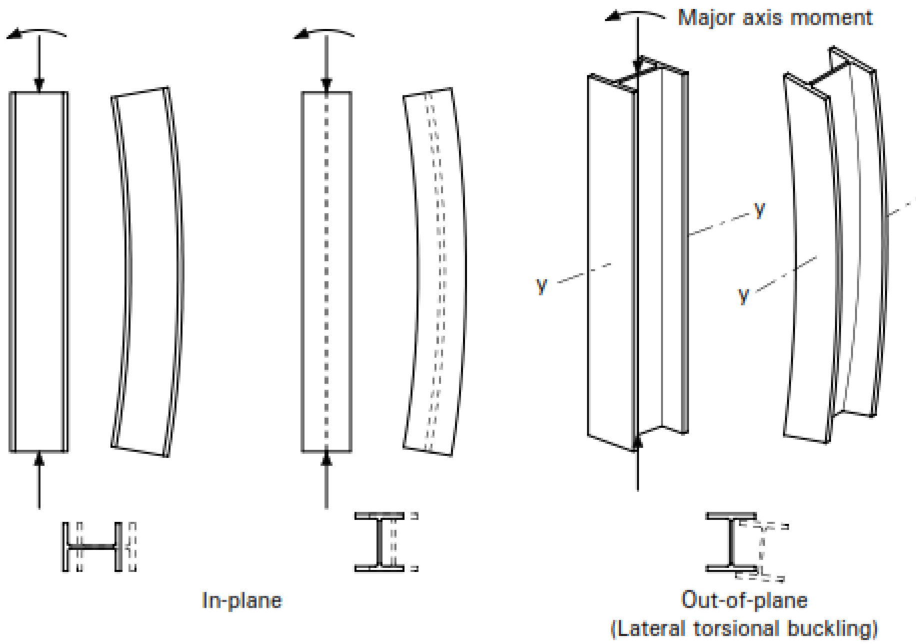


Figure 4.3 Buckling modes for members subject to axial compression and bending

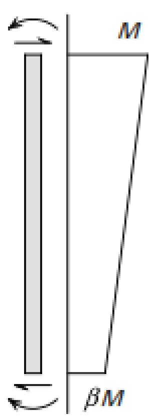
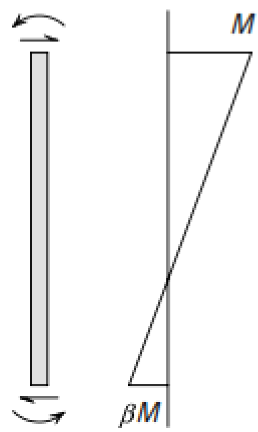
Equivalent uniform moment factors

The equivalent uniform moment factors *mx , my, mxy* and *mLT* take account of the shape of the bending moment diagram between restraints. These factors are required because the theory is based on members subject to a uniform moment, which is the worst case. Therefore, the uniform moment factors can conservatively be taken as 1.0 for all cases.

β=1 see table

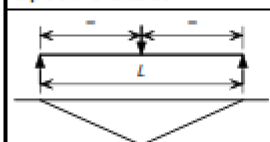
The equivalent uniform moment factors for flexural buckling (*mx, my,and mxy*) are obtained from Table 26 of BS 5950-1,. If there is bending about both axes, equivalent uniform moment factors are required for each axis and are likely to have different values

Table 26 Equivalent uniform moment factors for flexural buckling

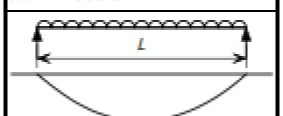
Segments with end moments only (m from formula for the general case)		β	m
β positive 	β negative 	1.0	1.00
		0.9	0.96
		0.8	0.92
		0.7	0.88
		0.6	0.84
		0.5	0.80
		0.4	0.76
		0.3	0.72
		0.2	0.68
		0.1	0.64
		0.0	0.60
		−0.1	0.58
		−0.2	0.56
		−0.3	0.54
		−0.4	0.52
		−0.5	0.50
		−0.6	0.48
		−0.7	0.46
		−0.8	0.44
		−0.9	0.42
		−1.0	0.40

Segments between intermediate lateral restraints

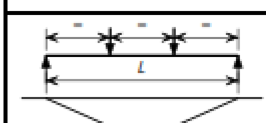
Specific cases



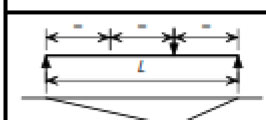
$m = 0.90$



$m = 0.95$

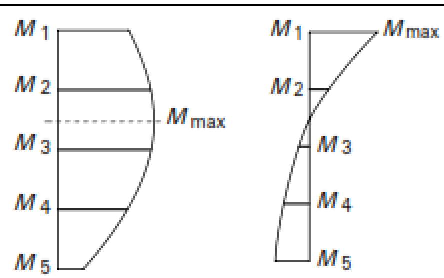


$m = 0.95$



$m = 0.80$

General case



$$m = 0.2 + \frac{0.1M_2 + 0.6M_3 + 0.1M_4}{M_{max}} \quad \text{but } m \geq \frac{0.8M_{24}}{M_{max}}$$

The moments M_2 and M_4 are the values at the quarter points and the moments M_3 is the value at mid length.

If M_2, M_3 and M_4 all lie on he same side of the axis, their values are all taken as positive. If they lie both side of the axis, the side leading to the larger value of m is taken as the positive side.

The values of M_{max} and M_{24} are always taken as positive. M_{max} is the maximum moment in the segment and M_{24} is the maximum moment in the central half of the segment.

Note: The applied point loads and the UDL provide no lateral restraint to the beam

5 Summary of design procedure

5.1 Tension members with moments

1. Select section and steel grade
2. Determine design strength — Table 9
3. Determine section classification — Table 11, Table 12
4. For Class 1 and Class 2 sections use gross section properties
5. For Class 3 semi-compact sections calculate the effective plastic modulus — [redacted]
6. For Class 4 slender sections calculate the effective elastic modulus — [redacted]
7. Evaluate the cross-section capacity interaction expressions. Check that the result does not exceed unity. — [redacted]
8. If the result is slightly greater than unity and the section is Class 1 or Class 2 calculate the reduced moment capacity in the presence of axial force and try using the more exact method for cross-section capacity. Check that the result does not exceed unity. — [redacted]
9. Check the member buckling resistance. Ignore the tensile axial load and design as an unrestrained beam. — [redacted]

5.2 Compression members with moments

1. Select section and steel grade	
2. Determine design strength	Table 9
3. Determine section classification	Table 11 Table 12
4. For Class 1 and Class 2 sections use gross section properties	
5. For Class 3 semi-compact sections calculate the effective plastic modulus	[redacted]
6. For Class 4 slender sections calculate the effective elastic modulus and the effective area if subject to compression	[redacted]
7. Evaluate the cross-section capacity interaction expressions. Check that the result does not exceed unity.	[redacted]
8. If the result is slightly greater than unity and the section is Class 1 or Class 2 calculate the reduced moment capacity in the presence of axial force and try using the more exact method for cross-section capacity. Check that the result does not exceed unity.	[redacted]

	BS 5950-1
9. Determine the compression resistance for buckling about the major axis and the minor axis.	[redacted]
10. Determine the buckling resistance moment and the maximum major axis moment within the segment length.	[redacted]
11. Determine values of the equivalent uniform moment factors.	Table 18 Table 26
12. Check the member buckling resistance. Evaluate the simplified method interaction expressions and check that the results do not exceed unity.	[redacted]
13. If a result is slightly greater than unity and the section is doubly symmetric try using the more exact method for member buckling resistance. Check that the result does not exceed unity.	[redacted]

7.5.3 Example of beam column design

A braced column 4.5 m long is subjected to the factored end loads and moments about the x–x axis, as shown in Figure 7.19(a). The column is held in position but only partially restrained in direction at the ends. Check that a 203 × 203 UC 52 in Grade 43 steel is adequate.

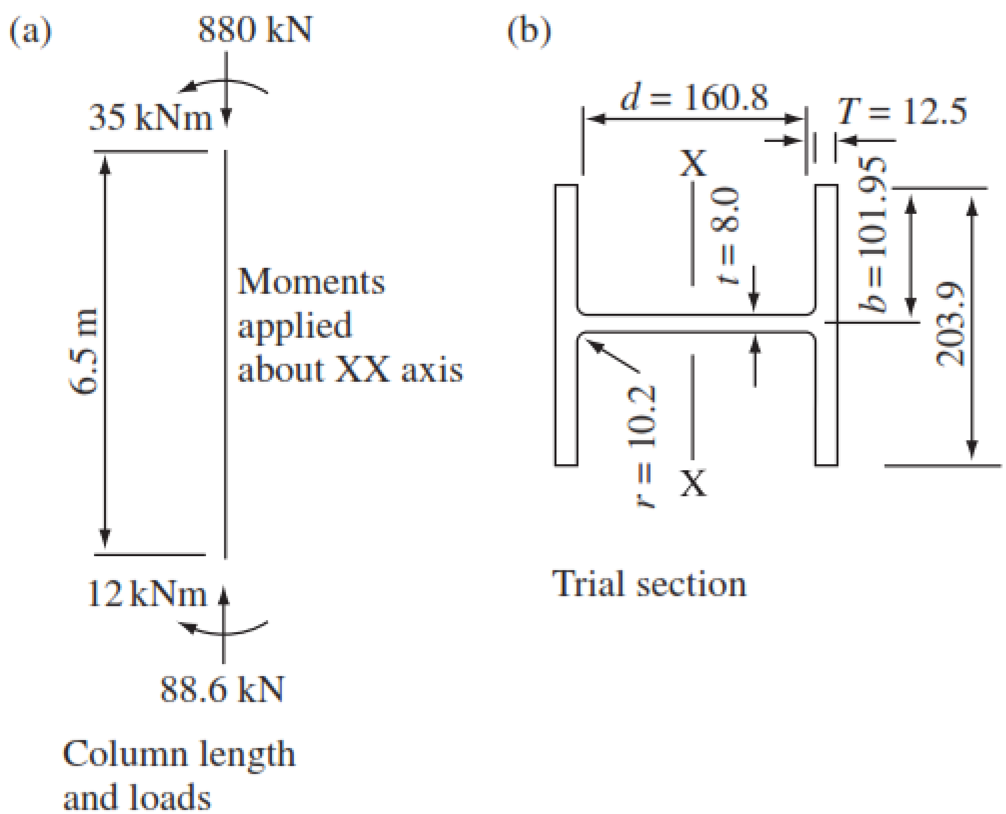


Figure 7.19 Beam column design example

$$\frac{F_c}{P_c} + \frac{m_x M_x}{p_y Z_x} + \cancel{\frac{m_y M_y}{p_y Z_y}} \le 1$$

$$\frac{F_c}{P_{cy}} + \frac{m_{LT} M_{LT}}{M_b} + \cancel{\frac{m_y M_y}{p_y Z_y}} \le 1$$

(1) Column-section classification

Design strength from Table 9 $p_y = 275\,\text{N/mm}^2$
Factor $\varepsilon = (275/p_y)^{0.5} = 1.0$
(see Figure 7.19(b))
Flange $b/T = 101.95/12.5 = 8.156 < 9.0$
Web $d/t = 160.8/8.0 = 20.1 < 40$

Referring to Table 11, the flanges are plastic and the web semi-compact.

(2) Cross-section capacity check

Section properties for 203 × 203 UC 52 are:

$$A = 66.4\,\text{cm}^2;\quad Z_x = 510\,\text{cm}^3;\quad r_y = 516\,\text{cm}$$
$$x = 15.8;\quad u = 0.848;\quad S_x = 568\,\text{cm}^3$$

Moment capacity about the x–x axis:

$$M_{cx} = 275 \times 568 \quad = 156.2\,\text{kN m}$$
$$< 1.2 \times 275 \times 510/10^3 = 168.4\,\text{kN m}$$

Interaction expression:

$$\frac{880 \times 10}{66.4 \times 275} + \frac{35}{156.2} = 0.48 + 0.22 = 0.7 < 1$$

The section is satisfactory with respect to local capacity.

$$= 0.48 + 0.22 = 0.7 < 1$$

(3) *Member buckling check*

The effective length from Table 22:

$L_E = 0.85 \times 4500 = 3825$
Slenderness $\lambda = 3825/51.6 = 74.1$

Table 22 — Nominal effective length L_E for a compression member[a]

a) non-sway mode		
Restraint (in the plane under consideration) by other parts of the structure		L_E
Effectively held in position at both ends	Effectively restrained in direction at both ends	$0.7L$
	Partially restrained in direction at both ends	$0.85L$
	Restrained in direction at one end	$0.85L$
	Not restrained in direction at either end	$1.0L$
b) sway mode		

From Table 23, select Table 24(c) for buckling about the y–y axis. From Table 24(c), compressive strength $p_y = 172.8\,\text{N/mm}^2$.
Referring to Table 13, the support conditions for the beam column are that it is laterally restrained and restrained against torsion but partially free to rotate in plan:

$$\text{Effective length } L_E = 0.85 \times 4500 = 3825\,\text{mm}$$
$$\text{Slenderness } \lambda = 74.1$$

Table 23 — Allocation of strut curve

Type of section	Maximum thickness (see note 1)	Axis of buckling	
		x-x	y-y
Hot-finished structural hollow section		a)	a)
Cold-formed structural hollow section		c)	c)
Rolled I-section	≤40 mm	a)	b)
	>40 mm	b)	c)
Rolled H-section	≤40 mm	b)	c)
	>40 mm	c)	d)
Welded I or H-section (see note 2 and 4.7.5)	≤40 mm	b)	c)

Table 24 — Compressive strength p_c (N/mm²) (*continued*)

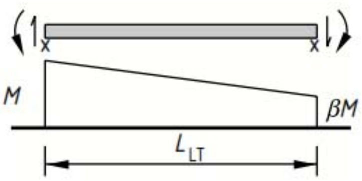
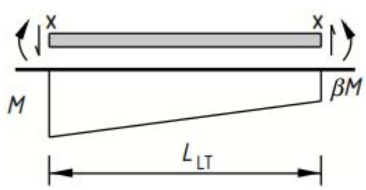
5) Values of p_c (N/mm²) with $\lambda < 110$ for strut curve c															
λ	Steel grade and design strength p_y (N/mm²)														
	S 275					S 355					S 460				
	235	245	255	265	275	315	325	335	345	355	400	410	430	440	460
15	235	245	255	265	275	315	325	335	345	355	398	408	427	436	455
20	233	242	252	261	271	308	317	326	336	345	387	396	414	424	442
25	226	235	245	254	263	299	308	317	326	335	375	384	402	410	428
30	220	228	237	246	255	289	298	307	315	324	363	371	388	396	413
35	213	221	230	238	247	280	288	296	305	313	349	357	374	382	397

The ratio of end moments:

$$\beta = 12/35 = 0.342$$

From Table 18 the equivalent uniform moment factor $m_x = 0.697$.

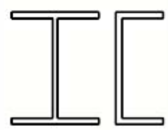
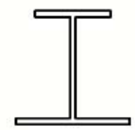
Table 18 — Equivalent uniform moment factor m_{LT} for lateral-torsional buckling
(continued overleaf)

Segments with end moments only (values of m_{LT} from the formula for the general case)	β	m_{LT}
β positive	1.0	1.00
	0.9	0.96
 	0.8	0.92
	0.7	0.88
	0.6	0.84
	0.5	0.80
	0.4	0.76
	0.3	0.72
	0.2	0.68
X Lateral restraint	0.1	0.64
β negative	0.0	0.60

$$\lambda_{LT} = uv\lambda$$

where $u = 0.848$ and denotes buckling parameter for H-section, $N = 0.5$ for uniform section with equal flanges, $x = 15.8$, the torsional index, $\lambda/x = 74.1/15.8 = 4.69$, $v = 0.832$, the slendemess factor from Table 19, $\lambda_{LT} = 0.848 \times 0.832 \times 74.1 = 52.2$

Table 19 — Slenderness factor v for sections with two plain flanges

λ/x	Unequal flanges, larger flange in compression  Compression Tension				Equal flanges 	Unequal flanges, smaller flange in compression Compression Tension 			
	η				η	η			
	0.9	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**	**0.3**	**0.2**	**0.1**
0.5	0.81	0.84	0.88	0.93	1.00	1.11	1.28	1.57	2.20
1.0	0.80	0.83	0.87	0.92	0.99	1.10	1.27	1.53	2.11

From Table 16, the bending strength:

$$p_b = 232.7\,\text{N/mm}^2$$

Table 16 — Bending strength p_b (N/mm²) for rolled sections

λ_{LT}	Steel grade and design strength p_y (N/mm²)														
	S 275					S 355					S 460				
	235	245	255	265	275	315	325	335	345	355	400	410	430	440	460
25	235	245	255	265	275	315	325	335	345	355	400	410	430	440	460
30	235	245	255	265	275	315	325	335	345	355	395	403	421	429	446
35	235	245	255	265	273	307	316	324	332	341	378	386	402	410	426
40	229	238	246	254	262	294	302	309	317	325	359	367	382	389	404
45	219	227	235	242	250	280	287	294	302	309	340	347	361	367	381
50	210	217	224	231	238	265	272	279	285	292	320	326	338	344	356
55	199	206	213	219	226	251	257	263	268	274	299	305	315	320	330
60	189	195	201	207	213	236	241	246	251	257	278	283	292	296	304
65	179	185	190	196	201	221	225	230	234	239	257	261	269	272	279

Buckling resistance moment:

$$M_b = 232.7 \times 568/10^3 = 132.1 \text{ kN m}$$

Interaction expression:

$$\frac{F_c}{A_g p_c} + \frac{m_x M_x}{M_b} + \frac{m_y M_y}{p_y Z_y} \leq 1$$

$$\frac{880 \times 10}{172.8 \times 66.4} + \frac{0.697 \times 35}{132.1} + 0 = 0.77 + 0.18 = 0.95 < 1.0$$

The section is also satisfactory with respect to overall buckling.

في بعض الاحيان يكون التحميل على العضوباكثر من اجهاد ولا يمكن اجهاد منهم تمثيله كتأثير مهيمن واحد. مثل هذه المشاكل تتطلب فهم الطريقة التي تتفاعل بها الاجهادات الهيكلية المختلفة مع بعضها البعض.   في أبسط في الحالات، قد لا يمثل هذا أكثر من مجرد جمع مباشر لعوامل لاجهادات" (على سبيل المثال المطبقة الحمل المحوري/المقاومة المحورية بالإضافة إلى قدرة الغضو).
بدلا من ذلك،في المشاكل المعقده تتطلب، دراسة متأنية للتفاعل بين مكونات الحمل الفردية والتشوهات الناتجة