Faculty of Engineering
Mechanical Engineering Department

Interest formulas and equivalence

Interest

- Interest is the manifestation of the time value of money, and it essentially represents "rent" paid for use of the money.
- Computationally, interest is the difference between an ending amount of money and the beginning amount.
- Interest paid or earned is determined by using the relation:

Interest = end amount - original amount

Interest rate

- When interest over a specific ***time unit*** is expressed as a percentage of the original amount, the result is called the *interest rate* or ***rate of return*** (ROR).

$$\text{Interest rate or rate of return} = \frac{\text{interest accrued per time unit}}{\text{original amount}} \times 100\%$$

- The time unit of the interest rate is called the ***interest period***. By far the most common interest period used to state an interest rate is *one* year.

Example 1

- An employee at LaserKinetics.com borrows $10,000 on May 1 and must repay a total of $10,700 exactly 1 year later. Determine the interest amount and the interest rate paid.

- **Solution**

$$\text{Interest paid} = \$10{,}700 - 10{,}000 = \$700$$

$$\text{Percent interest rate} = \frac{\$700}{\$10{,}000} \times 100\% = 7\% \text{ per year}$$

Example 2

a) Calculate the amount deposited 1 year ago to have $1000 now at an interest rate of 5% per year.
b) Calculate the amount of interest earned during this time period.

- **Solution**

a) The total amount accrued ($1000) is the sum of the original deposit and the earned interest. If X is the original deposit,

$$\text{Total accrued} = \text{original amount} + \text{original amount (interest rate)}$$
$$\$1000 = X + X(0.05) = X(1 + 0.05) = 1.05X$$

- The original deposit is

$$X = \frac{1000}{1.05} = \$952.38$$

Example 2

a) Calculate the amount deposited 1 year ago to have $1000 now at an interest rate of 5% per year.
b) Calculate the amount of interest earned during this time period.

- **Solution**

b) to determine interest earned:

$$\text{Interest} = \$1000 - 952.38 = \$47.62$$

Minimum Attractive Rate of Return (MARR)

- Engineering alternatives are evaluated upon the prognosis that a reasonable rate of return (ROR) can be realized. A reasonable rate must be established so that the accept or reject decision can be made.
- The reasonable rate, called the minimum attractive rate of return (MARR), must be higher than the cost of money used to finance the alternative, as well as higher than the rate that would be expected from a bank or safe (minimal risk) investment.
- The MARR is established by financial managers and is used as a criterion for accept/reject decisions.

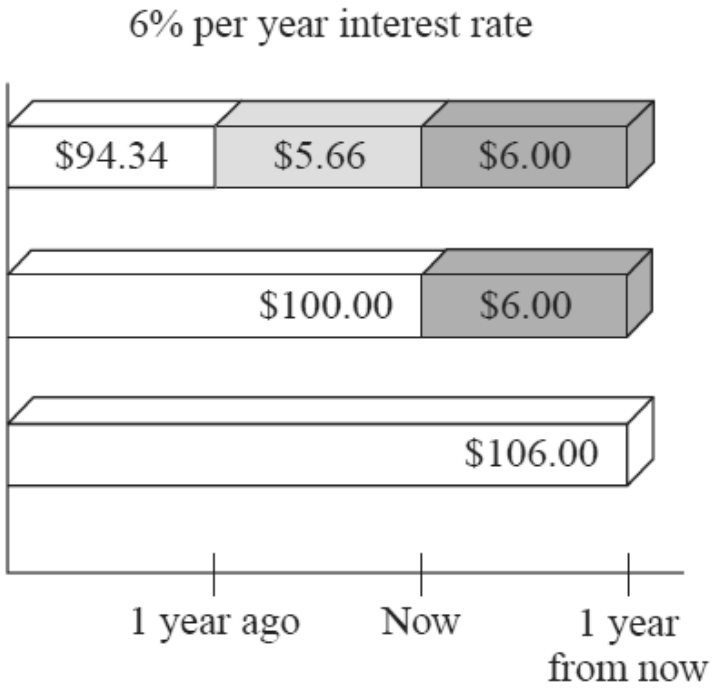
Equivalence

- Economic equivalence concept means that different sums of money at different times would be equal in economic value.

- For example, if the interest rate is 6% per year, $100 today (present time) is equivalent to $106 one year from today.

$$\frac{\$6}{\$100} \times 100\% = 6\% \text{ per year}$$

$$\frac{\$5.66}{\$94.34} \times 100\% = 6\% \text{ per year}$$



Example 3

- AC-Delco makes auto batteries available to General Motors dealers through privately owned distributorships. In general, batteries are stored throughout the year, and a 5% cost increase is added each year to cover the inventory carrying charge for the distributorship owner. Assume you own the City Center Delco facility. Make the calculations necessary to show which of the following statements are true and which are false about battery costs.

a. The amount of $98 now is equivalent to a cost of $105.60 one year from now.
b. A truck battery cost of $200 one year ago is equivalent to $205 now.
c. A $38 cost now is equivalent to $39.90 one year from now.
d. A $3000 cost now is equivalent to $2887.14 one year ago.
e. The carrying charge accumulated in 1 year on an investment of $2000 worth of batteries is $100.

Example 3

- **Solution**

a. Total amount accrued = 98(1.05) = \$102.90 ≠ \$105.60.
   Another way to solve this is as follows: Required original cost is 105.60/1.05 = \$100.57 ≠ \$98.
b. Required old cost is 205/1.05 = \$195.24 ≠ \$200.
c. The cost 1 year from now is \$38(1.05) = \$39.90.
d. Cost now is 2887.14(1.05) = \$3031.50 ≠ \$3000.
e. The charge is 5% per year interest, or \$2000(0.05) = \$100.

Simple Interest

- Simple interest is calculated using the principal only, ignoring any interest accrued in previous interest periods.
- The total simple interest over several periods is computed as

Interest = (principal)(number of periods)(interest rate)

Example 4

- HP borrowed money to do rapid prototyping for a new computer. The loan is $1000 for 3 years at 5% per year simple interest. How much money will HP repay at the end of 3 years?

- **Solution**

- The interest for each of the 3 years is:

$$\text{Interest per year} = 1000(.05) = \$50$$

- Total interest for 3 years is:

$$\text{Total interest} = 1000(3)(0.05) = \$150$$

- The amount due after 3 years is

$$\text{Total due} = \$1000 + 150 = \$1150$$

Example 4

- The $50,000 interest accrued in the first year and the $50,000 accrued in the second year do not earn interest. The interest due each year is calculated only on the $1000 principal.

End of Year	Amount Borrowed	Interest	Amount Owed	Amount Paid
0	$1000			
1	—	$50	$1050	$ 0
2	—	50	1100	0
3	—	50	1150	1150

Compound Interest

- For compound interest, the interest accrued for each interest period is calculated on the *principal plus the total amount of interest accumulated in all previous periods*.
- Thus, compound interest means interest on top of interest.
- Compound interest reflects the effect of the time value of money on the interest also.
- The interest for one period is calculated as

Interest = (principal + all accrued interest)(interest rate)

Example 5

- If HP borrows $1000 from a different source at 5% per year compound interest, compute the total amount due after 3 years. Compare the results of this and the previous example?
- **Solution**
- The interest and total amount due each year are computed separately.

Year 1 interest:	$\$1000(0.05) = \$50.00$
Total amount due after year 1:	$\$1000 + 50.00 = \$1050.00$
Year 2 interest:	$\$1050(0.05) = \$52.50$
Total amount due after year 2:	$\$1050 + 52.50 = \$1102.50$
Year 3 interest:	$\$1102.50(0.05) = \$55.13$
Total amount due after year 3:	$\$1102.50 + 55.13 = \$1157.63$

Example 5

- The repayment plan is the same as that for the simple interest example - no payment until the principal plus accrued interest is due at the end of year 3. An extra of $1157.63 - 1150 = $7.63 interest is paid compared to simple interest over the 3-year period.

End of Year	Amount Borrowed	Interest	Amount Owed	Amount Paid
0	$1000			
1	—	$50.00	$1050.00	$ 0
2	—	52.50	1102.50	0
3	—	55.13	1157.63	1157.63

Compound Interest

- Another and shorter way to calculate the total amount due after 3 years in Example 5 is to combine calculations rather than perform them on a year-by-year basis. The total due each year is as follows:

$$\begin{aligned} \text{Year 1:} \quad & \$1000(1.05)^1 = \$1050.00 \\ \text{Year 2:} \quad & \$1000(1.05)^2 = \$1102.50 \\ \text{Year 3:} \quad & \$1000(1.05)^3 = \$1157.63 \end{aligned}$$

- The year 3 total is calculated directly; it does not require the year 2 total.
- In general formula form:

Total due after a number of years = principal(1 + interest rate)$^{\text{number of years}}$