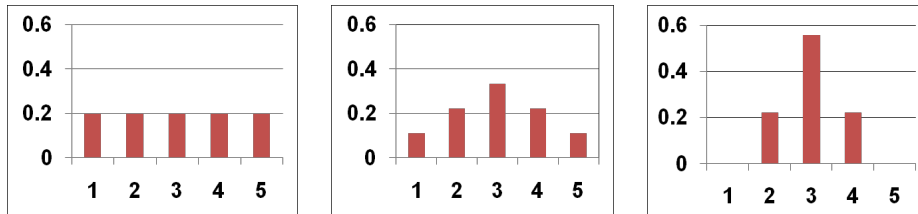# Variance, Bernoulli and Binomials

Today we are going to finish up our conversation of functions that we apply to random variables. Last time we talked about expectation, today we will cover variance. Then we will introduce two common, naturally occurring random variable types.

Variance

Consider the following 3 distributions (PMFs)

All three have the same expected value, $E[X] = 3$ but the "spread" in the distributions is quite different. Variance is a formal quantification of "spread".

If X is a random variable with mean μ then the variance of X, denoted $Var(X)$, is: $Var(X) = E[(X-\mu)2]$. When computing the variance often we use a different form of the same equation: $Var(X) = E[X^2] - E[X]^2$.

Here are some useful identities for variance:

- $Var(aX + b) = a^2 Var(X)$
- Standard deviation is the root of variance: $SD(X) = \sqrt{Var(X)}$

Example 1

Let X = value on roll of a 6 sided die. Recall that $E[X] = 7/2$. First lets calculate $E[X^2]$

$$E[X^2] = (1^2)\frac{1}{6} + (2^2)\frac{1}{6} + (3^2)\frac{1}{6} + (4^2)\frac{1}{6} + (5^2)\frac{1}{6} + (6^2)\frac{1}{6} = \frac{91}{6}$$

Which we can use to compute the variance:

$$Var(X) = E[X^2] - (E[X])^2$$
$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{35}{12}$$

Bernoulli

A Bernoulli random variable is random indicator variable (1 = success, 0 = failure) that represents whether or not an experiment with probability p resulted in success. Some example uses include a coin flip, random binary digit, whether a disk drive crashed or whether someone likes a Netflix movie.

Let X be a Bernoulli Random Variable $X \sim Ber(p)$.

$E[X] = p$
$Var(X) = p(1 - p)$

Binomial

A Binomial random variable is random variable that represents the number of successes in n successive independent trials of a Bernoulli experiment. Some example uses include # of heads in n coin flips, # distk drives crashed in 1000 computer cluster.

Let X be a Binomial Random Variable. $X \sim Bin(n,p)$ where p is the probability of success in a given trial.

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Example 2

Let X = number of heads after a coin is flipped three times. $X \sim Bin(3, 0.5)$. What is the probability of different outcomes?

$$P(X = 0) = \binom{3}{0} p^0 (1-p)^3 = \frac{1}{8}$$

$$P(X = 1) = \binom{3}{1} p^1 (1-p)^2 = \frac{3}{8}$$

$$P(X = 2) = \binom{3}{2} p^2 (1-p)^1 = \frac{3}{8}$$

$$P(X = 3) = \binom{3}{3} p^3 (1-p)^0 = \frac{1}{8}$$

Example 3

When sending messages over a network there is a chance that the bits will become corrupt. A Hamming Code allows for a 4 bit code to be encoded as 7 bits, and maintains the property that if 0 or 1 bit(s) are corrupted then the message can be perfectly reconstructed. You are working on the Voyager space mission and the probability of any bit being lost in space is 0.1. How does reliability change when using a Hamming code?

Image we use error correcting codes. Let $X \sim Bin(7, 0.1)$

$$P(X = 0) = \binom{7}{0} (0.1)^0 (0.9)^7 \approx 0.468$$

$$P(X = 1) = \binom{7}{1} (0.1)^1 (0.9)^6 = 0.372$$

$$P(X = 0) + P(X = 1) = 0.850$$

What if we didn't use error correcting codes? Let $X\ Bin(4, 0.1)$

$$P(X = 0) = \binom{4}{0} (0.1)^0 (0.9)^4 \approx 0.656$$

Using Hamming Codes improves reliability by 30%

Disclaimer: This handout was made fresh just for you. Notice any mistakes? Let Chris know.