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CS109

Extra Problems #3
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Practice Practice Problems #3 Solutions

We talked about problems 1 through 3 in class on Friday. See the lecture for solutions. Here are solutions to the two problems we didn't go over:

4.

a. Calculate the likelihood of each of the three scenarios:

i. The PDF of the uniform is:

$$f(x) = \frac{1}{0.9 - 0.1} = 1.25$$

Thus likelihood is 1.25^8

ii. The PDF of the uniform is:

$$f(x) = \frac{1}{1.0 - 0.0} = 1.0$$

Thus likelihood is 1^8

iii. Since some of the observed are outside the parameter range, their likelihood is 0. Since the likelihood of all data is a product, the resulting likelihood is also 0.

The maximum likelihood parameters for a normal are the min and max of the range.

b. Using Maximum Likelihood Estimators, we obtain the following parameters for the conditional distributions of X_1, X_2, and X_3:

$P(X_1 \mid Y = 0) \sim \text{Uni}(0.1, 0.7)$
$P(X_2 \mid Y = 0) \sim \text{Uni}(0.4. 0.8)$
$P(X_3 \mid Y = 0) \sim \text{Uni}(0.1, 0.6)$

$P(X_1 \mid Y = 1) \sim \text{Uni}(0.5, 0.9)$
$P(X_2 \mid Y = 1) \sim \text{Uni}(0.2, 0.7)$
$P(X_3 \mid Y = 1) \sim \text{Uni}(0.4, 0.8)$

c. We want to compute P(Y = 0 | test instance *i*) / P(Y = 1 | test instance *i*), and if this is greater than 1, we predict Y = 0 and otherwise we predict Y = 1.

Note that: P(Y = 0 | test instance *i*) / P(Y = 1 | test instance *i*)

$$= \frac{P(Y=0,\mathbf{X})}{P(\mathbf{X})} \Big/ \frac{P(Y=1,\mathbf{X})}{P(\mathbf{X})} = \frac{P(Y=0,\mathbf{X})}{P(Y=1,\mathbf{X})}$$

$$= P(\mathbf{X} \mid Y = 0)\, P(Y = 0) \,/\, P(\mathbf{X} \mid Y = 1)\, P(Y = 1)$$

Using the Naive Bayes assumption, we have:

P(**X** | Y = 0) P(Y = 0) / P(**X** | Y = 1) P(Y = 1)
= $P(X_1 | Y=0) P(X_2 | Y=0) P(X_3 | Y=0) P(Y=0) / P(X_1 | Y=1) P(X_2 | Y=1) P(X_3 | Y=1) P(Y=1)$

Here are the predictions for Y we make for each of the test instances:

P(Y = 0 | test instance 1)/P(Y = 1 | test instance 1)
= (1/0.6)(1/0.4)(1/0.5)(4/8)/(1/0.4)(1/0.5)(1/0.4)(4/8) = (5/3)(5/2)(2)/(5/2)(2)(5/2) = 2/3

Since this is < 1, we **classify test instance 1 as class Y = 1**

P(Y = 0 | test instance 2)/P(Y = 1 | test instance 2)
= (1/0.6)(1/0.4)(0)(4/8)/(1/0.4)(1/0.5)(1/0.4)(4/8) = (0)/(5/2)(2)(5/2) = 0

Since this is < 1, we **classify test instance 2 as class Y = 1**

P(Y = 0 | test instance 3)/P(Y = 1 | test instance 3)
= (1/0.6)(1/0.4)(1/0.5)(4/8)/(1/0.4)(1/0.5)(0)(4/8) = (5/3)(5/2)(2)/(0) = ∞

Since this is > 1, we **classify test instance 3 as class Y = 0**

5.

a. Let S be the event that Shakespeare wrote the document. Eyeball probability:

$$P(\text{Eyeball}|S) = \frac{\sum_i^k \text{contains}(\text{Eyeball}, D_i)}{|D|}$$

b. Let S be the event that Shakespeare wrote the document:

$$P(\text{S}|\text{Words}) = \frac{P(\text{Words}|\text{S})P(\text{S})}{P(\text{Words}|\text{S})P(\text{S}) + P(\text{Words}|\text{S}^C)P(\text{S}^C)}$$

Since the problem states "your prior belief is that the document is equally likely to be authored by Shakespeare or not by Shakespeare":
$P(S) = P(S^C) = 0.5$

Using the Naïve bayes assumption

$$P(\text{Words}|\text{S}) = \prod_i P(\text{Word}_i|\text{S})$$
$$P(\text{Words}|\text{S}^C) = \prod_i P(\text{Word}_i|\text{S}^C)$$

$P(\text{Word}_i|\text{S})$ can be calculated in the same way as part (a)
$P(\text{Word}_i|\text{S}^C)$ can be calculated using the same sum as part (a), but over the documents F.