# ACM-ICPC TEAM REFERENCE DOCUMENT

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#### 1 Data Structures

## 1.1 Disjoin Set Union

```
struct DSU {
   vector<int> par;
   vector<int> sz;
   DSU(int n) {
      FOR(i, 0, n) {
          par.pb(i);
          sz.pb(1);
   int find(int a) {
      return par[a] = par[a] == a ? a : find(par[a]);
   bool same(int a, int b) {
      return find(a) == find(b);
   void unite(int a, int b) {
      a = find(a);
      b = find(b);
      if(sz[a] > sz[b]) swap(a, b);
      sz[b] += sz[a];
      par[a] = b;
```

### 1.2 Fenwick 2D

```
 \begin{array}{l} struct \; Fenwick2D \; \{ \\ vector < vector < ll >> bit; \\ int \; n, \; m; \\ Fenwick2D (int \_n, \; int \_m) \; \{ \\ n = \_n; \; m = \_m; \\ bit = vector < vector < ll >> (n+1, \; vector < ll >(m+1, \; 0)); \\ \} \\ ll \; sum (int \; x, \; int \; y) \; \{ \\ ll \; ret = \; 0; \\ for \; (int \; i = \; x; \; i \; > \; 0; \; i \; -= \; i \; \& \; (-i)) \\ for \; (int \; j = \; y; \; j \; > \; 0; \; j \; -= \; j \; \& \; (-j)) \\ ret \; + \; bit[i][j]; \\ return \; ret; \\ \end{array}
```

```
 \begin{cases} & \text{ll sum(int } x1, \text{ int } y1, \text{ int } x2, \text{ int } y2) \ \{ & \text{return sum(} x2, \, y2) - \text{sum(} x2, \, y1 - 1) - \text{sum(} x1 - 1, \, y2) \ + \text{ sum(} x1 - 1, \, y1 - 1); \\ \} & \text{void add(int } x, \text{ int } y, \, \text{ll delta)} \ \{ & \text{for (int } i = x; \, i <= n; \, i += i \, \& \, (-i)) \\ & \text{for (int } j = y; \, j <= m; \, j \, += j \, \& \, (-j)) \\ & \text{bit[i][j]} \ += \text{delta;} \end{cases}
```

## 1.3 Fenwick Tree Point Update And Range Query

```
struct Fenwick {
    vector<ll> tree;
    int n;
    Fenwick(){}
    Fenwick(int _n) {
        n = \underline{n};
        tree = vector < ll > (n+1, 0);
    void add(int i, ll val) { // arr[i] += val
        for(; i \le n; i += i\&(-i)) tree[i] += val;
    ll get(int i) { // arr[i]
        return sum(i, i);
    ll sum(int i) { // arr[1]+...+arr[i]
        ll ans = 0;
        for(; i > 0; i -= i\&(-i)) ans += tree[i];
        return ans;
    \hat{l}l \text{ sum}(\text{int } l, \text{ int } r)  {// \text{arr}[l]+...+\text{arr}[r]
        return sum(r) - sum(l-1);
};
```

## 1.4 Fenwick Tree Range Update And Point Query

```
struct Fenwick {
    vector<ll> tree;
    vector<ll> arr;
    int n;
    Fenwick(vector<ll> _arr) {
        n = _arr.size();
        arr = _arr;
        tree = vector<ll>(n+2, 0);
```

```
} void add(int i, ll val) { // arr[i] += val for(; i <= n; i += i&(-i)) tree[i] += val; } void add(int l, int r, ll val) { // arr[l..r] += val add(l, val); add(r+1, -val); } ll get(int i) { // arr[i] ll sum = arr[i-1]; // zero based for(; i > 0; i -= i&(-i)) sum += tree[i]; return sum; // zero based } } .
```

#### 1.5 Fenwick Tree Range Update And Range Query

## 1.6 Implicit Treap

```
template <typename T>
struct Node {
    Node* l, *r;
    ll prio, size, sum;
    T val;
    bool rev;
```

```
Node() {}
   Node(T_val): l(nullptr), r(nullptr), val(_val), size(1), sum(_val), rev(false) {
      prio = rand() \cap (rand() << 15);
template <typename T>
struct ImplicitTreap {
   typedef Node<T>* NodePtr;
   int sz(NodePtr n) {
       return n ? n->size : 0;
   ll getSum(NodePtr n) {
       return n ? n->sum : 0;
   void push(NodePtr n) {
       if (n && n->rev) {
          n->rev = false;
          swap(n->l, n->r);
          if (n->1) n->1->rev = 1;
          if (n->r) n->r->rev = 1;
   void recalc(NodePtr n) {
       if (!n) return;
       n->size = sz(n->l) + 1 + sz(n->r);
       n->sum = getSum(n->l) + n->val + getSum(n->r);
   void split(NodePtr tree, ll key, NodePtr& l, NodePtr& r) {
       push(tree);
       if (!tree) {
          l = r = nullptr;
       else if (\text{key} \le \text{sz(tree->l)}) {
          split(tree->l, key, l, tree->l);
          r = tree;
          split(tree->r, key-sz(tree->l)-1, tree->r, r);
          l = tree;
       recalc(tree);
   void merge(NodePtr& tree, NodePtr l, NodePtr r) {
       push(l); push(r);
       if (!l || !r) {
           tree = 1 ? 1 : r;
       else if (l->prio > r->prio) {
          merge(l->r, l->r, r);
```

```
tree = 1;
   else {
       merge(r->l, l, r->l);
       tree = r;
   recalc(tree);
void insert(NodePtr& tree, T val, int pos) {
   if (!tree) {
       tree = new Node < T > (val);
       return;
   NodePtr L, R;
   split(tree, pos, L, R);
   merge(L, L, new Node<T>(val));
   merge(tree, L, R);
   recalc(tree);
void reverse(NodePtr tree, int l, int r) {
   NodePtr t1, t2, t3;
   split(tree, l, t1, t2);
   split(t2, r - l + 1, t2, t3);
   if(t2) t2 > rev = true;
   merge(t2, t1, t2);
   merge(tree, t2, t3);
void print(NodePtr t, bool newline = true) {
   push(t);
   if (!t) return;
   print(t->l, false);
   cout << t->val << " ";
   print(t->r, false);
   if (newline) cout << endl;
NodePtr fromArray(vector<T> v) {
   NodePtr t = nullptr;
   FOR(i, 0, (int)v.size()) {
      insert(t, v[i], i);
   return t;
ll calcSum(NodePtr t, int l, int r) {
   NodePtr L, R;
   split(t, l, L, R);
   NodePtr good;
   split(R, r - l + 1, good, L);
   return getSum(good);
```

```
}

};

/* Usage: ImplicitTreap<int> t;

Node<int> tree = t.fromArray(someVector); t.reverse(tree, l, r); ...

*/
```

#### 1.7 Segment Tree With Lazy Propagation

```
// Add to segment, get maximum of segment
struct LazySegTree {
   int n:
    vector<ll> t, lazy;
    LazySegTree(int n) {
       n = n; t = \text{vector} < \text{ll} > (4*n, 0); lazy = \text{vector} < \text{ll} > (4*n, 0);
    LazySegTree(vector<ll>& arr) {
       n = _n; t = \text{vector} < \text{ll} > (4*n, 0); lazy = vector < ll>(4*n, 0);
       build(arr, 1, 0, n-1); // same as in simple SegmentTree
    void push(int v) {
       t[v*2] += lazy[v];
       lazy[v*2] += lazy[v];
       t[v^*2+1] += lazy[v];
       lazy[v*2+1] += lazy[v];
       lazy[v] = 0;
    void update(int v, int tl, int tr, int l, int r, ll addend) {
       if (l > r)
           return;
       if (l == tl \&\& tr == r) {
           t[v] += addend;
           lazy[v] += addend;
       } else {
           push(v);
           int tm = (tl + tr) / 2;
           update(v*2, tl, tm, l, min(r, tm), addend);
           update(v*2+1, tm+1, tr, max(l, tm+1), r, addend);
           t[v] = max(t[v*2], t[v*2+1]);
   int query(int v, int tl, int tr, int l, int r) {
       if (l > r)
           return -OO;
       if (tl == tr)
           return t[v];
       push(v);
       int tm = (tl + tr) / 2;
       return max(query(v*2, tl, tm, l, min(r, tm)),
```

```
\begin{array}{c} {\rm query}(v^*2+1,\; tm+1,\; tr,\; max(l,\; tm+1),\; r));\\ \}; \end{array}
```

#### 1.8 Segment Tree

```
struct SegmentTree {
   int n;
   vector<ll> t:
   const ll IDENTITY = 0; // OO for min, -OO for max, ...
   ll f(ll a, ll b) {
      return a+b;
   SegmentTree(int _n) {
      n = _n; t = vector < ll > (4*n, IDENTITY);
   SegmentTree(vector<ll>& arr) {
      n = arr.size(); t = vector < ll > (4*n, IDENTITY);
      build(arr, 1, 0, n-1);
   void build(vector<ll>& arr, int v, int tl, int tr) {
      if(tl == tr) \{ t[v] = arr[tl]; \}
       else {
          int tm = (tl+tr)/2;
          build(arr, 2*v, tl, tm);
          build(arr, 2*v+1, tm+1, tr);
          t[v] = f(t[2*v], t[2*v+1]);
   // sum(1, 0, n-1, l, r)
   ll sum(int v, int tl, int tr, int l, int r) {
       if(l > r) return IDENTITY;
      if (l == tl \&\& r == tr) return t[v];
      int tm = (tl+tr)/2;
       return f(sum(2*v, tl, tm, l, min(r, tm)), sum(2*v+1, tm+1, tr, max(l, tm+1), r)
   // update(1, 0, n-1, i, v)
   void update(int v, int tl, int tr, int pos, ll newVal) {
      if(tl == tr) \{ t[v] = newVal; \}
       else {
          int tm = (tl+tr)/2;
          if(pos <= tm) update(2*v, tl, tm, pos, newVal);
          else update(2*v+1, tm+1, tr, pos, newVal);
          t[v] = f(t[2*v],t[2*v+1]);
```

#### 1.9 Treap

```
namespace Treap {
   struct Node {
       Node *l, *r;
       ll key, prio, size;
       Node() {}
       Node(ll key): key(key), l(nullptr), r(nullptr), size(1) {
          prio = rand() \cap (rand() << 15);
   typedef Node* NodePtr;
   int sz(NodePtr n) {
       return n ? n->size : 0;
   void recalc(NodePtr n) {
       if (!n) return;
       n->size = sz(n->l) + 1 + sz(n->r); // add more operations here as needed
   void split(NodePtr tree, ll key, NodePtr& l, NodePtr& r) {
       if (!tree) {
          l = r = nullptr;
       else if (key < tree->key) {
          split(tree->l, key, l, tree->l);
          r = tree;
       else {
          split(tree->r, key, tree->r, r);
          l = tree;
       recalc(tree);
   void merge(NodePtr& tree, NodePtr l, NodePtr r) {
       if (!l || !r) {
          tree = 1?1:r;
       else if (l->prio > r->prio) {
          merge(l->r, l->r, r);
          tree = 1;
          merge(r->l, l, r->l);
          tree = r;
       recalc(tree);
```

```
void insert(NodePtr& tree, NodePtr node) {
   if (!tree) {
       tree = node;
   else if (node->prio > tree->prio) {
       split(tree, node->key, node->l, node->r);
       tree = node;
       insert(node->key < tree->key ? tree->l : tree->r, node);
   recalc(tree);
void erase(NodePtr tree, ll key) {
   if (!tree) return;
   if (tree->key == key) {
       merge(tree, tree->l, tree->r);
   else {
       erase(key < tree->key ? tree->l : tree->r, key);
   recalc(tree);
void print(NodePtr t, bool newline = true) {
   if (!t) return;
   print(t->l, false);
   cout << t->key << " ";
   print(t->r, false);
   if (newline) cout << endl;
```

#### 1.10 Trie

```
struct Trie {
   const int ALPHA = 26;
   const char BASE = 'a';
   vector<vector<int>> nextNode;
   vector<int>> mark;
   int nodeCount;
   Trie() {
      nextNode = vector<vector<int>>(MAXN, vector<int>(ALPHA, -1));
      mark = vector<int>(MAXN, -1);
      nodeCount = 1;
   }
```

```
void insert(const string& s, int id) {
    int curr = 0;
    FOR(i, 0, (int)s.length()) {
        int c = s[i] - BASE;
        if(nextNode[curr][c] == -1) {
            nextNode[curr][c] = nodeCount++;
        }
        curr = nextNode[curr][c];
    }
    mark[curr] = id;
}
bool exists(const string& s) {
    int curr = 0;
    FOR(i, 0, (int)s.length()) {
        int c = s[i] - BASE;
        if(nextNode[curr][c] == -1) return false;
        curr = nextNode[curr][c];
    }
    return mark[curr] != -1;
}
```

#### 2 General

#### 2.1 Automatic Test

```
# Linux Bash
# gen, main and stupid have to be compiled beforehand
for((i=1;;++i)); do
    echo $i;
    ./gen $i > genIn;
    diff <(./main < genIn) <(./stupid < genIn) || break;
done

# Windows CMD
@echo off
FOR /L %%I IN (1,1,2147483647) DO (
    echo %%I
    gen.exe %%I > genIn
    main.exe < genIn > mainOut
    stupid.exe < genIn > stupidOut
    FC mainOut stupidOut || goto :eof
)
```

#### 2.2 C++ Template

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp> // gp_hash_table<int, int> == hash
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace ___gnu_pbds;
typedef long long ll;
typedef unsigned long long ull;
typedef long double ld;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef pair < double, double > pdd;
template <typename T> using min heap = priority queue<T, vector<T>, greater<
template <typename T> using max_heap = priority_queue<T, vector<T>, less<T
     >>;
template <typename T> using ordered_set = tree<T, null_type, less<T>,
     rb_tree_tag, tree_order_statistics_node_update>;
template <typename K, typename V> using hashmap = gp_hash_table<K, V>;
template<typename A, typename B> ostream& operator<<(ostream& out, pair<A, B
     > p) { out << "(" << p.first << ", " << p.second << ")"; return out;}
template<typename T> ostream& operator<<(ostream& out, vector<T> v) { out
     <<"["; for(auto& x : v) out << x <<", "; out <<"]";return out;}
template<typename T> ostream& operator<<(ostream& out, set<T> v) { out << "
      {"; for(auto& x : v) out << x << ", "; out << "}"; return out; }
template<typename K, typename V> ostream& operator<<(ostream& out, map<K,
     V> m) { out << "{"; for(auto& e : m) out << e.first << " -> " << e.second << ", "; out << "}"; return out; }
template<typename K, typename V> ostream& operator<<(ostream& out, hashmap
     \langle K, V \rangle m) { out \langle \langle "\{"; for(auto\& e: m) out << e.first <math>\langle \langle "-\rangle " \rangle << e.
     second << ", "; out << "}"; return out; }
#define FAST IO ios base::sync with stdio(false); cin.tie(NULL)
#define TESTS(t) int NUMBER OF TESTS; cin >> NUMBER OF TESTS; for(
     int t = 1; t \le NUMBER_OF_TESTS; t++)
#define FOR(i, begin, end) for (int i = (begin) - ((begin) > (end)); i != (end) - ((
     begin) > (end)); i += 1 - 2 * ((begin) > (end)))
#define sgn(a) ((a) > eps ? 1 : ((a) < -eps ? -1 : 0))
#define precise(x) fixed << setprecision(x)
#define debug(x) cerr << "> " << #x << " = " << x << endl;
#define pb push_back
#define rnd(a, b) (uniform_int_distribution<int>((a), (b))(rng))
#ifndef LOCAL
   #define cerr if(0)cout
   #define endl "\n"
mt19937 rng(chrono::steady clock::now().time since epoch().count());
clock_t ___clock___;
```

#### 2.3 Compilation

```
# Simple compile
g++ -DLOCAL -O2 -o main.exe -std-c++17 -Wall -Wno-unused-result -Wshadow main
.cpp
# Debug
g++ -DLOCAL -std=c++17 -Wshadow -Wall -o main.exe main.cpp -fsanitize=address
-fsanitize=undefined -fuse-ld=gold -D_GLIBCXX_DEBUG -g
```

## 3 Graphs

## 3.1 Bipartite Graph

```
class BipartiteGraph {
private:
    vector<int> _left, _right;
    vector<vector<int>> _adjList;
    vector<int> _ matchR, _matchL;
    vector<bool> _used;

bool _kuhn(int v) {
    if (_used[v]) return false;
    _used[v] = true;
    FOR(i, 0, (int)_adjList[v].size()) {
        int to = _adjList[v][i] - _left.size();
    }
}
```

```
if \; (\_matchR[to] == -1 \; || \; \_kuhn(\_matchR[to])) \; \{ \\
               _{\text{matchR[to]}} = v;
               _{\text{matchL}[v]} = to;
               return true;
       return false;
    void addReverseEdges() {
       FOR(i, 0, (int)_right.size()) {
           if ( \operatorname{matchR}[i] != -1) {
               \_adjList[\_left.size() + i].pb(\_matchR[i]);
   void _dfs(int p) {
       if ( used[p]) return;
        used[p] = true;
       for (auto x : adjList[p]) {
            _{dfs(x)}
    vector<pii> _buildMM() {
       vector<pair<int, int> > res;
       FOR(i, 0, (int)_right.size()) {
           if (\underline{\text{matchR}[i]} != -1) {
               res.push back(make pair( matchR[i], i));
       return res;
public:
   void addLeft(int x) {
        left.pb(x);
        _{adjList.pb({\{\}});}
        _matchL.pb(-1);
        _used.pb(false);
    void addRight(int x) {
       _{right.pb(x)};
       _{adjList.pb({\{\}});}
       _{\text{matchR.pb}(-1)};
       _used.pb(false);
    void addForwardEdge(int l, int r) {
        _{\text{adjList[l].pb(r + \_left.size());}}
   void addMatchEdge(int l, int r) {
       if(l != -1) \quad matchL[l] = r;
       if(r != -1) matchR[r] = 1;
    // Maximum Matching
```

```
vector<pii> mm() {
   _matchR = vector<int>(_right.size(), -1);
    matchL = vector < int > (left.size(), -1);
   // ^ these two can be deleted if performing MM on already partially matched
    _used = vector<bool>(_left.size() + _right.size(), false);
   bool path found;
   do {
       fill(_used.begin(), _used.end(), false);
       path\_found = false;
       FOR(i, 0, (int)_left.size()) {
           if (\underline{\mathrm{matchL}}[i] < 0 \&\& !\underline{\mathrm{used}}[i]) {
               path_found |= _kuhn(i);
   } while (path_found);
   return _buildMM();
// Minimum Edge Cover
// Algo: Find MM, add unmatched vertices greedily.
vector<pii> mec()
   auto ans = mm();
   FOR(i, 0, (int)\_left.size()) {
       if (\underline{\mathrm{matchL[i]}} != -1)
           for (auto x : _adjList[i]) {
               int ridx = x - left.size();
               if (\underline{\text{matchR}[\text{ridx}]} == -1) {
                   ans.pb(\{ i, ridx \});
                   \operatorname{matchR}[\operatorname{ridx}] = i;
   FOR(i, 0, (int) left.size()) {
       if(\underline{matchL[i]} == -1 \&\& (int)\underline{adjList[i].size()} > 0)
           int ridx = \_adjList[i][0] - \_left.size();
           matchL[i] = ridx;
           ans.pb(\{i, ridx\});
   return ans;
// Minimum Vertex Cover
// Algo: Find MM. Run DFS from unmatched vertices from the left part.
// MVC is composed of unvisited LEFT and visited RIGHT vertices.
pair<vector<int>, vector<int>> mvc(bool runMM = true) {
   if (runMM) mm();
    _addReverseEdges();
```

```
fill(_used.begin(), _used.end(), false);
   FOR(i, 0, (int)_left.size()) {
       if (\underline{\text{matchL}}[i] == -1) {
          _dfs(i);
   vector<int> left, right;
   FOR(i, 0, (int) left.size()) {
       if (! used[i]) left.pb(i);
   FOR(i, 0, (int)_right.size()) {
       if (_used[i + (int)_left.size()]) right.pb(i);
   return { left, right };
// Maximal Independant Vertex Set
// Algo: Find complement of MVC.
pair<vector<int>, vector<int>> mivs(bool runMM = true) {
   auto m = mvc(runMM);
   vector<br/>bool> containsL(_left.size(), false), containsR(_right.size(), false);
   for (auto x : m.first) containsL[x] = true;
   for (auto x : m.second) containsR[x] = true;
   vector<int> left, right;
   FOR(i, 0, (int) left.size())
       if (!containsL[i]) left.pb(i);
   FOR(i, 0, (int)_right.size()) {
       if (!containsR[i]) right.pb(i);
   return { left, right };
```

#### 3.2 Max Flow With Dinic

```
struct Edge {
   int f, c;
   int to;
   pii revIdx;
   int dir;
   int idx;
};
int n, m;
vector<Edge> adjList[MAX_N];
int level[MAX_N];
```

```
void addEdge(int a, int b, int c, int i, int dir) {
   int idx = adjList[a].size();
   int revIdx = adjList[b].size();
   adjList[a].pb(\{ 0,c,b, \{b, revIdx\}, dir,i \});
    adjList[b].pb(\{0,0,a,\{a,idx\},dir,i\});
bool bfs(int s, int t) {
    FOR(i, 0, n) level[i] = -1;
    level[s] = 0;
   queue<int> Q;
    Q.push(s);
    while (!Q.empty()) {
       auto t = Q.front(); Q.pop();
       for (auto x : adjList[t]) {
           if (level[x.to] < 0 \&\& x.f < x.c) {
               level[x.to] = level[t] + 1;
               Q.push(x.to);
    return level[t] >= 0;
int send(int u, int f, int t, vector<int>& edgeIdx) {
    if (u == t) return f;
    for (; edgeIdx[u] < adjList[u].size(); edgeIdx[u]++) {
       auto\& e = adjList[u][edgeIdx[u]];
       if (level[e.to] == level[u] + 1 \&\& e.f < e.c) {
           int curr flow = min(f, e.c - e.f);
           int next_flow = send(e.to, curr_flow, t, edgeIdx);
           if (\text{next\_flow} > 0) {
               e.f += next flow;
               adjList[e.revIdx.first][e.revIdx.second].f -= next_flow;
               return next flow;
    return 0;
int maxFlow(int s, int t) {
   int f = 0;
   while (bfs(s, t)) {
       vector < int > edgeIdx(n, 0);
       while (int extra = send(s, oo, t, edgeIdx)) {
          f += extra;
   return f;
void init() {
```

```
\begin{array}{l} {\rm cin} >> n >> m; \\ {\rm FOR}(i,\,0,\,m)\,\,\{\\ {\rm int}\,\,a,\,\,b,\,\,c; \\ {\rm cin} >> a >> b >> c; \\ {\rm a--;\,\,b--;} \\ {\rm addEdge}(a,\,\,b,\,\,c,\,\,i,\,\,1); \\ {\rm addEdge}(b,\,\,a,\,\,c,\,\,i,\,\,-1); \\ \,\,\} \end{array}
```

#### 3.3 Max Flow With Ford Fulkerson

```
struct Edge {
   int to, next;
   ll f, c;
   int idx, dir;
   int from;
int n, m;
vector<Edge> edges;
vector<int> first;
void addEdge(int a, int b, ll c, int i, int dir) {
   edges.pb({ b, first[a], 0, c, i, dir, a });
   edges.pb({ a, first[b], 0, 0, i, dir, b });
first[a] = edges.size() - 2;
   first[b] = edges.size() - 1;
void init() {
   cin >> n >> m;
   edges.reserve(4 * m);
   first = vector < int > (n, -1);
   FOR(i, 0, m) {
       int a, b, c;
       cin >> a >> b >> c;
       a--; b--;
       addEdge(a, b, c, i, 1);
       addEdge(b, a, c, i, -1);
int cur\_time = 0;
vector<int> timestamp;
ll dfs(int v, ll flow = OO) {
   if (v == n - 1) return flow;
   timestamp[v] = cur_time;
```

```
for (int e = first[v]; e != -1; e = edges[e].next) {
       if (edges[e].f < edges[e].c && timestamp[edges[e].to] != cur_time) {
           int pushed = dfs(edges[e].to, min(flow, edges[e].c - edges[e].f));
           if (pushed > 0) {
               edges[e].f += pushed;
edges[e ^ 1].f -= pushed;
               return pushed;
   return 0;
ll maxFlow() {
   cur time = 0;
   timestamp = vector < int > (n, 0);
   ll f = 0, add;
    while (true) {
       cur time++;
       add = dfs(0);
       if (add > 0) {
           f += add;
       élse {
           break;
   return f;
```

#### 3.4 Min Cut

```
init();
ll f = maxFlow(); // Ford-Fulkerson
cur_time++;
dfs(0);
set<int> cc;
for (auto e : edges) {
    if (timestamp[e.from] == cur_time && timestamp[e.to] != cur_time) {
        cc.insert(e.idx);
    }
} // (# of edges in min-cut, capacity of cut)
// [indices of edges forming the cut]
cout << cc.size() << " " << f << endl;
for (auto x : cc) cout << x + 1 << " ";</pre>
```

## 4 Math

## 4.1 Big Integer Multiplication With FFT

```
complex<ld> a[MAX_N], b[MAX_N];
complex<ld>fa[MAX_N], fb[MAX_N], fc[MAX_N];
complex<ld> cc[MAX_N];
string mul(string as, string bs) {
   int sgn1 = 1;
   int sgn 2 = 1;
   if (as[0] == '-') {
      sgn1 = -1;
       as = as.substr(1);
   if (bs[0] == '-') {
      sgn2 = -1;
      bs = bs.substr(1);
   int n = as.length() + bs.length() + 1;
   FFT::init(n):
   FOR(i, 0, FFT::pwrN) {
      a[i] = b[i] = fa[i] = fb[i] = fc[i] = cc[i] = 0;
   FOR(i, 0, as.size()) {
       a[i] = as[as.size() - 1 - i] - '0';
   FOR(i, 0, bs.size()) {
      b[i] = bs[bs.size() - 1 - i] - '0';
   FFT::fft(a, fa);
   FFT::fft(b, fb);
   FOR(i, 0, FFT::pwrN) {
      fc[i] = fa[i] * fb[i];
   // turn [0,1,2,...,n-1] into [0, n-1, n-2, ..., 1]
   FOR(i, 1, FFT::pwrN) {
      if (i < FFT::pwrN - i) {
          swap(fc[i], fc[FFT::pwrN - i]);
   FFT::fft(fc, cc);
   ll carry = 0;
   vector<int> v:
   FOR(i, 0, FFT::pwrN) {
      int num = round(cc[i].real() / FFT::pwrN) + carry;
      v.pb(num % 10);
      carry = num / 10;
   while (carry > 0) {
```

```
v.pb(carry % 10);
   carry /=10;
reverse(v.begin(), v.end());
bool start = false;
ostringstream ss;
bool allZero = true;
for (auto x : v) {
   if (x != 0) {
       allZero = false;
       break;
if (sgn1*sgn2 < 0 \&\& !allZero) ss << "-";
for (auto x : v) {
   if (x == 0 \&\& !start) continue;
   start = true;
   ss \ll abs(x);
if (!start) ss << 0;
return ss.str();
```

## 4.2 Chinese Remainder Theorem

Let's say we have some numbers  $m_i$ , which are all mutually coprime. Also, let  $M = \prod_i m_i$ . Then the system of congruences

$$\begin{cases} x \equiv a_1 \pmod{m_1} \\ x \equiv a_2 \pmod{m_2} \\ \dots \\ x \equiv a_k \pmod{m_k} \end{cases}$$

is equivalent to  $x \equiv A \pmod{M}$  and there exists a unique number A satisfying  $0 \le A \le M$ .

Solution for two:  $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2}$ . Let  $x = a_1 + km_1$ . Substituting into the second congruence:  $km_1 \equiv a_2 - a_1 \pmod{m_2}$ . Then,  $k = (m_1)_{m_2}^{-1}(a_2 - a_1) \pmod{m_2}$ . and we can easily find x. This can be extended to multiple equations by solving them one-by-one.

If the moduli are not coprime, solve the system  $y\equiv 0\pmod{\frac{m_1}{g}},y\equiv \frac{a_2-a_1}{g}\pmod{\frac{m_2}{g}}$  for y. Then let  $x\equiv gy+a_1\pmod{\frac{m_1m_2}{g}}$ .

#### 4.3 Euler Totient Function

## 4.4 Extended Euclidean Algorithm

```
// ax+by=gcd(a,b)
void solveEq(ll a, ll b, ll& x, ll& y, ll& g) {
   if(b==0) {
      x = 1:
      y = 0;
      g = a;
      return;
   ll xx, yy;
   solveEq(b, a%b, xx, yy, g);
   x = yy;
   y = xx-yy*(a/b);
// ax + by = c
bool solveEq(ll a, ll b, ll c, ll& x, ll& y, ll& g) {
   solveEq(a, b, x, y, g);
   if(c\%g != 0) return false;
   x *= c/g; y *= c/g;
   return true;
// Finds a solution (x, y) so that x >= 0 and x is minimal
bool solveEqNonNegX(ll a, ll b, ll c, ll& x, ll &y, ll& g) {
   if(!solveEq(a, b, c, x, y, g)) return false;
   ll k = x*g/b;
   x = x - k*b/g;
   y = y + k*a/g;
   if(x < 0) {
      x += b/g;
      y = a/g;
   return true;
```

All other solutions can be found like this:

$$x' = x - k \frac{b}{g}, y' = y + k \frac{a}{g}, k \in \mathbb{Z}$$

#### 4.5 Factorization With Sieve

```
// Use linear sieve to calculate minDiv
vector<pll> factorize(ll x) {
   vector<pll> res;
   ll prev = -1;
   ll cnt = 0;
   while(x != 1) {
       ll d = minDiv[x];
       if(d == prev) {
          cnt++;
       } else {
          if(prev != -1) res.pb(\{prev, cnt\});
          prev = d;
          cnt = 1:
       x /= d;
   res.pb({prev, cnt});
   return res;
```

#### 4.6 FFT With Modulo

```
 \begin{array}{l} bool \ is Generator(ll \ g) \ \{ \\ if \ (pwr(g, \ M-1)!=1) \ return \ false; \\ for \ (ll \ i=2; i^*i <= M-1; i++) \ \{ \\ if \ ((M-1) \ \% \ i=0) \ \{ \\ ll \ q=i; \\ if \ (is Prime(q)) \ \{ \\ ll \ p=(M-1) \ / \ q; \\ ll \ pp=pwr(g, p); \\ if \ (pp==1) \ return \ false; \\ \} \\ q=(M-1) \ / \ i; \\ if \ (is Prime(q)) \ \{ \\ ll \ p=(M-1) \ / \ q; \\ ll \ pp=pwr(g, p); \\ \end{array}
```

```
if (pp == 1) return false;
   return true;
namespace FFT {
   ll n;
   vector<ll> r;
   vector<ll> omega;
   ll logN, pwrN;
   void initLogN() {
      logN = 0;
      pwrN = 1;
      while (pwrN < n) {
          pwrN *= 2;
          logN++;
      n = pwrN;
   void initOmega() {
      ll g = 2;
      while (!isGenerator(g)) g++;
      ll G = 1;
      g = pwr(g, (M - 1) / pwrN);
      FOR(i, 0, pwrN) {
         omega[i] = G;
          G *= g;
          G \% = M;
   void initR() {
      r[0] = 0;
      FOR(i, 1, pwrN) {
         r[i] = r[i / 2] / 2 + ((i \& 1) << (logN - 1));
   void initArrays() {
      r.clear();
      r.resize(pwrN);
      omega.clear();
      omega.resize(pwrN);
   void init(ll n) {
      FFT::n = n;
      initLogN();
      initArrays();
```

#### 4.7 FFT

```
namespace FFT {
   int n;
   vector<int> r;
   vector<complex<ld>> omega;
   int logN, pwrN;
   void initLogN() {
      logN = 0;
      pwrN = 1;
      while (pwrN < n) {
         pwrN *= 2;
          logN++;
       n = pwrN;
   void initOmega() {
       FOR(i, 0, pwrN) {
          omega[\hat{i}] = { \cos(2 * i*PI / n), \sin(2 * i*PI / n) };
   void initR() {
      r[0] = 0;
```

```
FOR(i, 1, pwrN) {
      r[i] = \hat{r[i / 2]} / 2 + ((i \& 1) << (logN - 1));
void initArrays() {
   r.clear();
   r.resize(pwrN);
   omega.clear();
   omega.resize(pwrN);
void init(int n) {
   FFT::n = n;
   initLogN();
   initArrays();
   initOmega();
   initR();
void fft(complex<ld> a[], complex<ld> f[]) {
   FOR(i, 0, pwrN) {
       f[i] = a[r[i]];
   for (ll k = 1; k < pwrN; k *= 2) {
       for (ll i = 0; i < pwrN; i += 2 * k) {
          for (ll j = 0; j < k; j++) {
              auto z = omega[j*n / (2 * k)] * f[i + j + k];
              f[i + j + k] = f[i + j] - z;
              f[i + j] += z;
   }
```

#### 4.8 Linear Sieve

```
\begin{split} & \text{ll minDiv}[MAXN+1]; \\ & \text{vector} < \text{ll} > \text{primes}; \\ & \text{void sieve}(\text{ll n}) \{ \\ & \text{FOR}(k, 2, n+1) \{ \\ & \text{minDiv}[k] = k; \\ \} \\ & \text{FOR}(k, 2, n+1) \left\{ \\ & \text{if}(\text{minDiv}[k] == k) \right. \{ \\ & \text{primes.pb}(k); \\ & \text{} \} \end{split}
```

```
 \begin{array}{c} & \text{for(auto } p: primes) \ \{ \\ & \text{if(} p > minDiv[k]) \text{ break;} \\ & \text{if(} p^*k > n) \text{ break;} \\ & \text{minDiv[} p^*k] = p; \\ \} \\ \} \end{array}
```

## 4.9 Modular Inverse

```
bool invWithEuclid(ll a, ll m, ll& aInv) {
      ll x, y, g;
      if(!solveEqNonNegX(a, m, 1, x, y, g)) return false;
      aInv = x;
      return true;
}
// Works only if m is prime
ll invFermat(ll a, ll m) {
      return pwr(a, m-2, m);
}
// Works only if gcd(a, m) = 1
ll invEuler(ll a, ll m) {
      return pwr(a, phi(m)-1, m);
}
```

# 5 Strings

## 5.1 Hashing

```
struct HashedString {
   const ll A1 = 999999929, B1 = 1000000009, A2 = 1000000087, B2 = 1000000097;
   vector<|l> A1pwrs, A2pwrs;
   vector<|l> prefixHash;
   HashedString(const string& _s) {
        init(_s);
        calcHashes(_s);
   }
   void init(const string& s) {
        ll a1 = 1;
        ll a2 = 1;
        FOR(i, 0, (int)s.length()+1) {
            A1pwrs.pb(a1);
            A2pwrs.pb(a2);
        }
        restricted from the following formula for the following for the following formula for the following for the follow
```

```
\begin{array}{c} a1 = (a1^*A1)\%B1;\\ a2 = (a2^*A2)\%B2;\\ \\ \}\\ \\ void \ calcHashes(const \ string\& \ s) \ \\ \\ pll \ h = \{0, \, 0\};\\ \\ prefixHash.pb(h);\\ \\ for(char \ c : s) \ \\ \\ ll \ h1 = (prefixHash.back().first^*A1 + c)\%B1;\\ \\ ll \ h2 = (prefixHash.back().second^*A2 + c)\%B2;\\ \\ \\ prefixHash.pb(\{h1, \, h2\});\\ \\ \\ \}\\ \\ pll \ getHash(int \ l, \ int \ r) \ \\ \\ ll \ h1 = (prefixHash[r+1].first - prefixHash[l].first^*A1pwrs[r+1-l]) \% \ B1;\\ \\ \\ ll \ h2 = (prefixHash[r+1].second - prefixHash[l].second^*A2pwrs[r+1-l]) \% \ B2;\\ \\ \\ if(h1 < 0) \ h1 \ += B1;\\ \\ if(h2 < 0) \ h2 \ += B2;\\ \\ \\ return \ \{h1, \, h2\};\\ \\ \\ \}\\ \\ \}; \end{array}
```