ACM-ICPC Team Reference Document

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Contents

1 Data Structures

1.1 Disjoin Set Union

```
struct DSU {
    vector<int> par;
    vector<int> sz;

    DSU(int n) {
        FOR(i, 0, n) {
            par.pb(i);
            sz.pb(1);
        }
    }

    int find(int a) {
        return par[a] = par[a] == a ? a : find(par[a]);
    }

    bool same(int a, int b) {
        return find(a) == find(b);
    }

    void unite(int a, int b) {
        a = find(a);
        b = find(b);
        if(sz[a] > sz[b]) swap(a, b);
        sz[b] += sz[a];
        par[a] = b;
    }
};
```

1.2 Fenwick 2D

```
struct Fenwick2D {
    vector<vector<ll>> bit;
    int n, m;
    Fenwick2D(int _n, int _m) {
        n = _n; m = _m;
        bit = vector<vector<ll>>(n+1, vector<ll>(m+1, 0));
    }
    ll sum(int x, int y) {
        ll ret = 0;
        for (int i = x; i > 0; i -= i & (-i))
            for (int j = y; j > 0; j -= j & (-j))
                ret += bit[i][j];
        return ret;
    }
    ll sum(int x1, int y1, int x2, int y2) {
        return sum(x2, y2) - sum(x2, y1-1) - sum(x1-1, y2) + sum(x1-1, y1-1);
    }
    void add(int x, int y, ll delta) {
        for (int i = x; i <= n; i += i & (-i))
            for (int j = y; j <= m; j += j & (-j))
                bit[i][j] += delta;
    }
};
```

1.3 Fenwick Tree Point Update And Range Query

```
struct Fenwick {
    vector<ll> tree;
    int n;
    Fenwick(){}
    Fenwick(int _n) {
        n = _n;
        tree = vector<ll>(n+1, 0);
    }
    void add(int i, ll val) { // arr[i] += val
        for(; i <= n; i += i&(-i)) tree[i] += val;
    }
    ll get(int i) { // arr[i]
        return sum(i, i);
    }
    ll sum(int i) { // arr[1]+...+arr[i]
        ll ans = 0;
        for(; i > 0; i -= i&(-i)) ans += tree[i];
        return ans;
    }
    ll sum(int l, int r) {// arr[l]+...+arr[r]
        return sum(r) - sum(l-1);
    }
};
```

1.4 Fenwick Tree Range Update And Point Query

```
struct Fenwick {
    vector<ll> tree;
    vector<ll> arr;
    int n;
    Fenwick(vector<ll> _arr) {
        n = _arr.size();
        arr = _arr;
        tree = vector<ll>(n+2, 0);
```

```
    }
    void add(int i, ll val) { // arr[i] += val
        for(; i <= n; i += i&(-i)) tree[i] += val;
    }
    void add(int l, int r, ll val) {// arr[l..r] += val
        add(l, val);
        add(r+1, -val);
    }
    ll get(int i) { // arr[i]
        ll sum = arr[i-1]; // zero based
        for(; i > 0; i -= i&(-i)) sum += tree[i];
        return sum; // zero based
    }
};
```

1.5 Fenwick Tree Range Update And Range Query

```
struct RangedFenwick {
    Fenwick F1, F2; // support range query and point update
    RangedFenwick(int _n) {
        F1 = Fenwick(_n+1);
        F2 = Fenwick(_n+1);
    }
    void add(int l, int r, ll v) { // arr[l..r] += v
        F1.add(l, v);
        F1.add(r+1, -v);
        F2.add(l, v*(l-1));
        F2.add(r+1, -v*r);
    }
    ll sum(int i) { // arr[1..i]
        return F1.sum(i)*i-F2.sum(i);
    }
    ll sum(int l, int r) { // arr[l..r]
        return sum(r)-sum(l-1);
    }
};
```

1.6 Implicit Treap

```
template <typename T>
struct Node {
    Node* l, *r;
    ll prio, size, sum;
    T val;
    bool rev;
    Node() {}
    Node(T _val) : l(nullptr), r(nullptr), val(_val), size(1), sum(_val), rev(false) {
        prio = rand() ^ (rand() << 15);
    }
};
template <typename T>
struct ImplicitTreap {
    typedef Node<T>* NodePtr;
    int sz(NodePtr n) {
        return n ? n->size : 0;
    }
    ll getSum(NodePtr n) {
        return n ? n->sum : 0;
    }

    void push(NodePtr n) {
        if (n && n->rev) {
            n->rev = false;
            swap(n->l, n->r);
            if (n->l) n->l->rev ^= 1;
            if (n->r) n->r->rev ^= 1;
        }
    }

    void recalc(NodePtr n) {
        if (!n) return;
        n->size = sz(n->l) + 1 + sz(n->r);
        n->sum = getSum(n->l) + n->val + getSum(n->r);
    }

    void split(NodePtr tree, ll key, NodePtr& l, NodePtr& r) {
        push(tree);
        if (!tree) {
            l = r = nullptr;
        }
        else if (key <= sz(tree->l)) {
            split(tree->l, key, l, tree->l);
            r = tree;
        }
        else {
            split(tree->r, key-sz(tree->l)-1, tree->r, r);
            l = tree;
        }
        recalc(tree);
    }

    void merge(NodePtr& tree, NodePtr l, NodePtr r) {
        push(l); push(r);
        if (!l || !r) {
            tree = l ? l : r;
        }
        else if (l->prio > r->prio) {
            merge(l->r, l->r, r);
```

```
        tree = l;
    }
    else {
        merge(r->l, l, r->l);
        tree = r;
    }
    recalc(tree);
}

void insert(NodePtr& tree, T val, int pos) {
    if (!tree) {
        tree = new Node<T>(val);
        return;
    }
    NodePtr L, R;
    split(tree, pos, L, R);
    merge(L, L, new Node<T>(val));
    merge(tree, L, R);
    recalc(tree);
}

void reverse(NodePtr tree, int l, int r) {
    NodePtr t1, t2, t3;
    split(tree, l, t1, t2);
    split(t2, r - l + 1, t2, t3);
    if(t2) t2->rev = true;
    merge(t2, t1, t2);
    merge(tree, t2, t3);
}

void print(NodePtr t, bool newline = true) {
    push(t);
    if (!t) return;
    print(t->l, false);
    cout << t->val << " ";
    print(t->r, false);
    if (newline) cout << endl;
}

NodePtr fromArray(vector<T> v) {
    NodePtr t = nullptr;
    FOR(i, 0, (int)v.size()) {
        insert(t, v[i], i);
    }
    return t;
}

ll calcSum(NodePtr t, int l, int r) {
    NodePtr L, R;
    split(t, l, L, R);
    NodePtr good;
    split(R, r - l + 1, good, L);
    return getSum(good);
    }
};
/* Usage: ImplicitTreap<int> t;
Node<int> tree = t.fromArray(someVector); t.reverse(tree, l, r); ...
*/
```

1.7 Segment Tree With Lazy Propagation

```
// Add to segment, get maximum of segment
struct LazySegTree {
    int n;
    vector<ll> t, lazy;
    LazySegTree(int __n) {
        n = __n; t = vector<ll>(4*n, 0); lazy = vector<ll>(4*n, 0);
    }
    LazySegTree(vector<ll>& arr) {
        n = __n; t = vector<ll>(4*n, 0); lazy = vector<ll>(4*n, 0);
        build(arr, 1, 0, n-1); // same as in simple SegmentTree
    }
    void push(int v) {
        t[v*2] += lazy[v];
        lazy[v*2] += lazy[v];
        t[v*2+1] += lazy[v];
        lazy[v*2+1] += lazy[v];
        lazy[v] = 0;
    }
    void update(int v, int tl, int tr, int l, int r, ll addend) {
        if (l > r)
            return;
        if (l == tl && tr == r) {
            t[v] += addend;
            lazy[v] += addend;
        } else {
            push(v);
            int tm = (tl + tr) / 2;
            update(v*2, tl, tm, l, min(r, tm), addend);
            update(v*2+1, tm+1, tr, max(l, tm+1), r, addend);
            t[v] = max(t[v*2], t[v*2+1]);
        }
    }

    int query(int v, int tl, int tr, int l, int r) {
        if (l > r)
            return -OO;
        if (tl == tr)
            return t[v];
        push(v);
        int tm = (tl + tr) / 2;
        return max(query(v*2, tl, tm, l, min(r, tm)),
```

```
            query(v*2+1, tm+1, tr, max(l, tm+1), r));
    }
};
```

1.8 Segment Tree

```
struct SegmentTree {
    int n;
    vector<ll> t;
    const ll IDENTITY = 0; // OO for min, -OO for max, ...
    ll f(ll a, ll b) {
        return a+b;
    }
    SegmentTree(int _n) {
        n = _n; t = vector<ll>(4*n, IDENTITY);
    }
    SegmentTree(vector<ll>& arr) {
        n = arr.size(); t = vector<ll>(4*n, IDENTITY);
        build(arr, 1, 0, n-1);
    }
    void build(vector<ll>& arr, int v, int tl, int tr) {
        if(tl == tr) { t[v] = arr[tl]; }
        else {
            int tm = (tl+tr)/2;
            build(arr, 2*v, tl, tm);
            build(arr, 2*v+1, tm+1, tr);
            t[v] = f(t[2*v], t[2*v+1]);
        }
    }
    // sum(1, 0, n-1, l, r)
    ll sum(int v, int tl, int tr, int l, int r) {
        if(l > r) return IDENTITY;
        if (l == tl && r == tr) return t[v];
        int tm = (tl+tr)/2;
        return f(sum(2*v, tl, tm, l, min(r, tm)), sum(2*v+1, tm+1, tr, max(l, tm+1), r)
            );
    }
    // update(1, 0, n-1, i, v)
    void update(int v, int tl, int tr, int pos, ll newVal) {
        if(tl == tr) { t[v] = newVal; }
        else {
            int tm = (tl+tr)/2;
            if(pos <= tm) update(2*v, tl, tm, pos, newVal);
            else update(2*v+1, tm+1, tr, pos, newVal);
            t[v] = f(t[2*v],t[2*v+1]);
        }
    }
};
```

1.9 Treap

```
namespace Treap {
    struct Node {
        Node *l, *r;
        ll key, prio, size;
        Node() {}
        Node(ll key) : key(key), l(nullptr), r(nullptr), size(1) {
            prio = rand() ^ (rand() << 15);
        }
    };

    typedef Node* NodePtr;

    int sz(NodePtr n) {
        return n ? n->size : 0;
    }

    void recalc(NodePtr n) {
        if (!n) return;
        n->size = sz(n->l) + 1 + sz(n->r); // add more operations here as needed
    }

    void split(NodePtr tree, ll key, NodePtr& l, NodePtr& r) {
        if (!tree) {
            l = r = nullptr;
        }
        else if (key < tree->key) {
            split(tree->l, key, l, tree->l);
            r = tree;
        }
        else {
            split(tree->r, key, tree->r, r);
            l = tree;
        }
        recalc(tree);
    }

    void merge(NodePtr& tree, NodePtr l, NodePtr r) {
        if (!l || !r) {
            tree = l ? l : r;
        }
        else if (l->prio > r->prio) {
            merge(l->r, l->r, r);
            tree = l;
        }
        else {
            merge(r->l, l, r->l);
            tree = r;
        }
        recalc(tree);
```

```
    }

    void insert(NodePtr& tree, NodePtr node) {
        if (!tree) {
            tree = node;
        }
        else if (node->prio > tree->prio) {
            split(tree, node->key, node->l, node->r);
            tree = node;
        }
        else {
            insert(node->key < tree->key ? tree->l : tree->r, node);
        }
        recalc(tree);
    }

    void erase(NodePtr tree, ll key) {
        if (!tree) return;
        if (tree->key == key) {
            merge(tree, tree->l, tree->r);
        }
        else {
            erase(key < tree->key ? tree->l : tree->r, key);
        }
        recalc(tree);
    }

    void print(NodePtr t, bool newline = true) {
        if (!t) return;
        print(t->l, false);
        cout << t->key << " ";
        print(t->r, false);
        if (newline) cout << endl;
    }
}
```

1.10 Trie

```
struct Trie {
    const int ALPHA = 26;
    const char BASE = 'a';
    vector<vector<int>> nextNode;
    vector<int> mark;
    int nodeCount;
    Trie() {
        nextNode = vector<vector<int>>(MAXN, vector<int>(ALPHA, -1));
        mark = vector<int>(MAXN, -1);
        nodeCount = 1;
    }
    void insert(const string& s, int id) {
        int curr = 0;
        FOR(i, 0, (int)s.length()) {
            int c = s[i] - BASE;
            if(nextNode[curr][c] == -1) {
                nextNode[curr][c] = nodeCount++;
            }
            curr = nextNode[curr][c];
        }
        mark[curr] = id;
    }

    bool exists(const string& s) {
        int curr = 0;
        FOR(i, 0, (int)s.length()) {
            int c = s[i] - BASE;
            if(nextNode[curr][c] == -1) return false;
            curr = nextNode[curr][c];
        }
        return mark[curr] != -1;
    }
};
```

2 General

2.1 Automatic Test

```
# Linux Bash
# gen, main and stupid have to be compiled beforehand
for((i=1;;++i)); do
    echo $i;
    ./gen $i > genIn;
    diff <(./main < genIn) <(./stupid < genIn) || break;
done

# Windows CMD
@echo off
FOR /L %%I IN (1,1,2147483647) DO (
    echo %%I
    gen.exe %%I > genIn
    main.exe < genIn > mainOut
    stupid.exe < genIn > stupidOut
    FC mainOut stupidOut || goto :eof
)
```

2.2 C++ Template

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp> // gp_hash_table<int, int> == hash map
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
typedef long long ll;
typedef unsigned long long ull;
typedef long double ld;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef pair<double, double> pdd;
template <typename T> using min_heap = priority_queue<T, vector<T>, greater<T>>;
template <typename T> using max_heap = priority_queue<T, vector<T>, less<T>>;
template <typename T> using ordered_set = tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
template <typename K, typename V> using hashmap = gp_hash_table<K, V>;

template<typename A, typename B> ostream& operator<<(ostream& out, pair<A, B> p) { out << "(" << p.first << ", " << p.second << ")"; return out;}
template<typename T> ostream& operator<<(ostream& out, vector<T> v) { out << "["; for(auto& x : v) out << x << ", "; out << "]";return out;}
template<typename T> ostream& operator<<(ostream& out, set<T> v) { out << "{"; for(auto& x : v) out << x << ", "; out << "}"; return out; }
template<typename K, typename V> ostream& operator<<(ostream& out, map<K, V> m) { out << "{"; for(auto& e : m) out << e.first << " -> " << e.second << ", "; out << "}"; return out; }
template<typename K, typename V> ostream& operator<<(ostream& out, hashmap<K, V> m) { out << "{"; for(auto& e : m) out << e.first << " -> " << e.second << ", "; out << "}"; return out; }

#define FAST_IO ios_base::sync_with_stdio(false); cin.tie(NULL)
#define TESTS(t) int NUMBER_OF_TESTS; cin >> NUMBER_OF_TESTS; for(int t = 1; t <= NUMBER_OF_TESTS; t++)
#define FOR(i, begin, end) for (int i = (begin) - ((begin) > (end)); i != (end) - ((begin) > (end)); i += 1 - 2 * ((begin) > (end)))
#define sgn(a) ((a) > eps ? 1 : ((a) < -eps ? -1 : 0))
#define precise(x) fixed << setprecision(x)
#define debug(x) cerr << "> " << #x << " = " << x << endl;
#define pb push_back
#define rnd(a, b) (uniform_int_distribution<int>((a), (b))(rng))
#ifndef LOCAL
    #define cerr if(0)cout
    #define endl "\n"
#endif
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
clock_t __clock__;
void startTime() {__clock__ = clock();}
void timeit(string msg) {cerr << "> " << msg << ": " << precise(6) << ld(clock()-__clock__)/CLOCKS_PER_SEC << endl;}
const ld PI = asin(1) * 2;
const ld eps = 1e-14;
const int oo = 2e9;
const ll OO = 2e18;
const ll MOD = 1000000007;
const int MAXN = 1000000;

int main() {
    FAST_IO;
    startTime();

    timeit("Finished");
    return 0;
}
```

2.3 Compilation

```
# Simple compile
g++ -DLOCAL -O2 -o main.exe -std=c++17 -Wall -Wno-unused-result -Wshadow main.cpp
# Debug
g++ -DLOCAL -std=c++17 -Wshadow -Wall -o main.exe main.cpp -fsanitize=address -fsanitize=undefined -fuse-ld=gold -D_GLIBCXX_DEBUG -g
```

2.4 Ternary Search

```
double ternary_search(double l, double r) {
    while (r - l > eps) {
        double m1 = l + (r - l) / 3;
        double m2 = r - (r - l) / 3;
        double f1 = f(m1);
        double f2 = f(m2);
        if (f1 < f2)
            l = m1;
        else
            r = m2;
    }
    return f(l); //return the maximum of f(x) in [l, r]
}
```

3 Geometry

3.1 2d Vector

```
template <typename T>
struct Vec {
    T x, y;
    Vec(): x(0), y(0) {}
    Vec(T _x, T _y): x(_x), y(_y) {}
    Vec operator+(const Vec& b) {
        return Vec<T>(x+b.x, y+b.y);
    }
    Vec operator-(const Vec& b) {
        return Vec<T>(x-b.x, y-b.y);
    }
    Vec operator*(T c) {
        return Vec(x*c, y*c);
    }
    T operator*(const Vec& b) {
        return x*b.x + y*b.y;
    }
    T operator^(const Vec& b) {
        return x*b.y-y*b.x;
    }
    bool operator<(const Vec& other) const {
        if(x == other.x) return y < other.y;
        return x < other.x;
    }
    bool operator==(const Vec& other) const {
        return x==other.x && y==other.y;
    }
    bool operator!=(const Vec& other) const {
        return !(*this == other);
    }
    friend ostream& operator<<(ostream& out, const Vec& v) {
        return out << "(" << v.x << ", " << v.y << ")";
    }
    friend istream& operator>>(istream& in, Vec<T>& v) {
        return in >> v.x >> v.y;
    }
    T norm() { // squared length
        return (*this)*(*this);
    }
    ld len() {
        return sqrt(norm());
    }
    ld angle(const Vec& other) { // angle between this and other vector
        return acosl((*this)*other/len()/other.len());
    }
    Vec perp() {
        return Vec(-y, x);
    }
};
/* Cross product of 3d vectors: (ay*bz-az*by, az*bx-ax*bz, ax*by-ay*bx)
 */
```

3.2 Circle Circle Intersection

Let's say that the first circle is centered at $(0,0)$ (if it's not, we can move the origin to the center of the first circle and adjust the coordinates), and the second one is at (x_2, y_2). Then, let's construct a line $Ax + By + C = 0$, where $A = -2x_2, B = -2y_2, C = x_2^2 + y_2^2 + r_1^2 - r_2^2$. Finding the intersection between this line and the first circle will give us the answer. The only tricky case: if both circles are centered at the same point. We handle this case separately.

3.3 Circle Line Intersection

```
double r, a, b, c; // ax+by+c=0, radius is at (0, 0)
// If the center is not at (0, 0), fix the constant c to translate everything so that center
      is at (0, 0)
double x0 = -a*c/(a*a+b*b), y0 = -b*c/(a*a+b*b);
if (c*c > r*r*(a*a+b*b)+eps)
    puts ("no points");
else if (abs (c*c - r*r*(a*a+b*b)) < eps) {
    puts ("1 point");
    cout << x0 << ' ' << y0 << '\n';
}
else {
    double d = r*r - c*c/(a*a+b*b);
    double mult = sqrt (d / (a*a+b*b));
    double ax, ay, bx, by;
    ax = x0 + b * mult;
    bx = x0 - b * mult;
    ay = y0 - a * mult;
    by = y0 + a * mult;
    puts ("2 points");
    cout << ax << ' ' << ay << '\n' << bx << ' ' << by << '\n';
}
```

3.4 Common Tangents To Two Circles

```
struct pt {
    double x, y;

    pt operator- (pt p) {
        pt res = { x-p.x, y-p.y };
        return res;
    }
};
struct circle : pt {
    double r;
};
struct line {
    double a, b, c;
};
void tangents (pt c, double r1, double r2, vector<line> & ans) {
    double r = r2 - r1;
    double z = sqr(c.x) + sqr(c.y);
    double d = z - sqr(r);
    if (d < -eps) return;
    d = sqrt (abs (d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back (l);
}
vector<line> tangents (circle a, circle b) {
    vector<line> ans;
    for (int i=-1; i<=1; i+=2)
        for (int j=-1; j<=1; j+=2)
            tangents (b-a, a.r*i, b.r*j, ans);
    for (size_t i=0; i<ans.size(); ++i)
        ans[i].c -= ans[i].a * a.x + ans[i].b * a.y;
    return ans;
}
```

3.5 Convex Hull Gift Wrapping

```
vector<Vec<int>> buildConvexHull(vector<Vec<int>>& pts) {
    int n = pts.size();
    sort(pts.begin(), pts.end());
    auto currP = pts[0]; // choose some extreme point to be on the hull

    vector<Vec<int>> hull;
    set<Vec<int>> used;
    hull.pb(pts[0]);
    used.insert(pts[0]);
    while(true) {
        auto candidate = pts[0]; // choose some point to be a candidate

        auto currDir = candidate-currP;
        vector<Vec<int>> toUpdate;
        FOR(i, 0, n) {
            if(currP == pts[i]) continue;
            // currently we have currP->candidate
            // we need to find point to the left of this
            auto possibleNext = pts[i];
            auto nextDir = possibleNext - currP;
            auto cross = currDir ^ nextDir;
            if(candidate == currP || cross > 0) {
                candidate = possibleNext;
                currDir = nextDir;
            } else if(cross == 0 && nextDir.norm() > currDir.norm()) {
                candidate = possibleNext;
                currDir = nextDir;
            }
        }
        if(used.find(candidate) != used.end()) break;
        hull.pb(candidate);
        used.insert(candidate);
        currP = candidate;
    }
    return hull;
}
```

3.6 Convex Hull With Graham's Scan

```
// Takes in >= 3 points
// Returns convex hull in clockwise order
// Ignores points on the border
vector<Vec<int>> buildConvexHull(vector<Vec<int>> pts) {
    if(pts.size() <= 3) return pts;
    sort(pts.begin(), pts.end());
    stack<Vec<int>> hull;
    hull.push(pts[0]);
    auto p = pts[0];
    sort(pts.begin()+1, pts.end(), [&](Vec<int> a, Vec<int> b) -> bool {
        // p->a->b is a ccw turn
        int turn = sgn((a-p)^(b-a));
        //if(turn == 0) return (a-p).norm() > (b-p).norm();
        // ^ among collinear points, take the farthest one
        return turn == 1;
    });
    hull.push(pts[1]);
```

```
    FOR(i, 2, (int)pts.size()) {
        auto c = pts[i];
        if(c == hull.top()) continue;
        while(true) {
            auto a = hull.top(); hull.pop();
            auto b = hull.top();
            auto ba = a-b;
            auto ac = c-a;
            if((ba^ac) > 0) {
                hull.push(a);
                break;
            } else if((ba^ac) == 0) {
                if(ba*ac < 0) c = a;
                // ^ c is between b and a, so it shouldn't be added to the hull
                break;
            }
        }
        hull.push(c);
    }
    vector<Vec<int>> hullPts;
    while(!hull.empty()) {
        hullPts.pb(hull.top());
        hull.pop();
    }
    return hullPts;
}
```

3.7 Line

```
template <typename T>
struct Line { // expressed as two vectors
    Vec<T> start, dir;
    Line() {}
    Line(Vec<T> a, Vec<T> b): start(a), dir(b-a) {}

    Vec<ld> intersect(Line l) {
        ld t = ld((l.start-start)^l.dir)/(dir^l.dir);
        // For segment-segment intersection this should be in range [0, 1]
        Vec<ld> res(start.x, start.y);
        Vec<ld> dirld(dir.x, dir.y);
        return res + dirld*t;
    }
};
```

3.8 Number Of Lattice Points On Segment

Let's say we have a line segment from (x_1, y_1) to (x_2, y_2). Then, the number of lattice points on this segment is given by

$$gcd(x_2 - x_1, y_2 - y_1) + 1.$$

3.9 Pick's Theorem

We are given a lattice polygon with non-zero area. Let's denote its area by S, the number of points with integer coordinates lying strictly inside the polygon by I and the number of points lying on the sides of the polygon by B. Then:

$$S = I + \frac{B}{2} - 1.$$

4 Graphs

4.1 Bellman Ford Algorithm

```
struct Edge
{
    int a, b, cost;
};

int n, m, v; // v - starting vertex
vector<Edge> e;

/* Finds SSSP with negative edge weights.
 * Possible optimization: check if anything changed in a relaxation step. If not - you
        can break early.
 * To find a negative cycle: perform one more relaxation step. If anything changes - a
        negative cycle exists.
 */
void solve() {
    vector<int> d (n, oo);
    d[v] = 0;
```

```
    for (int i=0; i<n-1; ++i)
        for (int j=0; j<m; ++j)
            if (d[e[j].a] < oo)
                d[e[j].b] = min (d[e[j].b], d[e[j].a] + e[j].cost);
    // display d, for example, on the screen
}
```

4.2 Bipartite Graph

```
class BipartiteGraph {
private:
    vector<int> _left, _right;
    vector<vector<int>> _adjList;
    vector<int> _matchR, _matchL;
    vector<bool> _used;

    bool _kuhn(int v) {
        if (_used[v]) return false;
        _used[v] = true;
        FOR(i, 0, (int)_adjList[v].size()) {
            int to = _adjList[v][i] - _left.size();
            if (_matchR[to] == -1 || _kuhn(_matchR[to])) {
                _matchR[to] = v;
                _matchL[v] = to;
                return true;
            }
        }
        return false;
    }
    void _addReverseEdges() {
        FOR(i, 0, (int)_right.size()) {
            if (_matchR[i] != -1) {
                _adjList[_left.size() + i].pb(_matchR[i]);
            }
        }
    }
    void _dfs(int p) {
        if (_used[p]) return;
        _used[p] = true;
        for (auto x : _adjList[p]) {
            _dfs(x);
        }
    }
    vector<pii> _buildMM() {
        vector<pair<int, int> > res;
        FOR(i, 0, (int)_right.size()) {
            if (_matchR[i] != -1) {
                res.push_back(make_pair(_matchR[i], i));
            }
        }

        return res;
    }
public:
    void addLeft(int x) {
        _left.pb(x);
        _adjList.pb({});
        _matchL.pb(-1);
        _used.pb(false);
    }
    void addRight(int x) {
        _right.pb(x);
        _adjList.pb({});
        _matchR.pb(-1);
        _used.pb(false);
    }
    void addForwardEdge(int l, int r) {
        _adjList[l].pb(r + _left.size());
    }
    void addMatchEdge(int l, int r) {
        if(l != -1) _matchL[l] = r;
        if(r != -1) _matchR[r] = l;
    }
    // Maximum Matching
    vector<pii> mm() {
        _matchR = vector<int>(_right.size(), -1);
        _matchL = vector<int>(_left.size(), -1);
        // ^ these two can be deleted if performing MM on already partially matched
            graph
        _used = vector<bool>(_left.size() + _right.size(), false);


        bool path_found;
        do {
            fill(_used.begin(), _used.end(), false);
            path_found = false;
            FOR(i, 0, (int)_left.size()) {
                if (_matchL[i] < 0 && !_used[i]) {
                    path_found |= _kuhn(i);
                }
            }
        } while (path_found);

        return _buildMM();
    }

    // Minimum Edge Cover
    // Algo: Find MM, add unmatched vertices greedily.
    vector<pii> mec() {
        auto ans = mm();
        FOR(i, 0, (int)_left.size()) {
            if (_matchL[i] != -1) {
```

```
            for (auto x : _adjList[i]) {
                int ridx = x - _left.size();
                if (_matchR[ridx] == -1) {
                    ans.pb({ i, ridx });
                    _matchR[ridx] = i;
                }
            }
        }
    }
    FOR(i, 0, (int)_left.size()) {
        if (_matchL[i] == -1 && (int)_adjList[i].size() > 0) {
            int ridx = _adjList[i][0] - _left.size();
            _matchL[i] = ridx;
            ans.pb({ i, ridx });
        }
    }
    return ans;
}

// Minimum Vertex Cover
// Algo: Find MM. Run DFS from unmatched vertices from the left part.
// MVC is composed of unvisited LEFT and visited RIGHT vertices.
pair<vector<int>, vector<int>> mvc(bool runMM = true) {
    if (runMM) mm();
    _addReverseEdges();
    fill(_used.begin(), _used.end(), false);
    FOR(i, 0, (int)_left.size()) {
        if (_matchL[i] == -1) {
            _dfs(i);
        }
    }
    vector<int> left, right;
    FOR(i, 0, (int)_left.size()) {
        if (!_used[i]) left.pb(i);
    }
    FOR(i, 0, (int)_right.size()) {
        if (_used[i + (int)_left.size()]) right.pb(i);
    }
    return { left,right };
}

// Maximal Independant Vertex Set
// Algo: Find complement of MVC.
pair<vector<int>, vector<int>> mivs(bool runMM = true) {
    auto m = mvc(runMM);
    vector<bool> containsL(_left.size(), false), containsR(_right.size(), false);
    for (auto x : m.first) containsL[x] = true;
    for (auto x : m.second) containsR[x] = true;
    vector<int> left, right;
    FOR(i, 0, (int)_left.size()) {
        if (!containsL[i]) left.pb(i);
    }
    FOR(i, 0, (int)_right.size()) {
        if (!containsR[i]) right.pb(i);
    }
    return { left, right };
}

};
```

4.3 Dfs With Timestamps

```
vector<vector<int>> adj;
vector<int> tIn, tOut, color;
int dfs_timer = 0;

void dfs(int v) {
    tIn[v] = dfs_timer++;
    color[v] = 1;
    for (int u : adj[v])
        if (color[u] == 0)
            dfs(u);
    color[v] = 2;
    tOut[v] = dfs_timer++;
}
```

4.4 Finding Articulation Points

```
int n; // number of nodes
vector<vector<int>> adj; // adjacency list of graph

vector<bool> visited;
vector<int> tin, fup;
int timer;

void processCutpoint(int v) {
    // problem-specific logic goes here
    // it can be called multiple times for the same v
}

void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = fup[v] = timer++;
    int children=0;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            fup[v] = min(fup[v], tin[to]);
```

```
        } else {
            dfs(to, v);
            fup[v] = min(fup[v], fup[to]);
            if (fup[to] >= tin[v] && p!=-1)
                processCutpoint(v);
            ++children;
        }
    }
    if(p == -1 && children > 1)
        processCutpoint(v);
}

void findCutpoints() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    fup.assign(n, -1);
    for (int i = 0; i < n; ++i) {
        if (!visited[i])
            dfs (i);
    }
}
```

4.5 Finding Bridges

```
int n; // number of nodes
vector<vector<int>> adj; // adjacency list of graph

vector<bool> visited;
vector<int> tin, fup;
int timer;

void processBridge(int u, int v) {
    // do something with the found bridge
}

void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = fup[v] = timer++;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            fup[v] = min(fup[v], tin[to]);
        } else {
            dfs(to, v);
            fup[v] = min(fup[v], fup[to]);
            if (fup[to] > tin[v])
                processBridge(v, to);
        }
    }
}

// Doesn't work with multiple edges
// But multiple edges are never bridges, so it's easy to check
void findBridges() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    fup.assign(n, -1);
    bridges.clear();
    FOR(i, 0, n) {
        if (!visited[i])
            dfs(i);
    }
}
```

4.6 Lowest Common Ancestor

```
int n, l; // l == logN (usually about ~20)
vector<vector<int>> adj;

int timer;
vector<int> tin, tout;
vector<vector<int>> up;

void dfs(int v, int p)
{
    tin[v] = ++timer;
    up[v][0] = p;
    // wUp[v][0] = weight[v][u]; // <- path weight sum to 2^i-th ancestor
    for (int i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];
        // wUp[v][i] = wUp[v][i-1] + wUp[up[v][i-1]][i-1];

    for (int u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }

    tout[v] = ++timer;
}

bool isAncestor(int u, int v)
{
    return tin[u] <= tin[v] && tout[v] <= tout[u];
}

int lca(int u, int v)
```

```
{
    if (isAncestor(u, v))
        return u;
    if (isAncestor(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!isAncestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}

void preprocess(int root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<int>(l + 1));
    dfs(root, root);
}
```

4.7 Max Flow With Dinic 2

```
struct FlowEdge {
    int v, u;
    long long cap, flow = 0;
    FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(cap) {}
};

struct Dinic {
    const long long flow_inf = 1e18;
    vector<FlowEdge> edges;
    vector<vector<int>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;

    Dinic(int n, int s, int t) : n(n), s(s), t(t) {
        adj.resize(n);
        level.resize(n);
        ptr.resize(n);
    }

    void add_edge(int v, int u, long long cap) {
        edges.push_back(FlowEdge(v, u, cap));
        edges.push_back(FlowEdge(u, v, 0));
        adj[v].push_back(m);
        adj[u].push_back(m + 1);
        m += 2;
    }

    bool bfs() {
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int id : adj[v]) {
                if (edges[id].cap - edges[id].flow < 1)
                    continue;
                if (level[edges[id].u] != -1)
                    continue;
                level[edges[id].u] = level[v] + 1;
                q.push(edges[id].u);
            }
        }
        return level[t] != -1;
    }

    long long dfs(int v, long long pushed) {
        if (pushed == 0)
            return 0;
        if (v == t)
            return pushed;
        for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
            int id = adj[v][cid];
            int u = edges[id].u;
            if (level[v] + 1 != level[u] || edges[id].cap - edges[id].flow < 1)
                continue;
            long long tr = dfs(u, min(pushed, edges[id].cap - edges[id].flow));
            if (tr == 0)
                continue;
            edges[id].flow += tr;
            edges[id ^ 1].flow -= tr;
            return tr;
        }
        return 0;
    }

    long long flow() {
        long long f = 0;
        while (true) {
            fill(level.begin(), level.end(), -1);
            level[s] = 0;
            q.push(s);
            if (!bfs())
                break;
            fill(ptr.begin(), ptr.end(), 0);
            while (long long pushed = dfs(s, flow_inf)) {
                f += pushed;
            }
        }
        return f;
```

```
    }
};
```

4.8 Max Flow With Dinic

```
struct Edge {
    int f, c;
    int to;
    pii revIdx;
    int dir;
    int idx;
};

int n, m;
vector<Edge> adjList[MAX_N];
int level[MAX_N];

void addEdge(int a, int b, int c, int i, int dir) {
    int idx = adjList[a].size();
    int revIdx = adjList[b].size();
    adjList[a].pb({ 0,c,b, {b, revIdx} ,dir,i });
    adjList[b].pb({ 0,0,a, {a, idx} ,dir,i });
}

bool bfs(int s, int t) {
    FOR(i, 0, n) level[i] = -1;
    level[s] = 0;
    queue<int> Q;
    Q.push(s);
    while (!Q.empty()) {
        auto t = Q.front(); Q.pop();
        for (auto x : adjList[t]) {
            if (level[x.to] < 0 && x.f < x.c) {
                level[x.to] = level[t] + 1;
                Q.push(x.to);
            }
        }
    }
    return level[t] >= 0;
}

int send(int u, int f, int t, vector<int>& edgeIdx) {
    if (u == t) return f;
    for (; edgeIdx[u] < adjList[u].size(); edgeIdx[u]++) {
        auto& e = adjList[u][edgeIdx[u]];
        if (level[e.to] == level[u] + 1 && e.f < e.c) {
            int curr_flow = min(f, e.c - e.f);
            int next_flow = send(e.to, curr_flow, t, edgeIdx);
            if (next_flow > 0) {
                e.f += next_flow;
                adjList[e.revIdx.first][e.revIdx.second].f -= next_flow;
                return next_flow;
            }
        }
    }
    return 0;
}

int maxFlow(int s, int t) {
    int f = 0;
    while (bfs(s, t)) {
        vector<int> edgeIdx(n, 0);
        while (int extra = send(s, oo, t, edgeIdx)) {
            f += extra;
        }
    }
    return f;
}

void init() {
    cin >> n >> m;
    FOR(i, 0, m) {
        int a, b, c;
        cin >> a >> b >> c;
        a--; b--;
        addEdge(a, b, c, i, 1);
        addEdge(b, a, c, i, -1);
    }
}
```

4.9 Max Flow With Ford Fulkerson

```
struct Edge {
    int to, next;
    ll f, c;
    int idx, dir;
    int from;
};

int n, m;
vector<Edge> edges;
vector<int> first;

void addEdge(int a, int b, ll c, int i, int dir) {
    edges.pb({ b, first[a], 0, c, i, dir, a });
    edges.pb({ a, first[b], 0, 0, i, dir, b });
    first[a] = edges.size() - 2;
    first[b] = edges.size() - 1;
```

```
}

void init() {
    cin >> n >> m;
    edges.reserve(4 * m);
    first = vector<int>(n, -1);
    FOR(i, 0, m) {
        int a, b, c;
        cin >> a >> b >> c;
        a--; b--;
        addEdge(a, b, c, i, 1);
        addEdge(b, a, c, i, -1);
    }
}

int cur_time = 0;
vector<int> timestamp;

ll dfs(int v, ll flow = OO) {
    if (v == n - 1) return flow;
    timestamp[v] = cur_time;
    for (int e = first[v]; e != -1; e = edges[e].next) {
        if (edges[e].f < edges[e].c && timestamp[edges[e].to] != cur_time) {
            int pushed = dfs(edges[e].to, min(flow, edges[e].c - edges[e].f));
            if (pushed > 0) {
                edges[e].f += pushed;
                edges[e ^ 1].f -= pushed;
                return pushed;
            }
        }
    }
    return 0;
}

ll maxFlow() {
    cur_time = 0;
    timestamp = vector<int>(n, 0);
    ll f = 0, add;
    while (true) {
        cur_time++;
        add = dfs(0);
        if (add > 0) {
            f += add;
        }
        else {
            break;
        }
    }
    return f;
}
```

4.10 Min Cut

```
init();
ll f = maxFlow(); // Ford-Fulkerson
cur_time++;
dfs(0);
set<int> cc;
for (auto e : edges) {
    if (timestamp[e.from] == cur_time && timestamp[e.to] != cur_time) {
        cc.insert(e.idx);
    }
}
// (# of edges in min-cut, capacity of cut)
// [indices of edges forming the cut]
cout << cc.size() << " " << f << endl;
for (auto x : cc) cout << x + 1 << " ";
```

4.11 Number Of Paths Of Fixed Length

Let G be the adjacency matrix of a graph. Then $C_k = G^k$ gives a matrix, in which the value $C_k[i][j]$ gives the number of paths between i and j of length k.

4.12 Shortest Paths Of A Fixed Length

Define $A \bigodot B = C \iff C_{ij} = min_{p=1..n}(A_{ip} + B_{pj})$. Let G be the adjacency matrix of a graph. Also, let $L_k = G \bigodot \ldots \bigodot G = G^{\bigodot k}$. Then the value $L_k[i][j]$ denotes the length of the shortest path between i and j which consists of exactly k edges.

4.13 Strongly Connected Components

```
vector < vector<int> > g, gr; // adjList and reversed adjList
vector<bool> used;
vector<int> order, component;

void dfs1 (int v) {
    used[v] = true;
    for (size_t i=0; i<g[v].size(); ++i)
        if (!used[ g[v][i] ])
            dfs1 (g[v][i]);
```

```
    order.push_back (v);
}

void dfs2 (int v) {
    used[v] = true;
    component.push_back (v);
    for (size_t i=0; i<gr[v].size(); ++i)
        if (!used[ gr[v][i] ])
            dfs2 (gr[v][i]);
}

int main() {
    int n;
    // read n
    for (;;) {
        int a, b;
        // read edge a -> b
        g[a].push_back (b);
        gr[b].push_back (a);
    }

    used.assign (n, false);
    for (int i=0; i<n; ++i)
        if (!used[i])
            dfs1 (i);
    used.assign (n, false);
    for (int i=0; i<n; ++i) {
        int v = order[n-1-i];
        if (!used[v]) {
            dfs2 (v);
            // do something with the found component
            component.clear(); // components are generated in toposort-order
        }
    }
}
```

5 Math

5.1 Big Integer Multiplication With FFT

```
complex<ld> a[MAX_N], b[MAX_N];
complex<ld> fa[MAX_N], fb[MAX_N], fc[MAX_N];
complex<ld> cc[MAX_N];

string mul(string as, string bs) {
    int sgn1 = 1;
    int sgn2 = 1;
    if (as[0] == '-') {
        sgn1 = -1;
        as = as.substr(1);
    }
    if (bs[0] == '-') {
        sgn2 = -1;
        bs = bs.substr(1);
    }
    int n = as.length() + bs.length() + 1;
    FFT::init(n);
    FOR(i, 0, FFT::pwrN) {
        a[i] = b[i] = fa[i] = fb[i] = fc[i] = cc[i] = 0;
    }
    FOR(i, 0, as.size()) {
        a[i] = as[as.size() - 1 - i] - '0';
    }
    FOR(i, 0, bs.size()) {
        b[i] = bs[bs.size() - 1 - i] - '0';
    }
    FFT::fft(a, fa);
    FFT::fft(b, fb);
    FOR(i, 0, FFT::pwrN) {
        fc[i] = fa[i] * fb[i];
    }
    // turn [0,1,2,...,n-1] into [0, n-1, n-2, ..., 1]
    FOR(i, 1, FFT::pwrN) {
        if (i < FFT::pwrN - i) {
            swap(fc[i], fc[FFT::pwrN - i]);
        }
    }
    FFT::fft(fc, cc);
    ll carry = 0;
    vector<int> v;
    FOR(i, 0, FFT::pwrN) {
        int num = round(cc[i].real() / FFT::pwrN) + carry;
        v.pb(num % 10);
        carry = num / 10;
    }
    while (carry > 0) {
        v.pb(carry % 10);
        carry /= 10;
    }
    reverse(v.begin(), v.end());
    bool start = false;
    ostringstream ss;
    bool allZero = true;
    for (auto x : v) {
        if (x != 0) {
            allZero = false;
            break;
        }
    }
    if (sgn1*sgn2 < 0 && !allZero) ss << "-";
```

```
    for (auto x : v) {
        if (x == 0 && !start) continue;
        start = true;
        ss << abs(x);
    }
    if (!start) ss << 0;
    return ss.str();
}
```

5.2 Burnside's Lemma

Let G be a finite group that acts on a set X. For each g in G let X^g denote the set of elements in X that are fixed by g. Burnside's lemma asserts the following formula for the number of orbits:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

Example. Coloring a cube with three colors.

Let X be the set of 3^6 possible face color combinations. Let's count the sizes of the fixed sets for each of the 24 rotations:

- one 0-degree rotation which leaves all 3^6 elements of X unchanged
- six 90-degree face rotations, each of which leaves 3^3 elements of X unchanged
- three 180-degree face rotation, each of which leaves 3^4 elements of X unchanged
- eight 120-degree vertex rotations, each of which leaves 3^2 elements of X unchanged
- six 180-degree edge rotations, each of which leaves 3^3 elements of X unchanged

The average is then $\frac{1}{24}(3^6 + 6 \cdot 3^3 + 3 \cdot 3^4 + 8 \cdot 3^2 + 6 \cdot 3^3) = 57$. For n colors: $\frac{1}{24}(n^6 + 3n^4 + 12n^3 + 8n^2)$.

Example. Coloring a circular stripe of n cells with two colors.

X is the set of all colored striped (it has 2^n elements), G is the group of rotations (n elements - by 0 cells, by 1 cell, ..., by $(n-1)$ cells). Let's fix some K and find the number of stripes that are fixed by the rotation by K cells. If a stripe becomes itself after rotation by K cells, then its 1st cell must have the same color as its $(1+K \mod n)$-th cell, which is in turn the same as its $(1+2K \mod n)$-th cell, etc., until $mK \mod n = 0$. This will happen when $m = n/gcd(K,n)$. Therefore, we have $n/gcd(K,n)$ cells that must all be of the same color. The same will happen when starting from the second cell and so on. Therefore, all cells are separated into $gcd(K,n)$ groups, with each group being of one color, and that yields $2^{gcd(K,n)}$ choices. That's why the answer to the original problem is $\frac{1}{n}\sum_{k=0}^{n-1} 2^{gcd(k,n)}$.

5.3 Chinese Remainder Theorem

Let's say we have some numbers m_i, which are all mutually coprime. Also, let $M = \prod_i m_i$. Then the system of congruences

$$\begin{cases} x \equiv a_1 \pmod{m_1} \\ x \equiv a_2 \pmod{m_2} \\ \ldots \\ x \equiv a_k \pmod{m_k} \end{cases}$$

is equivalent to $x \equiv A \pmod{M}$ and there exists a unique number A satisfying $0 \le A \le M$.

Solution for two: $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2}$. Let $x = a_1 + km_1$. Substituting into the second congruence: $km_1 \equiv a_2 - a_1 \pmod{m_2}$. Then, $k = (m_1)^{-1}_{m_2}(a_2 - a_1) \pmod{m_2}$. and we can easily find x. This can be extended to multiple equations by solving them one-by-one.

If the moduli are not coprime, solve the system $y \equiv 0 \pmod{\frac{m_1}{g}}, y \equiv \frac{a_2 - a_1}{g} \pmod{\frac{m_2}{g}}$ for y. Then let $x \equiv gy + a_1 \pmod{\frac{m_1 m_2}{g}}$.

5.4 Euler Totient Function

```
// Number of numbers x < n so that gcd(x, n) = 1
ll phi(ll n) {
    if(n == 1) return 1;
    auto f = factorize(n);
    ll res = n;
    for(auto p : f) {
        res = res - res/p.first;
    }
    return res;
}
```

5.5 Extended Euclidean Algorithm

```
// ax+by=gcd(a,b)
void solveEq(ll a, ll b, ll& x, ll& y, ll& g) {
    if(b==0) {
        x = 1;
        y = 0;
        g = a;
        return;
    }
    ll xx, yy;
    solveEq(b, a%b, xx, yy, g);
    x = yy;
    y = xx-yy*(a/b);
}
// ax+by=c
bool solveEq(ll a, ll b, ll c, ll& x, ll& y, ll& g) {
    solveEq(a, b, x, y, g);
    if(c%g != 0) return false;
    x *= c/g; y *= c/g;
    return true;
}
// Finds a solution (x, y) so that x >= 0 and x is minimal
bool solveEqNonNegX(ll a, ll b, ll c, ll& x, ll &y, ll& g) {
    if(!solveEq(a, b, c, x, y, g)) return false;
    ll k = x*g/b;
    x = x - k*b/g;
    y = y + k*a/g;
    if(x < 0) {
        x += b/g;
        y -= a/g;
    }
    return true;
}
```

All other solutions can be found like this:

$$x' = x - k\frac{b}{g}, y' = y + k\frac{a}{g}, k \in \mathbb{Z}$$

5.6 Factorization With Sieve

```
// Use linear sieve to calculate minDiv
vector<pll> factorize(ll x) {
    vector<pll> res;
    ll prev = -1;
    ll cnt = 0;
    while(x != 1) {
        ll d = minDiv[x];
        if(d == prev) {
            cnt++;
        } else {
            if(prev != -1) res.pb({prev, cnt});
            prev = d;
            cnt = 1;
        }
        x /= d;
    }
    res.pb({prev, cnt});
    return res;
}
```

5.7 FFT With Modulo

```
bool isGenerator(ll g) {
    if (pwr(g, M - 1) != 1) return false;
    for (ll i = 2; i*i <= M - 1; i++) {
        if ((M - 1) % i == 0) {
            ll q = i;
            if (isPrime(q)) {
                ll p = (M - 1) / q;
                ll pp = pwr(g, p);
                if (pp == 1) return false;
            }
            q = (M - 1) / i;
            if (isPrime(q)) {
                ll p = (M - 1) / q;
                ll pp = pwr(g, p);
                if (pp == 1) return false;
```

```
            }
        }
    }
    return true;
}

namespace FFT {
    ll n;
    vector<ll> r;
    vector<ll> omega;
    ll logN, pwrN;

    void initLogN() {
        logN = 0;
        pwrN = 1;
        while (pwrN < n) {
            pwrN *= 2;
            logN++;
        }
        n = pwrN;
    }

    void initOmega() {
        ll g = 2;
        while (!isGenerator(g)) g++;
        ll G = 1;
        g = pwr(g, (M - 1) / pwrN);
        FOR(i, 0, pwrN) {
            omega[i] = G;
            G *= g;
            G %= M;
        }
    }

    void initR() {
        r[0] = 0;
        FOR(i, 1, pwrN) {
            r[i] = r[i / 2] / 2 + ((i & 1) << (logN - 1));
        }
    }

    void initArrays() {
        r.clear();
        r.resize(pwrN);
        omega.clear();
        omega.resize(pwrN);
    }

    void init(ll n) {
        FFT::n = n;
        initLogN();
        initArrays();
        initOmega();
        initR();
    }

    void fft(ll a[], ll f[]) {
        for (ll i = 0; i < pwrN; i++) {
            f[i] = a[r[i]];
        }
        for (ll k = 1; k < pwrN; k *= 2) {
            for (ll i = 0; i < pwrN; i += 2 * k) {
                for (ll j = 0; j < k; j++) {
                    auto z = omega[j*n / (2 * k)] * f[i + j + k] % M;
                    f[i + j + k] = f[i + j] - z;
                    f[i + j] += z;
                    f[i + j + k] %= M;
                    if (f[i + j + k] < 0) f[i + j + k] += M;
                    f[i + j] %= M;
                }
            }
        }
    }
}
```

5.8 FFT

```
namespace FFT {
    int n;
    vector<int> r;
    vector<complex<ld>> omega;
    int logN, pwrN;

    void initLogN() {
        logN = 0;
        pwrN = 1;
        while (pwrN < n) {
            pwrN *= 2;
            logN++;
        }
        n = pwrN;
    }

    void initOmega() {
        FOR(i, 0, pwrN) {
            omega[i] = { cos(2 * i*PI / n), sin(2 * i*PI / n) };
        }
    }

    void initR() {
        r[0] = 0;
        FOR(i, 1, pwrN) {
```

```
            r[i] = r[i / 2] / 2 + ((i & 1) << (logN - 1));
        }
    }

    void initArrays() {
        r.clear();
        r.resize(pwrN);
        omega.clear();
        omega.resize(pwrN);
    }

    void init(int n) {
        FFT::n = n;
        initLogN();
        initArrays();
        initOmega();
        initR();
    }

    void fft(complex<ld> a[], complex<ld> f[]) {
        FOR(i, 0, pwrN) {
            f[i] = a[r[i]];
        }
        for (ll k = 1; k < pwrN; k *= 2) {
            for (ll i = 0; i < pwrN; i += 2 * k) {
                for (ll j = 0; j < k; j++) {
                    auto z = omega[j*n / (2 * k)] * f[i + j + k];
                    f[i + j + k] = f[i + j] - z;
                    f[i + j] += z;
                }
            }
        }
    }
}
```

5.9 Formulas

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$; $\sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$; $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$; $\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$; $\sum_{i=a}^{b} c^i = \frac{c^{b+1}-c^a}{c-1}, c \neq 1$; $\sum_{i=1}^{n} a_1 + (i-1)d = \frac{n(a_1+a_n)}{2}$; $\sum_{i=1}^{n} a_1 r^{i-1} = \frac{a_1(1-r^n)}{1-r}, r \neq 1$; $\sum_{i=1}^{\infty} ar^{i-1} = \frac{a_1}{1-r}, |r| \leq 1$.

5.10 Linear Sieve

```
ll minDiv[MAXN+1];
vector<ll> primes;

void sieve(ll n){
    FOR(k, 2, n+1){
        minDiv[k] = k;
    }
    FOR(k, 2, n+1) {
        if(minDiv[k] == k) {
            primes.pb(k);
        }
        for(auto p : primes) {
            if(p > minDiv[k]) break;
            if(p*k > n) break;
            minDiv[p*k] = p;
        }
    }
}
```

5.11 Modular Inverse

```
bool invWithEuclid(ll a, ll m, ll& aInv) {
    ll x, y, g;
    if(!solveEqNonNegX(a, m, 1, x, y, g)) return false;
    aInv = x;
    return true;
}
// Works only if m is prime
ll invFermat(ll a, ll m) {
    return pwr(a, m-2, m);
}
// Works only if gcd(a, m) = 1
ll invEuler(ll a, ll m) {
    return pwr(a, phi(m)-1, m);
}
```

5.12 Simpson Integration

```
const int N = 1000 * 1000; // number of steps (already multiplied by 2)

double simpsonIntegration(double a, double b){
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) {
        double x = a + h * i;
        s += f(x) * ((i & 1) ? 4 : 2);
```

```
    }
    s *= h / 3;
    return s;
}
```

6 Strings

6.1 Aho Corasick Automaton

```
// alphabet size
const int K = 70;

// the indices of each letter of the alphabet
int intVal[256];
void init() {
    int curr = 2;
    intVal[1] = 1;
    for(char c = '0'; c <= '9'; c++, curr++) intVal[(int)c] = curr;
    for(char c = 'A'; c <= 'Z'; c++, curr++) intVal[(int)c] = curr;
    for(char c = 'a'; c <= 'z'; c++, curr++) intVal[(int)c] = curr;
}

struct Vertex {
    int next[K];
    vector<int> marks;
    // ^ this can be changed to int mark = -1, if there will be no duplicates
    int p = -1;
    char pch;
    int link = -1;
    int exitLink = -1;
    // ^ exitLink points to the next node on the path of suffix links which is marked
    int go[K];

    // ch has to be some small char
    Vertex(int _p=-1, char ch=(char)1) : p(_p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};

vector<Vertex> t(1);

void addString(string const& s, int id) {
    int v = 0;
    for (char ch : s) {
        int c = intVal[(int)ch];
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].marks.pb(id);
}

int go(int v, char ch);

int getLink(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(getLink(t[v].p), t[v].pch);
    }
    return t[v].link;
}

int getExitLink(int v) {
    if(t[v].exitLink != -1) return t[v].exitLink;
    int l = getLink(v);
    if(l == 0) return t[v].exitLink = 0;
    if(!t[l].marks.empty()) return t[v].exitLink = l;
    return t[v].exitLink = getExitLink(l);
}

int go(int v, char ch) {
    int c = intVal[(int)ch];
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(getLink(v), ch);
    }
    return t[v].go[c];
}

void walkUp(int v, vector<int>& matches) {
    if(v == 0) return;
    if(!t[v].marks.empty()) {
        for(auto m : t[v].marks) matches.pb(m);
    }
    walkUp(getExitLink(v), matches);
}

// returns the IDs of matched strings.
// Will contain duplicates if multiple matches of the same string are found.
vector<int> walk(const string& s) {
    vector<int> matches;
    int curr = 0;
    for(char c : s) {
```

```
        curr = go(curr, c);
        if(!t[curr].marks.empty()) {
            for(auto m : t[curr].marks) matches.pb(m);
        }
        walkUp(getExitLink(curr), matches);
    }
    return matches;
}
/* Usage:
 * addString(strs[i], i);
 * auto matches = walk(text);
 * .. do what you need with the matches - count, check if some id exists, etc ..
 * Some applications:
 * - Find all matches: just use the walk function
 * - Find lexicographically smallest string of a given length that doesn't match any of
      the given strings:
 * For each node, check if it produces any matches (it either contains some marks or
      walkUp(v) returns some marks).
 * Remove all nodes which produce at least one match. Do DFS in the remaining
      graph, since none of the remaining nodes
 * will ever produce a match and so they're safe.
 * - Find shortest string containing all given strings:
 * For each vertex store a mask that denotes the strings which match at this state.
      Start at (v = root, mask = 0),
 * we need to reach a state (v, mask=2^n-1), where n is the number of strings in the
      set. Use BFS to transition between states
 * and update the mask.
 */
```

6.2 Hashing

```
struct HashedString {
    const ll A1 = 999999929, B1 = 1000000009, A2 = 1000000087, B2 = 1000000097;
    vector<ll> A1pwrs, A2pwrs;
    vector<pll> prefixHash;
    HashedString(const string& _s) {
        init(_s);
        calcHashes(_s);
    }
    void init(const string& s) {
        ll a1 = 1;
        ll a2 = 1;
        FOR(i, 0, (int)s.length()+1) {
            A1pwrs.pb(a1);
            A2pwrs.pb(a2);
            a1 = (a1*A1)%B1;
            a2 = (a2*A2)%B2;
        }
    }
    void calcHashes(const string& s) {
        pll h = {0, 0};
        prefixHash.pb(h);
        for(char c : s) {
            ll h1 = (prefixHash.back().first*A1 + c)%B1;
            ll h2 = (prefixHash.back().second*A2 + c)%B2;
            prefixHash.pb({h1, h2});
        }
    }
    pll getHash(int l, int r) {
        ll h1 = (prefixHash[r+1].first - prefixHash[l].first*A1pwrs[r+1-l]) % B1;
        ll h2 = (prefixHash[r+1].second - prefixHash[l].second*A2pwrs[r+1-l]) % B2;
        if(h1 < 0) h1 += B1;
        if(h2 < 0) h2 += B2;
        return {h1, h2};
    }
};
```

6.3 KMP

```
// Knuth-Morris-Pratt algorithm
vector<int> findOccurences(const string& s, const string& t) {
    int n = s.length();
    int m = t.length();
    string S = s + "#" + t;
    auto pi = prefixFunction(S);
    vector<int> ans;
    FOR(i, n+1, n+m+1) {
        if(pi[i] == n) {
            ans.pb(i-2*n);
        }
    }
    return ans;
}
```

6.4 Prefix Function Automaton

```
// aut[oldPi][c] = newPi
vector<vector<int>> computeAutomaton(string s) {
    const char BASE = 'a';
    s += "#";
    int n = s.size();
    vector<int> pi = prefixFunction(s);
    vector<vector<int>> aut(n, vector<int>(26));
    for (int i = 0; i < n; i++) {
```

```
        for (int c = 0; c < 26; c++) {
            if (i > 0 && BASE + c != s[i])
                aut[i][c] = aut[pi[i-1]][c];
            else
                aut[i][c] = i + (BASE + c == s[i]);
        }
    }
    return aut;
}
vector<int> findOccurs(const string& s, const string& t) {
    auto aut = computeAutomaton(s);
    int curr = 0;
    vector<int> occurs;
    FOR(i, 0, (int)t.length()) {
        int c = t[i]-'a';
        curr = aut[curr][c];
        if(curr == (int)s.length()) {
            occurs.pb(i-s.length()+1);
        }
    }
    return occurs;
}
```

6.5 Prefix Function

```
// pi[i] is the length of the longest proper prefix of the substring s[0..i] which is also a
    suffix
// of this substring
vector<int> prefixFunction(const string& s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}
```

6.6 Suffix Array

```
vector<int> sortCyclicShifts(string const& s) {
    int n = s.size();
    const int alphabet = 256; // we assume to use the whole ASCII range
    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for (int i = 0; i < n; i++)
        cnt[s[i]]++;
    for (int i = 1; i < alphabet; i++)
        cnt[i] += cnt[i-1];
    for (int i = 0; i < n; i++)
        p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i-1]])
            classes++;
        c[p[i]] = classes - 1;
    }
    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0)
                pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for (int i = 0; i < n; i++)
            cnt[c[pn[i]]]++;
        for (int i = 1; i < classes; i++)
            cnt[i] += cnt[i-1];
        for (int i = n-1; i >= 0; i--)
            p[--cnt[c[pn[i]]]] = pn[i];
        cn[p[0]] = 0;
        classes = 1;
        for (int i = 1; i < n; i++) {
            pair<int, int> cur = {c[p[i]], c[(p[i] + (1 << h)) % n]};
            pair<int, int> prev = {c[p[i-1]], c[(p[i-1] + (1 << h)) % n]};
            if (cur != prev)
                ++classes;
            cn[p[i]] = classes - 1;
        }
        c.swap(cn);
    }
    return p;
}
vector<int> constructSuffixArray(string s) {
    s += "$"; // <- this must be smaller than any character in s
    vector<int> sorted_shifts = sortCyclicShifts(s);
    sorted_shifts.erase(sorted_shifts.begin());
    return sorted_shifts;
}
```