Blind Frequency-Dependent I/Q Imbalance Compensation for Direct-Conversion Receivers

Yingming Tsai, Chia-Pang Yen, and Xiaodong Wang

Abstract—**Frequency-dependent I/Q imbalance is one of the major impairments in the direct-conversion receivers (DCR) for high-speed wideband wireless systems. We propose two new blind methods for compensating frequency-dependent I/Q imbalance. The first one is a time-domain approach. Specifically we develop a blind identifiability condition based on which a cost function and a gradient descent search algorithm are proposed for blind I/Q imbalance compensation. The second blind method is a frequency-domain approach for OFDM systems. Here we provide blind estimators for the frequency-selective I/Q imbalance parameters, which once obtained, the I/Q imbalance can then be compensated by a simple single-tap matrix filter inversion. We provide extensive simulation results to demonstrate the performance of the proposed algorithms.**

Index Terms—**Direct-conversion receiver (DCR), frequency-dependent I/Q imbalance, OFDM, blind identifiability, gradient descent, second-order statistics.**

I. Introduction

THE evolution of wireless communication systems has been driving the design and implementation of modern radio transceivers. The next-generation wireless networks will support high data-rate applications, which require efficient and low-cost wideband radio design for the terminals. The direct conversion receiver (DCR) has become a major approach to achieving compact and low cost transceiver design in wideband radio [1]. In DCR, the received signal is quadrature down-converted from RF directly to a baseband signal. One of the problems of DCR is that the downconverter circuits can easily produce a phase and amplitude imbalance between the in-phase (I) and quadrature-phase (Q) signals. Furthermore, to reject interference in wideband radio systems, the low-pass filters of the I- and Q-branches in DCR shown in Fig. 1(a) require sharp cut-off frequency (implying higher-order design). Furthermore, the non-ideal characteristics of the low-pass filters will cause coefficient discrepancies in the transfer functions, causing the I/Q imbalance parameters to be frequency-dependent and further complicating the design of DCR in wideband systems. Due to the inevitable mismatch between the in-phase and quadrature signal paths/components, cross-talk or interference will occur between the mirror-frequencies upon down conversion to baseband. Thus, the I/Q imbalance degrades the effective signal-to-interference power ratio and causes performance degradation. The impact of I/Q imbalance is more severe to systems employing high-order modulations and high coding rates. Therefore, effective I/Q imbalance compensation is essential for the design of high data-rate systems employing the DCR.

Some existing I/Q imbalance estimation and compensation methods are based on making use of training signals, e.g., [2], [3]. However, the transmission of training signals costs extra radio resources and thus blind compensation methods are of great interest. Several blind compensation methods for frequency-dependent I/Q imbalance have been developed. In particular, in [4], a method is developed that exploits a second-order statistical characteristic of the signal, namely, the properness property; and an adaptive compensation based on the least-mean-square (LMS) algorithm is proposed. However, the convergence of the LMS algorithm is typically slow and the steady-state performance exhibits a high variance due to the stochastic nature of the LMS algorithm, which makes it less attractive for high-speed applications. In [5], a compensation method is proposed based on the multichannel blind deconvolution (MBD) algorithm [6]. This scheme requires the probability density function (pdf) of the transmitted signal to be given. The methods proposed in this paper, however, do not require to know the pdf of the transmitted signal, but only assume that the transmitted signal is a proper and white process, which is a mild condition that is met by practical communication signals. Here we give a necessary and sufficient condition for perfect frequency-dependent I/Q imbalance compensation, and, based on this condition, we propose a time-domain compensation method.

The OFDM transmission technique is by now widely adopted by the next-generation cellular systems. Most of the existing frequency-dependent I/Q imbalance compensation algorithms for OFDM systems are based on special pilot patterns [7], [8]. In this paper, we also propose a blind I/Q imbalance compensation method for OFDM systems. The proposed method first estimates the I/Q imbalance parameters for each subcarrier based on the second-order statistics of the received signal on that subcarrier, and then applies the compensation filter that are calculated based on the estimated parameters.

The remainder of the paper is organized as follows. In Section II, the frequency-dependent I/Q imbalance signal model is described and the compensation problem is formulated. In Section III, we give a condition for perfect I/Q imbalance compensation and based on which we propose a new blind time-domain I/Q imbalance compensation method. In Section IV, we develop a blind frequency-domain I/Q imbalance

Manuscript received May 4, 2009; revised October 12, 2009; accepted January 1, 2010. The associate editor coordinating the review of this paper and approving it for publication was J. Lu.

Y. Tsai and C.-P. Yen are with InterDigital, Inc., 2 Huntington Quadrangle, 4th Floor, South Wing, Melville, NY 11747-4508 (e-mail: {Yingming.Tsai, Chia-Pang.Yen}@InterDigital.com).

X. Wang is with the Electrical Engineering Dept., Columbia Univ., New York, NY 10027 (e-mail: wangx@ee.columbia.edu).

Digital Object Identifier 10.1109/TWC.2010.06.090651.

compensation method for OFDM systems. Simulation results are provided in Section V and finally Section VI concludes the paper.

II. System Descriptions

A. I/Q Imbalance Signal Model

A typical block diagram of the RF front-end for DCR is given in Fig. 1(a) and its equivalent mathematical model in Fig. 1(b). The received RF signal with a central frequency w_c is expressed as

$$r(t) = 2\mathcal{R}e\left\{s(t)e^{jw_c t}\right\} = s(t)e^{jw_c t} + s^*(t)e^{-jw_c t}, \quad (1)$$

where $s(t) = s_I(t) + js_Q(t)$ is the baseband received signal and * denotes the complex conjugate. In general we have

$$s(t) = h(t,\tau) \otimes d(t),$$

where $d(t)$ is the transmitted data signal, $h(t,\tau)$ is the time-varying channel response and $\otimes$ denotes the convolution. The received RF signal $r(t)$ is directly down-converted by a local oscillator signal $u_{LO}(t)$ with mismatched I and Q branches [9]. Denote γ and ϕ as the mismatched amplitude and phase, respectively. Then the local oscillator signal $u_{LO}(t)$ of an imbalanced quadrature demodulator is given by

$$u_{LO}(t) = \cos(w_c t) - j\gamma \sin(w_c t + \phi). \quad (2)$$

The down-converted signal $x(t) = x_I(t) + jx_Q(t)$ is then expressed as (3) where $\mathcal{LPF}_I$ and $\mathcal{LPF}_Q$ denote the low-pass filters for I and Q branches, respectively. The frequency response of $\mathcal{LPF}_I$ and $\mathcal{LPF}_Q$ are denoted as $G_I(f)$ and $G_Q(f)$ respectively. From (3), the frequency-domain I- and Q-branch signals after low-pass filtering are given respectively as

$$\begin{aligned} X_I(f) &= \frac{1}{2}G_I(f)\left[S(f) + S^*(-f)\right], \\ \text{and} \quad X_Q(f) &= \frac{\gamma}{2j}G_Q(f)\left[S(f)e^{-j\phi} - S^*(-f)e^{j\phi}\right]. \end{aligned} \quad (4)$$

Then the received baseband signals $X(f)$ is

$$\begin{aligned} X(f) = &\underbrace{\frac{G_I(f) + \gamma G_Q(f)e^{-j\phi}}{2}}_{G_1(f)} S(f) \\ &+ \underbrace{\frac{G_I(f) - \gamma G_Q(f)e^{j\phi}}{2}}_{G_2(f)} S^*(-f). \end{aligned} \quad (5)$$

Hence the time-domain down-converted signal is given by

$$\begin{aligned} x(t) &= g_1(t) \otimes s(t) + g_2(t) \otimes s^*(t), \\ \text{with} \quad g_1(t) &= \frac{g_I(t) + \gamma g_Q(t)e^{-j\phi}}{2}, \\ \text{and} \quad g_2(t) &= \frac{g_I(t) - \gamma g_Q(t)e^{j\phi}}{2}. \end{aligned} \quad (6)$$

It is seen that the I/Q imbalance causes the received baseband signal $s(t)$ distorted by its image signal $s^*(t)$. To evaluate the signal distortion caused by the I/Q imbalance, we define the analog front-end image-reject ratio (IRR) [9] measured in decibels (dB) as

$$\mathrm{IRR}(f) = 10\log_{10}\frac{|G_1(f)|^2}{|G_2(f)|^2}. \quad (7)$$

After sampling $x(t)$ with a sampling interval T_s, the received discrete-time down-converted baseband signal becomes

$$x[n] = x(nT_s) = g_1[n] \otimes s[n] + g_2[n] \otimes s^*[n]. \quad (8)$$

B. Second-order Signal Statistics

The autocorrelation function (ACF) of a discrete-time complex random signal $x[n]$ is defined as $R_x[m] = \mathbb{E}\{x[n]x^*[n-m]\}$, where $\mathbb{E}\{\cdot\}$ denotes expectation. Another second-order statistic, the complementary autocorrelation function (CACF) is defined as $C_x[m] = \mathbb{E}\{x[n]x[n-m]\}$. A complex random signal $x[n]$ is proper if its CACF is equal to zero for all lag m [10], i.e., $C_x[m] = 0, \forall\, m$. In this paper, we assume that the transmitted data signal $d[n]$ is a zero-mean proper white process, i.e.,

$$\begin{aligned} R_d[m] = \mathbb{E}\{d[n]d^*[n-m]\} &= \sigma_d^2\delta[m], \quad (9) \\ \text{and} \quad C_d[m] = \mathbb{E}\{d[n]d[n-m]\} &= 0, \ \forall m, \quad (10) \end{aligned}$$

where $\delta[m]$ is the Dirac delta function. Let $s[n]$ be the received signals after the time-varying channel, i.e.,

$$s[n] = h[n,l] \otimes d[l] = \sum_l h[n,l]d[n-l]. \quad (11)$$

In this paper, we assume that the time-varying channel $h[n,l]$ is wide-sense stationary uncorrelated scattering (WSSUS), i.e., its ACF is expressed as [11]

$$\begin{aligned} &\mathbb{E}\{h[n_1,l_1]h^*[n_2,l_2]\} \\ &= \beta(l_1)J_0\left(2\pi f_D T_s(n_1-n_2)\right)\delta[l_1-l_2], \end{aligned} \quad (12)$$

where $\beta(l_1)$ is a function of l_1; f_D is the maximum Doppler shift; and $J_0()$ is the zero-order Bessel function of the first kind. We next show that the received signal $s[n]$ in (11) remains white and proper. First, its ACF can be evaluated as (13). Moreover, the CACF of $s[n]$ is given by (14).

III. Time-domain I/Q Imbalance Compensation

Assume that the filters $g_1[n]$ and $g_2[n]$ in (8) are approximated by FIR filters of length P and define

$$\mathbf{G}[p] = \begin{bmatrix} g_1[p] & g_2[p] \\ g_2^*[p] & g_1^*[p] \end{bmatrix}, \quad p = 0, \ldots, P-1.$$

Denote further $\mathbf{x}[n] = \begin{bmatrix} x[n] \ x^*[n] \end{bmatrix}^T$ and $\mathbf{s}[n] = \begin{bmatrix} s[n] \ s^*[n] \end{bmatrix}^T$. Then (8) can be rewritten in the form of the following conjugate signal model

$$\mathbf{x}[n] = \sum_{p=0}^{P-1} \mathbf{G}[p]\mathbf{s}[n-p]. \quad (15)$$

A. Identifiability

The objective of the I/Q imbalance compensation is to filter the signal $x[n]$ such that the output is a scaled and delayed version of $s[n]$ or $s^*[n]$. Mathematically, the goal is to find a separation filter of length L of the following form

$$\mathbf{W}[l] = \begin{bmatrix} w_{11}[l] & w_{12}[l] \\ w_{12}^*[l] & w_{11}^*[l] \end{bmatrix}, \quad l = 0, 1, \ldots, L-1, \quad (16)$$

$$
\begin{aligned}
x(t) &= \mathcal{LPF}_I\left\{r(t)\cos(w_c t)\right\} - j\mathcal{LPF}_Q\left\{\gamma r(t)\sin(w_c t+\phi)\right\} \\
&= \mathcal{LPF}_I\left\{\frac{1}{2}\left[s(t)+s^*(t)\right]+\frac{1}{2}\left[s(t)e^{2jw_c t}+s^*(t)e^{-2jw_c t}\right]\right\}+ \\
&\quad j\mathcal{LPF}_Q\left\{\frac{\gamma}{2j}\left[s(t)e^{-j\phi}-s^*(t)e^{j\phi}\right]+\frac{\gamma}{2j}\left[s(t)e^{j(2w_c t+\phi)}-s^*(t)e^{-j(2w_c t+\phi)}\right]\right\},
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
R_s[m] &= \sum_{l_1}\sum_{l_2}\mathbb{E}\left\{h(n,l_1)h^*(n-m,l_2)\right\}\mathbb{E}\left\{d[n-l_1]d^*[n-m-l_2]\right\} \\
&= \sigma_d^2\sum_{l}\mathbb{E}\left\{h(n,m+l)h^*(n-m,l)\right\} = \underbrace{\sigma_d^2\sum_{l}\beta(l)J_0(2\pi f_D T_s m)}_{\sigma_s^2}\,\delta[m].
\end{aligned} \tag{13}
$$

$$
C_s[m] = \sum_{l_1}\sum_{l_2}\mathbb{E}\{h(n,l_1)h(n-m,l_2)\}\underbrace{\mathbb{E}\{d[n-l_1]d[n-m-l_2]\}}_{C_d[m+l_2-l_1]=0} = 0, \tag{14}
$$

such that the filter output $\mathbf{y}[n] = \left[y[n]\ y^*[n]\right]^T$ is given by

$$
\mathbf{y}[n] = \sum_{l=0}^{L-1}\mathbf{W}[l]\mathbf{x}(n-l) = \lambda\mathbf{P}\mathbf{s}[n-n_0], \tag{17}
$$

where $\mathbf{P}$ is a permutation matrix, λ is a scaling factor, and n_0 is a delay.

We further define the following stacked signal model in (18) where $M \geq L-1$. Then we have

$$
\underline{\mathbf{y}}[n] = \underline{\mathbf{W}}\,\underline{\mathbf{x}}[n]. \tag{19}
$$

We have the following result on the identifiability for the I/Q imbalance problem.

Proposition 1: We have $\mathbf{y}[n] = \lambda\mathbf{P}\mathbf{s}[n-n_0]$ if and only if $\mathbb{E}\left\{\underline{\mathbf{y}}[n]\underline{\mathbf{y}}^H[n]\right\} = \kappa\mathbf{I}$, for some constant κ.

Proof: We first show the necessary condition. From (20) and since $\mathbf{y}[n] = [y[n]\ y^*[n]]^T$, we have

$$
\mathbf{R}_{\mathbf{y}}[m] = \begin{bmatrix} R_y[m] & C_y[m] \\ C_y^*[m] & R_y^*[m] \end{bmatrix}.
$$

Then (20) implies the following:

$$
\begin{aligned}
C_y[m] &\triangleq \mathbb{E}\left\{y[n]y[n-m]\right\} = 0,\ \forall m, &(21)\\
\text{and}\ R_y[m] &\triangleq \mathbb{E}\left\{y[n]y^*[n-m]\right\} = \kappa\delta[m]. &(22)
\end{aligned}
$$

We can now proceed to show that if $y[n]$ satisfies (21)-(22) then $\mathbf{y}[n] = \lambda\mathbf{P}\mathbf{s}[n-\tau]$. From (15) and (17), we can write

$$
y[n] = f_1[n]\otimes s[n] + f_2[n]\otimes s^*[n], \tag{23}
$$

where $f_1[n]$ and $f_2[n]$ are composite filters derived from $\{\mathbf{W}[l]\}$ and $\{\mathbf{G}[l]\}$. Substituting (23) into (21), and by using (13)-(14), we have

$$
\begin{aligned}
C_y[m] &= \sigma_s^2\Big(f_1[-m]\otimes f_2[m] + f_1[m]\otimes f_2[-m]\Big) \\
&= \sigma_s^2\Big(h[-m]+h[m]\Big) = 0,
\end{aligned} \tag{24}
$$

where $h[n] \triangleq f_1[n]\otimes f_2[-n]$. Note that (24) holds only when $h[n] = 0, \forall n$ or $h[n]$ is anti-symmetric and thus non-causal. We can exclude the later case for non-casual filters do not exist in practice. Hence, we can conclude that either $f_1[n] = 0$ or $f_2[n] = 0$. Let us first assume $f_2[n] = 0$. Then $y[n]$ in (23) becomes

$$
y[n] = f_1[n]\otimes s[n]. \tag{25}
$$

Substituting (25) into (22), we obtain

$$
R_y[m] = \sigma_s^2\left(f_1^*[-m]\otimes f_1[m]\right) = \kappa\delta[m]. \tag{26}
$$

Now taking the Fourier transform of $f_1^*[-m]\otimes f_1[m]$ we have $\|F_1(\omega)\|^2 = \frac{\kappa}{\sigma_s^2}$, which means $f_1[n]$ is an all-pass filter. Since $f_1[n]$ is an FIR filter, we must have $f_1[n] = \lambda_1\delta[n-n_1]$ for some delay n_1 and some scaling λ_1. Hence $y[n] = \lambda_1 s[n-n_1]$ or equivalently $\mathbf{y}[n] = \lambda_1\cdot\mathbf{I}\cdot\mathbf{s}[n-n_1]$.

Similarly, if $f_1[n] = 0$ then we will have $y[n] = \lambda_2 s^*[n-n_2]$ or equivalently $\mathbf{y}[n] = \lambda_2\cdot\mathbf{J}\cdot\mathbf{s}[n-n_2]$, where $\mathbf{J} \triangleq \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Now we proceed to prove the sufficiency. If $\mathbf{y}[n] = \lambda\mathbf{P}\mathbf{s}[n-n_0]$, then (20) becomes (27). Since $s[n]$ is white and proper, we have $\mathbf{R}_{\mathbf{s}}[m] = \sigma_s^2\delta[m]\mathbf{I}$. Therefore, (27) is a diagonal matrix. □

Remark: The output of the blind I/Q imbalance compensation filter is a delayed and scaled version of $s[n]$ or $s^*[n]$, which can be resolved as follows. For the delay estimation, many synchronization techniques can be used [12]. The scale ambiguity can be lumped into the effective channel gain and removed by channel estimation. As for resolving the ambiguity of conjugation, cyclic redundancy check (CRC) which is common in modern commercial wireless communication systems can be used.

B. Blind I/Q Imbalance Compensation Algorithm

In order to obtain the I/Q imbalance compensation filter $\mathbf{W}[l]$, based on Proposition 1, we define the following cost function by setting $\kappa = 1$

$$
c(\{\mathbf{W}[l]\}) \triangleq \left\|\mathbf{R}_{\underline{\mathbf{y}}} - \mathbf{I}\right\|_F^2. \tag{28}
$$

$$\begin{aligned}\underline{\mathbf{y}}[n] &= \left[\mathbf{y}^T[n], \mathbf{y}^T[n-1], \ldots, \mathbf{y}^T[n-M+1]\right]^T, \\ \underline{\mathbf{x}}[n] &= \left[\mathbf{x}^T[n], \mathbf{x}^T[n-1], \ldots, \mathbf{x}^T[n-(M+L)+1]\right]^T, \\ \underline{\mathbf{W}} &= \begin{bmatrix} \mathbf{W}[0] & \mathbf{W}[1] & \ldots & \mathbf{W}[L-1] & 0 & \ldots & 0 \\ 0 & \mathbf{W}[0] & \mathbf{W}[1] & \ldots & \mathbf{W}[L-1] & 0 & \ddots \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & \ddots & \mathbf{W}[0] & \mathbf{W}[1] & \ldots & \mathbf{W}[L-1] \end{bmatrix},\end{aligned} \tag{18}$$

$$\mathbb{E}\left\{\underline{\mathbf{y}}[n]\underline{\mathbf{y}}[n]^H\right\} = \begin{bmatrix} \mathbf{R_y}[0] & \mathbf{R_y}[1] & \ldots & \mathbf{R_y}[M-1] \\ \mathbf{R_y}[-1] & \mathbf{R_y}[0] & \ldots & \mathbf{R_y}[M-2] \\ \vdots & \vdots & \ddots & \ddots \\ \mathbf{R_y}[-(M-1)] & \mathbf{R_y}[-(M-2)] & \ldots & \mathbf{R_y}[0] \end{bmatrix} = \kappa\mathbf{I}. \tag{20}$$

$$\mathbb{E}\left\{\underline{\mathbf{y}}[n]\underline{\mathbf{y}}[n]^H\right\} = \lambda^2 \begin{bmatrix} \mathbf{P}\mathbf{R_s}[0]\mathbf{P} & \mathbf{P}\mathbf{R_s}[1]\mathbf{P} & \cdots & \mathbf{P}\mathbf{R_s}[M-1]\mathbf{P} \\ \mathbf{P}\mathbf{R_s}[-1]\mathbf{P} & \mathbf{P}\mathbf{R_s}[0]\mathbf{P} & \cdots & \mathbf{P}\mathbf{R_s}[M-2]\mathbf{P} \\ \vdots & & \ddots & \\ \mathbf{P}\mathbf{R_s}[-M+1]\mathbf{P} & \cdots & \cdots & \mathbf{P}\mathbf{R_s}[0]\mathbf{P} \end{bmatrix}. \tag{27}$$

We can then obtain $\{\mathbf{W}[l]\}$ by minimizing the above cost function, i.e.,

$$\{\mathbf{W}^{\mathrm{opt}}[l]\} = \min_{\{\mathbf{W}[l]\}} c(\{\mathbf{W}[l]\}). \tag{29}$$

We will use the gradient descent search method [6] to solve (29). In what follows we denote the (i,j)-th 2×2 submatrix of a matrix $\mathbf{A}$ as $[\mathbf{A}]_{ij} \triangleq \begin{bmatrix} a_{2(i-1)+1,2(j-1)+1} & a_{2(i-1)+1,2(j-1)+2} \\ a_{2(i-1)+2,2(j-1)+1} & a_{2(i-1)+2,2(j-1)+2} \end{bmatrix}$. Using the identity $\|\mathbf{A}\|_F^2 = \mathrm{tr}(\mathbf{A}\mathbf{A}^H) = \sum_i \mathrm{tr}([\mathbf{A}\mathbf{A}^H]_{ii}) = \sum_i \sum_l \mathrm{tr}([\mathbf{A}]_{il}[\mathbf{A}^H]_{li})$, (28) can be expressed as (30) - (32). Using (31) the term $\mathrm{tr}\left([\mathbf{R}_{\underline{\mathbf{y}}}]_{ij}[\mathbf{R}_{\underline{\mathbf{y}}}]_{ij}^H\right)$ in (30) is equal to (33). Similarly, the term $\mathrm{tr}\left(\left([\mathbf{R}_{\underline{\mathbf{y}}}]_{ii} - \mathbf{I}\right)\left([\mathbf{R}_{\underline{\mathbf{y}}}]_{ii} - \mathbf{I}\right)^H\right)$ in (30) is expressed as (34).

The following gradient descent iteration can be employed to solve (29) [6], [13],

$$\mathbf{W}[m]^{(k+1)} = \mathbf{W}[m]^{(k)} - \mu\left(\frac{\partial c(\{\mathbf{W}[l]\})}{\partial \mathbf{W}^*[m]^{(k)}}\right), \tag{35}$$

where $\mathbf{W}[m]^{(k)}$ is the m-th tap matrix of the I/Q imbalance filter at the k-th iteration, and μ is the learning rate.

From (33)-(34), it is seen that the cost function (30) is composed of terms in the forms of $\mathbf{W}[i]\mathbf{A}\mathbf{W}^H[j]\mathbf{W}[k]\mathbf{B}\mathbf{W}^H[l]$ and $\mathbf{W}[i]\mathbf{A}\mathbf{W}^H[j]$, where $\mathbf{A}$ and $\mathbf{B}$ are 2×2 submatrices from $\mathbf{R}_{\underline{\mathbf{x}}}$. For example, when $i = 2$, $j = 2$, $k_1 = k_2 = 2$, $l_1 = l_2 = 2$, one of the terms in (34) is $[\mathbf{W}]_{22}\left[\mathbf{R}_{\underline{\mathbf{x}}}\right]_{22}\left[\mathbf{W}^H\right]_{22}[\mathbf{W}]_{22}\left[\mathbf{R}_{\underline{\mathbf{x}}}^H\right]_{22}\left[\mathbf{W}^H\right]_{22} = \mathbf{W}[0]\mathbf{R_x}[0]\mathbf{W}^H[1]\mathbf{W}[0]\mathbf{R_x}^H[0]\mathbf{W}^H[1]$.

Define $\delta_{i,m} = 1$, if $i = m$ and 0 otherwise. Define further a matrix transformation operator $\mathfrak{T}(\mathbf{B}) = \mathfrak{T}\left(\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\right) \triangleq \begin{bmatrix} b_{22} & b_{12} \\ b_{21} & b_{11} \end{bmatrix}$. Then the gradient in (35) consists of the following terms (see Appendix A for derivations)

$$\begin{aligned} &\frac{\partial}{\partial \mathbf{W}^*[m]} \mathrm{tr}\left(\mathbf{W}[i]\mathbf{A}\mathbf{W}^H[j]\mathbf{W}[k]\mathbf{B}\mathbf{W}^H[l]\right) \\ &\quad = \delta_{i,m}\mathfrak{T}\left(\mathbf{A}\mathbf{W}^H[j]\mathbf{W}[k]\mathbf{B}\mathbf{W}^H[l]\right) \\ &\quad + \delta_{j,m}\mathbf{W}[k]\mathbf{B}\mathbf{W}^H[l]\mathbf{W}[i]\mathbf{A} \\ &\quad + \delta_{k,m}\mathfrak{T}\left(\mathbf{B}\mathbf{W}^H[l]\mathbf{W}[i]\mathbf{A}\mathbf{W}^H[j]\right) \\ &\quad + \delta_{l,m}\mathbf{W}[i]\mathbf{A}\mathbf{W}^H[j]\mathbf{W}[k]\mathbf{B}, \end{aligned} \tag{36}$$

and

$$\begin{aligned} &\frac{\partial}{\partial \mathbf{W}^*[m]} \mathrm{tr}\left(\mathbf{W}[i]\mathbf{A}\mathbf{W}^H[j]\right) \\ &\quad = \delta_{i,m}\mathfrak{T}\left(\mathbf{A}\mathbf{W}^H[j]\right) + \delta_{j,m}\mathbf{W}[i]\mathbf{A}. \end{aligned} \tag{37}$$

Note that the matrices $\mathbf{A}$ and $\mathbf{B}$ in (36)-(37) correspond to the autocorrelation matrix of $\mathbf{x}[n]$, i.e., $\mathbf{R_x}[m] = \mathbb{E}\{\mathbf{x}[n]\mathbf{x}^H[n-m]\}$, and they are estimated using the time-average of the signal samples $\mathbf{x}[n]$.

IV. I/Q Imbalance Compensation for OFDM Systems

In this section, we consider blind I/Q imbalance compensation in OFDM systems. For an OFDM system with N subcarriers, (5) becomes

$$\begin{aligned} X[k] &= G_1[k]S[k] + G_2[k]S^*[-k], \\ &\quad k = -N, \ldots, -1, 1, \ldots N. \end{aligned} \tag{38}$$

In (38) $S[k] = H[k]D[k]$ where $H[k]$ and $D[k]$ are the channel frequency response and the transmitted symbol at the k-th subcarrier, respectively. As before, we assume the channel satisfies the WSSUS property and the data symbols $D[k]$ are zero-mean and uncorrelated. Then the received signals satisfy $\mathbb{E}\{S[k]S^*[k]\} = \sigma_s^2$ and $\mathbb{E}\{S[k]S[-k]\} = 0$. It is seen

$$c(\{\mathbf{W}[l]\}) = \sum_{i=1}^{L}\sum_{j\neq i}^{L} \mathrm{tr}\left([\mathbf{R}_{\underline{\mathbf{y}}}]_{ij}[\mathbf{R}_{\underline{\mathbf{y}}}]_{ij}^{H}\right) + \sum_{i=1}^{L}\mathrm{tr}\left(\left([\mathbf{R}_{\underline{\mathbf{y}}}]_{ii}-\mathbf{I}\right)\left([\mathbf{R}_{\underline{\mathbf{y}}}]_{ii}-\mathbf{I}\right)^{H}\right), \tag{30}$$

$$\text{with } [\mathbf{R}_{\underline{\mathbf{y}}}]_{ij} = \left[\mathbf{W}\mathbb{E}\left\{\underline{\mathbf{x}}\underline{\mathbf{x}}^{H}\right\}\mathbf{W}^{H}\right]_{ij}, \tag{31}$$

$$\text{and } \mathbb{E}\left\{\underline{\mathbf{x}}\underline{\mathbf{x}}^{H}\right\} = \begin{bmatrix} \mathbf{R}_{\mathbf{x}}[0] & \mathbf{R}_{\mathbf{x}}[1] & \dots & \mathbf{R}_{\mathbf{x}}[M+P-1] \\ \mathbf{R}_{\mathbf{x}}[-1] & \mathbf{R}_{\mathbf{x}}[0] & \dots & \mathbf{R}_{\mathbf{x}}[M+P-2] \\ \vdots & \vdots & \ddots & \ddots \\ \mathbf{R}_{\mathbf{x}}[-(M+P-1)] & \mathbf{R}_{\mathbf{x}}[-(M+P-2)] & \dots & \mathbf{R}_{\mathbf{x}}[0] \end{bmatrix}. \tag{32}$$

$$\begin{aligned} &\mathrm{tr}\left([\mathbf{R}_{\underline{\mathbf{y}}}]_{ij}[\mathbf{R}_{\underline{\mathbf{y}}}]_{ij}^{H}\right) \\ &= \mathrm{tr}\left([\mathbf{W}\mathbf{R}_{\underline{\mathbf{x}}}\mathbf{W}^{H}\mathbf{W}\mathbf{R}_{\underline{\mathbf{x}}}^{H}\mathbf{W}^{H}]_{ij}\right) \\ &= \mathrm{tr}\left(\sum_{k_1=j}^{j+L-1}\sum_{k_2=j}^{j+L-1}\sum_{l_1=i}^{i+L-1}\sum_{l_2=i}^{i+L-1}[\mathbf{W}]_{il_1}\left[\mathbf{R}_{\underline{\mathbf{x}}}\right]_{l_1k_1}\left[\mathbf{W}^{H}\right]_{k_1j}[\mathbf{W}]_{k_2j}\left[\mathbf{R}_{\underline{\mathbf{x}}}^{H}\right]_{l_2k_2}\left[\mathbf{W}^{H}\right]_{il_2}\right). \end{aligned} \tag{33}$$

$$\begin{aligned} &\mathrm{tr}\left(\left([\mathbf{R}_{\underline{\mathbf{y}}}]_{ii}-\mathbf{I}\right)\left([\mathbf{R}_{\underline{\mathbf{y}}}]_{ii}-\mathbf{I}\right)^{H}\right) \\ &= \mathrm{tr}\left(\sum_{k_1=j}^{j+L-1}\sum_{k_2=j}^{j+L-1}\sum_{l_1=i}^{i+L-1}\sum_{l_2=i}^{i+L-1}[\mathbf{W}]_{il_1}\left[\mathbf{R}_{\underline{\mathbf{x}}}\right]_{l_1k_1}\left[\mathbf{W}^{H}\right]_{k_1i}[\mathbf{W}]_{k_2i}\left[\mathbf{R}_{\underline{\mathbf{x}}}^{H}\right]_{l_2k_2}\left[\mathbf{W}^{H}\right]_{il_2}\right) \\ &\quad -\mathrm{tr}\left(\sum_{k=j}^{j+L-1}\sum_{l=i}^{i+L-1}[\mathbf{W}]_{il}\left[\mathbf{R}_{\underline{\mathbf{x}}}\right]_{lk}\left[\mathbf{W}^{H}\right]_{ki}\right) \\ &\quad -\mathrm{tr}\left(\sum_{k=j}^{j+L-1}\sum_{l=i}^{i+L-1}[\mathbf{W}]_{ki}\left[\mathbf{R}_{\underline{\mathbf{x}}}\right]_{lk}\left[\mathbf{W}^{H}\right]_{il}\right) + \mathbf{I}. \end{aligned} \tag{34}$$

that the I/Q imbalance causes the signal at k-th subcarrier corrupted by that at the $-k$-th subcarrier. Using (5), we can write

$$G_1[k] = \frac{G_I[k]}{2}\left(1+G_d[k]e^{-j\phi}\right), \tag{39}$$

$$\text{and } G_2[k] = \frac{G_I[k]}{2}\left(1-G_d[k]e^{j\phi}\right), \tag{40}$$

where $G_d[k] \triangleq \gamma G_Q[k]/G_I[k]$. Then using (38) we have (41). In (41) we have used the fact that since the low pass filter $g_I[n]$ has real impulse response, therefore, $G_I[k] = G_I^*[-k]$. It is seen from (41) that the common factor $G_I[k]$ in $G_1[k]$ and $G_2[k]$ can be absorbed into the channel. Hence the effect of the I/Q imbalance is equivalent to a one-tap (vector) channel model and if an estimate of the tap matrix $\mathbf{G}[k]$ is available, then the I/Q imbalance can be compensated for by simply inverting $\mathbf{G}[k]$.

In order to estimate $\mathbf{G}[k]$ in (41), we need to estimate $G_d[k]$ and ϕ. Writing $G_d[k]$ in the polar form, i.e., $G_d[k] = \alpha[k]e^{j\beta[k]}$, then we have the following estimators for $\alpha[k]$, $\beta[k]$, whose derivations are given in Appendix B.

$$\hat{\alpha}[k] = \sqrt{\frac{\mathbb{E}\left\{|X[k]-X^*[-k]|^2\right\}}{\mathbb{E}\left\{|X[k]+X^*[-k]|^2\right\}}}, \tag{42}$$

$$\text{and } \hat{\beta}[k] = \arctan\left\{-\frac{\mathbb{E}\left\{|X[k]|^2-|X[-k]|^2\right\}}{2\mathcal{I}m\left\{\mathbb{E}\left\{X[k]X[-k]\right\}\right\}}\right\}. \tag{43}$$

The phase mismatch ϕ can be estimated at each subcarrier by

$$\hat{\phi}[k] = \frac{\mathbb{E}\left\{|X[k]-X^*[-k]|^2\right\}}{\mathbb{E}\left\{|X[k]+X^*[-k]|^2\right\}\mathcal{I}m\{G_d[k]\}}. \tag{44}$$

The final estimate of ϕ is then given by the average over all subcarriers, i.e., $\hat{\phi} = \frac{1}{N}\sum_{k=-N}^{N}\hat{\phi}[k]$.

In practice, the expectation operator $\mathbb{E}\{\cdot\}$ in (42)-(44) is replaced by a time-average operation over OFDM symbols. For example, suppose we collect U OFDM symbols $\{X[k,t], k = -N,\dots,N;\ t = 1,\dots,U\}$. Then $\mathbb{E}\{|X[k]|^2\} \cong \frac{1}{U}\sum_{t=1}^{U}|X[k,t]|^2$. In practice, the frequency-selectivity is typically mild and we can assume the second-order statistics in (42)-(44) are approximately equal over some consecutive subcarriers. Hence we group K subcarriers when computing the average, i.e., $\mathbb{E}\{|X[k_0]|^2\} \cong \frac{1}{KT}\sum_{k=k_0-K/2}^{k_0+K/2-1}\sum_{t=1}^{U}|X[k,t]|^2$. By such grouping in the frequency domain, we can reduce the number of OFDM symbols used for averaging and thus reduce the estimation latency.

$$\begin{bmatrix} X[k] \\ X^*[-k] \end{bmatrix} = \underbrace{\begin{bmatrix} 1+G_d[k]e^{-j\phi} & 1-G_d[k]e^{j\phi} \\ 1-G_d^*[-k]e^{-j\phi} & 1+G_d^*[-k]e^{j\phi} \end{bmatrix}}_{\mathbf{G}[k]} \begin{bmatrix} \frac{1}{2}G_I[k]H[k] & \\ & \frac{1}{2}G_I^*[-k]H^*[-k] \end{bmatrix} \begin{bmatrix} D[k] \\ D^*[-k] \end{bmatrix}. \quad (41)$$

In this way, we can estimate the parameters $\{\alpha[k], \beta[k], k = K, 2K, 3K, \ldots\}$. Then the parameters corresponding to other subcarriers can be obtained using interpolations. In particular, in our implementations, a cubic spline interpolation [14] is used to estimate $\alpha[k]$, and a second-order polynomial interpolation is used to obtain smoother estimates of the phase terms $\beta[k]$.

V. Simulation Results

In this section we present simulation results to demonstrate the performance of the proposed time-domain and frequency-domain blind I/Q imbalance compensation algorithms

A. Simulation Setup

We have simulated an LTE OFDM system [15] with the following system parameters: $N = 1024$ subcarriers, guard interval $N_{cp} = 72$, subcarrier spacing $\Delta f = 15$kHz, sampling interval $T_s = 0.651\mu$s and therefore the OFDM symbol duration is $T = NT_s = 0.67ms$. The time-varying channel is assumed to be WSSUS and is modeled as a tapped delay line model with exponential delay power profile [16]. The length of the mobile channel is $L_h = 32$ and the exponential decay parameter $\psi = 0.1$ is defined as the amplitude variance of the last path assuming the first path has unit variance. The channel is then normalized to have unit power. The Doppler spectrum is assumed to follow Jake's model [17] and a normalized maximum Doppler shift $f_D T = 0.22$ is used in the simulations. Since our focus is on the performance of the proposed I/Q imbalance compensation algorithms, the channel is assumed to be perfectly known.

In the simulation, we consider two different sets of I/Q imbalance parameters. Each of the imbalance filters $g_I[n]$ and $g_Q[n]$ is split into two filters (cf. Fig. 1(b)), one is the desired low pass filters $g_{nom}[n]$ and the others are $g_I^{'}[n]$ and $g_Q^{'}[n]$, the filters that captures the non-ideal characteristics introduced during the manufacturing process [9]. In practice, the non-ideal characteristics is not serve. Therefore, the lengths of $g_I^{'}[n]$ and $g_Q^{'}[n]$ are short. Note that since what matters is the mismatched response $g_{mis}[n] = \text{IDFT}\{\frac{G_I(e^{jw})}{G_Q(e^{jw})}\} = \text{IDFT}\{\frac{G_I^{'}(e^{jw})}{G_Q^{'}(e^{jw})}\}$ [7], in the simulations, we consider only $g_I^{'}[n]$ and $g_Q^{'}[n]$. In the following, we will just use $g_I[n]$ and $g_Q[n]$ for simplicity.

Case 1: The gain mismatch and phase mismatch in (2) are $\gamma = 1.03$ and $\phi = 3°$, respectively; and the I- and Q-branch LPFs are $g_I(z) = 0.01 + z^{-1} + 0.01z^{-2}$ and $g_Q(z) = 0.01 + z^{-1} + 0.2z^{-2}$, respectively [5]. In this case, the uncompensated $\text{IRR}(f) = 10\log_{10}\frac{|G_1(f)|^2}{|G_2(f)|^2}$ is approximate 20dB (cf. Fig. 3(a)). Based on these parameters, the 3-taps convolutive mixture matrices in (15) are $\mathbf{G}[0] = \begin{bmatrix} 0.0101 - 0.0003i & -0.0001 - 0.0003i \\ -0.0001 + 0.0003i & 0.0101 + 0.0003i \end{bmatrix}$, $\mathbf{G}[1] = \begin{bmatrix} 1.0143 - 0.0270i & -0.0143 - 0.0270i \\ -0.0143 + 0.0270i & 1.0143 - 0.0270i \end{bmatrix}$, $\mathbf{G}[2] = \begin{bmatrix} 0.1079 - 0.0054i & -0.0979 - 0.0054i \\ -0.0979 + 0.0054i & 0.1079 - 0.0054i \end{bmatrix}$.

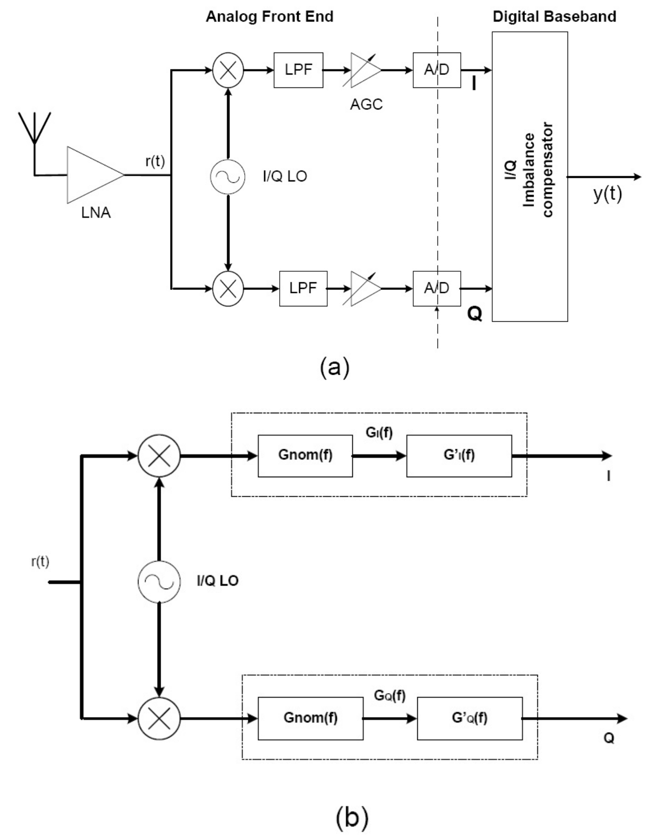

Fig. 1. (a) The direct-conversion receiver with I/Q imbalance compensation. (b) The effective model of a direct-conversion receiver with I/Q imbalance.

Case 2: The gain mismatch and phase mismatch are the same as before. But the I- and Q-branch LPFs are $g_I(z) = 0.98 + 0.03z^{-1}$ and $g_Q(z) = 1.0 - 0.005z^{-1}$, respectively [18]. The 2-taps convolutive mixture matrices in (15) are $\mathbf{G}[0] = \begin{bmatrix} 1.0043 - 0.0270i & -0.0243 - 0.0270i \\ -0.0243 + 0.0270i & 1.0043 + 0.0270i \end{bmatrix}$, $\mathbf{G}[1] = \begin{bmatrix} 0.0124 + 0.0001i & 0.0176 + 0.0001i \\ 0.0176 - 0.0001i & 0.0124 - 0.0001i \end{bmatrix}$.

In both cases, the modulation is 64-QAM and the received SNR is 25dB.

B. Performance of Time-domain Blind Compensation Algorithm

The performance of the proposed time-domain blind I/Q imbalance compensation algorithm is presented in this subsection for the OFDM system described above. However, note that this method can be applied to other non-OFDM systems as well. We set $\mu = 0.001$ in (35). For Case 1 the compensation filter length is $L = 3$ and the initial values are $\mathbf{w}[0] = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, $\mathbf{w}[1] = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ and $\mathbf{w}[2] = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$. For Case 2 the compensation filter length is $L = 2$ and the initial values are $\mathbf{w}[0] = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ and $\mathbf{w}[1] = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$. The above choices of the initial taps are justified by the fact that the LPFs on the I- and Q- branches are both close to ideal, i.e, their impulse responses are close to $\delta(n)$. In practice, the gradient descent procedure (35) is stopped when the norm of gradient is below a certain threshold or the number of iterations exceeds a certain value. Here the threshold is set to be 0.0005 and maximum number of iterations is 3000. In Fig. 2, the norm of the gradient, $\left\|\mathbf{W}^{(k+1)} - \mathbf{W}^{(k)}\right\|_F = \sum_{l=0}^{L-1} \left\|\mathbf{W}[l]^{(k+1)} - \mathbf{W}[l]^{(k)}\right\|_F$ is plotted against the number of iterations k. It is seen that the proposed time-domain algorithm takes about 1100 iterations to converge for Case 1 and 900 iterations for Case 2. This is because Case 1 has more severe I/Q imbalance. We also compare our proposed time-domain algorithm with the following adaptive algorithm proposed in [4].

$$
\begin{aligned}
y[n] &= x[n] + \tilde{\mathbf{w}}_n^T \mathbf{x}^*[n], \\
\mathbf{w}_{n+1} &= \mathbf{w}_n - \boldsymbol{\lambda} \odot \mathbf{y}[n] y[n], \\
\tilde{\mathbf{w}}_{n+1} &= \alpha \tilde{\mathbf{w}}_n + (1-\alpha)\mathbf{w}_{n+1}, \qquad (45)
\end{aligned}
$$

where $\mathbf{w}_n = [w_1[n], w_2[n], \ldots, w_L[n]]^T$ denotes the L-tap compensator at time n; $\mathbf{x}[n] = [x[n],\ x[n-1], \ldots, x[n-L+1]]^T$ contains the received signal samples (8); $\mathbf{y}[n] = [y[n], y[n-1], \ldots, y[n-L+1]]^T$; $\boldsymbol{\lambda}$ contains the step-sizes for all taps and $\odot$ denotes the element-wise product; α is a smoothing factor. The length of compensated filter L is set to the same as our proposed method. As in [4], the step-sizes are set as $\boldsymbol{\lambda} = 10^{-4}[1, 0.5, 0.5]^T$ for Case 1 and $\boldsymbol{\lambda} = 10^{-4}[1, 0.5]^T$ for Case 2; and the smoothing parameter $\alpha = 0.999$. Note that our method performs block processing and the algorithm in [4] is a sequential LMS-type approach. Although the optimal step size and smoothing parameter suggested in [4] are used in the simulation for performance evaluation, the convergence of the method [4] is still varying between 7000 to 20000 samples. In Fig. 3, the metric $\|\tilde{\mathbf{w}}_{n+1} - \tilde{\mathbf{w}}_n\|^2 = \sum_{l=0}^{L-1} |\tilde{w}_l[n+1] - \tilde{w}_l[n]|^2$ is plotted as a function of iteration number n. In order to have a fair comparison, we used the same number of samples for both methods, i.e., 20000 time-domain samples (about 20 OFDM symbols) are used for Case 1 and 10000 samples (about 10 OFDM symbols) for Case 2 in each experiment. Thus the latency of both algorithms are identical. The reason that Case 1 uses more samples is again due to its more severe I/Q imbalance. Figures 4(a) and 5(a) show the IRR performance defined in (7) for both methods, obtained by averaging individual IRR of 100 experiments. For the proposed time-domain method, the IRR can be computed as follow. From (15) and (17), $y[n] = c_1[n] \otimes s[n] + c_2[n] \otimes s^*[n]$ where $c_1[n] = w_{11}[n] \otimes g_1[n] + w_{12}[n] \otimes g_2^*[n]$ and $c_2[n] = w_{11}[n] \otimes g_2[n] + w_{12}[n] \otimes g_1^*[n]$. Therefore, $\mathrm{IRR}(f) = 10\log_{10} \frac{|C_1(f)|^2}{|C_2(f)|^2}$ where $C_i(f)$ is the frequency response of $c_i[n]$. Similarly, the IRR of the method in [4] can be computed as $\mathrm{IRR}(f) = 10\log_{10} \frac{|Q_1(f)|^2}{|Q_2(f)|^2}$, where $q_1[n] = g_1[n] + g_2^*[n] \otimes w[n]$, and $q_2[n] = g_2[n] + g_1^*[n] \otimes w[n]$. It is seen that the IRR is

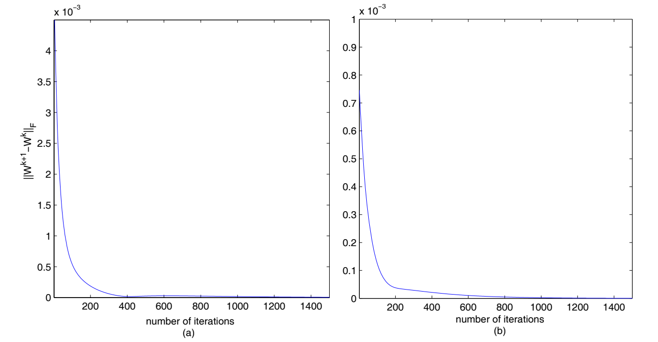

Fig. 2. Convergence of the proposed time-domain blind I/Q imbalance compensation method. (a) Case 1. (b) Case 2.

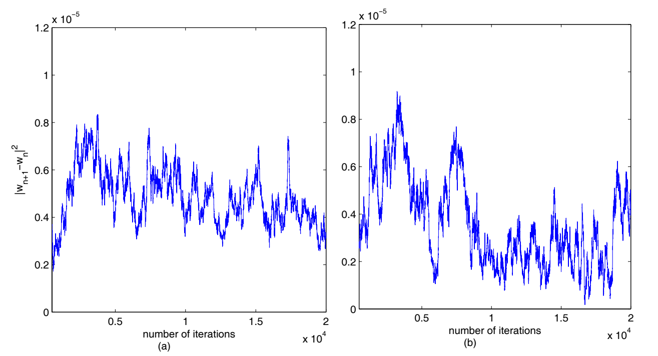

Fig. 3. Convergence of the LMS method in [4]. (a) Case 1. (b) Case 2.

improved by around 20-25dB by the proposed I/Q imbalance compensation method. Compared with the method in [4], our proposed algorithm offers approximately 10dB gain for Case 1 and a few dB gain for Case 2. Therefore, the proposed time-domain method is shown to have better and more stable performance at the cost of higher complexity. The computation complexity in (35) involves $(2L)^7$ of 2×2 matrix multiplications and L^4 of 2×2 matrix additions for each coefficient update, where L is the compensation filter length. Figures 4(b) and 5(b) show the symbol error rate (SER) performance using 64-QAM modulation for both methods. Again it is seen that the proposed method can more effectively mitigate the impairment caused by the I/Q imbalance.

C. Performance of Frequency-domain Blind Compensation Algorithm

We now consider the performance of the frequency-domain blind I/Q imbalance compensation method for OFDM systems developed in Section IV. Recall that when computing the second-order moments that are needed for parameter estimation, we make use of a group of K consecutive subcarriers and U OFDM symbols. In the simulations, we fixed the total number samples for average, i.e., $KU = 30000$. Since in the frequency-domain approach, each subcarrier has its own one-tap compensation filter, and there is only one sample at each subcarrier in each OFDM symbol, the estimation latency is higher compared with the time-domain approach in terms of number of samples. However, the advantages of the frequency-domain method include 1) when only certain frequency bands are of interest, we can just compute the

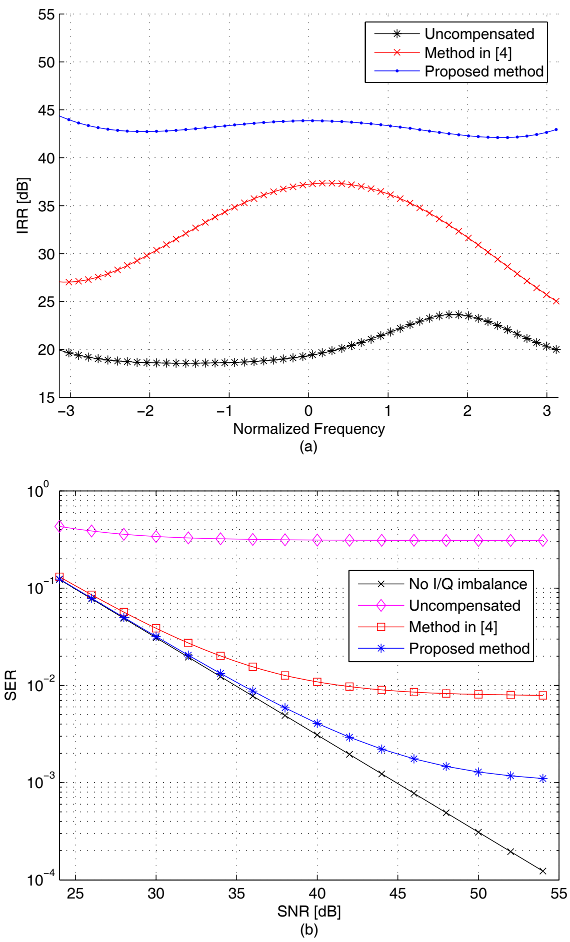

Fig. 4. The IRR and SER performance of the proposed time-domain blind I/Q imbalance compensation method for Case 1. (a) IRR performance. (b) SER performance.

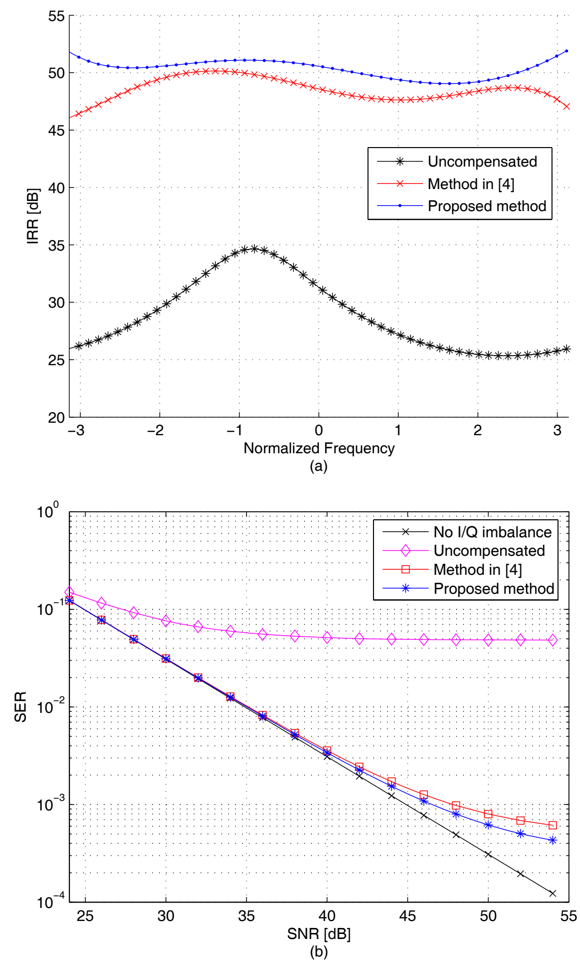

Fig. 5. The IRR and SER performance of the proposed time-domain blind I/Q imbalance compensation method for Case 2. (a) IRR performance. (b) SER performance.

corresponding compensation filters; and 2) the compensation filters are in closed-form and involve no iterations. Note that $K = 1$ corresponds to no subcarrier grouping for which case interpolation is still applied to smooth the estimates. In Figs. 6 and 7, we show the SER performance comparisons among different choices of the subcarrier grouping parameter K. It is seen that for $K = 2, 4, 8$ the performance is the same. For $K = 16$, a performance loss is seen around SER=10^{-3}. This is because the I/Q imbalance parameters can no longer be considered constant when the group size K is too large. Therefore there is a trade-off between the system performance and the processing latency. For both cases, $K = 8$ is a good choice for reducing the latency while still maintaining the good performance. The proposed frequency-domain method is based on parameters extraction, there is no iteration is required. Therefore, the complexity is dominated by estimation of auto-correlation. Assume U symbols are used in time average, in our simulation $U = 30000$ when $K = 1$. For (42), to estimate $\alpha[k]$ for a given k, in computing the denominator $\mathbb{E}\left\{|X[k] + X^*[-k]|^2\right\}$, it takes U complex additions and U complex multiplications. Similarly, in computing the numerator, $\mathbb{E}\left\{|X[k] - X^*[-k]|^2\right\}$, it takes U complex additions and U complex multiplications. Hence, to estimate $\alpha[k]$, for all k, $4NU$ complex additions and $4NU$ multiplications are needed. In computing $\beta[k]$ for all k, NU complex additions and $3NU$ multiplications are needed. At last, for $\phi[k]$ in (44), the autocorrelation computed in (42), (43) can be reused.

VI. Conclusions

We have proposed two blind approaches to compensating the frequency-dependent I/Q imbalance for wideband direct-conversion receivers. One is a time-domain method for general systems and the other is a frequency-domain method that is specifically for OFDM systems. For the time-domain method, a blind identifiability condition is given based on which a cost function for compensating the I/Q imbalance is proposed; and a gradient-descent algorithm is derived to obtain the compensating filter. For the frequency-domain method, we have developed estimators for the frequency-dependent I/Q imbalance parameters based on the second-order statistics of the received signal; the compensation filter can then be obtained in closed-form given these estimated parameters. Simulation results show that the proposed approaches can effectively mitigate the I/Q imbalance and maintain the high performance of the receiver.

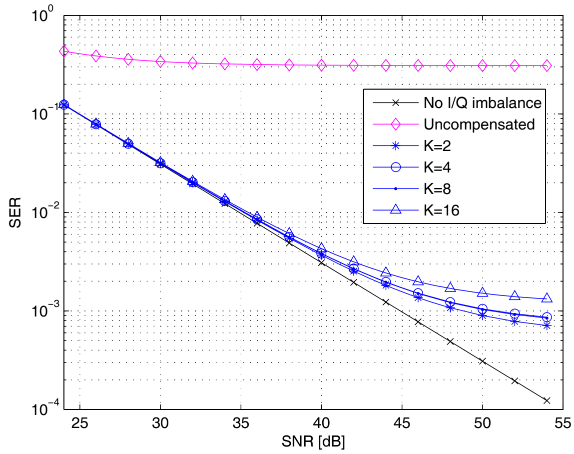

Fig. 6. The SER performance of the proposed frequency-domain blind I/Q imbalance compensation method for OFDM systems for Case 1.

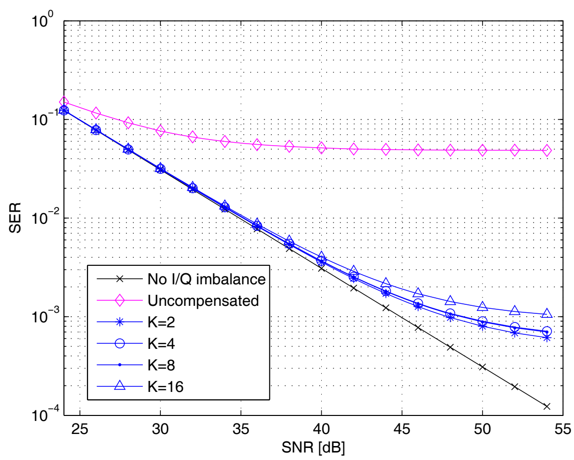

Fig. 7. The SER performance of the proposed frequency-domain blind I/Q imbalance compensation method for OFDM systems for Case 2.

Appendix A: Derivations of (36)-(37)

The derivatives with respect to $w = x + jy$ and $w^* = x - jy$ of $f(w)$ are called the formal partial derivatives of f at $w \in \mathbb{C}$ [19], and they are defined as

$$\frac{\partial}{\partial w} f(w) = \frac{1}{2}\Big[\frac{\partial}{\partial x} f(w) - j\frac{\partial}{\partial y} f(w)\Big],$$
$$\frac{\partial}{\partial w^*} f(w) = \frac{1}{2}\Big[\frac{\partial}{\partial x} f(w) + j\frac{\partial}{\partial y} f(w)\Big].$$

Alternatively, when computing $\frac{\partial}{\partial w} f(w)$ and $\frac{\partial}{\partial w^*} f(w)$, w and w^* can be treated as independent variables, i.e., $\frac{\partial w}{\partial w^*} = \frac{\partial w^*}{\partial w} = 0$.

Recall that the filter coefficient matrices are of the form $\mathbf{W} = \begin{bmatrix} w_1 & w_2 \\ w_2^* & w_1^* \end{bmatrix}$. The derivative is thus defined as [13], [19] $\frac{\partial}{\partial \mathbf{W}^*} f(\mathbf{W}) = \begin{bmatrix} \frac{\partial f}{\partial w_1^*} & \frac{\partial f}{\partial w_2^*} \\ \frac{\partial f}{\partial w_2} & \frac{\partial f}{\partial w_1} \end{bmatrix}$. The terms that are needed in computing the gradients in (36) - (37) are given as (46).

Appendix B: Derivations of (42)-(44)

Note that the LPFs for both I- and Q-branches are real filters. We therefore have $G_I(f) = G_I^*(-f)$, $G_Q(f) = G_Q^*(-f)$ [20] and

$$G_d^*(-f) = \frac{\gamma G_Q^*(-f)}{G_I^*(-f)} = G_d(f). \tag{47}$$

Using (5) and (47), we have

$$\begin{aligned} G_1[k] + G_2^*[-k] &= G_1^*[-k] + G_2[k] = G_I[k], \\ G_1[k] - G_2^*[-k] &= G_I[k]G_d[k]e^{-j\phi}, \\ G_2[k] - G_1^*[-k] &= -G_I[k]G_d[k]e^{j\phi}. \end{aligned} \tag{48}$$

Then we can write (49).

Similarly we evaluate the following second-order statistics (50) - (53) and (54). Recall that $G_d[k] = \alpha[k]e^{j\beta[k]}$. Using (53) and (54), the phase estimate of $\beta[k]$ is expressed in (43). Using (49), (50), the amplitude estimate $\alpha[k]$ is given by (42). From (43), (42) and (53), we can estimate the mismatch phase ϕ using (44).

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$$\frac{\partial}{\partial \mathbf{W}^*}\mathrm{tr}(\mathbf{WA}) = \frac{\partial}{\partial \mathbf{W}^*}\mathrm{tr}(\mathbf{AW}) = \frac{\partial}{\partial \mathbf{W}^*}\mathrm{tr}\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix} w_1 & w_2 \\ w_2^* & w_1^*\end{bmatrix}\right)$$
$$= \begin{bmatrix} \frac{\partial(a_{11}w_1+a_{12}w_2^*+a_{21}w_2+a_{22}w_1^*)}{\partial w_1^*} & \frac{\partial(a_{11}w_1+a_{12}w_2^*+a_{21}w_2+a_{22}w_1^*)}{\partial w_2^*} \\ \frac{\partial(a_{11}w_1+a_{12}w_2^*+a_{21}w_2+a_{22}w_1^*)}{\partial w_2} & \frac{\partial(a_{11}w_1+a_{12}w_2^*+a_{21}w_2+a_{22}w_1^*)}{\partial w_1}\end{bmatrix} = \begin{bmatrix} a_{22} & a_{12} \\ a_{21} & a_{11}\end{bmatrix} \triangleq \mathfrak{T}(\mathbf{A}).$$

$$\frac{\partial}{\partial \mathbf{W}^*}\mathrm{tr}(\mathbf{AW}^H) = \frac{\partial}{\partial \mathbf{W}^*}\mathrm{tr}\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix} w_1^* & w_2^* \\ w_2 & w_1\end{bmatrix}\right)$$
$$= \begin{bmatrix} \frac{\partial(a_{11}w_1^*+a_{12}w_2+a_{21}w_2^*+a_{22}w_1)}{\partial w_1^*} & \frac{\partial(a_{11}w_1^*+a_{12}w_2+a_{21}w_2^*+a_{22}w_1)}{\partial w_2^*} \\ \frac{\partial(a_{11}w_1^*+a_{12}w_2+a_{21}w_2^*+a_{22}w_1)}{\partial w_2} & \frac{\partial(a_{11}w_1^*+a_{12}w_2+a_{21}w_2^*+a_{22}w_1)}{\partial w_1}\end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22}\end{bmatrix} = \mathbf{A}. \quad (46)$$

$$\begin{aligned}
&\mathbb{E}\left\{|X[k] + X^*[-k]|^2\right\} \\
&= \mathbb{E}\left\{|(G_1[k] + G_2^*[-k])S[k] + (G_1^*[-k] + G_2[k])S^*[-k]|^2\right\} \\
&= \mathbb{E}\left\{|G_I[k](\,S[k] + S^*[-k]\,)|^2\right\} \\
&= |G_I[k]|^2\left(\mathbb{E}\left\{|S[k]|^2\right\} + \mathbb{E}\left\{|S[-k]|^2\right\} + \mathbb{E}\left\{S[k]S[-k]\right\} + \mathbb{E}\left\{S^*[k]S^*[-k]\right\}\right) \\
&= 2\,|G_I[k]|^2\,\sigma_s^2.
\end{aligned} \quad (49)$$

$$\begin{aligned}
\mathbb{E}\left\{|X[k] - X^*[-k]|^2\right\} & \\
&= \mathbb{E}\left\{|(G_1[k] - G_2^*[-k])S[k] + (G_1^*[-k] - G_2[k])S^*[-k]|^2\right\} \\
&= 2\,|G_I[k]|^2\,|G_d[k]|^2\,\sigma_s^2, \quad (50)\\
\mathbb{E}\left\{|X[k]|^2\right\} &= |G_1[k]|^2\,\mathbb{E}\left\{|S[k]|^2\right\} + |G_2[k]|^2\,\mathbb{E}\left\{|S[-k]|^2\right\} + \\
&\quad G_1[k]G_2^*[k]\mathbb{E}\left\{S[k]S[-k]\right\} + G_1^*[k]G_2[k]\mathbb{E}\left\{S^*[k]S^*[-k]\right\} \\
&= (|G_1[k]|^2 + |G_2[k]|^2)\sigma_s^2, \quad (51)\\
\mathbb{E}\left\{|X[-k]|^2\right\} &= |G_1[-k]|^2\,\mathbb{E}\left\{|S[-k]|^2\right\} + |G_2[-k]|^2\,\mathbb{E}\left\{|S[k]|^2\right\} + \\
&\quad G_1[-k]G_2^*[-k]\mathbb{E}\left\{S[k]S[-k]\right\} + G_1^*[-k]G_2[-k]\mathbb{E}\left\{S^*[k]S^*[-k]\right\} \\
&= (|G_1[-k]|^2 + |G_2[-k]|^2)\sigma_s^2, \quad (52)
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}\left\{|X[k]|^2 - |X[-k]|^2\right\} \\
&= \mathbb{E}\left\{|X[k]|^2\right\} - \mathbb{E}\left\{|X[-k]|^2\right\} \\
&= \frac{1}{4}\,|G_I[k]|^2\left(\left|1 + G_d[k]e^{-j\phi}\right|^2 - \left|1 + G_d^*[k]e^{-j\phi}\right|^2 - \left|1 - G_d[k]e^{j\phi}\right|^2 + \left|1 + G_d^*[k]e^{j\phi}\right|^2\right)\sigma_s^2 \\
&= 2\,|G_I[k]|^2\,\mathcal{I}m\{G_d[k]\}\sin(\phi).
\end{aligned} \quad (53)$$

$$\begin{aligned}
\mathbb{E}\left\{X[k]X[-k]\right\} &= G_1[k]G_2[-k]\mathbb{E}\left\{|S[k]|^2\right\} + G_1[-k]G_2[k]\mathbb{E}\left\{|S[-k]|^2\right\} + \\
&\quad G_1[k]G_1[-k]\mathbb{E}\left\{S[k]S[-k]\right\} + G_2[k]G_2[-k]\mathbb{E}\left\{S^*[k]S^*[-k]\right\} \\
&= \left(G_1[k]G_2[-k] + G_1[-k]G_2[k]\right)\sigma_s^2 \\
&= |G_I[k]|^2\left[\frac{1}{2}\left(1 - |G_d[k]|^2\right) - j\mathcal{R}e\{G_d[k]\}\sin(\phi)\right]\sigma_s^2.
\end{aligned} \quad (54)$$

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Yingming Tsai (M'05) received the M.S. degree in computer science from Syracuse University and M. Eng. degree in electrical engineering from Cornell University in 1994 and 1996, respectively. He is currently pursuing Ph.D. degree in electrical engineering at the Columbia University, New York.

From 1996 to 2000, he worked as member of technical staff at Lucent Technologies. Later on, he joined Wiscom Technologies and worked as principle DSP engineer during 2000 to 2002. Since 2002, he has been working at InterDigital, Inc. in Melville, New York, where he has worked on 3GPP WCDMA and Long Term Evolution (LTE) systems. His research interests are in signal processing and wireless communications. He holds several US patents.

Chia-Pang Yen (S'02 M'07) was born in Taiwan, 1972. He received the B.S. and M.S. degree in naval architecture and ocean engineering from National Taiwan University, Taiwan, and the M.S. and Ph.D. degree in electrical engineering from Polytechnic University (now Polytechnic Institute of New York University), Brooklyn, New York in 2003 and 2007 respectively. Since 2007, he has been with InterDigital, Inc. in Melville, New York. Dr. Yen's research interests are in signal processing and wireless communications.

Xiaodong Wang (S'98-M'98-SM'04-F'08) received the Ph.D. degree in Electrical Engineering from Princeton University. He is a Professor in the Department of Electrical Engineering at Columbia University. Dr. Wang's research interests fall in the general areas of computing, signal processing and communications, and has published extensively in these areas. Among his publications is a book entitled "Wireless Communication Systems: Advanced Techniques for Signal Reception," published by Prentice Hall in 2003. His current research interests include wireless communications, statistical signal processing, and genomic signal processing. Dr. Wang received the 1999 NSF CAREER Award, and the 2001 IEEE Communications Society and Information Theory Society Joint Paper Award. He has served as an Associate Editor for the IEEE Transactions on Communications, the IEEE Transactions on Wireless Communications, the IEEE Transactions on Signal Processing, and the IEEE Transactions on Information Theory. Dr. Wang is a Fellow of the IEEE.