Blind Separation of Complex I/Q Independent Sources With Phase Recovery

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***Abstract*—Blind source separation (BSS) techniques allow recovery of individual source signals from observed mixtures, exploiting only the assumption of mutual independence of sources. Generally, complex signals are recovered with an arbitrary phase rotation. In this letter, we propose two BSS algorithms to separate complex sources that have independent in-phase and quadrature (I/Q) parts. The proposed algorithms allow source phase recovery as well as separation. Simulation results demonstrate that the algorithms are effective in recovering source phases without affecting source separation.**

***Index Terms*—Adaptive signal processing, blind source separation (BSS), phase recovery.**

I. Introduction

BLIND SOURCE SEPARATION (BSS) methods allow recovery of individual unobserved signals or sources from their observed unknown mixtures. BSS techniques generally exploit only the statistical independence between source signals. Therefore, sources may be recovered in arbitrary order and can have arbitrary scale factors, which will result in sign ambiguity for real sources and phase ambiguity for complex sources [1]–[3]. In this letter, we propose two constrained versions of a well-known BSS algorithm, for in-phase and quadrature (I/Q) independent sources, to separate sources and recover source phases.

The simplest BSS model assumes a vector $\boldsymbol{s}_k$ of n unknown independent source signals $s_1(k), s_2(k), \ldots, s_n(k)$ and a vector $\boldsymbol{x}_k$ of n observed linear mixtures $x_1(k), x_2(k), \ldots, x_n(k)$, expressed in matrix form as

$$\boldsymbol{x}_k = \boldsymbol{A}\boldsymbol{s}_k, \quad k = 1, 2, \cdots \tag{1}$$

where $\boldsymbol{A}$ is an $n \times n$ unknown mixing matrix. The problem is to recover the source vector $\boldsymbol{s}_k$ from the observed data $\boldsymbol{x}_k$, based on the assumed independence between the sources and possibly on some prior information about the probability distribution of the sources [1]. The BSS problem can be formulated as the computation of an $n \times n$ separation matrix $\boldsymbol{B}$ such that the components of the output

$$\boldsymbol{y}_k = \boldsymbol{B}\boldsymbol{x}_k = \boldsymbol{B}\boldsymbol{A}\boldsymbol{s}_k \tag{2}$$

are possibly reordered, amplitude scaled, and phase shifted versions of the components of $\boldsymbol{s}_k$. Perfect separation and estimation requires the condition $\boldsymbol{B}\boldsymbol{A} = \boldsymbol{I}$.

The simple and useful Equivariant Adaptive Separation via Independence (EASI) algorithm was proposed by Cardoso and Laheld [3]. It updates the estimate of the $\boldsymbol{B}$ matrix for each new observation vector, the estimate $\boldsymbol{B}_{k+1}$ after the kth observation having the form

$$\boldsymbol{B}_{k+1} = \boldsymbol{B}_k - \lambda_k \left[\boldsymbol{y}_k\boldsymbol{y}_k^H - \boldsymbol{I} + \boldsymbol{g}(\boldsymbol{y}_k)\boldsymbol{y}_k^H - \boldsymbol{y}_k\boldsymbol{g}(\boldsymbol{y}_k)^H\right]\boldsymbol{B}_k. \tag{3}$$

Here, $\boldsymbol{g}(\boldsymbol{y}_k) = [g(y_1(k))\ g(y_2(k))\ \cdots\ g(y_n(k))]^T$ is a component-wise nonlinear function. The scalar function g is generally a complex-valued function of a complex argument.

II. Constrained I/Q BSS

In conventional BSS using an algorithm such as the EASI algorithm, the recovered complex source signals may have undetermined component order and phase. While component order may be recoverable using some side information in practice (e.g., sources with different signalling constellations or some frame-level information), phase rotation is undesirable in communications signalling. In such applications, we can usually assume that the complex signals have independent I/Q components [4]–[6]. Note that this assumption holds for equally likely points in square quadrature amplitude modulation (QAM) constellations but not for phase shift keying (PSK) constellations. If channel coding is involved, strict I/Q independence may not hold, although approximate I/Q independence may be a reasonable assumption. To recover the phase rotation of each source at the time of source separation, a *constrained* BSS technique can be defined to separate sources as well as recover source phases for I/Q-independent sources. The basic idea of this constrained I/Q BSS algorithm is as follows: Since the I/Q components of each source are statistically independent, they are also independent of I/Q components of other sources, and the original n complex source mixtures can be considered as $2n$ real source mixtures. These $2n$ real source mixtures are composed of mutually independent I/Q parts of the n independent complex sources. Source separation techniques can then be applied to separate the $2n$ independent real signals. Because of the order indeterminacy of BSS methods, special constraints are required so that the separated signals retain correct I/Q association.

Without loss of generality, consider the case of two independent complex sources with independent I/Q components. The two complex source mixtures can be considered as four real

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mixtures of the I/Q parts of the two complex sources. The BSS model can be expressed as

$$\begin{bmatrix} x_{1r}(k) \\ x_{1i}(k) \\ x_{2r}(k) \\ x_{2i}(k) \end{bmatrix} = \begin{bmatrix} a_{11r} & -a_{11i} & a_{12r} & -a_{12i} \\ a_{11i} & a_{11r} & a_{12i} & a_{12r} \\ a_{21r} & -a_{21i} & a_{22r} & -a_{22i} \\ a_{21i} & a_{21r} & a_{22i} & a_{22r} \end{bmatrix} \begin{bmatrix} s_{1r}(k) \\ s_{1i}(k) \\ s_{2r}(k) \\ s_{2i}(k) \end{bmatrix} \tag{4}$$

where subscripts r and i represent the real and imaginary parts of the variable, respectively. The output using a separating matrix must then ideally be of the inverting form

$$\begin{bmatrix} y_{1r}(k) \\ y_{1i}(k) \\ y_{2r}(k) \\ y_{2i}(k) \end{bmatrix} = \begin{bmatrix} b_{11r}(k) & -b_{11i}(k) & b_{12r}(k) & -b_{12i}(k) \\ b_{11i}(k) & b_{11r}(k) & b_{12i}(k) & b_{12r}(k) \\ b_{21r}(k) & -b_{21i}(k) & b_{22r}(k) & -b_{22i}(k) \\ b_{21i}(k) & b_{21r}(k) & b_{22i}(k) & b_{22r}(k) \end{bmatrix} \times \begin{bmatrix} x_{1r}(k) \\ x_{1i}(k) \\ x_{2r}(k) \\ x_{2i}(k) \end{bmatrix}. \tag{5}$$

By imposing the constraint given in (5) on the structure of the real separating matrix $\boldsymbol{B}_k$ at each iteration of an adaptive BSS algorithm seeking the four real independent source signals, the correct I/Q association can be obtained.

At each iteration of an *unconstrained* adaptive BSS algorithm, an updated real separating matrix is first obtained, denoted as $\boldsymbol{B}_{u,k+1}$ with the entries $b_{uij}(k+1)$ $(i, j = 1, 2, 3, 4)$. To enforce the constraint of (5), we impose the modification in (6), shown at the bottom of the page. The resulting constrained separation matrix is used in the next iteration.

It can be shown that by using the constrained I/Q BSS, we can maintain the correct I/Q association for each I/Q-independent source, with only $\pi/2$-phase ambiguity possible during the whole process of simultaneous source separation and phase recovery. The reason for the remaining $\pi/2$-phase ambiguity is that a sign indeterminacy remains for the separated source components. Details are given in [7]. The $\pi/2$ ambiguity can be handled by differential encoding in communications.

III. Constrained Nonlinear Function for EASI Algorithm

As discussed above, constrained I/Q BSS in the EASI algorithm can separate sources and recover source phases. Alternatively, we can apply the I/Q independence property implicitly in the choice of the component scalar complex-valued nonlinear function g in the EASI algorithm to achieve source phase recovery. We call this the *constrained nonlinear function method*. In this method, the scalar complex-valued nonlinear function g for the EASI algorithm is constrained to have the following decomposition:

$$g\left(y_l(k)\right) = \tilde{g}\left(y_{lr}(k)\right) + j\tilde{g}\left(y_{li}(k)\right), \quad 1 \leq l \leq n \tag{7}$$

where $\tilde{g}$ is a real-valued function of a real argument. Note that in the original EASI algorithm [3], the function $g(y_l(k))$ for complex sources was defined to be of the phase-preserving form

$$g\left(y_l(k)\right) = y_l f\left(|y_l|\right) \tag{8}$$

where f is a real-valued function. We will show that the constraints on the real separating matrix given in (5) are essentially equivalent to the use of our constrained nonlinear function, given in (7), for the EASI algorithm.

Without loss of generality, we consider a single complex signal with independent I/Q components observed with some unknown phase rotation. The received signal can be considered as two real linear mixtures of two independent real sources, i.e., $\boldsymbol{s}_k = [s_r(k) \;\; s_i(k)]^T$. We seek the proper separating matrix to obtain independent output components for $\boldsymbol{y}_k = [y_r(k) \; y_i(k)]^T$. Using the constrained I/Q BSS EASI algorithm of the previous section with real-valued scalar function $g = \tilde{g}$ operating on real observations (I and Q components), updates of the real 2×2 separating matrix are formed as

$$\begin{aligned}
b_{11}&(k+1) \\
&= b_{22}(k+1) = \frac{\left(b_{u11}(k+1) + b_{u22}(k+1)\right)}{2} \\
&= b_{11}(k) - \lambda_k\left[\left(\frac{y_r^2(k)}{2} + \frac{y_i^2(k)}{2} - 1\right) b_{11}(k)\right. \\
&\quad \left. + \left(\tilde{g}\left(y_r(k)\right) y_i(k) - y_r(k)\tilde{g}\left(y_i(k)\right)\right) b_{21}(k)\right] \\
b_{21}&(k+1) \\
&= -b_{12}(k+1) = \frac{\left(b_{u21}(k+1) - b_{u12}(k+1)\right)}{2} \\
&= b_{21}(k) - \lambda_k\left[-\left(\tilde{g}\left(y_r(k)\right) y_i(k) - y_r(k)\tilde{g}\left(y_i(k)\right)\right) b_{11}(k)\right. \\
&\quad \left. + \left(\frac{y_r^2(k)}{2} + \frac{y_i^2(k)}{2} - 1\right) b_{21}(k)\right].
\end{aligned} \tag{9}$$

Now, suppose that we use the constrained nonlinear function method to seek the complex-valued 1×1 separating matrix to obtain complex output $y(k) = y_r(k) + jy_i(k)$. With the constrained nonlinear function $g(y(k)) = \tilde{g}(y_r(k)) + j\tilde{g}(y_i(k))$, we obtain the updated separating matrix $\boldsymbol{B}_{k+1}$ as

$$\begin{aligned}
\boldsymbol{B}_{k+1} &= b_r(k+1) + jb_i(k+1) \\
b_r(k+1) &= b_r(k) - \lambda_k \\
&\quad \times \left[\left(y_r^2(k) + y_i^2(k) - 1\right) b_r(k)\right. \\
&\quad \left. +2\left(\tilde{g}\left(y_r(k)\right) y_i(k) - y_r(k)\tilde{g}\left(y_i(k)\right)\right) b_i(k)\right] \\
b_i(k+1) &= b_i(k) - \lambda_k \\
&\quad \times \left[-2\left(\tilde{g}\left(y_r(k)\right) y_i(k) - y_r(k)\tilde{g}\left(y_i(k)\right)\right) b_r(k)\right. \\
&\quad \left. + \left(y_r^2(k) + y_i^2(k) - 1\right) b_i(k)\right].
\end{aligned} \tag{10}$$

Comparing (9) and (10), we note that they are very similar, differing only by scale factors. The constraint (7) on the nonlinear function of the EASI algorithm for complex observations allows

$$\boldsymbol{B}_{k+1} = \begin{bmatrix} \frac{b_{u11}(k+1)+b_{u22}(k+1)}{2} & -\frac{b_{u21,k+1}-b_{u12,k+1}}{2} & \frac{b_{u13}(k+1)+b_{u24}(k+1)}{2} & -\frac{b_{u23}(k+1)-b_{u14}(k+1)}{2} \\ \frac{b_{u21}(k+1)-b_{u12}(k+1)}{2} & \frac{b_{u11,k+1}+b_{u22,k+1}}{2} & \frac{b_{u23}(k+1)-b_{u14}(k+1)}{2} & \frac{b_{u13}(k+1)+b_{u24}(k+1)}{2} \\ \frac{b_{u31}(k+1)+b_{u42}(k+1)}{2} & -\frac{b_{u41,k+1}-b_{u32,k+1}}{2} & \frac{b_{u33}(k+1)+b_{u44}(k+1)}{2} & -\frac{b_{u43}(k+1)-b_{u34}(k+1)}{2} \\ \frac{b_{u41}(k+1)-b_{u32}(k+1)}{2} & \frac{b_{u31,k+1}+b_{u42,k+1}}{2} & \frac{b_{u43}(k+1)-b_{u34}(k+1)}{2} & \frac{b_{u33}(k+1)+b_{u44}(k+1)}{2} \end{bmatrix}. \tag{6}$$

the algorithm to recover the sources without phase rotation, as in the case of the constrained I/Q BSS scheme of Section II. The constrained nonlinear function approach does not require doubling of the dimension (from complex to real) and does not impose a separate constraint on the $\boldsymbol{B}_k$ updates.

Although [3] provides an analysis of the source separation performance of the EASI algorithm using a phase-preserving g function, it does not consider the constrained g function structure of (7). However, our simulation results suggest that the source separation performance (convergence rate and steady-state error) of the constrained nonlinear function method is very similar to that of the original EASI method for complex sources.

IV. Simulation Results

In the simulations, we considered the transmission of mixtures of one 4-QAM, one 16-QAM, one 8-PSK, and one 16-PSK independent symbol sequences each of unit variance, mixed by an arbitrary 4×4 complex mixing matrix. The signal-to-noise ratio (SNR) before source mixing was 25 dB for each source. Three BSS schemes were applied: the conventional EASI algorithm, constrained I/Q method, and constrained nonlinear function method. In all three cases, the normalized version of the EASI algorithm was used, with $\boldsymbol{B}_k$ updates given as

$$\boldsymbol{B}_{k+1}=\boldsymbol{B}_k-\lambda_k\left[\frac{\boldsymbol{y}_k\boldsymbol{y}_k^H-\boldsymbol{I}}{1+\lambda_k\boldsymbol{y}_k^H\boldsymbol{y}_k}+\frac{\boldsymbol{g}(\boldsymbol{y}_k)\boldsymbol{y}_k^H-\boldsymbol{y}_k\boldsymbol{g}(\boldsymbol{y}_k)^H}{1+\lambda_k\left|\boldsymbol{y}_k^H\boldsymbol{g}(\boldsymbol{y}_k)\right|}\right]\boldsymbol{B}_k. \tag{11}$$

The normalized version improves the stability of the algorithm by preventing updates from being overly influenced by samples that are spurious for the statistical model that the choice of $\boldsymbol{g}$ is based on. The step size for the three schemes was set to $\lambda_k = 0.005$. For the conventional EASI algorithm, the complex nonlinear function $g(x) = x|x|^2$ was used. For the constrained I/Q BSS, the EASI algorithm was used on real and imaginary components (eight-dimensional) with $g(x) = x^3$. For the constrained nonlinear complex function scheme, the nonlinear function $g(x) = (\mathrm{Re}(x))^3 + j(\mathrm{Im}(x))^3$ was used in the original four-dimensional complex setting.

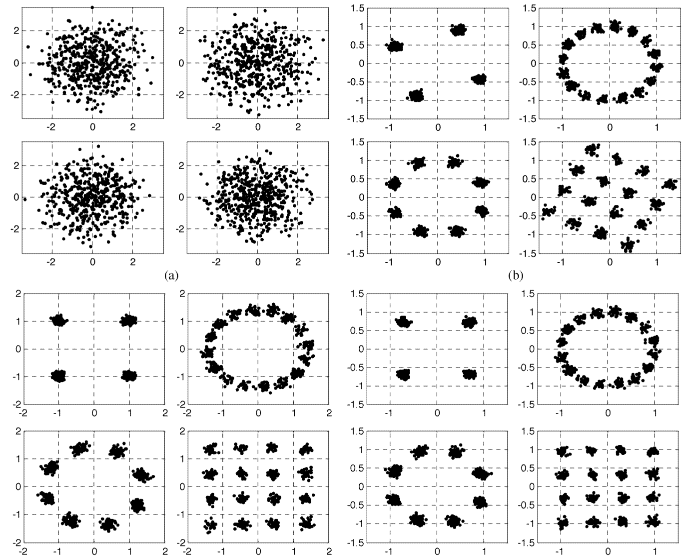
Fig. 1 illustrates the constellation of the observed mixtures and the separated signals with the three schemes after 1000 iterations. The conventional EASI algorithm separates the four source signals [see Fig. 1(b)], but each source has arbitrary phase rotation. As seen in Figs. 1(c) and (d), both the constrained I/Q and constrained nonlinear function methods not only separate the four source signals but also recover the 4-QAM and 16-QAM signal phases.

In order to compare the performance of the three BSS schemes, we used the so-called *intersignal interference* (ISI) as a performance measurement. Define a global system matrix $\boldsymbol{C}_k$ as

$$\boldsymbol{C}_k = \boldsymbol{B}_k\boldsymbol{A}. \tag{12}$$

The ISI for the ith separated source signal at time k is defined as

$$\mathrm{ISI}(i) = \frac{\sum_l |c_{il}(k)|^2 - |c_i(k)|^2_{\max}}{|c_i(k)|^2_{\max}} \tag{13}$$



(a) (b) (c) (d)

Fig. 1. Separating the mixtures of one 4-QAM, one 16-QAM, one 8-PSK, and one 16-PSK signals without frequency offset. (a) Four complex observed mixtures. (b) Separated constellation using the normalized EASI algorithm. (c) Separated constellation using constrained I/Q BSS method. (d) Separated constellation using constrained nonlinear function approach.

where $c_{il}(k)$ is the (i,l)th entry of matrix $\boldsymbol{C}_k$, and $|c_i(k)|_{\max}$ is the maximum absolute value of the ith row of matrix $\boldsymbol{C}_k$. The ISI gives a measure of the deviation of $\boldsymbol{C}_k$ from the identity matrix $\boldsymbol{I}$ or from a permutation of $\boldsymbol{I}$ after scaling. In order to get a comparable 4×4 system matrix for the constrained I/Q method as those for the other two schemes, we recovered the 4×4 complex separating matrix from the 8×8 real-valued separating matrix based on the matrix structure of (5). Fig. 2 depicts the ISI plots of the conventional EASI, constrained I/Q, and constrained nonlinear function methods from the simulations. The three schemes show very similar ISI performance, even though the two constrained BSS schemes also recover I/Q independent source phases.

We also considered the effect of source frequency offset on the performance of the three BSS schemes. Fig. 3 shows the input mixtures and output constellations with the three BSS schemes after 1000 iterations, for small frequency offsets Δf_1, Δf_2, Δf_3, and Δf_4 with $\Delta f_1 T = 2 \times 10^{-4}$ for the 4-QAM source, $\Delta f_2 T = -2 \times 10^{-4}$ for the 16-QAM source, $\Delta f_3 T = -10^{-4}$ for the 8-PSK source, and $\Delta f_4 T = 10^{-4}$ for the 16-PSK source, where T is the symbol period. The conventional EASI algorithm separates the sources but does not track the frequency offsets. The two constrained BSS schemes do compensate the frequency offset of the I/Q independent sources and obtain the QAM constellations, although frequency offsets remain for the I/Q nonindependent PSK sources. As seen in Figs. 3(c) and (d), there are residual phase shifts for the QAM sources. As shown in [4], the residual phase shifts are due to residual frequency tracking errors. The ability of the schemes to remove the phase shifts depends on the value of the step size relative to the frequency offset. For large frequency offsets, we have found that although the constrained BSS schemes cannot compensate for the offsets, source separation itself is not affected. In addition,

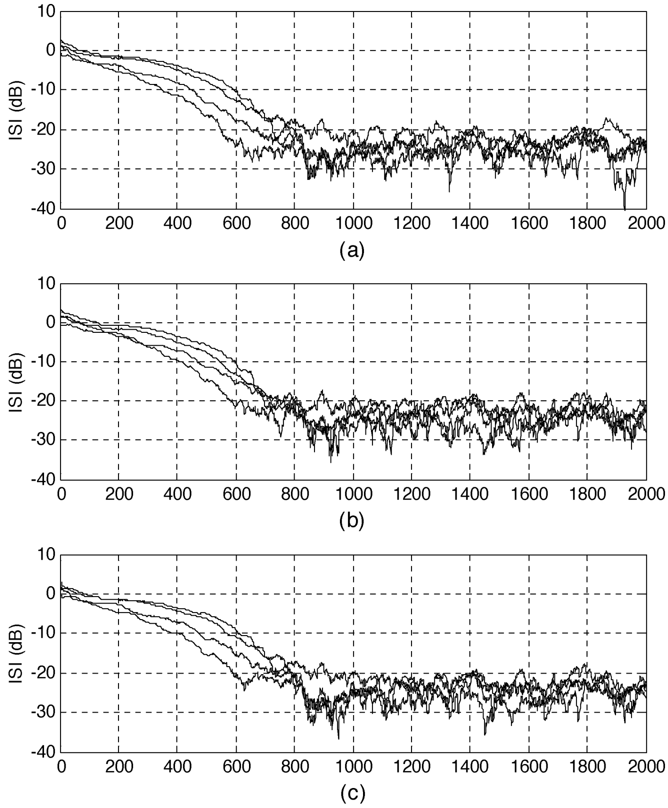


Fig. 2. Intersignal interference for separating the mixtures of one 4-QAM, one 16-QAM, one 8-PSK, and one 16-PSK signals without frequency offset. (a) Conventional normalized EASI algorithm. (b) Constrained I/Q BSS method. (c) Constrained nonlinear function approach.

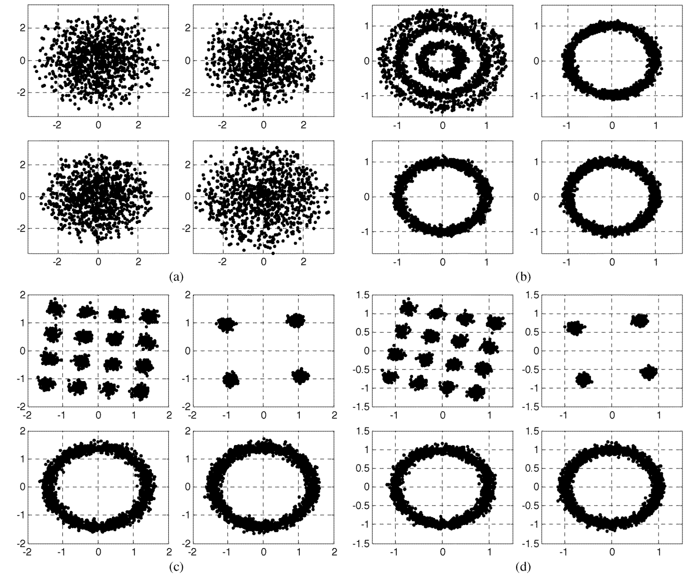


Fig. 3. Separating the mixtures of one 4-QAM, one 16-QAM, one 8-PSK, and one 16-PSK signals with small frequency offset. (a) Four complex observed mixtures. (b) Separated constellation using the normalized EASI algorithm. (c) Separated constellation using constrained I/Q BSS method. (d) Separated constellation using constrained nonlinear function approach.

we have found that the ISI performance with the frequency offsets we have used is very close to that shown in Fig. 3. Therefore, we can conclude that compared to the conventional EASI BSS scheme, the constrained BSS schemes have the advantage of recovering source phases for stationary constellations and compensating small frequency offsets, for I/Q independent sources, without affecting source separation performance.

V. Conclusion

In this letter, we proposed two BSS algorithms that recover phase rotation or compensate small frequency offsets of I/Q independent complex sources without affecting the performance of source separation. One algorithm applied the BSS technique to real-valued signals (real and imaginary parts) from the received complex signals. The other algorithm used a constrained nonlinear function in the conventional EASI algorithm. The constrained nonlinear function method does not require extra processing for the separating matrix as required for the constrained I/Q method. It has almost the same computational complexity as the conventional EASI algorithm, which cannot recover signal phase rotation. Therefore, compared to the conventional EASI algorithm, the constrained EASI nonlinear function method has the advantage of phase recovery without extra cost.

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