Optimal Block Adaptive I/Q Mismatch Compensation Based on Circularity

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***Abstract*- Wireless systems frequently employ I/Q modulation techniques to achieve spectral efficiency for high data rate applications. However, the main drawback of I/Q downconversion is the amplitude and phase imbalances between the analog components in the I and Q branches of the receiver. The resulting I/Q mismatch is unavoidable for practical quadrature receivers and can be frequency-dependent in nature. In this paper, a novel Optimal Block Adaptive algorithm based on the circularity property is presented for frequency-dependent I/Q imbalance compensation. The proposed technique, called OBA-C, is based on the assumption that the received baseband signal deviates from circularity in the presence of I/Q mismatch. OBA-C uses the complex Taylor series expansion to optimally update the adaptive filter coefficients at each iteration, until the circularity of the received signal is restored. Simulation results confirm the remarkable improvement in I/Q mismatch compensation and convergence speed of the proposed technique as compared to another recently proposed circularity based method.**

Keywords- adaptive filter, I/Q imbalance, image-frequency interference, Taylor series expanstion .

I. INTRODUCTION

In today's competitive market, wireless receivers are required to satisfy rigorous design constraints such as low cost, low power, and small size, in addition to specific bandwidth and performance requirements [1, 2]. The quadrature or I/Q receiver structure is often adopted, and it offers significant potential for the development of compact, yet flexible multimode radio systems. Such receivers use two independent channels to generate the in-phase (I) and the quadrature-phase (Q) components of the received signal [3]. Many high bit rate modulation schemes such as Quadrature Phase Shift Keying (QPSK), Quadrature Amplitude Modulation (QAM), etc, can be employed with quadrature receivers. However, imperfections of the analog components in the I and Q branches of the receiver Front-End (FE) cause the so-called I/Q imbalance problem [4, 5]. These mismatches are unavoidable and limit the image frequency attenuation to 20-40 dB in practical receivers [6]. Research on DSP-based, both blind and data-aided I/Q imbalance compensation techniques continues to receive significant attention [7-9]. In [6], a blind approach was proposed for frequency selective I/Q imbalance compensation based on the circularity property of complex baseband communication signals. However, the limitations of this method were slow convergence and dependence on the proper choice of convergence factor or step size. Furthermore, an inappropriate step size may lead to divergence.

In this paper, an adaptive blind frequency-dependent I/Q imbalance compensation technique is proposed, which is named Optimal Block Adaptive I/Q imbalance compensation based on Circularity (OBA-C). Similar to [6], the proposed algorithm employs the concept that under the influence of I/Q mismatch, the received baseband signal loses its circularity. However, to avoid manually selecting a proper step size as in [6], the presented algorithm employs the Taylor series expansion to optimally update the adaptive filter coefficients at each iteration. In this manner, the adaptive filter compensates for the I/Q mismatch by restoring the circularity of the received signal. Computer simulations show that the proposed OBA-C technique demonstrates fast convergence, while maintaining excellent Image Rejection Ratio (IRR) over a wide signal bandwidth.

The rest of the paper is organized as follows: Section II presents the basic I/Q signal model under frequency-dependent mismatch. An overview of second-order statistics is given in Section III. In Section IV, the formulation of the proposed OBA-C I/Q mismatch compensation is presented. Simulation results are given in Section V, followed by conclusions in Section VI.

II. FREQUENCY DEPENDENT

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I/Q MISMATCH SIGNAL MODEL

In Fig. 1, a block diagram of a quadrature receiver with I/Q Mismatch is shown. Following the notations given in [6], $H_{norm}\,(f)$ denotes the same nominal frequency responses of both branches, while $H_I(f)$ and $H_Q(f)$ represent the imperfections in the I and Q filter responses that differ from the nominal response. As mentioned before, all imperfections and finite tolerances of the analog FE components (filters, mixers, amplifiers, and ADC) contribute to the I/Q mismatch. One obvious imbalance source is the nonideal local oscillator (LO), which generates the signals $\cos(\omega_{LO}t)$ and $-g{\cdot}\sin(\omega_{LO}t{+}\phi)$ instead of the designed $\cos(\omega_{LO}t)$ and $-\sin(\omega_{LO}t)$ in the I/Q mixer stage, where g and ϕ denote the amplitude and phase imbalance parameters of the local oscillator, respectively. Another imbalance source is modeled as the difference between impulse responses of the linear filters $H_I(f)$ and $H_Q(f)$, caused by other front-end components. This imbalance models the frequency-dependent part of the I/Q mismatch.

With the ideal baseband equivalent of the received signal denoted by $z(t)$, the corresponding baseband equivalent of the imbalanced RF signal can be expressed as

$$x(t) = g_1(t) * z(t) + g_2(t) * z^*(t) \tag{1}$$

where the impulse responses $g_1(t)$ and $g_2(t)$ are given by:

$$g_1(t) = \left[h_I(t) + g \cdot exp(j\emptyset) \cdot h_Q(t)\right]/2 \tag{2}$$

$$g_2(t) = \left[h_I(t) - g \cdot exp(j\emptyset) \cdot h_Q(t)\right]/2 \tag{3}$$

Under perfect I/Q balance without any mismatch, $g = 1$, $\emptyset = 0$, $h_I(t) = h_Q(t) = h(t)$ and thus $g_1(t) = h(t)$ and $g_2(t) = 0$. In this scenario, $x(t) = h(t) * z(t)$, hence the baseband signal does not contain image interference $z^*(t)$.

In general, complex conjugation in the time domain corresponds to complex conjugation and mirroring in the frequency domain. Thus, if the Fourier Transform (FT) of $z(t)$ is $Z(f)$, then the FT of $z^*(t)$ is $Z^*(-f)$. Therefore, applying the FT to (1), we obtain

$$X(f) = G_1(f) \cdot Z(f) + G_2(f) \cdot Z^*(-f) \tag{4}$$

where $G_1(f)$ and $G_2(f)$ are the FT of $g_1(t)$ and $g_2(t)$ in (2) and (3), respectively.

Based on (4), the image-frequency Rejection Ratio (IRR) of the whole analog front-end can now be defined as

$$IRR_{FE}(f) = \frac{|G_1(f)|^2}{|G_2(f)|^2} \tag{5}$$

The obtainable image frequency attenuation is limited by analog component matching to the 20-40 dB range. From the discussion above, it is clear that the conjugate term $Z^*(-f)$ in the frequency domain, or $z^*(t)$ in the time domain, is the source of image interference. Hence, in order to improve the image rejection ratio, this conjugate signal term should be removed or mitigated.

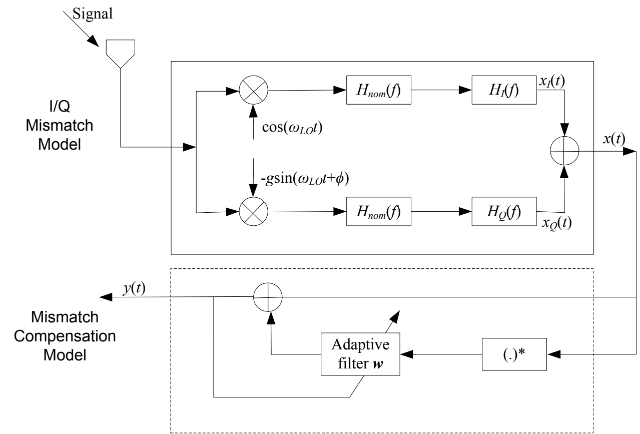

Fig. 1. Generalized I/Q imbalance model for the analog front-end and the proposed OBA-C I/Q mismatch compensation structure

III. SECOND-ORDER STATISTICS: PROPERNESS AND CIRCULARITY

The proposed OBA-C I/Q imbalance compensation algorithm utilizes a property of the ideal baseband equivalent signal, called circularity, which is a special case of properness. A second-order stationary signal $s(t)$ is defined to be proper if its complementary autocorrelation function (CACF) equals zero i.e.:

$$c_s(\tau) = E\{s(t)s(t-\tau)\} = 0 \;\; \forall\tau \tag{6}$$

In addition, a complex random signal $s(t)$ is defined to be circular if

$$c_s(0) = E\{s^2(t)\} = 0 \tag{7}$$

Thus, properness is a more general and stronger version of second-order circularity, which implies that proper signals are always circular, but a circular signal can be improper.

For linear I/Q modulation schemes, the circularity assumption of the ideal baseband equivalent $z(t)$ can be proved as follows: $z(t)$ can be expressed as $z(t) = z_I(t) + jz_Q(t)$, and its CACF at $\tau = 0$ is $c_z(0) = E\{z^2(t)\} = E\{z_I{}^2(t) - z_Q{}^2(t) + 2jz_I(t)z_Q(t)\}$. In this expression, the I and Q parts of $z(t)$ are always uncorrelated with each other, which yields $E\{2jz_I(t)z_Q(t)\} = 0$. In addition, they have the same power, resulting in $E\{z_I{}^2(t) - z_Q{}^2(t)\} = 0$. Then,

$$c_z(0) = E\{z^2(t)\} = \\ E\{z_I{}^2(t) - z_Q{}^2(t)\} + E\{2jz_I(t)z_Q(t)\} = 0 \tag{8}$$

As the result of the CACF becomes 0, the ideal baseband equivalent $z(t)$ is circular.

It is obvious that the observed I/Q mismatched baseband signal $x(t)$ is not circular, as its CACF is of the form

$$c_x(0) = E\{x^2(t)\} = \cdots$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_1(\lambda_3)\, g_2(\lambda_2)\gamma_z(\lambda_2 - \lambda_1) d\lambda_1 d + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_1(\lambda_3)\, g_2(\lambda_4)\gamma_z(\lambda_3 - \lambda_4) d\lambda_3 d\lambda_4 \neq 0 \quad (9)$$

Here, $\gamma_z(\tau) = E\{z(t)z^*(t-\tau)\} \neq 0$ is the ordinary autocorrelation function (ACF) of $z(t)$, and $g_1(t), g_2(t) \neq 0$.

Intuitively, our I/Q mismatch compensation strategy is to restore the circularity of the received signal, using a digital signal processing approach.

IV. OBA-C I/Q IMBALANCE COMPENSATION

A block diagram of OBA-C I/Q mismatch compensation is illustrated in Fig. 1. The observed complex signal is compensated using a Wide Linear (WL) adaptive filter compensator structure, which can be formulated as:

$$y(t) = x(t) + \boldsymbol{w}^T(t)\boldsymbol{x}^*(t) \quad (10)$$

where $y(t)$ is the recovered signal, $\boldsymbol{w}(t)$ is the weight vector at time *t* and $\boldsymbol{x}(t)$ is the vector containing $x(t)$ and the preceding *N*-1 inputs, given by

$$\boldsymbol{w}(t) = [w_1(t), w_2(t), \dots w_N(t)]^T \quad (11)$$

$$\boldsymbol{x}(t) = [x(t), x(t-1), \dots x(t-N+1)]^T \quad (12)$$

Here, *N* is the number of independent coefficients (taps) in the adaptive filter.

Therefore, the mismatch compensation problem is reduced to finding an optimum and blind update rule for the adaptive filter coefficients. The objective is to make the output of the compensator circular again, ideally at the next iteration, that is

$$E\{y^2(t+1)\} = 0 \quad (13)$$

Using Taylor series expansion, $y^2(t+1)$ can be expressed in terms of the compensator output $y(t)$ and the values of the coefficients $\boldsymbol{w}(t)$ in the current iteration, as follows:

$$y^2(t+1) = y^2(t) + \sum_{n=1}^{N} \frac{\partial y^2(t)}{\partial w_n(t)} \Delta w_n(t) + \frac{1}{2!} \sum_{m=1}^{N} \sum_{k=1}^{N} \frac{\partial^2 y^2(t)}{\partial w_m(t) \partial w_k(t)} \Delta w_m(t) \Delta w_k(t) + \cdots \quad (14)$$

where $\Delta w_n(t) = w_n(t+1) - w_n(t)$ $n = 1,2,\dots,N$. (15)

For computational convenience, the derivatives higher than 1st order can be ignored, if we limit the change in weight coefficients to small values. Then, taking the expectation of $y^2(t+1)$ yields:

$$E\{y^2(t+1)\} \approx E\{y^2(t)\} + E\{\sum_{n=1}^{N} \frac{\partial y^2(t)}{\partial w_n(t)} \Delta w_n(t)\}$$

$$= E\{y^2(t)\} + \Delta \boldsymbol{w}^T(t) \cdot E\{2y(t) \cdot \boldsymbol{x}^*(t)\} \quad (16)$$

Substituting (16) in (13), we obtain $\Delta \boldsymbol{w}(t)$:

$$\Delta \boldsymbol{w}^{\boldsymbol{T}}(t) = -E\{y^2(t)\} \cdot pseudoinverse(E\{2y(t) \cdot \boldsymbol{x}^*(t)\} \quad (17)$$

From linear algebra, we know that the pseudo-inverse A^+ of an m×n matrix A is calculated as $A^+ = (A^H A)^{-1} A^H$ [10]. Here, $pseudoinverse(E\{2y(t) \cdot \boldsymbol{x}^*(t)\})$ is a $1 \times N$ vector, and $E\{y^2(t)\}$ is a scalar, which yields a $1 \times N$ vector for $\Delta \boldsymbol{w}^T(t)$.

It can be inferred from (13), that the expectation operator cannot be ignored, otherwise the recovered signal will approach 0 after convergence, which is obviously incorrect. Thus, a block based technique is chosen instead of an iterative technique to estimate the expectation operation, which estimates expectation values based on the data within a processing block (frame).

Hence, the proposed OBA-C I/Q mismatch compensation algorithm is described as follows. Assume that the algorithm starts from the t^th input sample with $\boldsymbol{w}(t)$ available.

1. Calculate recovered signal y(t)

$$y(t) = x(t) + \boldsymbol{w}^T(t)\boldsymbol{x}^*(t)$$

2. Derive $\Delta \boldsymbol{w}^T(t)$

2.1 For a block size L, estimate $E\{2y(t) \cdot \boldsymbol{x}^*(t)\}$ within a block.

Set $C = E\{2y(t) \cdot \boldsymbol{x}^*(t)\} = \sum_{l=0}^{L-1} y(t-l) \cdot \boldsymbol{x}^*(t-l)$

2.2 Compute

$$pseudoinverse(E\{2y(t) \cdot \boldsymbol{x}^*(t)\}) = (C^H C)^{-1} C^H$$

2.3 Calculate $E\{y^2(t)\}$ within a block.

$$E\{y^2(t)\} = \sum_{l=0}^{L-1} y^2(t-l)$$

2.4 Compute $\Delta \boldsymbol{w}^T(t)$ using equation (17).

3. Update $\boldsymbol{w}(t+1)$

$$\boldsymbol{w}(t+1) = \boldsymbol{w}(t) + \Delta \boldsymbol{w}(t)$$

4. Check the convergence of the algorithm

Calculate the IRR over the image band. If it is not increasing, stop the iteration and terminate, otherwise $t = t + 1$, then go to step 2.

V. SIMULATIONS

In this section, the performance of the proposed technique is compared to another circularity based algorithm in [6], using the compensation block diagram in Fig. 1. The performance measure is the overall IRR, which is the sum of the analog FE image attenuation and the gain from digital signal processing. By combining (1) and (10), the total IRR as a function of frequency is obtained as follows:

$$IRR_{TOT}(f) = \frac{|G_1(f) + W(f) G_2^*(-f)|^2}{|G_2(f) + W(f) G_1^*(-f)|^2} \quad (18)$$

The desired source signal is a QPSK signal modulated at -3 MHz intermediate frequency, with 25% roll-off raised-cosine pulse-shaping. The symbol rate is 3.84 MHz, yielding roughly a 5 MHz channel bandwidth. The front-end sampling rate is 4×3.84=15.36 MHz. The local oscillator (LO) mismatch levels are 3% in amplitude and 3^0 in phase, and the branch filter non-idealities are $H_I(z) = 0.98 + 0.03z^{-1}$ and $H_Q(z) = 1.0 - 0.005z^{-1}$. The input SNR = 30dB and the block size for the proposed OBA-C I/Q imbalance compensator is 18.

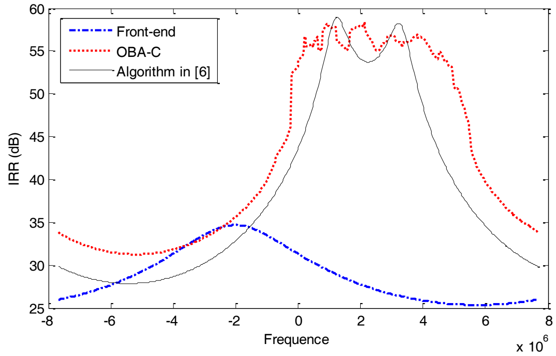

Fig. 2 IRR before/after Compensation

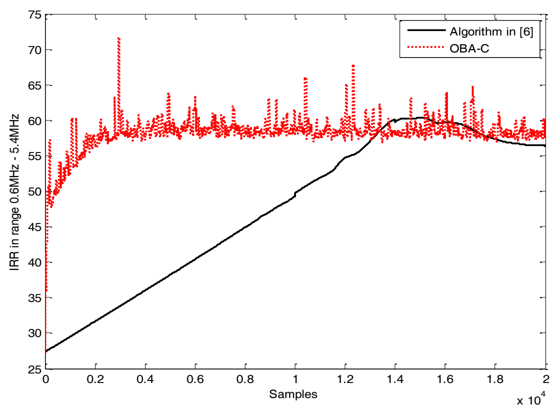

Fig. 3 Convergence Speed

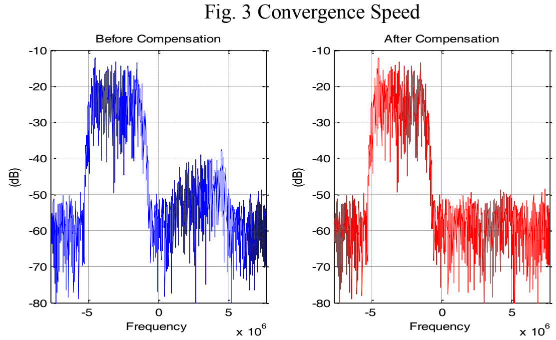

Fig. 4 Frequency Spectral before/after Compensation

Fig. 2 shows the achieved IRR before and after compensation with a 3-tap adaptive filter, averaged over 1000 Monte Carlo simulation runs. In Fig. 3, the IRR over the image frequency band (0.6-5.4 MHz) vs. the sample number is plotted. Fig. 4 shows the signal spectrum before and after compensation using the OBA-C I/Q imbalance compensator.

From Fig. 2, it can be clearly seen that the OBA-C I/Q imbalance compensation algorithm demonstrates excellent IRR while maintaining wider image rejection bandwidth than the algorithm in [6]. Fig. 3 indicates that the presented method yields a considerable improvement in convergence speed of the order of 10. Fig. 4 illustrates that after OBA-C I/Q compensation, the image interference signal is significantly attenuated.

VI. CONCLUSIONS

In this paper, a novel fast-converging circularity based approach for I/Q mismatch compensation is presented, called OBA-C. The OBA-C employs the circularity property of the desired baseband signal in conjunction with Taylor's series expansion to optimally adjust the taps of the compensator or adaptive filter at each iteration. In this manner, the circularity of the received signal is restored and the I/Q mismatch is compensated. The performance of the OBA-C I/Q imbalance compensation technique is tested using computer simulations, and compared to a recently proposed algorithm. Simulation results show that the proposed algorithm yields high image rejection performance with a significant improvement in convergence speed.

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