IMPACTS OF I/Q IMBALANCE ON QPSK-OFDM-QAM DETECTION

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Abstract - The impacts of the I/Q imbalance in the quadrature down-converter on the performance of a QPSK-OFDM-QAM system are studied. Either amplitude or phase imbalance introduces inter-channel interference (ICI). In addition to the ICI, there is a cross-talk between in-phase and quadrature channels in each and every sub-carrier when both amplitude and phase imbalances are present. The BER (Bit Error Ratio) performance of QPSK sub-carriers are also calculated to illustrate the impacts of the I/Q imbalance. It is observed that with the amplitude imbalance less than 1 dB and phase imbalance less than 5 degrees, the degradation of BER performance is less than 0.5 dB for BER>10^{-6}.

1. Introduction

An OFDM-QAM (Orthogonal Frequency Division Multiplex-Quadrature Amplitude Modulation) signal as specified in the ETSI-DAB [1] system requires a quadrature down-converter to translate the RF (or IF) signal into in-phase and quadrature baseband signals. It can be done by mixing the RF/IF signal with an in-phase local signal, $\cos(2\pi f_L t)$, and its 90-degree phase-shift replica, $-\sin(2\pi f_L t)$, respectively. (In this paper, we assume that the frequency of the local oscillator is ideally locked to the center frequency of the incoming signal.) A block diagram of a quadrature down-converter is depicted in Fig. 1. Ideally, the two local carriers should have the same signal level and a phase difference of exact 90 degrees. However, a practical quadrature down-converter rarely has equal signal level in its two arms. Moreover, the phase shift is not exact 90 degrees either. In this paper, we discuss the impacts of the amplitude and phase imbalances of the local carriers on the performance of a QPSK-OFDM-QAM signal. In a QPSK-OFDM-QAM signal, each sub-carrier is (differential) QPSK modulated; QPSK-modulated sub-carriers are then orthogonally multiplexed by using the frequency division technique. An example of QPSK-OFDM-QAM signals is the DAB signal as specified in [1].

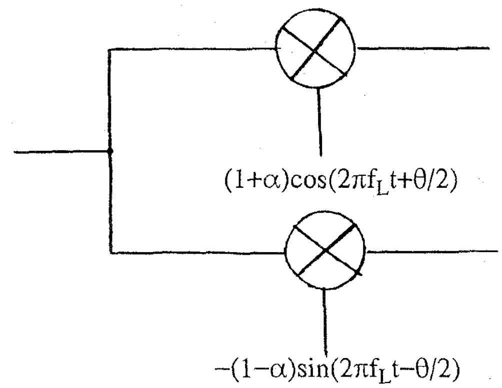

Fig. 1 Block diagram of a quadrature demodulator with amplitude and phase imbalances.

We try to draw a picture of how amplitude and phase imbalances affect the performance of a QPSK-OFDM-QAM system in this paper. The results we obtain can serve as a guideline for the specification of down-converter designs. A general formula for the demodulated signal is derived in Section II. The inter-channel interference and cross-talk generated by the I/Q imbalance are analyzed in Section III. In Section IV, we calculate the BER performance degradation due to the I/Q imbalance assuming that each sub-carrier is QPSK-modulated. The last section summarizes the results.

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2. System Analyses

The complex envelope of an OFDM signal [2] can be represented by the discrete frequency inverse Fourier transformation of the complex symbols carried in the signal, that is

$$b(t)=\frac{1}{N}\sum_{k=-N/2}^{N/2-1} a_k e^{j2\pi k f_s t} \tag{1}$$

where

N is the number of multiplex channels (or number of sub-carriers in an OFDM-QAM signal)
f_s is the channel spacing between two adjacent sub-carriers
a_k is the complex symbol of the k+N/2+1st sub-carrier

Note that N is typically chosen as a power of 2; the actual number of active channels may be less than N. For an inactive channel, a_k=0. The RF waveform (of an OFDM-QAM or multi-carrier signal) is

$$s(t)=Re\left\{b(t)e^{j2\pi f_c t}\right\} \tag{2}$$

where *Re* represents the real part and f_c is the carrier frequency.

After adding the additive Gaussian noise, the received signal is

$$r(t)=s(t)+n(t) \tag{3}$$

Here we assume that n(t) has been properly band-limited. For mathematical convenience, it is useful to define

$$r_c(t)=[b(t)+m(t)]e^{j2\pi f_c t} \tag{3a}$$

where $m(t)=n(t)e^{-j2\pi f_c t}$ is the complex envelope of the noise. If the frequency and phase of the local carrier is perfectly locked to the carrier of the incoming signal and the phase shift between two arms is 90 degrees, the down-converted in-phase and quadrature baseband signals are

$$i(t)=Re\left\{b(t)+m(t)\right\} \tag{4a}$$

and

$$q(t)=Im\left\{b(t)+m(t)\right\} \tag{4b}$$

respectively. The complex input signal to the OFDM demodulator is

$$i(t)+jq(t)=r_c(t)e^{-j2\pi f_c t}=b(t)+m(t) \tag{4c}$$

Assuming that there is a phase imbalance of θ and an amplitude imbalance of β in dB, the in-phase local carrier is $(1+\alpha)\cos(2\phi_L t-\theta/2)$ and the quadrature carrier is $-(1-\alpha)\sin(2\pi f_L t+\theta/2)$, where

$$\alpha=\frac{10^{\beta/20}-1}{10^{\beta/20}+1} \tag{5}$$

With perfect AFC (Automatic Frequency Control), i.e. $f_L=f_c$, the down-converted in-phase and quadrature signals are

$$\hat{i}(t)=(1+\alpha)[i(t)\cos(\theta/2)-q(t)\sin(\theta/2)] \tag{6a}$$

and

$$\hat{q}(t)=(1-\alpha)[q(t)\cos(\theta/2)-i(t)\sin(\theta/2)] \tag{6b}$$

respectively. The input complex signal to the OFDM demodulator is

$$\hat{b}(t)=\hat{i}(t)+j\hat{q}(t) \tag{7}$$

Plug (6a), (6b), and (4c) into (7), we have

$$\hat{b}(t)=[\cos(\theta/2)+j\alpha\sin(\theta/2)][b(t)+m(t)] \\ +[\alpha\cos(\theta/2)-j\sin(\theta/2)][b^*(t)+m^*(t)] \tag{7a}$$

where * denotes complex conjugate. Plug (1) into (7a) and after some manipulation, we have

$$\hat{b}(t)=\frac{1}{N}\sum_{k=-N/2}^{N/2-1}[(c_k+\alpha c^*_{-k})\cos(\theta/2) \tag{8}$$

$$+j(\alpha c_k - c^*_{-k})\sin(\theta/2)]e^{j2\pi k f_s t}$$

where $c_k = a_k + \eta_k$, and η_k is the DFT component of the complex envelope of the noise, m(t). After the OFDM demultiplex, which is simply a DFT, the demultiplexed complex symbol of sub-carrier k is

$$\hat{a}_k = (c_k + \alpha c^*_{-k})\cos(\theta/2) + j(\alpha c_k - c^*_{-k})\sin(\theta/2) \quad (9)$$

3. Performance Analyses

In this section, we investigate the effects of the I/Q imbalance in a noise-free channel, i.e. n(t)=0, and consequently m(t)=0 and $\eta_k=0$ for all k. In a noise-free channel, (9) is reduced to

$$\hat{a}_k = (a_k + \alpha a^*_{-k})\cos(\theta/2) + j(\alpha a_k - a^*_{-k})\sin(\theta/2) \quad (9a)$$

3.1 Phase Imbalance

First, we assume that $\alpha=0$, i.e. there is no amplitude imbalance, only the phase imbalance is present. When $\alpha=0$, (9a) is reduced to

$$\hat{a}_k = a_k\cos(\theta/2) - ja^*_{-k}\sin(\theta/2) \quad (9b)$$

From (9b), we observe that the in-phase (quadrature) channel of sub-channel k is interfered by the quadrature (in-phase) channel of sub-channel -k. The signal-to-ICI power ratio is

$$\text{SICI(phase imbalance)} = 20\log\cot(\theta/2)\text{(dB)} \quad (10)$$

The signal to ICI power ratio when the phase imbalance is present is depicted in Fig. 2. We observe that for the SICI (phase imbalance) to be greater than 20, 30, and 40 dB, the phase imbalance is required to be less than 11.42, 3.62, and 1.14 degrees respectively.

3.2 Amplitude Imbalance

Let us now assume that the phase shift between two arms is exact 90 degrees. However, we assume that there is an amplitude mismatch between in-phase and quadrature signals. The demodulated complex symbol in (9a) is reduced to

$$\hat{a}_k(t) = a_k + \alpha a^*_{-k} \quad (9c)$$

From (9c), we observe that the in-phase (quadrature) channel of sub-channel k is interfered by the in-phase (quadrature) channel of sub-channel -k. The signal-to-ICI power ratio is

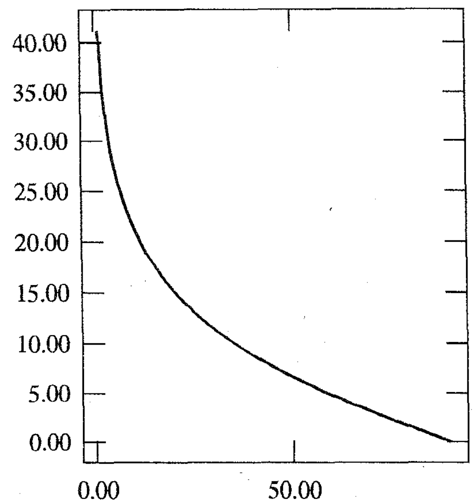

Fig. 2 Signal to Inter-Channel Interference power ratio (SICI) as a function of the phase imbalance when there is no amplitude imbalance. X-axis: θ in degree, Y-axis: SICI in dB.

$$\text{SICI(amplitude imbalance)} = 20\log\frac{1}{|\alpha|}\text{(dB)} \quad (11)$$

The signal to ICI power ratio when the amplitude imbalance is present is depicted in Fig. 3. For more than 20, 30, and 40 dB signal-to-ICI ratio, it is required that the amplitude imbalance to be less than 1.74, 0.54, and 0.17 dB respectively.

3.3 Combined Effects

We further consider the combined effects of amplitude and phase imbalances. It can be observed from (9a) that

(1) The desired signal b(t) is attenuated by $\cos(\theta/2)$ as in the phase imbalance case.

(2) There is an inter-channel interference due to the phase imbalance, $-j\sin(\theta/2)b^*(t)$.

(3) There is an inter-channel interference due to the amplitude imbalance, $\alpha\cos(\theta/2)b^*(t)$. Note that the ICI is also attenuated by $\cos(\theta/2)$ as the desired signal.

(4) There is an interference term, $j\alpha\sin(\theta/2)b(t)$, due to combined amplitude and phase imbalances. The term generates an I/Q cross-talk as can be observed from (9a).

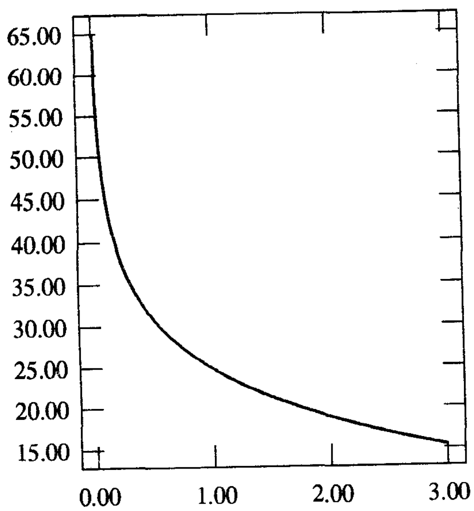

Fig. 3 Signal to Inter-Channel Interference power ratio (SICI) as a function of the amplitude imbalance when there is no phase imbalance. X-axis: β in dB, Y-axis: SICI in dB.

3.4 Noise Enhancement

Before we start to calculate BER performances, we need to consider the effect of the I/Q imbalance on the noise power prior to detection. First, from Fig. 1, it can be readily observed that the temporal average noise power at the detector input of in-phase and quadrature channels are amplified or attenuated by $(1+\alpha)^2$ and $(1-\alpha)^2$ respectively. However, what we are most interested in is the noise power appears at the detector input after the OFDM demultiplex. From (9), it can be observed that the noise power, η_k, are amplified by $1+\alpha^2$ at both in-phase and quadrature channels of every and each sub-carrier.

4. BER Degradation

In Sect. 3, we calculate the maximum imbalance allowed based on the SICI power ratio. However, the required SICI power ratio depends on the sub-carrier modulation format and the operating condition. In this section, we assume that the sub-carrier modulation is coherent QPSK and calculate the BER degradation due to the I/Q imbalance. From the results, we can specify the maximum imbalance allowed if the BER degradation is specified by the system design.

Let us consider the case where only the phase imbalance is present first. If a_k is a QPSK symbol, the BER performance can be calculated by calculating the BER of the real part of a_k conditioned on the imaginary part of a_{-k} first. There are only two possibilities of the imaginary part of a_{-k}, both with probability 0.5. It either has the same sign as that of $Re\{a_k\}$ and makes a constructive interference or is opposite to $Re\{a_k\}$ and makes a destructive interference. The BER so calculated is also applied to the imaginary part and equals to the BER of the QPSK-modulated sub-carrier. Since every sub-carrier has the same QPSK modulation, the calculated BER is also equal to the BER of the OFDM symbol and the OFDM-QAM system in a an Additive White Gaussian Noise (AWGN) channel.

The BER's for phase imbalances of 20, 10, and 5 degrees are shown in Fig. 4. These correspond to SICI power ratio of 15, 21, and 27 dB respectively. If the phase imbalance is less than 10 degrees, the degradation is less than 0.5 dB at BER=10^{-4}; even at BER=10^{-6}, the degradation is only slightly greater than 0.5 dB. The degradation is greater than 1 dB for 20-degree imbalance. Note that the phase imbalance does not affect the noise power prior to the detection. Our simulations results are in good agreement with the analytical results. However, they are not shown here for the clarity of presentation.

When there is only amplitude imbalance, the BER can be calculated in the same way. (Now when calculating the BER of the real part of a_k, we must condition it on the real part of a_{-k}.) The

BER performances for amplitude imbalances of 1, 2, and 3 dB are shown in Fig. 5. For 2 dB imbalance, the degradation is 0.6 to 0.8 dB for 10^{-6}<BER<10^{-4}. However, for 3 dB imbalance, the degradation is greater than 1 dB even for BER=10^{-4}. With the amplitude imbalance, the noise power at the in-phase channel is amplified by $(1+\alpha)^2$ times. The noise power at the quadrature channel is attenuated by $(1-\alpha)^2$. However, after the OFDM demultiplex, i.e. the DFT operation, the noise power at each channel, either in-phase or quadrature, of every and each sub-carrier is amplified by $1+\alpha^2$ times. Our simulations results are very close to the analytical results.

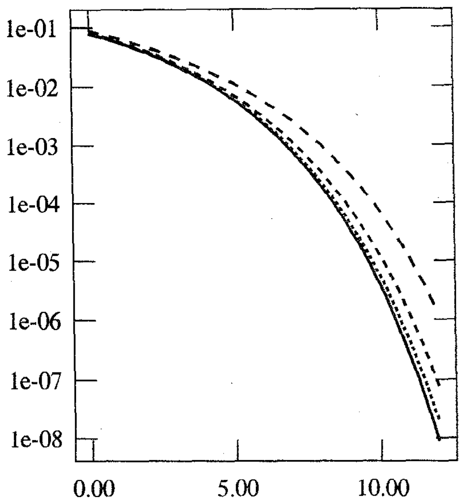

Fig. 4 Bit Error Ratio (P(e)) as functions of E_b/N_o with the phase imbalance θ as a parameter. X-axis: E_b/N_o in dB, Y-axis: P(e). From left to right: θ= 0, 5, 10, and 20 degrees.

If there are both amplitude and phase imbalances, we need to calculate the BER of *Re* $\{a_k\}$ conditioned on *Re* $\{a_{-k}\}$, *Im* $\{a_k\}$, and *Im* $\{a_{-k}\}$. The BER degradation for amplitude imbalance of 1 or 2 dB and phase imbalance of 5 or 10 degrees is depicted in Fig. 6. Except for 2 dB and 10 degree imbalances, the degradation is less than 1 dB at BER=10^{-4}. For BER=10^{-6}, the degradation barely reaches 0.5 dB for 1 dB and 5 degree imbalances. In Fig. 7, we show the comparisons between simulations and analytical results. It can be observed that simulation results are very close to analytical results. Simulations for other amplitude and phase imbalances are also in good agreement with analytical results but are not shown for the clarity of presentation.

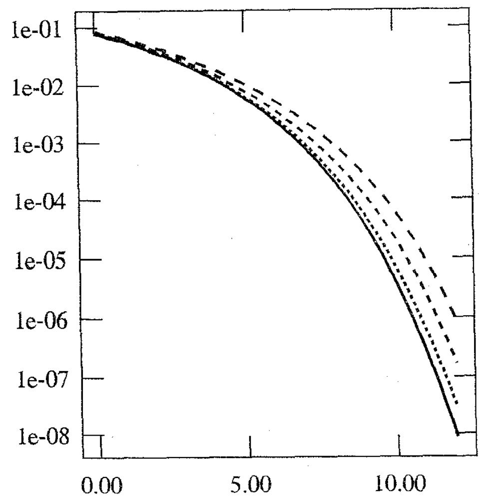

Fig. 5 Bit Error Ratio (P(e)) as functions of E_b/N_o with the amplitude imbalance β as a parameter. X-axis: E_b/N_o, Y-axis: P(e). From left to right: β= 0, 1, 2, and 3 dB.

5. Conclusion

In this paper, we analyze the impacts of amplitude and phase imbalances that are always present in a practical quadrature down-converter. We derive the close-forms of signal to ICI (Inter-Channel Interference) and I/Q cross-talk power ratio. The BER degradation is also calculated for a variety of imbalances. For 1-dB amplitude imbalance and 5-degree phase imbalance, the BER degradation can be kept below 0.5 dB for BER<10^{-6}. The impacts of the I/Q imbalance on the automatic frequency control (AFC) and carrier recovery are next topics that we shall investigate for an OFDM-QAM system.

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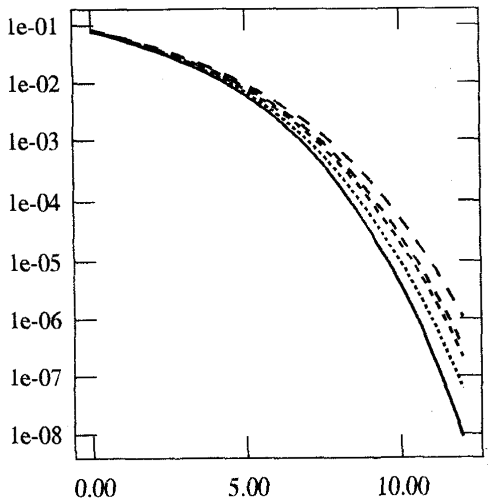

Fig. 6 Bit Error Ratio (P(e)) as functions of E_b/N_o with amplitude and phase imbalances as parameters. X-axis: E_b/N_o in dB, Y-axis: P(e). From left to right: β= 0 dB, θ=0°; β= 1 dB, θ = 5°; β= 1 dB, θ= 10°; β=2 dB, θ= 5°; β = 2 dB, θ=10°.

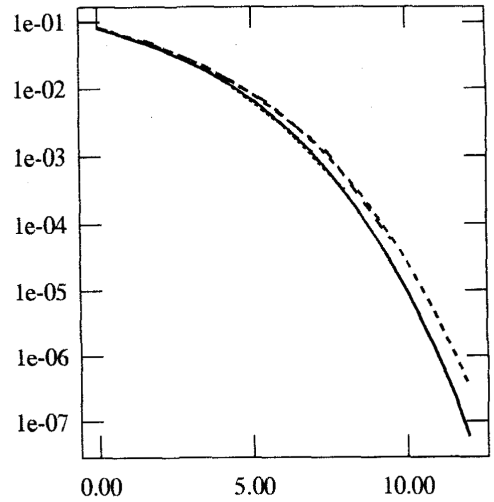

Fig. 7 Comparisons between analytical and simulations results for β= 1 dB, θ = 5° and β = 2 dB, θ=10°.

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