Efficient Mitigation of Frequency-Selective I/Q Imbalance in OFDM Receivers

Lauri Anttila, Mikko Valkama, and Markku Renfors
Department of Communications Engineering
Tampere University of Technology
P.O. Box 553, FI-33101 Tampere, Finland

***Abstract*—I/Q imbalance is one of the main practical obstacles in the implementation of direct-conversion receivers. This paper presents novel DSP-based techniques for the estimation and compensation of frequency-selective receiver I/Q imbalances in OFDM systems. The estimation is based on a special pilot or preamble structure, and frequency-domain smoothing is utilized to effectively reduce the effect of noise. Reliable estimation is attained with a minimum of two OFDM symbols. Further, it is shown that estimation of frequency-selective I/Q imbalance and the frequency-selective radio channel can be decoupled, which is beneficial from the computational complexity point of view. Simulation analysis shows impressive performance with fast convergence.**

***Index Terms*—Direct-conversion radio, I/Q imbalance, RF impairments, Multicarrier systems, OFDM**

I. Introduction

THE wireless radio has quickly become a commodity, a mass-market product, pushing the industry to seek for *low-cost* transceiver solutions. On the other hand, the transceiver architecture must be *flexible* in order to support the growing number of wireless standards. The direct-conversion radio architecture (see [1],[2]) is currently considered the most viable solution to meet these conflicting requirements. [1],[2]

There are still a few practical implementation related issues to be solved before the direct-conversion transceiver can be deployed to transmit and receive wideband signals with high-order modulations and/or high dynamic range. The most notable physical impairment related to the direct-conversion architecture is I/Q imbalance, which is synonymous to gain and phase mismatches between the two analog signal branches of the transceiver (the I and Q). These mismatches are due to random variations in the fabrication processes of the components (mixers, phase splitters, low-pass filters, amplifiers, ADC's, and DAC's), and are unavoidable in any practical implementation. I/Q imbalances create *mirror-frequency interference*, which, depending on the exact channelization structure, is seen as either in-band interference or adjacent channel interference [1]-[7]. In case of multicarrier systems such as OFDM, mirror-frequency interference is effectively seen as *mirror-carrier interference*. Typically, the image-rejection ratios (IRR) in modern direct-conversion implementations are in the order of 25-40 dB [1],[2]. This article presents DSP-based algorithms for compensating the effects of *frequency-selective* receiver I/Q imbalances in OFDM type multicarrier systems. From the overall link point of view, we assume that the transmitter is perfect, i.e., possible transmitter I/Q imbalances are already corrected at the source, leaving the receiver only its own I/Q imbalance to deal with.

I/Q imbalance estimation and compensation in OFDM systems has been studied quite extensively during the current decade (see, e.g., [4]-[7]). However, most of the work assumes that the I/Q imbalances are independent of frequency (e.g., [4],[7]). The works in [5],[6] take the more realistic stand and consider the imbalances to be frequency-dependent. In addition to the above pilot-based methods, several blind algorithms exist for compensating I/Q imbalance, many of which are directly applicable to OFDM systems (see [3] and the references therein). However, the convergence times of many of the blind techniques can be unacceptably long in some cases.

In this paper, efficient pilot-aided algorithms are developed for estimating and compensating frequency-selective receiver I/Q imbalance in OFDM systems. The techniques solve a simple 4-by-4 linear equation for each subcarrier *pair* to find the effective channel matrix for that pair, and then apply novel least squares (LS) and weighted least squares (WLS) frequency-domain smoothing to alleviate the effect of noise. The techniques are fast, since only a minimum of two multicarrier symbols are needed for reliable estimation, i.e., to push the mirror-frequency interference below the noise level. This will be illustrated through link-level simulations in Section IV.

Some notational practices: vectors are in bold lower-case, matrices in bold upper-case; superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote the conjugate, transpose and conjugate transpose operators, respectively; convolution is indicated by $*$, and the statistical expectation operator is $\mathrm{E}[\cdot]$. Throughout the paper, we use two-sided subcarrier indexing, i.e., physically opposite (mirror) subcarriers are indexed k and $-k$.

II. Signal and System Models

A. Frequency-Selective Receiver I/Q Imbalance

Let us denote by g and ϕ the relative amplitude and phase imbalances at the I/Q mixer, respectively, and assume that they are referred to the Q branch. Further, denoting the overall frequency-response *difference* between the I and Q branches by $H(f)$, with the corresponding impulse response being $h(t)$, the observed I/Q signal becomes [3],[5]

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$$x(t) = g_1(t) * z(t) + g_2(t) * z^*(t)\,. \tag{1}$$

Here $z(t)$ denotes the received signal with perfect I/Q balance, and the *imbalance filters* are given by [3]

$$\begin{aligned} g_1(t) &\triangleq [\delta(t) + g\exp(-j\phi)h(t)]/2 \\ g_2(t) &\triangleq [\delta(t) - g\exp(j\phi)h(t)]/2 \end{aligned} \tag{2}$$

where $\delta(t)$ denotes an impulse function. The frequency-independent imbalance model is a special case of (1) with $h(t) = \delta(t)$, and thus the observed signal becomes $x(t) = K_1 z(t) + K_2 z^*(t)$, with $K_1 = [1 + g\exp(-j\phi)]/2$ and $K_2 = [1 - g\exp(j\phi)]/2$ [3].

In the frequency domain, the model in (1) transforms into

$$X(f) = G_1(f)Z(f) + G_2(f)Z^*(-f)\,, \tag{3}$$

which reveals clearly the mirror-frequency interference due to the conjugate signal term in (1).

B. Receiver I/Q Imbalance in Multicarrier Systems

In the following, we assume that the OFDM symbol incorporates a cyclic prefix (CP), and that the length of the cascade impulse response of the radio channel and the receiver I/Q imbalance (filters $g_1(t)$ and $g_2(t)$ in (1)) does not exceed the length of the CP. For now, we assume that perfect symbol timing has been obtained, and that there is no carrier frequency offset (CFO) between the modulator and the demodulator. Thus, a simplified subcarrier-wise baseband model for the link can be used. Modifications to the system model and the estimators under CFO will be given in a future publication.

First, let us denote by S_k the transmitted symbol on subcarrier *k*. The signal then propagates through the multipath channel with complex gain H_k. The ideal received signal on subcarrier *k*, without receiver imbalance, is then

$$Z_k = H_k S_k + N_k\,. \tag{4}$$

Above, the noise contribution N_k is assumed to be zero-mean complex circular Gaussian, and statistically independent of the noise contributions of all other subcarriers.

Then, including the receiver I/Q imbalance, denoted here by $\{G_{1,k}, G_{2,k}\}$, the imbalanced received signal on subcarrier *k* becomes

$$\begin{aligned} X_k &= G_{1,k} Z_k + G_{2,k} Z_{-k}^* \\ &= G_{1,k} H_k S_k + G_{2,k} H_{-k}^* S_{-k}^* + N_k' \end{aligned}. \tag{5}$$

The noise component $N_k' = G_{1,k}N_k + G_{2,k}N_{-k}^*$ is still zero-mean complex circular Gaussian, but now the mirror-carrier noise terms N_k' and N_{-k}' are clearly dependent since $\mathrm{E}[N_k'\, N_{-k}'] = (G_{1,k}G_{2,-k} + G_{1,-k}G_{2,k})\sigma_N^2$ ($\sigma_N^2 = \mathrm{E}[|N_k|^2]$).

Now a vector-matrix model of a single received mirror-subcarrier pair can be written as

$$\begin{bmatrix} X_k \\ X_{-k}^* \end{bmatrix} = \begin{bmatrix} G_{1,k}H_k & G_{2,k}H_{-k}^* \\ G_{2,-k}^*H_k & G_{1,-k}^*H_{-k}^* \end{bmatrix} \begin{bmatrix} S_k \\ S_{-k}^* \end{bmatrix} + \begin{bmatrix} G_{1,k} & G_{2,k} \\ G_{2,-k}^* & G_{1,-k}^* \end{bmatrix} \begin{bmatrix} N_k \\ N_{-k}^* \end{bmatrix} \tag{6}$$

or $\mathbf{x}_k = \mathbf{G}_k^{\mathrm{tot}}\mathbf{s}_k + \mathbf{G}_k\mathbf{n}_k$, where $\mathbf{G}_k^{\mathrm{tot}}$ can be factorized as

$$\mathbf{G}_k^{\mathrm{tot}} = \begin{bmatrix} G_{1,k} & G_{2,k} \\ G_{2,-k}^* & G_{1,-k}^* \end{bmatrix} \begin{bmatrix} H_k & 0 \\ 0 & H_{-k}^* \end{bmatrix} = \mathbf{G}_k\mathbf{H}_k\,. \tag{7}$$

C. I/Q Imbalance Compensation and Channel Equalization

The equalization or compensation of the imbalance and the radio channel can be performed either jointly or separately, as explained in the following. The actual estimation techniques of the imbalance parameters will be introduced in Section III.

1) Joint Equalization

Once an estimate of the imbalance/channel matrix $\hat{\mathbf{G}}_k^{\mathrm{tot}}$ is obtained, there are several possibilities for performing joint equalization/detection. A joint zero-forcing (ZF) equalizer for receiver I/Q imbalance and the channel arises directly from (6) as

$$\hat{\mathbf{s}}_k = (\hat{\mathbf{G}}_k^{\mathrm{tot}})^{-1}\mathbf{x}_k\,. \tag{8}$$

However, the ZF equalizer is known to be far from optimal, having problems with noise enhancement and the possible ill-conditioning of $\hat{\mathbf{G}}_k^{\mathrm{tot}}$. The optimum detector is the joint Maximum Likelihood (ML) detector, but its complexity is often restrictive, especially with larger constellations and/or large number of subcarriers. We will not explore channel equalization or detection techniques any further, since they are out of the scope of this paper. Instead, we will concentrate on I/Q imbalance compensation only.

2) Decoupled Equalization

Alternatively to (8), the compensation of the different impairments can be done successively, as is clear from the factorization of $\mathbf{G}_k^{\mathrm{tot}}$ in (7). The compensation in this case is done in reversed order of appearance, i.e., receiver imbalance first and then the channel. The decoupled compensation naturally calls for an estimation scheme which can decouple the estimation of the two impairments as well. In Section III, it will be shown that I/Q imbalance and channel estimation can indeed be decoupled perfectly.

Contrary to the case of joint compensation, the optimal compensator for *I/Q imbalance only* is indeed the zero-forcing inverse transformation $\mathbf{G}_k^{-1}\mathbf{x}_k$. Optimality follows from the fact that the noise is also transformed by $\mathbf{G}_k$, and thus multiplying (6) from the left by $\mathbf{G}_k^{-1}$, the signal model is reduced back to (4). Notice also that the imbalance matrix $\mathbf{G}_k$ is guaranteed to invert due to its structure. More precisely, in any practical imbalance scenario we have $|G_{1,i}| \approx 1$ and $|G_{2,i}| \ll |G_{1,i}|$, and the determinant of $\mathbf{G}_k$ is $\det(\mathbf{G}_k) = G_{1,k}G_{1,-k}^* - G_{2,k}G_{2,-k}^* \neq 0$.

Thus, compensating for receiver I/Q imbalance first with linear processing will not sacrifice the optimality of the overall detector. The channel equalizer or detector can then be tailored to the specific channel conditions and available computational resources, giving more flexibility in the receiver design.

The decoupling of the estimation and compensation parameters is beneficial also from the computational burden point of view. The computational savings come from the fact that once the I/Q imbalance is estimated and compensated, the model in (5)-(6) for the *future symbols* reduces to a subcarrier-wise model. This is feasible since the I/Q imbalance is constant or only very slowly varying with time, and thus it generally needs to be estimated much less frequently than the channel. The complexity of channel estimation as well as equalization will then be halved (compared to (8)), meaning clear savings in computations in the long run.

D. Simplified Compensation Structure for I/Q Imbalance

Instead of using $\mathbf{G}_k^{-1}\mathbf{x}_k$, we adopt the *simplified compensator structure*, written out here for subcarrier *k* as

$$\begin{aligned} Y_k &= X_k + W_k X_{-k}^* \\ &= G_{1,k}H_kS_k + G_{2,k}H_{-k}^*S_{-k}^* \\ &\quad + W_k(G_{2,-k}^*H_kS_k + G_{1,-k}^*H_{-k}^*S_{-k}^*) \\ &= (G_{1,k} + W_kG_{2,-k}^*)H_kS_k + (G_{2,k} + W_kG_{1,-k}^*)H_{-k}^*S_{-k}^* \end{aligned} \tag{9}$$

Above, the noise is neglected for brevity. Now, for compensating the mirror-frequency interference, it is enough that the contribution of S_{-k}^* on Y_k is nulled, i.e., W_k should be chosen such that $G_{2,k} + W_kG_{1,-k}^* = 0$. This yields, for subcarrier *k*, the ideal compensator coefficient

$$W_k^{\mathrm{id}} = -\frac{G_{2,k}}{G_{1,-k}^*}. \tag{10}$$

Substituting (10) into (9), the ideally compensated signal on subcarrier *k* becomes

$$\begin{aligned} Y_k &= X_k + W_k^{\mathrm{id}}X_{-k}^* \\ &= (G_{1,k} - \frac{G_{2,k}G_{2,-k}^*}{G_{1,-k}^*})H_kS_k \approx G_{1,k}H_kS_k = H_k^{\mathrm{m}}S_k \end{aligned} \tag{11}$$

The approximation on the bottom row is justified since $|G_{1,i}| \approx 1$ and $|G_{2,i}| \ll |G_{1,i}|$, and therefore the first entry inside the parentheses is much larger compared to the second entry. Thus ideally, the compensation removes the mirror signal component completely, leaving a small additional linear distortion $G_{1,k}$ to the signal. This is in practice merged with the channel coefficient, and handled jointly with it in the equalization or detection stages. Notice that now since Y_k depends on S_k only, the *modified channel response* $H_k^{\mathrm{m}} = G_{1,k}H_k$ can be estimated in a *per-subcarrier manner* for the future symbols, as in any traditional OFDM receiver. As discussed already earlier, computational burden is then halved compared to calculating (8) for each mirror subcarrier pair.

III. I/Q Imbalance Estimation

In the following, efficient pilot-based imbalance parameter estimators are proposed. An underlying assumption in the formulations is that the joint imbalance/channel response stays constant for the duration of two consecutive multicarrier symbols. This assumption is not restrictive in most practical scenarios with low to medium mobility, since the coherence times in current and future mobile systems are anyway in the order of tens of multicarrier symbols.

A. Calculation of the Raw Estimates

First, we write (6) in another, more convenient form as

$$\begin{bmatrix} X_k \\ X_{-k}^* \end{bmatrix} = \begin{bmatrix} S_k & S_{-k}^* & 0 & 0 \\ 0 & 0 & S_k & S_{-k}^* \end{bmatrix} \begin{bmatrix} G_{1,k}H_k \\ G_{2,k}H_{-k}^* \\ G_{2,-k}^*H_k \\ G_{1,-k}^*H_{-k}^* \end{bmatrix} + \begin{bmatrix} N_k' \\ N_{-k}'^* \end{bmatrix}. \tag{13}$$

The above matrix equation is clearly not uniquely solvable. However, by considering two consecutive multicarrier symbols, (13) can be extended to

$$\begin{bmatrix} X_k^{(1)} \\ X_{-k}^{(1)*} \\ X_k^{(2)} \\ X_{-k}^{(2)*} \end{bmatrix} = \begin{bmatrix} S_k^{(1)} & S_{-k}^{(1)*} & 0 & 0 \\ 0 & 0 & S_k^{(1)} & S_{-k}^{(1)*} \\ S_k^{(2)} & S_{-k}^{(2)*} & 0 & 0 \\ 0 & 0 & S_k^{(2)} & S_{-k}^{(2)*} \end{bmatrix} \begin{bmatrix} G_{1,k}H_k \\ G_{2,k}H_{-k}^* \\ G_{2,-k}^*H_k \\ G_{1,-k}^*H_{-k}^* \end{bmatrix} + \begin{bmatrix} N_k'^{(1)} \\ N_{-k}'^{(1)*} \\ N_k'^{(2)} \\ N_{-k}'^{(2)*} \end{bmatrix} \text{ or}$$

$$\bar{\mathbf{x}}_k = \mathbf{S}_k\mathbf{g}_k^{\mathrm{tot}} + \bar{\mathbf{n}}_k, \tag{14}$$

where superscript (*i*), *i*=1,2 denotes the multicarrier symbol (time) index. Now, let the pilot symbols in the *first* OFDM symbol ($S_k^{(1)}$ and $S_{-k}^{(1)}$) be randomly taken from some alphabet. If we then choose the symbols in the *second* OFDM symbol properly, e.g., as

$$S_k^{(2)} = S_k^{(1)} \text{ and } S_{-k}^{(2)} = -S_{-k}^{(1)}, \tag{15}$$

the data matrix in (14) can be guaranteed to invert. The estimate of the parameter vector can then be obtained as

$$\hat{\mathbf{g}}_k^{\mathrm{tot}} = \mathbf{S}_k^{-1}\bar{\mathbf{x}}_k \tag{16}$$

The vector $\hat{\mathbf{g}}_k^{\mathrm{tot}}$ contains estimates of the elements of the imbalance/channel matrix $\mathbf{G}_k^{\mathrm{tot}}$. $\mathbf{G}_k^{\mathrm{tot}}$ or its inverse $(\mathbf{G}_k^{\mathrm{tot}})^{-1}$ can now be reconstructed from $\hat{\mathbf{g}}_k^{\mathrm{tot}}$ and used in (8) for the joint imbalance/channel equalization, or as a basis for joint ML detection. Alternatively, channel and imbalance can be equalized successively, if their estimation can be decoupled, as was discussed in Section II C. Such a decoupled estimation method will be described next.

From (16), assuming no noise, it is straightforward to solve the compensator coefficients for the mirror subcarrier pair as

$$\begin{aligned} \hat{W}_k &= -\frac{\hat{\mathbf{g}}_k^{\mathrm{tot}}(2)}{\hat{\mathbf{g}}_k^{\mathrm{tot}}(4)} = -\frac{G_{2,k}H_{-k}^*}{G_{1,-k}^*H_{-k}^*} = -\frac{G_{2,k}}{G_{1,-k}^*} = W_k^{\mathrm{id}} \\ \hat{W}_{-k} &= -\left(\frac{\hat{\mathbf{g}}_k^{\mathrm{tot}}(3)}{\hat{\mathbf{g}}_k^{\mathrm{tot}}(1)}\right)^* = -\frac{G_{2,-k}H_k^*}{G_{1,k}^*H_k^*} = -\frac{G_{2,-k}}{G_{1,k}^*} = W_{-k}^{\mathrm{id}} \end{aligned}. \tag{17}$$

Thus, the ideal imbalance compensator coefficients are obtainable. The estimates of the modified channel response

$H_k^{\mathrm{m}} = G_{1,k}H_k$ are also obtained directly from $\hat{\mathbf{g}}_k^{\mathrm{tot}}$ as $\hat{H}_k^{\mathrm{m}} = \hat{\mathbf{g}}_k^{\mathrm{tot}}(1)$ and $\hat{H}_{-k}^{\mathrm{m}} = \hat{\mathbf{g}}_k^{\mathrm{tot}}(4)^*$. Thus, imbalance and channel estimation can indeed be *decoupled*.

The pilot symbol structure in (14)-(15) could be realized by transmitting two consecutive training/preamble symbols with the given structure, or by making the possible scattered pilot symbols (assuming they are on mirror subcarriers) to follow the structure. It is also possible to add more multicarrier symbols to the matrix equation (14), if the parameter vector can be assumed to stay more or less constant over the corresponding time interval. In this case, the structure of the data matrix would not be restricted by (15). The solution to this kind of an over-determined matrix equation is essentially the least-squares (LS) solution of the parameter vector. This formulation was used in [4], but in the *frequency-independent* I/Q imbalance case. Using more data in the estimation stage of course entails better noise averaging, but also increases the computational burden as well as the latency of the estimator. Furthermore, in mobile channels, assuming the parameter vector (i.e., the channel) to stay constant for longer periods of time may be unrealistic.

B. Coefficient Estimate Smoothing

Here we derive efficient smoothing transformations for the raw compensator estimates in (17). As was briefly discussed earlier, additive noise is disturbing the estimation. The raw estimates of the parameters may in fact be quite noisy and unreliable as such, due to the relatively small amount of data (two OFDM symbols) used in the estimation. However, the impulse response lengths of $g_1(t)$ and $g_2(t)$ are in practice quite restricted, since the frequency-dependence of true I/Q imbalances is relatively smooth. This indicates that considerable coefficient estimate smoothing is possible in the frequency domain, as will be described in the following.

Let us define the frequency-domain *raw estimate vector* as

$$\hat{\mathbf{W}}_{\mathrm{raw}} \triangleq [\hat{W}_0, \hat{W}_1, \ldots, \hat{W}_{N/2-1}, \hat{W}_{-N/2}, \hat{W}_{-N/2+1}, \ldots, \hat{W}_{-1}]^{\mathrm{T}}, \quad (18)$$

where *N* is the number of subcarriers. Then, assume that the raw estimate curve springs from a time domain impulse response of limited length, say *M*, plus additive zero-mean noise. The data model for the frequency-domain raw estimate vector in (18) is therefore

$$\hat{\mathbf{W}}_{\mathrm{raw}} = \mathbf{F}_M \mathbf{w} + \boldsymbol{\varepsilon} \quad (19)$$

where $\boldsymbol{\varepsilon}$ is a zero-mean noise component, and the matrix $\mathbf{F}_M$ is comprised of the first *M* columns of the *N*-by-*N* discrete Fourier transform (DFT) matrix, formally given as

$$\mathbf{F}_M = \left\{ \exp(-j\tfrac{2\pi mn}{N}) \right\}_{n=0,\ldots,N-1, m=0,\ldots,M-1}.$$

Now, the objective is to find the impulse response vector $\mathbf{w}$ (of length *M*) which minimizes the sum of errors squared, i.e.,

$$\min_{\mathbf{w} \in \mathbb{C}^M} \left\| \hat{\mathbf{W}}_{\mathrm{raw}} - \mathbf{F}_M \mathbf{w} \right\|^2. \quad (20)$$

The solution to the LS problem in (20) is well known, and corresponds to multiplying (19) from the left by the pseudo-inverse of $\mathbf{F}_M$, yielding the impulse response (of length *M*)

$$\hat{\mathbf{w}}_{\mathrm{LS}} = \left(\mathbf{F}_M^{\mathrm{H}} \mathbf{F}_M \right)^{-1} \mathbf{F}_M^{\mathrm{H}} \hat{\mathbf{W}}_{\mathrm{raw}} = \tfrac{1}{N} \mathbf{F}_M^{\mathrm{H}} \hat{\mathbf{W}}_{\mathrm{raw}}. \quad (21)$$

Equation (21) actually corresponds to calculating the full inverse DFT of $\hat{\mathbf{W}}_{\mathrm{raw}}$, and then truncating the result to *M* taps. The smoothened frequency-domain coefficients are then obtained by multiplying once again from the left by $\mathbf{F}_M$, as

$$\hat{\mathbf{W}}_{\mathrm{LS}} = \mathbf{F}_M \hat{\mathbf{w}}_{\mathrm{LS}} = \tfrac{1}{N} \mathbf{F}_M \mathbf{F}_M^{\mathrm{H}} \hat{\mathbf{W}}_{\mathrm{raw}} = \mathbf{S}_{\mathrm{LS}} \hat{\mathbf{W}}_{\mathrm{raw}}, \quad (22)$$

where $\mathbf{S}_{\mathrm{LS}} = \tfrac{1}{N} \mathbf{F}_M \mathbf{F}_M^{\mathrm{H}}$ is now defined the frequency-domain LS smoothing matrix.

In practice, some of the raw compensator estimates in (17) and (18) may be badly biased due to bad channel and/or noise. These poor estimates are more likely obtained on those subcarriers with a *weak mirror subcarrier*, as is evident from the definitions in (17). In other words, $\hat{W}_k$'s get more unreliable as H_{-k}^* 's become smaller, since received symbols on subcarrier *–k* become more noisy. When H_{-k}^* is small, the denominator in (16) ($\mathbf{g}(4)$) can be close to zero, yielding huge outliers which will bias the LS solution. Another problem with LS smoothing is that it has no mechanism to deal with possible *null subcarriers*, i.e., the empty subcarriers for which estimates are not available. These observations lead us to a weighted LS solution, which will be described next.

From the above discussion it is obvious that the weighting function should be such that it gives more weight to those raw estimates $\hat{W}_k$ which have a strong mirror carrier. Further, it should give very small weight to the null subcarriers. Thus intuitively, the simplest weighting function is the frequency-mirrored power spectrum of the received signal. The power spectrum can be estimated for subcarrier *k*, e.g., from the two received training/pilot symbols as

$$P_k = \frac{1}{2}\left(\left| X_k^{(1)} \right|^2 + \left| X_k^{(2)} \right|^2 \right). \quad (22)$$

Some more sophisticated spectrum estimation methods could also be used, but this choice is both simple and effective. Then we define the diagonal weighting matrix as

$$\mathbf{P} \triangleq \mathrm{diag}(P_0, P_{-1}, \ldots, P_{-N/2+1}, P_{-N/2}, P_{N/2-1} \ldots, P_1). \quad (23)$$

The original LS problem is transformed into the weighted LS problem by multiplying (19) from the left by the square-root of the weighting matrix, i.e.,

$$\min_{\mathbf{w} \in \mathbb{C}^M} \left\| \mathbf{P}^{1/2} \hat{\mathbf{W}}_{\mathrm{raw}} - \mathbf{P}^{1/2} \mathbf{F}_M \mathbf{w} \right\|^2, \quad (24)$$

where $\mathbf{P}^{1/2} = \mathrm{diag}(\sqrt{P}_0, \ldots \sqrt{P}_{-N/2}, \sqrt{P}_{N/2-1}, \ldots, \sqrt{P}_1)$. The solution is obtained similarly as in (21), by multiplying the transformed set of equations from the left by the pseudo-inverse

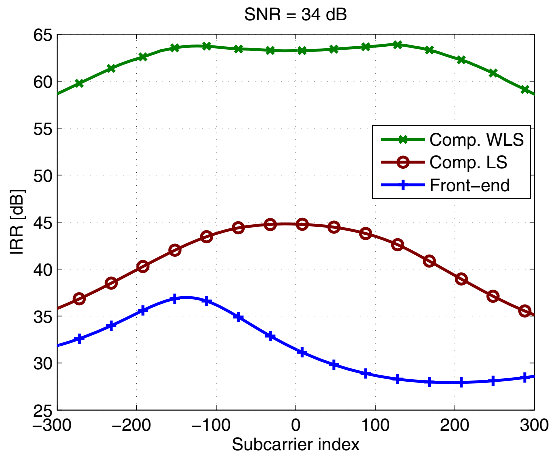

Fig. 1. Uncompensated and compensated image rejection ratios with 600/1024 loading, 64-QAM modulation, and SNR=34 dB.

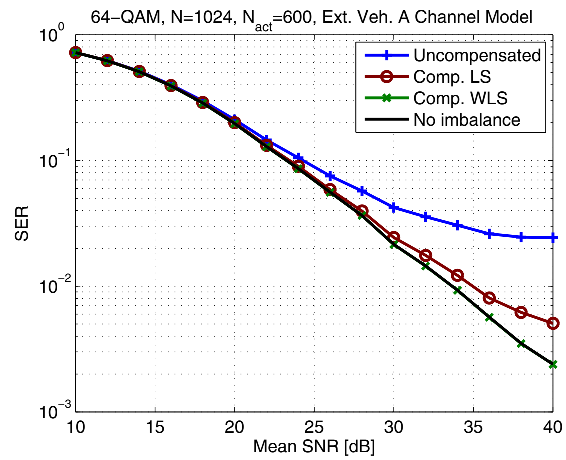

Fig. 2. Uncoded symbol error rates for 64-QAM OFDM with 600 active subcarriers out of a total of 1024. Extended Vehicular A channel model.

of $\mathbf{P}^{1/2}\mathbf{F}_M$, yielding the impulse response

$$\hat{\mathbf{w}}_{\mathrm{WLS}} = \left(\mathbf{F}_M^{\mathrm{H}}\mathbf{P}\mathbf{F}_M\right)^{-1}\mathbf{F}_M^{\mathrm{H}}\mathbf{P}\hat{\mathbf{W}}_{\mathrm{raw}}\,. \tag{25}$$

The frequency-domain coefficients are again obtained as

$$\hat{\mathbf{W}}_{\mathrm{WLS}} = \mathbf{F}_M\left(\mathbf{F}_M^{\mathrm{H}}\mathbf{P}\mathbf{F}_M\right)^{-1}\mathbf{F}_M^{\mathrm{H}}\mathbf{P}\hat{\mathbf{W}}_{\mathrm{raw}} = \mathbf{S}_{\mathrm{WLS}}\hat{\mathbf{W}}_{\mathrm{raw}}\,, \tag{26}$$

where $\mathbf{S}_{\mathrm{WLS}} = \mathbf{F}_M\left(\mathbf{F}_M^{\mathrm{H}}\mathbf{P}\mathbf{F}_M\right)^{-1}\mathbf{F}_M^{\mathrm{H}}\mathbf{P}$ is the WLS smoothing matrix.

IV. Simulation Results

The performance of the proposed I/Q imbalance compensation schemes is assessed through link-level simulations. In the first simulation, image rejection ratio (IRR) is simulated in an OFDM-based system with 64-QAM modulation in the Extended Vehicular A channel model [8]. Channel is assumed quasi-static, i.e., constant for two consecutive OFDM symbols. Subcarrier spacing is 15 kHz, and out of the total 1024 subcarriers, 600 are active. Mean SNR is 34 dB, which corresponds to a SER of 10^{-2} without I/Q imbalance (see Fig. 2). The IRR of the analog I/Q stages is frequency-selective, and varying between 28-37 dB within the signal band (bottom curve in Fig.1.). As evidenced by Fig.1, the IRRs are clearly improved by using the proposed algorithms. WLS smoothing performs especially well, improving the IRR by 25-35 dB within the signal band. The big difference between WLS and LS is partly due to the ability of WLS to put zero weights on the empty subcarriers in the spectrum edges.

Fig. 2 presents simulated SERs of the system described above. The WLS smoothing technique is performing very well. It is able to push the mirror-frequency interference below the noise level at all SNR, evidenced by the overlapping of the "no imbalance" and the "WLS compensation" curves. The LS technique is also able to clearly improve the situation with respect to the uncompensated case.

V. Conclusions

In this paper, two novel, efficient techniques were developed for estimating and compensating frequency-selective receiver I/Q imbalances in OFDM-based systems. The methods are fast, giving reliable estimates of the imbalance parameters in two OFDM symbols with the proposed pilot design. The weighted least squares based estimator performs especially well, being able to compensate for all the significant effects of I/Q imbalance.

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