A STARMA Model for Wind Power Space-Time Series

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***Abstract*--This paper had analyzed and modeled the wind power characteristic from the perspective of space-time series. Firstly, measured data of wind power had been analyzed for the coupled spatial-temporal correlation. Then, a spatial relation matrix, which was used to describe the location of wind farms, had been embedded into the Space-Time Auto Regressive Moving Average (STARMA) model in order to reflect the spatial-temporal correlation of multi-dimensional wind power series. Simulation results showed that this model has restored not only temporal autocorrelation, but also time shifting characteristic of spatial correlation of the original wind power series, which essentially reflected the coupled spatial-temporal characteristic of real wind power series. This model can be used to produce huge amount of simulated wind power data, which have same characteristics with real wind power data, and can provide basics for the planning and operation of wind power integration systems.**

***Index Terms*--wind power, space-time series, STARMA, spatial-temporal correlation**

I. INTRODUCTION

As characteristics of wind power have great differences from other conventional power sources, it may bring new challenges on the planning and operation of power systems. Generally, the Monte Carlo (MC) is one of the most common used method in wind power integration researches, which simulate a large amount of wind power scenarios to calculate the statistical index of power system operation [1], [2]. As wind power model is the basis of this kind of research, and has great impacts on the simulation results of power systems, wind power modeling has been the fundamental work in wind power integration studies.

Currently, analyzing and modeling of wind power characteristics have mainly adopted the statistical method based on historical data. By using the probability distribution, spatial-temporal correlation and other statistical indicators, wind power characteristics can be restored [3]. Among the common wind power statistical models, time series model is most extensively used in wind power integration study due to the excellent simulation performance for time correlation [4].

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However, the existing wind power time series model is mainly based on one dimension series, and only the autocorrelation of wind power in time dimension is restored. When multiple adjacent wind farms are connected to the same regional power system, spatial correlations are also significant among the power outputs of different wind farms. Therefore, temporal and spatial correlation should be simultaneously taken into account in the modeling of multi-dimensional wind power series. Spatial correlation is expressed in different ways in multi-dimensional time series modeling: Paper [5] uses the correlation coefficient matrix to simulate spatial correlation, and the simplified Vector Autoregressive model (VAR) to establish a model for multi-dimensional wind power series. In paper [6], power series of each wind farm is established by Markov chain model, and spatial correlation between wind farms is described by the Coupla function. Paper [7] uses the Principal Component Analysis (PCA) method to generate uncorrelated space base vectors of the original multi-dimensional power series, and then spatial correlation between each dimension of the wind power series is implicitly restored by the linear combination of these base vectors.

It is necessary to point that although the above methods realizes the restoration of spatial and temporal correlation in different ways, but they all ignore the variation properties of spatial correlation at time dimension. In other words, they only consider the spatial correlation of wind power series on one time section, ignoring the coupling characteristics of these two kind of correlations. In order to comprehensively reflect the impacts of wind power on power systems, coupled spatial and temporal correlation in multi-dimensional wind power series should be considered.

Space-Time Autoregressive Moving Average (STARMA) model is developed from the Autoregressive Moving Average (ARMA) model by embedding the description of spatial location in the autoregressive process of space-time series [8]. As ARMA model has already been widely used in the temporal correlation modeling for wind power [9], STARMA model is applied in this paper to restore the coupled spatial-temporal correlation of wind power series to illustrate its applicability. With this model, characteristics accounted in wind power integration studies can be extend from temporal correlation to coupled spatial-temporal correlation.

This paper is organized as follows: Spatial-temporal correlations are analyzed comprehensively with measured wind power data in section II, and the STARMA model details along with its modeling procedure are briefly described in section III. An application case with real data from wind farms is then conducted in section IV, and finally conclusions are addressed Section V.

II. COUPLED SPATIAL-TEMPORAL CORRELATION

A. Wind Power Data Description

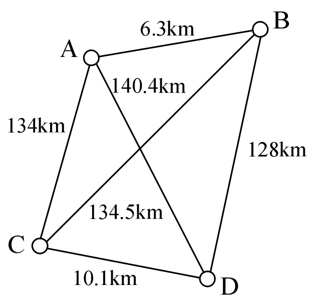

Figure 1. Location of wind farm A, B, C and D

Measured wind power data of four real existing wind farms (named as A, B, C and D) in China are used for wind power analyzing and modeling in this paper. The relative location of these wind farms is shown in figure 1, and wind power series of each wind farm is shown in figure 2. For simplicity, time horizon of the wind power data is one month and time granularity is 15 minutes.

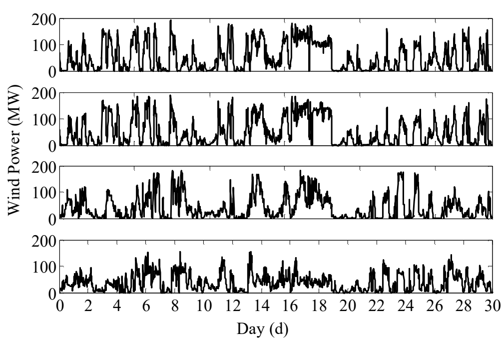

Figure 2. Measured power series of each wind farm in June, 2013

B. Coupled Spatial-Temporal Correlation Analysis

As indicated in figure 2, the tendencies of four wind power series are intuitively correlated with each other. Obviously, this kind of correlation is caused by the spatial relationship of each wind farm. For further analysis on the spatial and time correlation of wind power space-time series, autocorrelation and cross-correlation functions represented by equations (1) and (2) are used [10]. As autocorrelation and cross-correlation functions can be easily calculated, they have been extensively used in time series analysis. But it is necessary to point out that being different from the commonly used cross-correlation coefficient, cross-correlation function calculate the correlation coefficients for space-time series with a set of time lags, which therefore can reveal the variation pattern of cross-correlations over time dimension.

$$R_{xx}(k) = \frac{\mathrm{E}[(x(\mathrm{t}) - \overline{x(\mathrm{t})}) \cdot (x(\mathrm{t}+k) - \overline{x(\mathrm{t})})]}{\sigma_{x(\mathrm{t})}^2} \tag{1}$$

$$R_{xy}(k) = \frac{\mathrm{E}[(x(\mathrm{t}) - \overline{x(\mathrm{t})}) \cdot (y(\mathrm{t}+k) - \overline{y(\mathrm{t})})]}{\sigma_{x(\mathrm{t})} \cdot \sigma_{y(\mathrm{t})}} \tag{2}$$

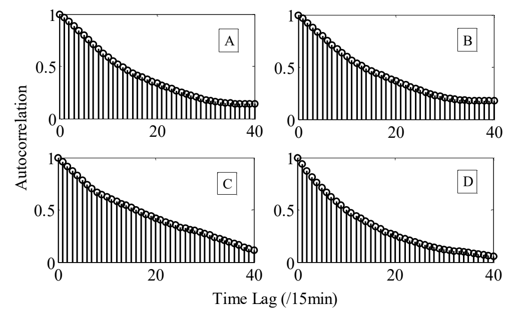

Figure 3. Autocorrelation curve of measured power series

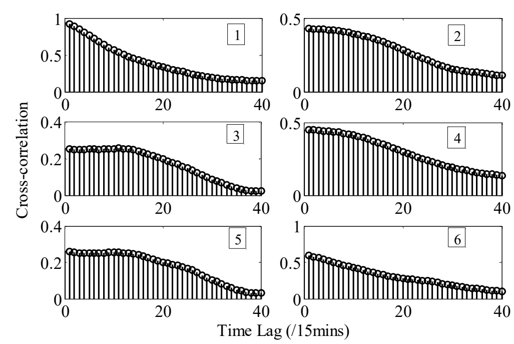

Figure 4. Cross-correlation curve of measured power series

Autocorrelation and cross-correlation curves of measured wind power data are calculated and shown in figure 3 and 4. In figure 3, autocorrelation curves of each wind power series gradually decrease over time, which fits the research results for single wind farm in [11]. For spatial correlation, figure 4 shows the cross-correlation between each pair of the four wind farms (subfigures 1-6 represent for cross-correlations between wind farm A and B, A and C, A and D, B and C, B and D, C and D, respectively.). As wind farm A and B, C and D have a relatively close spatial distance, the corresponding cross-correlation values are larger than other values in figure 5. Besides, with the increase of time lags, the cross-correlation values between each pair wind farms are attenuated.

In conclusion, specific variation patterns over time dimension does exist in the spatial correlation of wind farms, and these variation patterns reflects the essential characteristics of coupled spatial-temporal correlation of wind power. This kind of correlation characteristic has complex effects on the operation of power system. Therefore, the coupled spatial-temporal correlation between the wind power outputs of each wind farm become strong when these wind farms are close on geographical location, and it is necessary to analyze and model these wind farms as a whole.

III. PRINCIPLE OF STARMA MODEL

Space-Time Autoregressive Moving Average (STARMA) model restores the coupled spatial-temporal correlation characteristics of space-time series data in a concise form, and has been successfully applied in the fields of economy, transportation and environment [12], [13]. In order to simulate the coupled spatial-temporal correlation of the multi-dimension wind power series, the STARMA model is introduced into the modeling process of wind power.

A. The Spatial Weight Matrix

The STARMA model uses spatial relation matrix to quantitatively measure the spatial proximity of space-time series. Assuming there are N polygons in the study area, and there is a spatial relation between any two polygons, then a total of N*N spatial relations are required to store in the spatial relation matrix. The concrete form of spatial relation matrix W is shown in (3):

$$W=\begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ w_{n1} & w_{n2} & \cdots & w_{nn} \end{bmatrix} \tag{3}$$

According to different rules for spatial relations, various forms of spatial relation matrix can be defined, in which spatial adjacency matrix and spatial weight matrix are most common used. In order to describe the spatial relations more extensively, the hierarchy of spatial neighborhood should also be considered. First-order neighborhood refers to those areas closest to the target space unit, named as space units with time lag 1. In this way, second-order and other higher order neighborhood can also be classified. In addition, each space unit is defined as its zero order neighborhood, and the zero order space relation matrix can be defined as unit matrix.

In this work, the spatial correlations value between each wind farm are considered to be greatly depended on their distance, therefore the spatial weight matrix defined by spatial distance between wind farms can be used to describe the spatial relationships of wind power space-time series.

B. Space–Time Autocorrelation Function

In the STARMA model, space-time autocorrelation function (STACF) is used to measure the spatial-temporal correlation of time series. According to the definition in [14], STACF can be written as follows:

$$\rho_{h0}(k)=\frac{\gamma_{h0}(k)}{\sigma_h(0)\cdot\sigma_0(0)}=\frac{\mathrm{Cov}([W^{(h)}\boldsymbol{X}(\mathrm{t})],[W^{(0)}\boldsymbol{X}(\mathrm{t}+k)])}{\sqrt{\mathrm{var}(W^{(h)}\boldsymbol{X}(\mathrm{t}))\cdot\mathrm{var}(W^{(0)}\boldsymbol{X}(\mathrm{t}))}} \tag{4}$$

In above equation, $\rho_{h0}(k)$ represents for STACF value with time lag k and space lag h, while $W^{(h)}$ and $W^{(0)}$ represent for spatial weight matrix with space lag h and 0 respectively. Note that $W^{(0)}$ is a unit matrix. Since the STACF function has two independent variables of time lag and spatial lag, it can be used to characterize the coupled correlations on the dimensions of time and space.

C. STARMA Modeling Procedure

With the definitions of spatial weight matrix and STACF for space-time series, the STARMA model can be written in a form similar to ARMA model as in (5):

$$\boldsymbol{X}(\mathrm{t})=\sum_{k=1}^{p}\sum_{l=0}^{\lambda_k}\varphi_{kl}W_l\boldsymbol{X}(\mathrm{t}-k)-\sum_{k=1}^{q}\sum_{l=0}^{m_k}\theta_{kl}W_l\boldsymbol{\varepsilon}(\mathrm{t}-k)+\boldsymbol{\varepsilon}(\mathrm{t}) \tag{5}$$

In above equation, $\boldsymbol{X}(\mathrm{t})$ and $\boldsymbol{\varepsilon}(\mathrm{t})$ represent for the space-time series and normally distributed random error, respectively. Parameter φ_{kl} represents for the autoregressive parameter at time lag k and space lag h, while θ_{kl} represents for the moving average parameter at time lag k and space lag h. Superscript p is the autoregressive order, and q is the moving average order, while λ_k is the spatial order of the kth autoregressive term and m_k is the spatial order of the kth moving average term.

$$\rho_h(k)=\frac{T}{T-k}\cdot\frac{\sum_{i=1}^{N}\sum_{t=1}^{T-k}[W^{(h)}x_i(t)]\cdot[W^{(0)}x_i(t+k)]}{\sqrt{\sum_{i=1}^{N}\sum_{t=1}^{T-k}[W^{(h)}x_i(t)]^2\cdot\sum_{i=1}^{N}\sum_{t=1}^{T-k}[W^{(0)}x_i(t)]^2}} \tag{6}$$

$$\hat{\rho}_h(k)=\sum_{k=1}^{q}\sum_{h=1}^{m_k}\varphi_{kh}\hat{\rho}_{h-1}(k) \tag{7}$$

Referring to the order identification method of the ARMA model in [15], the autoregressive order p and the moving average order q of STARMA model can also be determined by an examination of STACF and STPACF. The STACF of the sample data can be calculated by formula (6), and STPACF can be obtained by solving the Yule-Waller equations (7). The detailed model identification steps are given in Ref. [16]. With the determined order of STARMA model, model parameters in (5) can be estimated by using the least-square estimation (LSE) and maximum likelihood estimation (MLE), and then the complete STARMA model is obtained.

It can be seen from the structural of STARMA model that spatial relation matrix is coupled in both autoregressive and moving average process, which means that spatial relation will be accounted along with temporal relation in the same way. With this model, the description of spatial correlation for wind power series can be extended from spatial correlation coefficient on single time section to spatial correlation function over time dimension, ,thus restoring the essential characteristics of coupled spatial-temporal correlation for wind power series.

IV. Modeling of Wind Power Space-Time Series

The STARMA model introduced in section III provides a practical method for modeling the coupled spatial-temporal correlation for space-time series. In this section, the STARMA model will be used for modeling the multi-dimensional wind power series. According to the modeling procedures described in section III, spatial weight matrix for wind farms should be constructed first. Application case data described in section II is also used for STARMA modeling in this section, and spatial weight matrix weighted by the distance between wind farms is used for spatial relationship description. More generally, executing row normalization, the first-order spatial weight matrix can be obtained as shown in (8):

$$W^1=\begin{array}{c} \\ \mathrm{A} \\ \mathrm{B} \\ \mathrm{C} \\ \mathrm{D} \end{array}\begin{array}{c}\begin{array}{cccc} \mathrm{A} & \mathrm{B} & \mathrm{C} & \mathrm{D} \end{array} \\ \begin{bmatrix} 0 & 0.916 & 0.041 & 0.043 \\ 0.912 & 0 & 0.045 & 0.043 \\ 0.062 & 0.069 & 0 & 0.869 \\ 0.066 & 0.065 & 0.869 & 0 \end{bmatrix}\end{array} \tag{8}$$

In addition, zero-order space weight matrix for wind farms is taken as a unit matrix, and as the number of wind farms in this case is small, higher order space weight matrixes are not considered in this paper.

A. Wind Power Series Preparation

In a similar way as ARMA model, the STARMA model also iteratively combine the variables in sample series with independent white noise variables to generate the simulated space-time series, causing the STARMA model can only simulate the stationary and normal series. Hence, in the practical modeling of real wind power series with STARMA model, series transformation method should be adopted to stabilize and normalized the original series. Referring to [11], this paper also adopts the normal transformation method by using multiple cumulative distribution function (CDF) curves, and then the transformed normal series of wind power are shown in figure 5. It can be seen that the original series have been transformed into basic stationary series, and verification of the probability distribution function (PDF) also indicates that the transformed series is stationary and normally distributed.

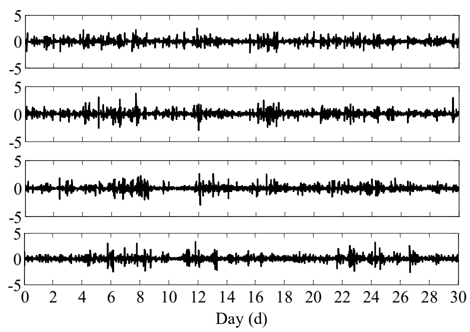

Figure 5. Wind power series after stationary and normally transformation

B. Multisite Wind Power Modeling

After the original wind power space-time series have been transformed into stationary and normal series, the STARMA model can then be used to model the transformed series. The space-time autocorrelation functions (ST-ACF) and the space-time partial correlation functions (ST-PACF) of transformed series are calculated by the equations (6) and (7) with results shown in figure 6.

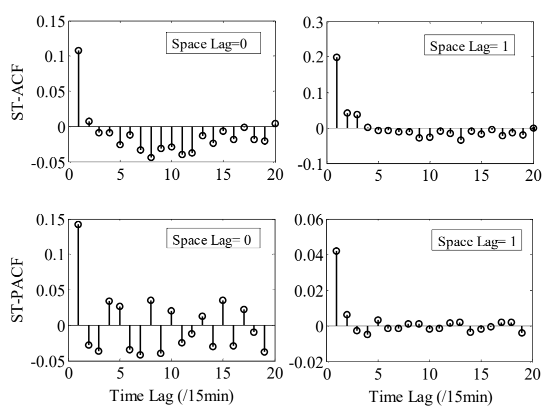

Figure 6. ST-ACF and ST-PACF of measured wind power series

It can be seen from figure 6 that the ST-ACF values of the transformed series appear to cut off at time lag one for space lag zero and one, while the ST-PACF values appear to tail off at time lag one for space lag zero and one. According to the model identification rules in Ref. [16], the form of wind power space-time series model can be preliminarily determined as STARMA(1,1), and detailed formula of this STARMA model can be written as follows:

$$x_i(t) = \varphi_{10} x_i(t-1) + \varphi_{11} W_1 x_i(t-1) - \theta_{10}\varepsilon_i(t-1) - \theta_{11} W_1 \varepsilon_i(t-1) + \varepsilon_i(t) \tag{9}$$

After determining the order of STARMA model, parameters of the model can be estimated with the sample data. In this paper, nonlinear least square estimation (NLSE) [17] is used to estimate the parameters of the model, and the final formula is obtained as follows:

$$x_i(t) = 0.829 \cdot x_i(t-1) + 0.139 \cdot W_1 x_i(t-1) + 0.012 \cdot \varepsilon_i(t-1) + 0.030 \cdot W_1 \varepsilon_i(t-1) + \varepsilon_i(t) \tag{10}$$

Using this model, stationary and normal series can be generated, with the coupled spatial-temporal correlation characteristics of the original data restored. And then these normal series can be transformed to enforce the actual marginal distributions of sample data by the inverse process of normal transformation, getting the final simulated wind power space-time series as shown in figure 7.

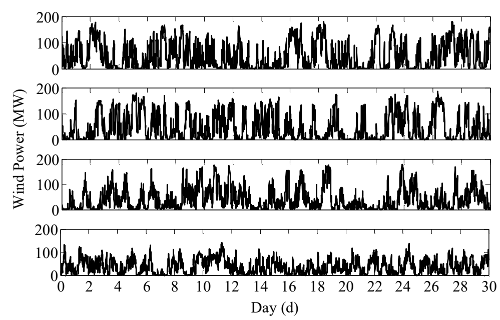

Figure 7. Wind power series after stationary and normally transformation

C. Results Analysis

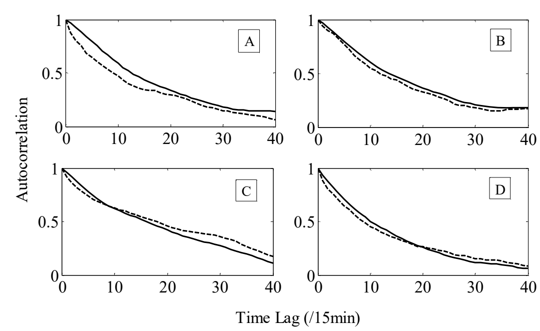

Figure 8. Comparison of autocorrelation for original and simulated series

Comparison results on the statistical index of the original series and the simulated series are shown in figure 8 and 9. For figure 8, the autocorrelation function curves of the simulated wind power space-time series have a good agreement with the sample data, which are consistent with the simulation performance of ARMA model in Ref. [11]. For the restoration of coupled spatial-temporal correlation for wind power series, the cross-correlation function curves of the original series and the simulated series are compared in Figure 9, which also appear to have a good agreement.

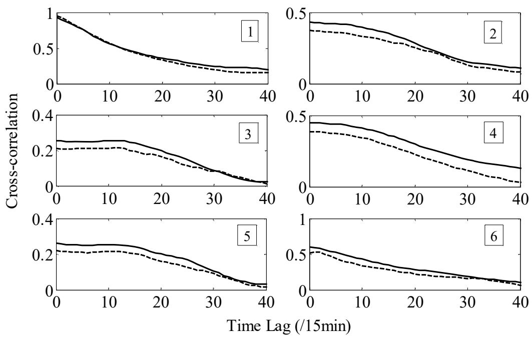

Figure 9. Comparison of cross-correlation for original and simulated series

Comparison results of autocorrelation curves in Figure 8 indicate that the STARMA model has restored the temporal correlation of wind power series very well. In fact, looking into the structure of STARMA and ARMA model, it can be found that the autoregressive and moving average process in the two models are similar, leading to the same restoration performance in temporal correlation. As for spatial correlation in the situation of multiple wind farms, the cross-correlation function curves of the simulated series and original series in Figure 9 are very close, which indicates that the simulated and original wind power space-time series has the same spatial correlation characteristics.

Further analysis on the structure of STARMA model reveals that for the situation of multiple wind farms, the spatial weight matrix is coupled into the temporal correlation modeling process through multiplying with the autoregressive and moving average term, which ensures the variation properties of spatial correlation in the simulated series automatically fit with that in the original series. On the contrary, these variation properties in spatial correlation cannot be guaranteed if each wind farm is model independently. All these results and analysis have demonstrated that the STARMA model accurately simulate the coupled spatial-temporal correlation in the multi-dimensional wind power space-time series in a unified modeling framework.

V. CONCLUSION

In order to restore the coupled spatial-temporal correlation of wind power space-time series, this paper had adopted the space-time autoregressive moving average model (STARMA) into the wind power modeling. Spatial weight matrix was used to reflect the spatial position of each wind farm, and then nested into the autoregressive and moving average process of the model to simulate the coupled spatial-temporal correlation in wind power space-time series. The test results with real measured data show that the proposed wind power series model can not only simulate the temporal correlation, but also ensure the variation properties of spatial correlation between wind farms, meaning that the STARMA model has perfectly restored the coupled spatial-temporal correlation characteristics for wind power space-time series.

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