Data-Driving Robust Method Apply to AC OPF with Uncertain Wind Power

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***Abstract*-- The goal of power system optimal power flow (OPF) is to minimize the line loss while satisfying the system operation constraints. This paper present a data-driving robust method to AC-OPF with uncertain wind power. It proposed a tighter uncertainty set base on available historical data and goodness-of-fit statistics for wind power. The Data-Driving Robust AC-OPF formulation with uncertain wind power was transformed a linear programming problem using SOCP technique and duality properties. Numerical results on the IEEE-14 bus and IEEE-118 bus with uncertain wind power reveals that the proposed method can guarantees the system economy well without loss of robustness as compared to those from a recent robust method.**

***Index Terms*—Data-driving method, Optimal power flow (OPF), robust optimization, uncertainty, wind power.**

I. Nomenclature

$\mathcal{B}$	Set of bus
$\mathcal{L}$	Set of transmission lines
$\mathcal{G}$	Generation unit
$\mathcal{W}$	Wind power unit
$Y \in \mathbb{C}^{\lvert\mathcal{B}\rvert\times\lvert\mathcal{B}\rvert}$	Nodal admittance matrix $Y_{ij} = G_{ij} + jB_{ij}$
V_i	Voltage at bus i $V_i = e_i + jf_i$ $i \in \mathcal{B}$
p_i^g	Real power output of the generator at bus i
q_i^g	Reactive power output of the generator at bus i
$p_i^{w.r}$	Predicted real output of the wind farm at bus i
$p_i^{w.ac}$	Actual output of the wind farm at bus i
C_{2i}'	Coefficient for the evaluation of the expected cost function
γ_i	Rate of change for each unit's output $i \in \mathcal{G}$
ζ	Vector of uncertain parameter
Λ	Covariance matrix of the uncertain prediction errors
Δp_i^w	Difference between the actual value and the predicted values at bus $i \in \mathcal{W}$
N_i	Sample size of the data for the i^{th} parameter
min/max	Minimum/maximum value of a quantity

II. Introduction

THE development and utilization of renewable energy is an inevitable trend in the worldwide. Wind power technology has become more mature and less costly. The operation of the power system has also changed with large-scale uncertainty and intermittent renewable energy access [1]. The optimal power flow problem (OPF) is a large-scale, multi-constrained and nonlinear optimization problem. Traditional method solved the OPF with renewable energy by a deterministic optimization theory [2]. Due to the uncertainty of renewable energy (RES), the deterministic optimal power flow will no longer apply to the analysis of power system operation.

According to the different modeling methods of uncertain variables, the uncertain optimization methods can be divided into three types: stochastic programming, fuzzy programming and robust optimization [3]. Fuzzy programming uses fuzzy variables to describe uncertain factors. Reference [4] uses fuzzy sets to represent constraints and uses fuzzy membership functions to define the satisfaction of constraints. However, the membership function of uncertain factors is often subject to subjective arbitrariness through limited data samples and the experience of decision makers themselves. Stochastic programming has been most extensively explored in the past ten years [5-6]. These studies have shown that the stochastic model can improve the expected performance of OPF decisions under system uncertainties. However, it is difficult to get the probability distribution of intermittent power output accurately by stochastic programming, and a large number of scene samples need to be collected for obtain a relatively reliable probability distribution. This difficulty is usually addressed by advanced scenario selection algorithms [7], and decomposition techniques [8-9].

Robust optimization minimizes the worst-case total cost over all possible realizations within a deterministic uncertainty set. It describes the uncertainty of a variable (rather than a probability distribution) in the form of a "set" so that the constraint can be satisfied if the uncertainty variable takes

978-1-5386-7703-2/18/$31.00 ©2018 IEEE

values of all possible values in a known set. Ben-Tal [10], El-Ghaoui and Bertsimas [11-12] describes the uncertain parameters with ellipsoid sets and ball-box sets. Bertsimas [13] described uncertainty set by an arbitrary norm to improve the conservatism of the solution. Jabr [14-15] modeled the wind power and uncertain demand by adjustable robust optimization framework, proposed an affinely adjustable robust DC-OPF formulation Bai [16] presents a robust optimization-based AC optimal power flow model with uncertain wind power generation which considering transmissions losses and overcome DC power flow may lead to an infeasible OPF solution. Despite this works are well thought out of the uncertainty of renewable energy, the worst-case realization is sometimes too pessimistic for modeling system uncertainties, resulting in over-conservative solutions.

The construction of the uncertainty set directly affects the conservatism of the robust optimal solution. With poorly chosen uncertain sets, robust models may be too conservative or even computationally intractable. As an application of big data, the data-driving optimization theory gets wide attention and research [17-18]. Reference [19] investigate data-driving optimization methods that are suited to dispatch power systems with uncertain balancing reserves provided by load control. In this paper, we attempt to apply the data-driving robust optimization method to the Alternating Current OPF(ACOPF) with wind power where using the full historical-data information and based on goodness-of-fit statistics known in the statistics literature as $\phi -$ divergence to construct the uncertain wind power sets. Then we conduct computational experience to compare the data-driving RO with general RO. Results show that the results of data-driving RO lead to tighter uncertainty sets compared to the existing safe approximation methods.

The reminder of this paper is organized as follow. In section III, the model of ACOPF with renewable wind power is given. Section IV proposes the robust programming with history data and the algorithm structure of constructing an uncertain set with historical data of wind power. Section V carries out numerical simulation to demonstrate the effectiveness of our approach. The conclusion is in section VI.

III. ACOPF with Uncertain Wind Power

Consider a typical power system, having $\mathcal{B}$buses, $\mathcal{L}$ lines, $\mathcal{G}$ dispatchable generators and $\mathcal{W}$ wind power generators. Let $C_i(p_i^g)$ denotes the original cost function of generate i.The power generation consists of the base-point value $p_{i(base)}^g$, which is the output corresponding to nominal conditions, and the change in generation is corresponding to fluctuations in the output of the wind generators [15]. Therefore, the objective function the ACOPF model is:

$$\sum_{i\in\mathcal{G}} C_i\left(p_{i(base)}^g\right) + \sum_{i\in\mathcal{G}} C'_{2i}\gamma_i^2 \quad (1)$$

The participation factor γ_i^2 is the rate of change of the generator output corresponding to the change in total controllable generation which is fixed by equation (2):

$$\frac{(1/C_{2i})}{\sum_{j\in g}(1/C_{2j})} \quad (2)$$

$c'_{2i} = \left[\sum_{j=1}^{\mathcal{W}}\sum_{k=1}^{\mathcal{W}} \Lambda_{(j,k)}\right] \times c_{2i},\ i \in$, the sum of all participation factors should be set to one, where γ is variable.

$$\sum_{i\in\mathcal{G}} \gamma_i = 1 \quad (3)$$

The objective of the deterministic ACOPF is to minimize (1) subject to the constraints (4)-(9):

1) Power flow constraints

$$p_i^g + p_i^{w.r} - p_i^d = G_{ii}(e_i^{\,2} + f_i^{\,2}) + \sum_{j\in\delta(i)}\left[G_{ij}(e_ie_j + f_if_j) - B_{ij}(e_if_j - e_jf_i)\right] \quad (4)$$

$$q_i^g - q_i^d = -B_{ii}(e_i^{\,2} + f_i^{\,2}) + \sum_{j\in\delta(i)}\left[-B_{ij}(e_ie_j + f_if_j) - G_{ij}(e_if_j - e_jf_i)\right] \quad (5)$$

2) Bounds of active power of AC line flow

$$\left|G_{ij}(e_i^2 + f_i^2 - e_ie_j - f_if_j) + B_{ij}(e_if_j - e_jf_i)\right| \le p_l^{max} \quad (6)$$

3) Bounds of voltage and generator

$$(V_i^{min})^2 \le V_i^2 \le (V_i^{max})^2 \quad (7)$$

$$q_i^{g.min} \le q_i^g \le q_i^{g.max} \quad (8)$$

$$p_i^{g.min} \le p_i^g \le p_i^{g.max} \quad (9)$$

The above deterministic ACOPF is a nonconvex nonlinear optimization problem with thousands of buses, generators and loads. The optimal solution is difficult to obtain and easy to fall into local optimum. From reference [20]. The strong second order cone programming (SOCP) relaxation is used to transform nonconvex optimization problem (NP) to a convex linear convex problem. Therefore, the reformulated SOCP-ACOPF is as below:

$$\begin{cases} p_i^g + p_i^{w.r} - p_i^d = G_{ii}c_{ii} + \sum_{j\in\delta(i)}\left[G_{ij}c_{ij} - B_{ij}s_{ij}\right] \\ q_i^g - q_i^d = -B_{ii}c_{ii} + \sum_{j\in\delta(i)}\left[-B_{ij}c_{ij} - G_{ij}s_{ij}\right] \\ \left|G_{ij}(c_{ii} - c_{ij}) + B_{ij}s_{ij}\right| \le p_l^{max} \\ c_{ij} = c_{ji} \\ s_{ij} = -s_{ji} \\ c_{ij}^2 + s_{ij}^2 + \left(\frac{c_{ii}-c_{jj}}{2}\right)^2 \le \left(\frac{c_{ii}+c_{jj}}{2}\right)^2 \\ (7)-(9) \end{cases} \quad (10)$$

IV. Robust ACOPF With Current Approach And Data-Driving Framework

The optimal ACOPF with distributed wind power system is a typical robust optimization problem. Define the vector of decision variables $x = [p_i^g, q_i^g, c_{ii}, c_{ij}, s_{ij}, \gamma]$ and vector of uncertain parameter ζ.For the intuitive representation of data-driving RACOPF (Robust Alternating Current Optimization Power Flow), a general robust optimization model is defined as follow:

$$\begin{cases} \min_{x\in R^n} \sup_{\zeta\in U} f_0(x,\zeta) \\ s.t. \sup_{\zeta\in U} f_i(x,\zeta) \preccurlyeq 0 (i = 1, \dots, m) \end{cases} \quad (11)$$

Then an uncertain ball-box set U is constructed for the wind power.

$$U(p_i^{w.r}, \Delta p_i^w) := \{p_i^w = p_i^{w.r} + \Delta p_i^w : \\ s.t. \|\Delta p_i^w\|_\infty < \Delta\overline{p}_i^w, i \in \mathcal{W} \quad (12)$$

And wind power generation p_i^w vector belonging to the uncertain set $[p_i^{w.r} - r_i\Delta\overline{p}_i^w, p_i^{w.r} + c\Delta\overline{p}_i^w]$ $\|r_i\|$, the constraint (12) could be converted into an equality constraint with uncertain parameters.

$$p_i^{g.min} \le p_i^g + \gamma_i \sum_{i\in\mathcal{W}} \Delta p_i^w r_i \le p_i^{g.max} \quad (13)$$

With the dual norm theory, for the constraint (13), there:

$$\max_{\|r_i\|\leqslant P} \gamma_i \sum_{i\in\mathcal{W}} \Delta p_i^w r_i = \|\gamma_i \sum_{i\in\mathcal{W}} \Delta p_i^w\|_{P^*} \quad (14)$$

Where $\|\cdot\|_{P^*}$ denotes the dual P^* -norm and $1/P + 1/P^* = 1$[25], (13) can be replaced by:

$$p_i^{g.min} \le p_i^g + \|\gamma_i \sum_{i\in\mathcal{W}} \Delta p_i^w\|_{P^*} \le p_i^{g.max} \quad (15)$$

Therefore, the ACOPF for a given ζ can be written in terms with:

$$\text{minimize}_{p_{i(base)}^g, \gamma_i^2} \sum_{i\in\mathcal{G}} C_i\left(p_{i(base)}^g\right) + \sum_{i\in\mathcal{G}} C'_{2i}\gamma_i^2 \quad (16)$$

Subject to:

$$\begin{cases} p_i^g + p_i^{w.r} - p_i^d = G_{ii}c_{ii} + \sum_{j\in\delta(i)}\left[G_{ij}c_{ij} - B_{ij}s_{ij}\right] \\ q_i^g - q_i^d = -B_{ii}c_{ii} + \sum_{j\in\delta(i)}\left[-B_{ij}c_{ij} - G_{ij}s_{ij}\right] \\ \left|G_{ij}\left(c_{ii} - c_{ij}\right) + B_{ij}s_{ij}\right| \le p_l^{max} \\ c_{ij} = c_{ji} \\ s_{ij} = -s_{ji} \\ c_{ij}^2 + s_{ij}^2 + \left(\frac{c_{ii}-c_{jj}}{2}\right)^2 \le \left(\frac{c_{ii}+c_{jj}}{2}\right)^2 \\ (7)-(8),(15) \end{cases} \quad (17)$$

For the robust ACOPF, it merges the steps of estimating the random parameters and finding a solution that remains feasible for any realization of the uncertain coefficients within prescribed uncertainty set. It may be too conservative or even computationally intractable.

A data-driving method for ACOPF problem

The data-driving robust ACOPF with uncertain wind power is to utilize the history data of wind power to construct a well-chosen uncertainty set, whose solutions perform as well or better than other approaches.

Generally, contrast to robust model (17) the data-driving robust is developed to solve optimization problems with uncertain parameters in the following forms:

$$\begin{cases} \min_{x\in R^n} \sup_{\zeta\in U} f_0(x,\zeta) \\ s.t. P_{r.\zeta_{\zeta\in Z}}\left(f_i(x,\zeta) \leqslant 0\right) \geqslant \beta \ (i = 1,\dots,m) \end{cases} \quad (18)$$

Where β is the prescribed probability, and the property of (18) is less stringent than ask $P_{r.\zeta}(\zeta \in Z) \geqslant \beta$. For the giving historical data of wind power Δp_i^w, there are m RDG, scale Δp_i^w to $\left[\Delta \underline{p}_i^w, \Delta \overline{p}_i^w\right] i \in \{1,\dots,m\}$. The support of Δp_i^w is divided into k_i cells, n is equivalent to

$$n = \frac{\Delta \overline{p}_i^w - \Delta \underline{p}_i^w}{0.1} \quad (19)$$

The frequency of cell $j \in k_i$ for the i^{th} uncertain Δp_i^w is denoted by $r_j^{(i)}$. An aggregate cell is indexed by $(j_1, j_2, \dots, j_m)$, where $j_i \in k_i$ for all $i \in \{1,\dots,m\}$ is denoted C_j. And because of the independence, the probability of the uncertain parameters in cell $(j_1, j_2, \dots, j_m)$ is equivalent to

$$r_{j_1,j_2\dots,j_m} = \prod_{i=1}^m r_{j_i}^{(i)} \ \forall j_i \in k_i,\ \forall i \in \{1,\dots,m\} \quad (20)$$

And the frequency of cell $(j_1, j_2, \dots, j_m)$ is given by:

$$\ell_{j_1,j_2\dots,j_m} = \prod_{i=1}^m \ell_{j_i}^{(i)} \quad (21)$$

Then construct a $(1-\alpha)$ confidence set for r using the empirical estimate ℓ and goodness-of-fit.

$$\begin{cases} r_j^{(i)} \in \mathbb{R}^{k_1k_2\dots k_m}: r_j^{(i)} \ge 0, \sum_{i=1}^m r_j^{(i)} = 1, (17), \forall i \in \{1,\dots,m\} \\ I_\emptyset(r,\ell) \le \frac{\emptyset''(1)}{2N_1N_2\dots N_m}\chi^2_{(k_1-1)(k_2-1)\dots(k_m-1),1-\alpha} \end{cases} \quad (22)$$

For the problem (16), we would choose C_S that approximate the uncertainty region C_j.The uncertainty region for all the cells in $S \in k_i$ is giving by

$$C_S = \cup_{j\in S} C_j \quad (23)$$

Let Z be C_S and x be any feasible solution for the approximation (16), and we have

$$P_{r,\zeta}\left(f_i(x,\zeta) \le 0\right) \ge \mu(S,\alpha)\ (i = 1,\dots,m) \quad (24)$$

Satisfy the confidence level of $(1-\alpha)$, $\mu(S,\alpha)$ is defined as:

$$\begin{cases} \mu(S,\alpha) = \sum_{(j_1,j_2,\dots,j_m)\in S} r_{j_1,j_2\dots,j_m} \\ s.t.\ (22) \end{cases} \quad (25)$$

The aim is to find tight S that approximate the uncertainty region by C_S such that $\mu(S,\alpha) \ge \beta$, and (22) holds with a $(1-\alpha)$ confidence level. Then the data-driving robust optimization algorithm is presented in below:

Algorithm: data-driving robust optimization algorithm

1:**Initialization:** set probability bound β, uncertain set Z is an ball-box set, $Z = \{\Delta p_i^w \in R^l, \|\Delta p_i^w\|_\infty < \Delta \overline{p}_i^w, \|\Delta p_i^w\|_2 < \varepsilon\}$, and ε=0.confidence region $(1-\alpha)$, choose a iteration step size τ, historical data of Δp_i^w, probability bound β, initial ball uncertain set parameters ω

2:Solve the robust counterparts of the giving problem with the ellipsoid uncertain set and find the optimal solutions x^*

3:Calculate a uncertain region C_S

4:Calculate the safe approximation of constraint (16) with a better probability bound by using uncertain C_S replace Z.

5:Calculate $\mu(S,\alpha)$

if $\mu(S,\alpha) \ge \beta$ then $Z = \{\zeta \in [-1,1]: f_i(x^*,\zeta) \leqslant 0\}$

and terminate the algorithm

else go to step 2

6:Set ω = ω + τ and go to step 1.

V. Case Study And Numerical Results

In this section, all simulations are done with a personal computer with 2.4-GHz CPU and 8 GB of RAM. The proposed method is implemented in MATLAB R2014a by using YALMIP R20150626 package as a modeling software and GUROBI 7.0[21] as a solver.

The formulation analysis in this section is divided into three parts, section A is reported first to show the results of this proposed method with different values of the probability bound β. Section B reports comparison of optimal decision in DDOPF with forecast wind power, RCACOPF with general norms and RACOPF use method of this paper. The security and robustness comparison results of the data-driving formulation with other method are given in section C.

A RACCOPF with history data

Two tests that the IEEE 14-bus and the IEEE 118-bus system are used for simulation, and the data of the test system are derived from the file of MATPOWER[22].The historical data of wind power forecast and actual output are provided by an Irish National Power Grid Corp, was recorded from September 2014 to July 2015. The IEEE 14-bus network has two wind power plants at node 7, 11 with each having a capacity of 40MW. The IEEE 118-bus network has two wind farms at node 21, 87, 105 with each having a capacity of 300MW. The wind power penetrations relative to the total load for the IEEE 14-bus and IEEE 118-bus are 29.87%, 25.02% respectively.

The power base of the IEEE14-buse system is 100MW, and the uncertain values of difference between actual and predicted $\Delta p_i^w \in [-0.7,0.7]$. The number of observations in the associated cells and the sample size is 40993, hence the frequency of a cell can be calculated by dividing the number of observations. The associated cell to be the sample size in TABLE I.

TABLE I
FREQUENCIES OF THE DEVIATION OF WIND POWER

Δp_i^w	14-bus	
	freq. (Δp_1^w)	freq. (Δp_2^w)
[-0.7 -0.6]	0.0093	0.01
[-0.6 -0.5]	0.0172	0.0197
[-0.5 -0.4]	0.0274	0.0291
[-0.4 -0.3]	0.0493	0.0462
[-0.3 -0.2]	0.0762	0.0693
[-0.2 -0.1]	0.1202	0.1165
[-0.1 0]	0.1785	0.1822
[0 0.1]	0.2384	0.2445
[0.1 0.2]	0.1325	0.1566
[0.2 0.3]	0.071	0.0631
[0.3 0.4]	0.0365	0.0326
[0.3 0.5]	0.0214	0.0174
[0.5 0.6]	0.0135	0.0081
[0.6 0.7]	0.0086	0.0047

The joint uncertainty set of Δp_1^w and Δp_2^w has 196(14×14) cells and the frequency of a cell is found by multiplying the frequencies of the associated intervals by Δp_1^w, Δp_2^w. For 118-bus, there are three uncertain parameters (Δp_1^w, Δp_2^w, Δp_3^w), the total number of cells in the joint uncertainty space is 2744(10^3). Using the data-driving approximation method to find the tightest uncertain set $\mathcal{Z}$ for different values of the probability bound β and numerical results are listed in TABLE II and TABLE III.

TABLE II
RESULT FOR FORMULATION FOR THE IEEE14-bus

β	Nom	0.6	0.7	0.8	0.9	0.99	fc
BB_ω	0	0.23	0.32	0.41	0.82	0.87	1
$\mu(S,\alpha)$	0.66	0.66	0.74	0.83	0.93	0.99	1
obj	195. 575	195. 585	195. 657	195. 723	195. 842	195. 912	196. 018

TABLE III
RESULTS FOR FORMULATION FOR THE IEEE118-bus

β	Nom	0.6	0.7	0.8	0.9	0.99	fc
BB_ω	0	0.47	0.58	0..67	0.81	0.89	1
$\mu(S,\alpha)$	0.42	0.65	0.72	0.83	0.91	0.99	1
obj	106. 258	106. 729	107. 281	107. 593	107. 827	108. 031	108. 246

Where in the above TABLE, we use $\chi-$distance as the $\phi-$divergence function when α =0.001.The second column (nom) is a nominal problem, which indicates a tightest uncertain set. Relative to the last column (fc), it is the worst-case solution with full space uncertain set. The results in TABLE II and TABLE III reveal the optimal objective function values with a different β, and we can see the final uncertainty set C_S yields significantly better probability bounds than the robust full space uncertain set.

A. Data-driving RACOPF Versus RACOPF

Table 3 compares the optimal objective function value of the data-driving model and general robust model. Because of considering the uncertainty, data-driving ACOPF model and the general robust ACOPF model are worse than the deterministic ACOPF. In the test system, we using IEEE14-bus network with a 29.87% REG penetration. And in the RACOPF case, robust counterparts of original linear optimization with an ellipsoid uncertain set. And the deterministic ACOPF is where uncertain wind power is determined. And parameter β=0.95, α =0.001. To evaluate the performance of the proposed data-driving ACOPF model, define the index variables as follows:

$$\mathrm{C}_{diff} = \frac{C_{dr}-C_{base}}{C_{dr}} \tag{25}$$

Where C_{base} equal to the objective of data-driven model minus the target of the deterministic model. C_{dr} is the difference between the RACOPF model objective and the data-driving RACOPF model objective. A bigger value of C_{diff} indicates that the data-driving RACOPF has a better economy.

TABLE IV
RESULTS OF DETERMINISTIC ACOPF, DATA-DRIVING ACOPF AND RACOPF

	Deterministic ACOPF	Data-driving ACROPF	r_i	RACOPF	C_{diff}
obj	194.3415	195.7949	20%	196.2847	25.2%
			30%	196.4829	32.1%
			50%	196.8924	43.1 %
			70%	197.2615	50.2%
			90%	197.5721	55.1%
			100%	197.9847	60.1%

As show in TABLE IV, the value of the objective function gradually increase with the increase of r_i.When parameter $r_i =$ 100% , uncertain sets contain all possible values of the uncertain wind power, and obtain the most uneconomical feasible solutions. Where data-driving RACOPF yields significantly better economic than the general RACOPF by 60.1% in the most conservative case (r_i =100%). The RACOPF is more robust than the data-driving RACOPF.

B. Safety check

In the section A, we could obtain a scheduling decision. We use operation data to check the robustness and economy of decision making in section B. that is, the output data of wind power generation is known. We use RACOPF and data driven RACOPF's decision to carry out power flow calculation. The convergence of power flow calculation indicates that the model's decision can satisfy all the stability and stability constraints. The higher convergence rate indicates that the model can better deal with the uncertainty of wind power output.

In this section, the prediction error of wind power is assumed to be normal distribution, and Monte Carlo method is used to simulate the randomness of wind power active power output. 5000 scenarios of the uncertain wind power generation are generated by using a multivariate normal distribution with zeros means and a covariance matrix $\Lambda_{wind} = \Lambda * (\Delta p_i^w r_i)^2$ [16]. Then the power flow is calculated with the optimal decision quantity as the fixed value. In 5000 sets of experimental samples, the test group for optimal power flow convergence showed no violation of any constraints, defined as a successful test group, and the success rates are equal to the number of successful test groups /5000.

TABLE V gives a comparison of the success rates of the data-driving RACOPF and the RACOPF in the case of r_i=0~100%. We can see the success rates of ROPF increased with the parameter r_i, and there is a maximum uncertain set contained all possible values of the uncertain wind power, which could maintain 100% success rate. But according to the Table 3, when r_i=100%, it has the most uneconomical solution. Where in the data-driving RACOPF method, we set β=0.99, α =0.001. The optimal decision based on the data-driving RACOPF model is almost free from the disturbance of uncertain wind power. It could see that the data-driving RACOPF model has a good convergence rate without loss of economy, which means that the model is well balanced with the economy and the safety of the system. Compared with the RACOPF model, the data driven RACOPF has good economy on the premise of ensuring the security and stability of the system.

TABLE V
PERCENTAGE SUCCESS RATE OF ROPF WITH DATA-DRIVING ROPF

r_i		0.2	0.4	0.6	0.8	1.0
14	R	69.3%	78.5%	89.2%	93.6%	100%
	D	99.1%				
118	R	68.5%	79.1%	89.6%	93.9%	100%
	D	99.3%				

R:RACOPF; D: data-driving RACOPF; 14:14-bus; 118: 118-bus

VI. CONCLUSION

This paper presents an optimal power flow model of power system with renewable energy based on robust data driven optimization. We use the history data of wind power and goodness-of-fit statistics base on $\phi -$ divergence to construct a tighter uncertain set. The proposed model is transformed using dual norm theory and second-order cone programing, and solved by Start-of-the-Art Mathematical. The results shows a modest increase in expected cost, but it can restore feasibility for all realizations within the wind power output uncertainty set. The numerical results show that it improve the economy when ensure the safe operation of the system. It is superior to choosing uncertain set via ball-box and including them as deterministic constraints in OPF as discussed in recent research. The characteristics of robust optimization that are too conservative are improved.

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