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# Game-Theoretic Models for Generative Learning

## PhD Thesis Proposal

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### Abstract

Machine learning has achieved great success in object recognition and classification problems based on the rapid development of deep neural networks. However, there's still a big gap between computer and human intelligence. Discriminative learning attempts to infer knowledge from data by modeling the conditional distribution of class labels given data samples, while generative learning tries to estimate the full data distribution and synthesizes new samples. It can control the generated samples by changing its class domain or modifying its visual appearance. My thesis will focus on the latter problem and investigate generative learning from a game-theoretic perspective. We first formulate the problem as a distributionally robust game with payoff uncertainty, and then develop a robust optimization algorithm to solve the Nash equilibrium. Meanwhile, we will study the distance metrics that measure the similarity between distributions, which is a major issue in many generative learning problems. We then propose a conditional generative model to solve the style transfer problem in image and speech processing. The disentangled representations of domain-specific style information and domain-invariant content information are modeled by autoencoders and domain classifiers. The encoders, decoders and classifiers form a distributionally robust game with competitive and collaborative agent coalitions. Finally, we plan to test our approach on real-world applications including conditional image synthesis and emotional speech conversion.

## 1 Introduction

In the last decade, machine learning has achieved great success with the rapid development of deep neural networks. Recent algorithms beat humans on ImageNet Challenge [79], the largest benchmark for image recognition. On the other side, people use computers to mimic the creative power of humans. A stream of papers [75][76][77][78][19] claimed their ability to generate lifelike pictures, text and music that can fool the human evaluators as well as machine classifiers. I identify three levels of artificial intelligence: memorization, recognition and creativity. Computers are approaching humans on the first two levels, and now going to the third.

Apart from object recognition and classification, people want to learn the mechanism of data generation and synthesize new samples with desired properties. Discriminative learning tries to infer knowledge from data, while generative learning attempts to learn the full distribution of data and generate new samples. My research focuses on the latter problem.

In probability theory, discriminative models make predictions by learning a conditional distribution  $p(y|x)$ , and generative models synthesize new data by drawing samples from  $p(x)$ . Both distributions are estimated from a limited set of observations, which could be noisy and incomplete. The former problem is easier since  $y$  is usually in the low dimension space. Sometimes people only use a small portion of the data and ignore the other parts. For example in support vector machines (SVM) only points near the decision surface have influence on the classifier. The latter problem is harder because generative models need to estimate the full data distribution  $p(x)$ . Modeling the high-dimensional random variable  $x$  is difficult. As the number of configurations can grow exponentially with the number of dimensions, there are not enough training examples for each dimension. Another concern

42 is the computation challenge; many algorithms involve operations that grow exponentially with the  
43 number of dimensions.

44 Generative models are widely used in unsupervised learning. A major limitation of the current  
45 learning algorithms is that they rely on large amounts of well-labelled data to achieve good accuracy.  
46 However, it is not available in many industrial applications. For this reason, people adapt a pretrained  
47 model on a source dataset to a similar target dataset. The model may perform poorly as it is specialized  
48 to the source domain. Domain adaptation strategies can solve this problem. Let  $x_1 \in X_1$  be source  
49 domain data with associated labels  $y_1$ ,  $x_2 \in X_2$  be target domain data with unknown labels  $f(x_2)$ . A  
50 mapping from the source domain to the target domain can be established by the conditional generative  
51 model  $p_{x_2|x_1}$ . The classifier  $f$  in the target domain is learnt by the category knowledge from the  
52 source domain.

53 Apart from traditional generative models like Gaussian mixture model (GMM) and Naive Bayes,  
54 neural network models exhibit increasing importance in modeling high dimension data. It is amazing  
55 if machines can generate artistic work in painting, music and sculpture. There are three kinds of  
56 dominant approaches in deep generative learning: generative adversarial networks (GAN), variational  
57 auto-encoders (VAE) and autoregressive models. In general, training deep generative models is hard  
58 and time consuming. The high dimensional training data and complex objective structures lead to  
59 many problems in optimization, such as instability, saturation, and mode collapse. Moreover, some  
60 models fail to provide enough diversity in the generated examples or just memorize the training set.

61 In this research, we plan to develop a new game-theoretic framework for generative learning. We  
62 first address the problem of data synthesis. The goal is to produce lifelike artificial examples that  
63 are indistinguishable from the training data. This involves the problem of computing the statistical  
64 distance between two datasets: the original real sample set and the generated fake sample set. Several  
65 distance metrics will be investigated to measure the similarity between distributions. As an important  
66 part of this research, We will compare Wasserstein distance [?] with the most prevalent information-  
67 based metrics such as Kullback-Leibler [?] and Jensen-Shannon divergence [?]. We plan to develop  
68 a practical method to approximately calculate the distance between distributions. Both theoretical  
69 analysis and experimental simulations will be provided.

70 We then propose a conditional generative model to solve the unsupervised style transfer problem in  
71 image and speech processing. The objective is to learn a translation model between domains that can  
72 modify the domain-specific style features and preserve the domain-invariant content information. The  
73 disentangled representations of style and content are modeled by autoencoders and domain classifiers.

74 We formulate the problem as a distributionally robust game with payoff uncertainty. The encoders,  
75 decoders and classifiers form competitive and collaborative agent coalitions in this game. They  
76 optimize towards their self objectives as well as the common interest with other players. Agents  
77 with similar objectives form a group and work together against the others in order to optimize their  
78 expected payoffs. The optimization process is not deterministic since the payoff function of each  
79 player depends on the actions of other players. The distributionally robust Nash equilibrium is  
80 achieved by solving a minimax optimization problem, in which each agent tries to maximize its  
81 worst-case payoff. An iterative learning algorithm is introduced to solve the robust Nash equilibrium.

82 We will first verify the theoretical results through simulations, and then apply our approach on  
83 real datasets of images and speech. The agents are implemented with deep convolution neural  
84 networks to capture the spatial and temporal correlations in high dimension data. We plan to work on  
85 several tasks such as object detection, vehicle tracking, image synthesis and voice conversion. This  
86 research contributes to the areas of distributionally robust game, deep generative learning, stochastic  
87 optimization and time-series data analysis.

88 The following chapters first describes the already completed work: the theory part of distributionally  
89 robust games (Chapter 2), the investigation of distance metrics (Chapter 3), learning algorithms  
90 (Chapter 4). After that the progress on the current work of style transfer is described in Chapter 5.  
91 Finally, the thesis outline and research plan are given in Chapter 6 and Chapter 7.

## 92 2 Distributionally Robust Games

93 One fundamental problem in generative learning is to estimate the full data distribution given finite  
94 noisy observations. Suppose the training samples  $x$  are drawn from a distribution  $m$ , and the  
95 generative model produces new data  $x'$  by sampling from an estimated distribution  $m'$ . The objective  
96 is to minimize the statistical distance between  $m$  and  $m'$  so that the synthesized data is similar to  
97 the real ones. Since the comparison is made on two statistical objects instead of two individual  
98 sample points, [there is no deterministic objective to use](#). In this chapter, we formulate the problem  
99 as a distributionally robust game with payoff uncertainty, and then solve it by stochastic robust  
100 optimization.

101 We first introduce the concept of distributionally robust games, and then define the objective function  
102 based on the statistical notions of  $f$ -divergence between two distributions. The complexity of the  
103 problem is analyzed and reduced by means of triality theory. We propose stochastic Bregman learning  
104 algorithms to solve the robust Nash equilibria. The algorithm is proved to have doubly logarithmic  
105 convergence time with respect to the precision of the minimax value in potential convex games. In  
106 simulation, the theoretical findings are illustrated in convex setting and its limitations are tested with  
107 a non-convex non-concave example. Finally, we apply this approach to train a generative model.

### 108 2.1 Introduction

#### 109 2.1.1 Distribution Uncertainty Set

#### 110 2.1.2 Related Work

### 111 2.2 Problem Formulation

#### 112 2.2.1 From unsupervised learning to Generative Model

#### 113 2.2.2 Game Theoretic Framework for Learning

#### 114 2.2.3 Definition

#### 115 2.2.4 The Existence of Distributionally Robust Nash Equilibria

### 116 2.3 Minimax Robust Game

#### 117 2.3.1 From Duality to Triality Theory

#### 118 2.3.2 Dimension Reduction

#### 119 2.3.3 Evaluation

### 120 2.4 Case Study: Learning a Generative Adversarial Model

## 121 3 Wasserstein Metric

122 All headings should be lower case (except for first word and proper nouns), flush left, and bold.

123 First-level headings should be in 12-point type.

### 124 3.1 Introduction

#### 125 3.1.1 Optimal Transportation Problem

#### 126 3.1.2 Definition

127 Second-level headings should be in 10-point type.

### 128 3.2 From KL divergence to Wasserstein Metric

129 Third-level headings should be in 10-point type.

### 130 **3.3 Other Metrics: L1, L2, Maximum Mean Discrepancy**

### 131 **3.4 Dynamic Optimal Transport**

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133 with the text, with the heading followed by 1 em of space.

### 134 **3.5 Case Study: A Toy Example**

## 135 **4 Learning Algorithms**

136 These instructions apply to everyone.

### 137 **4.1 Bregman Learning under f-divergence**

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139 long as you maintain internal consistency. As to the format of the references themselves, any style is  
140 acceptable as long as it is used consistently.

141 The documentation for `natbib` may be found at

142 `http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf`

143 Of note is the command `\citet`, which produces citations appropriate for use in inline text. For  
144 example,

145 `\citet{hasselmo}` investigated\dots

146 produces

147 Hasselmo, et al. (1995) investigated...

148 If you wish to load the `natbib` package with options, you may add the following before loading the  
149 `nips_2018` package:

150 `\PassOptionsToPackage{options}{natbib}`

151 If `natbib` clashes with another package you load, you can add the optional argument `nonatbib`  
152 when loading the style file:

153 `\usepackage[nonatbib]{nips_2018}`

### 154 **4.2 Distributionally Robust Optimization**

155 Footnotes should be used sparingly. If you do require a footnote, indicate footnotes with a number<sup>1</sup>  
156 in the text. Place the footnotes at the bottom of the page on which they appear. Precede the footnote  
157 with a horizontal rule of 2 inches (12 picas).

158 Note that footnotes are properly typeset *after* punctuation marks.<sup>2</sup>

### 159 **4.3 Train a Deep Generative Model**

160 All artwork must be neat, clean, and legible. Lines should be dark enough for purposes of reproduction.  
161 The figure number and caption always appear after the figure. Place one line space before the figure  
162 caption and one line space after the figure. The figure caption should be lower case (except for first  
163 word and proper nouns); figures are numbered consecutively.

164 You may use color figures. However, it is best for the figure captions and the paper body to be legible  
165 if the paper is printed in either black/white or in color.

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<sup>1</sup>Sample of the first footnote.

<sup>2</sup>As in this example.



Figure 1: Sample figure caption.

Table 1: Sample table title

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Name	Description	Size ( $\mu\text{m}$ )
Dendrite	Input terminal	$\sim 100$
Axon	Output terminal	$\sim 10$
Soma	Cell body	up to $10^6$

#### 166 4.4 Case Study: Unsupervised Learning for Clustering

167 All tables must be centered, neat, clean and legible. The table number and title always appear before  
168 the table. See Table 1.

169 Place one line space before the table title, one line space after the table title, and one line space after  
170 the table. The table title must be lower case (except for first word and proper nouns); tables are  
171 numbered consecutively.

172 Note that publication-quality tables *do not contain vertical rules*. We strongly suggest the use of the  
173 booktabs package, which allows for typesetting high-quality, professional tables:

174 <https://www.ctan.org/pkg/booktabs>

175 This package was used to typeset Table 1.

#### 176 4.5 Case Study: Generative Modeling for Image Synthesis

## 5 Style Transfer as a Minimax Game

### 5.1 Introduction

Style transfer originally means rendering an image in different styles. This meaning can be extended to other kinds of data like music and speech. More generally, it refers to mapping data from one domain to another while keeping its semantic (underlying) content / the domain-invariant knowledge. For example, transfer photographs to artistic paintings, convert one person’s voice to another, or translate music to imitate different instruments. Another case is transfer learning. It adapts a pre-trained model in source domain to classify samples in target domain where labeled data is limited.

Let  $x_1 \in \mathbb{X}_1$  be samples in the source domain and  $x_2 \in \mathbb{X}_2$  be samples in the target domain. The goal of style transfer is to learn a mapping function  $\mathcal{T} : \mathbb{X}_1 \rightarrow \mathbb{X}_2$  such that the generated output  $x'_{2 \leftarrow 1} = \mathcal{T}(x_1)$  is indistinguishable from the real samples drawn from the target domain. The optimal mapping  $\mathcal{T}^*$  transforms  $x_1$  to  $x'_{2 \leftarrow 1}$  such that  $x'_{2 \leftarrow 1} \stackrel{d}{=} x_2$ . Semantic content should be preserved during the transformation.

We propose a game-theoretic approach to learn the mapping. The domain-invariant content information and domain-specific style information are decomposed by disentangled representation learning. For high dimensional data like image and speech waveform, we employ autoencoders to independently model the high-level semantic content and the low-level style information. The learning problem is formulated as a distributionally robust game with cooperative agents and payoff uncertainty. In this game, several groups of players run with different objectives. The intergroup competition and intragroup collaboration enable the players to learn from each other and optimize their worst-case performance.

This work has a wide range of applications. In visual and performing arts, it’s inspiring to automatically generate artificial paintings with user-specified style or play synthetic music with desired timbre and musical instrument. In informatics, it’s useful to transform speaker identity by modifying his voice to sound like another person. It is also possible to learn and mimic animal’s vocalization and study the feedback on the artificially generated sound. For case study, we apply our approach in two scenarios: image style transfer and emotional voice conversion.

### 5.2 Motivation

In machine learning, discriminative models predict labels from data by learning a conditional distribution  $p(y|x)$ , while generative models synthesize new data with desired labels by drawing samples from estimated distribution  $p(x|y)$ . From a perspective of probabilistic modelling, style transfer learns two conditional distributions  $p(x_1|x_2)$  and  $p(x_2|x_1)$ . When paired data is available, it is easy to infer from the joint distribution  $p(x_1, x_2)$ . For nonparallel data, the problem is ill-posed because the joint solution is not unique given two marginal distributions  $p(x_1), p(x_2)$ .

To solve this problem, additional constraints are required. Some researchers proposed to keep a particular part of the data unchanged, e.g., pixel intensity, gradient or object boundaries [2][3]; others suggested to preserve some properties of the data, such as semantic features or class labels [4].

Zhu et al. [10] proposed a very straightforward constraint called cycle-consistency. It assumes that if a sample is translated from source domain to target domain and then translated back, it should be unchanged. Choi et al. [11] generalized it to perform multiple-domain translation using a single generative model. However, domain transfer is not a one-to-one mapping, but many-to-many. In some cases, the cycle-consistency constraint is too strong to provide enough diversity in the translated outputs.

Based on a similar idea, Liu et al. [8] developed the UNIT framework by making a fully shared latent space assumption, in which corresponding images across domains can be mapped to a same latent code in shared-latent space. This assumption implies the cycle-consistency constraint. Xun et al. [9] extended it to a partially shared latent space assumption, where each example is generated from a shared content code and a domain-specific style code. Images are translated across domains by replacing the style code.

Some approaches [11][12][13] assume there exists a transformation  $\mathcal{T}$  such that the source and target data can be matched in a new representation  $p(\mathcal{T}(x_1)) = p(\mathcal{T}(x_2))$ . The types of transformation

includes projections, affine transform, and non-linear mapping defined by neural networks. The objective is to minimize the gap between two transformed distributions. Several metrics used to compare distributions have been discussed in chapter 3. One requirement is that  $p(x_1)$  and  $p(x_2)$  share a common support, otherwise the optimization process is instable due to vanishing gradient issues.

Another idea is to directly assume the mapping  $\mathcal{T} : x_2 = \mathcal{T}(x_1)$  to be the optimal transport between  $p(x_1)$  and  $p(x_2)$ . The optimal solution  $\mathcal{T}^*$  minimizes the global transportation cost (Wasserstein distance) between the source and target distributions. As a by-product, minimizing the transportation cost gives the alignment of corresponding samples as well as their labels. It can be used to solve the domain adaptation problem [14] by finding the joint distribution optimal transport between  $p(x_1, y_1)$  and  $p(x_2, f(x_2))$ .  $f$  is the classifier in the target domain, which can be inferred from the aligned data-label pairs.

While many approaches have been proposed, learning the mapping between different domains is still an open problem. Our work is motivated to support the development of more robust and comprehensible generative models for style transfer.

### 5.3 Related Work

There are several topics closely related to this work, but with different definitions.

**Deep Generative Modeling** The original objective of this topic is to generate new samples from scratch by learning complicated data distributions in an unsupervised way. At test time, it takes random noise as input and outputs realistic samples. In some cases, it can also take in conditional information to produce user-specified output. There are three main frameworks of deep generative modeling: generative adversarial networks (GANs) [15], variational auto-encoders (VAEs) [16], and auto-regressive models [17].

GANs build the generative model on the top of a discriminative network to force the output to be indistinguishable from the real samples. This model works pretty well for generating images with impressive visual quality [18] and high resolution [19]. Variations under this framework include conditional GAN [20] that generate samples conditioned on class labels, LAPGAN [21] that generates images in a coarse-to-fine fashion, WGAN-GP [22] that enables stable training of GANs without hyperparameter tuning.

VAEs use an encoder-decoder framework to model data in a latent space and optimize the reconstruction loss plus a regularizer. The generative process has two steps of sampling: first draw latent variables from  $p(z)$  and then draw datapoints from the conditional distribution  $p(x|z)$ . At test time, the encoder part is discarded and the decoder takes random noise as input to generate new samples. However, the reconstructed samples are blurry. This is because the VAE decoder assumes  $p(x|z)$  to be an isotropic Gaussian, which leads to the use of L2 loss. To remedy this, VAE-GAN [23] suggests learning the loss through a GAN discriminator.

Auto-regressive model is quite different from the above two. It aims at modeling time-varying processes by assuming that the value of a time series depends on its previous values and a stochastic term. For a sequential data sample  $x = (x_1, x_2, \dots, x_T)$ , the joint distribution  $p(x)$  is factorised as a product of conditional distributions

$$p(x) = \prod_{t=1}^T p(x_t | x_1, \dots, x_{t-1}) \quad (1)$$

This idea is quite straightforward for modeling audio sequence [24], but it also works for images. In PixelRNN [25], each image is written as a sequence, in which pixels are taken row by row from the image. The two-dimensional spatial autocorrelation of pixels is modeled by one-dimensional temporal correlations. Since the generation process is sequential, it requires a lot of GPU memory and computation time (200K updates over 32 GPUs) even after some modifications [26].

**Image Style Transfer** There are two types of style transfer problems: example-based style transfer where the style comes from one image, and domain-based style transfer where the style is learnt from a collection of images in a specific domain. The former problem originates from nonphoto-realistic rendering (NPR) [27] in computer graphic, and has the similar meaning of realistic image

manipulation. The goal is to edit image in a user-specified way and keep it as realistic as possible. Practical issues include texture synthesis and transfer [28], photo manipulation of shape and color [29], photorealistic image stylization [30], etc. In general, the output should be similar to the input in high-level structures and varies in low-level details such as color and texture.

Recently, Gatys et al. [31] claimed the image content and style information are separable in Convolutional Neural Network representations. They introduced a method [32] to separate and recombine content and style of natural images by matching feature correlations (Gram matrix) in different convolutional layers. However, their synthesis process is slow (an hour for a 512\*512 image). Moreover, the style from a single image is ambiguous and may not capture the general theme of an entire domain of images.

The second problem, also named as image-to-image translation, learns a mapping to transfer images from one domain to another. For example, super-resolution [39] maps low-dimensional images to high-dimension, colorization [40] maps gray images to color; other cases include day to night, dog to cat, young to old, summer to winter, photographs to paintings, aerial photos to maps [30,34,35,36,37,38,41]. The mapping can be learnt in a supervised or unsupervised manner. In supervised settings [33,42,43], corresponding image pairs across domains are available for training. In unsupervised settings [2,4,8,9,10], there's no paired data and the training set only contains independent set of images for each domain. Our work is under the unsupervised settings because it is more applicable, and the training data is almost free and unlimited.

**Domain Adaptation** Most recognition algorithms are developed and evaluated on the same data distributions, e.g, the public datasets ImageNet, MS-COCO, CIFAR-10, MNIST. In real applications, people often confront performance degradation when apply a classifier trained on a source domain to a target domain.

In unsupervised domain adaptation, source domain has labeled data  $\mathcal{D}_s = \{x_i^s, y_i^s\}_{i=1}^{n_s}$  while target domain contains data without labels  $\mathcal{D}_t = \{x_i^t\}_{i=1}^{n_t}$ . The goal is to learn a classifier  $f : x_i^t \mapsto y_i^t$  for the unseen target samples by exploring the knowledge learnt from the source domain. Domain adaptation algorithms attempts to transfer knowledge across domains by solving the domain shift problem, i.e., the data-label distributions  $p(x^s, y^s)$  and  $p(x^t, y^t)$  are different.

There are many approaches to address this issue. One is to extract transferable features that are invariant across domains [45,46], or learn representative hash codes [47] to find a common latent space where the classifier can be used without considering the data's origin. Another trend is to learn the transformation between domains [48] to align the source and target datapoints through barycentric mapping, and train a classifier on the transferred source data. Courty [49] and Damodaran[14] proposed to look for a transformation that matches the data-label joint distributions  $p(x^s, y^s)$  in source domain to its equivalent version  $p(x^t, y^t)$  in target domain. The predictive function  $f$  is learnt by minimizing the optimal transport loss between the distributions  $p(x^s, y^s)$  and  $p(x^t, f(x^t))$ . As a by-product, minimizing the optimal transport cost is equivalent to mapping a source domain sample to a target domain sample with similar semantic content, and this is the domain transfer problem.

**Voice Conversion** Voice conversion (VC) aims to change a speaker's voice to make it sounds like spoken by another person. It is a special case of voice transformation (VT), whose goal is to modify human speech without changing its content. VC transforms speaker identity by replacing speaker-dependent components of the signal while maintaining the linguistic information. Speech quality and speaker similarity are two important factors to evaluate a VC system. There are a bunch of VC applications, such as movie dubbing, personalized TTS (Text To Speech) systems, speaker accent or emotion transformation, speaking-aid devices, call quality enhancement, etc.

Most VC frameworks involve three steps: feature extraction, feature conversion, waveform generation. In speech analysis, waveform signals are encoded into feature representations that are easy to control and modify. Spectral envelope, mel-cepstrum, fundamental frequency ( $f_0$ ), formant frequencies and bandwidths are the most widely used features to represent speech in short-time segments. To capture contextual information across frames, implicit methods such as hidden Markov models (HMMs), Long Short-Term Memory (LSTM) and recurrent neural networks (RNNs) [63] were developed.

The main work in VC is to transform the source feature sequences to target feature sequences that capture the speaker identity. Most traditional VC systems perform frame-by-frame mapping under the assumption that speech segments are independent from each other. Some recent models such as HMM



and RNN incorporate speech dynamics implicitly. There are four typical approaches to learn the mapping function: codebook mapping (e.g., Vector quantization (VQ) [56]), mixed linear mappings (e.g., Gaussian mixture model (GMM) [57]), neural network mapping (e.g., RBM, DNN, RNN [55]), and exemplar-based mapping (e.g., non-negative matrix factorization (NMF) [58]). Beyond these, an autoregressive neural network model called WaveNet [24] was proposed. It can directly learn the mapping based on raw audio and generate speech waveforms conditioning on the speaker identity.

There are various assumptions in speech analysis and waveform generation. Source-filter models assume speech to be generated by excitation signals passing through a vocal tract, and encode speech waveforms as acoustic features that represent sound source and vocal tract independently. However, the original phase information will lose under this assumption. At conversion time, the converted target features are passed through a vocoder based on the source filter model to reconstruct the waveform. Quality degradation may happen due to the inaccurate assumption. Iterative phase reconstruction algorithm Griffin-Lim [59] was adopted to alleviate this issue. Harmonic plus noise models (HNM) [54] assume speech to be a combination of a noise component and a harmonic component, i.e., sinusoidal waves with frequencies relevant to pitch. Speech is parameterized by the fundamental frequency  $f_0$  and a spectrum which consists of a lower band of harmonic and a higher band of noise. Other assumptions include stationary speech signal, frame-by-frame mapping, time-invariant linear filter, etc. Recently, Tamamori et. al [53] proposed a speaker-dependent WaveNet vocoder that does not require explicit modeling of excitation signals and those assumptions.

In terms of conversion conditions, VC can be categorized into parallel and non-parallel, text-dependent and text-independent systems [50]. In parallel systems, the training corpus consists of paired recordings from the source and target speakers with same linguistic contents. The shared acoustic features can be used to train the mapping model. To get parallel feature sequences of equal length, a time-alignment step must be included to remove the temporal differences in the recordings, for example, the dynamic time warping (DTW) [56] algorithm. Phoneme transcriptions are also useful for time alignment. Non-parallel system does not require sentences with the same linguistic contents. It is much more useful and practical because non-parallel speech data is easier to collect and therefore can get larger training sets. There are several ways to learn the mapping without paired data: (1) use unit selection [60] to choose matched linguistic feature pairs; (2) build pseudo parallel sentences on extra automatic speech recognition (ASR) modules [61]; (3) extract speaker-independent features in shared latent space [62]; (4) use unpaired image-to-image translation approaches [8][9][33].

Parallel, text-dependent systems are supposed to have better performance. However, parallel utterance pairs are difficult to get. Most parallel VC systems require time alignment to extract parallel source-target features. The misalignment in automatic time alignment algorithms often leads to degradation in speech quality, while manual correction is arduous. Recently, the winner of VC Challenge 2018 [51] showed their algorithm can achieve similar results in both parallel and non-parallel settings. It first uses a lot of external speech data with phonetic transcriptions to train a speaker-independent content-posterior-feature extractor, followed by a speaker-dependent LSTM-RNN to predict fundamental frequency  $f_0$  and STRAIGHT spectral features [52], and then reconstruct the waveforms with a speaker-dependent WaveNet vocoder [53]. Moreover, Kaneko et.al [64] and Fang et. al [54] claimed their nonparallel, text-independent VC algorithms based on CycleGAN [10] perform comparable to or better than the state-of-the-art parallel approaches.

## 5.4 Method

We propose a general approach for style transfer. Let  $x_i \in \mathbb{X}_i$  be a datapoint sampled from domain  $i$ . The goal is to learn a conversion model  $p(x_j|x_i)$  that maps  $x_i$  to domain  $j$  ( $j \neq i$ ) by changing its style and preserving the original content. For unpaired training data, we have marginal distributions  $p(x_i), p(x_j)$  instead of the joint  $p(x_i, x_j)$ , so it requires additional constraints to determine  $p(x_j|x_i)$ . Inspired by disentangled representation learning in [32], we assume that each example  $x_i \in \mathbb{X}_i$  can be decomposed into a content code  $c \in \mathcal{C}$  that encodes domain-invariant information and a style code  $s_i \in \mathcal{S}_i$  that encodes domain-dependent information.  $\mathcal{C}$  is shared across domains and contains the information we want to preserve.  $\mathcal{S}_i$  is domain-specific and contains the information we want to change. In conversion stage, we extract the content code of  $x_i$  and recombine it with a style code randomly sampled from  $\mathcal{S}_j$ . A generative adversarial network (GAN) [15] is added to ensure that the converted samples are indistinguishable from the real ones.

Figure 2 shows the autoencoder model of style transfer with a partially shared latent space. Any pair of corresponding datapoints  $(x_i, x_j)$  is assumed to have a shared latent code  $c \in \mathcal{C}$  and domain-specific style codes  $s_i \in \mathcal{S}_i, s_j \in \mathcal{S}_j$ . The generative model of domain  $i$  is an autoencoder that consists of a deterministic decoder  $x_i = G_i(c_i, s_i)$  and two encoders  $c_i = E_i^c(x_i), s_i = E_i^s(x_i)$ . The encoder  $E_i = (E_i^c, E_i^s)$  and decoder  $G_i$  are inverse operations such that  $E_i = G_i^{-1}$ . To generate converted sample  $x'_{j \leftarrow i}$ , we just extract and recombine the content code of  $x_i$  with the style code of domain  $j$ .

$$\begin{aligned} x'_{i \leftarrow j} &= G_i(c_j, s_i) = G_i(E_j^c(x_j), s_i) \\ x'_{j \leftarrow i} &= G_j(c_i, s_j) = G_j(E_i^c(x_i), s_j) \end{aligned} \quad (2)$$

It should be noted that the style code  $s_i$  is not inferred from one example, but learnt from the entire target domain  $j$ , which is a major difference from [32]. This is because the style extracted from a single example is ambiguous and may not capture the general characteristics of the target domain. To restrict the mapping  $p(x_j|x_i)$ , we use a constraint that is slightly different from the cycle consistency [10]. It assumes that an example converted to another domain and converted back should be unchanged, i.e.,  $x''_{i \leftarrow j \leftarrow i} = x_i$ . Instead, we apply a semi-cycle consistency in the latent space by assuming that only the latent codes remain unchanged  $E_i^c(x'_{i \leftarrow j}) = c_i$  and  $E_i^s(x'_{i \leftarrow j}) = s_i$ . The relaxed constraint provides diversity to the generated samples.

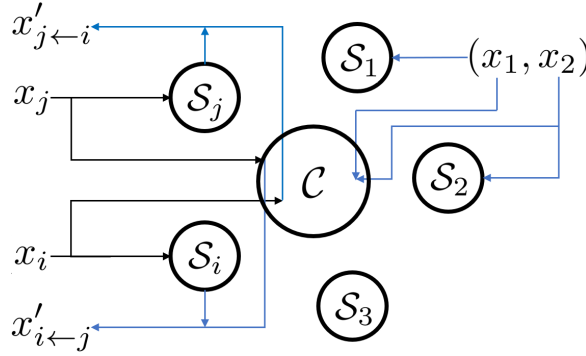


Figure 2: Autoencoder model with partially shared latent space. Sample  $x_i$  is encoded into a domain-specific space  $\mathcal{S}_i$  and a shared content space  $\mathcal{C}$ . Corresponding datapoints  $(x_1, x_2)$  are encoded in the same content code. Converted sample  $x'_{j \leftarrow i}$  is generated by recombining the original content code of  $x_i$  and the style code randomly sampled from  $\mathcal{S}_j$ .

We formulate the learning problem as a distributionally robust game. Each domain  $i$  has four agents  $E_i^c, E_i^s, G_i, D_i$ , in which  $D_i$  is a discriminator with two objectives. One is to distinguish between real samples and machine-generated samples, the other is to classify domain labels. On the other side, the generators  $G_i$  have two purposes: synthesize realistic samples and convert them into domain  $i$ . For example in voice conversion, the synthesized speech is evaluated on both the naturalness of its quality and the correctness of speaker’s identity.

#### 5.4.1 Two-domain transformation

There are  $4n$  agents for  $n$  domains. For simplicity, we first investigate the conversion model between two domains. When  $n = 2$ , there are 8 agents:  $E_1^c, E_1^s, G_1, D_1$  for domain  $\mathbb{X}_1$  and  $E_2^c, E_2^s, G_2, D_2$  for domain  $\mathbb{X}_2$ . Each agent has a different objective, and the utility function of one agent depends on the action of other agents. Collaboration and competition exist among them. So this is a distributionally robust game with cooperative agents and uncertain utility functions. Based on the analysis above, there are five modules need to learn:

- ① domain-invariant content encoder
- ② domain-specific style encoders
- ③ real/fake discriminator
- ④ domain classifier  $D_i$
- ⑤ fake samples generator

The encoders and decoders form a group to synthesize converted samples in the target domain. Another group is the discriminator and classifier. They work on the opposite side to distinguish between real/fake samples and predict the domain label. The intergroup competition and intragroup collaboration are listed in table 2.

How to define the game?

When Nash equilibrium reaches, the autoencoder  $(E_i^{c*}, E_i^{s*}, G_i^*)$  minimizes the reconstruction error  $L_{rec}^x \rightarrow 0, L_{rec}^c \rightarrow 0, L_{rec}^s \rightarrow 0$ . The GAN network  $(D_i^*, G_i^*)$  and adversarial loss  $L_{GAN}^{x_i}$  converge at saddle points that minimize the distance between  $p(x_i)$  and  $p(x'_{j \leftarrow i})$ . The classifier  $D_i^{cls*}$  correctly predicts the domain category of both real and fake samples  $D_i^{cls*}(x_i) = i, D_i^{cls*}(x'_{i \leftarrow j}) = i$ .

Table 2: Cooperative game

Intergroup competition	Learning module	Objective
$E_1^s, E_2^s$	②	$\min L_{cyc}^s$
$D_1, D_2$	④	$\min L_{cls}^x$
Intragroup collaboration	Learning module	Objective
$E_1^c, E_1^s, G_1$	<i>Autoencoder</i> <sub>1</sub>	$\min L_{rec}^{x_1}$
$E_2^c, E_2^s, G_2$	<i>Autoencoder</i> <sub>2</sub>	$\min L_{rec}^{x_2}$
$G_1, G_2$	⑤	$\min L_{GAN}^x$
$D_1, D_2$	③	$\max L_{GAN}^x$
$E_1^c, E_2^c$	①	$\min L_{cyc}^c$

The collaboration means, agents in different domains can help each other as they may have common interests. For instance, a sample can be used to update the real/fake discriminator even if its class label is missing. Except for the autoencoders  $E_i^c, E_i^s, G_i$ , other coalitions are cross-domain.  $(G_1, G_2)$  works on data synthesis,  $D_1, D_2$  works on real/fake discrimination,  $E_1^c, E_2^c$  extracts high-level content information that we want to preserve.

We jointly train the encoders, decoders and GAN’s discriminators with multiple objectives. To keep encoder and decoder as inverse operations, a reconstruction loss is applied in the direction  $x_i \rightarrow (c_i, s_i) \rightarrow x'_i, (i, j \in 1, 2)$ . Sample  $x_i$  should not be changed after encoding and decoding.

$$L_{rec}^{x_i} = \mathbb{E}_{x_i}(\|x_i - x'_i\|_1), \quad x'_i = G_i(E_i^c(x_i), E_i^s(x_i)) \quad (3)$$

In our model, the latent space is partially shared. Thus the cycle consistency constraint [10] is not preserved, i.e.,  $x''_{1 \leftarrow 2 \leftarrow 1} \neq x_1$ . We apply a semi-cycle loss in the coding direction  $c_1 \rightarrow x'_{2 \leftarrow 1} \rightarrow c'_{2 \leftarrow 1}$  and  $s_2 \rightarrow x'_{2 \leftarrow 1} \rightarrow s'_{2 \leftarrow 1}$ .

$$\begin{aligned} L_{cyc}^{c_1} &= \mathbb{E}_{c_1, s_2}(\|c_1 - c'_{2 \leftarrow 1}\|_1), \quad c'_{2 \leftarrow 1} = E_2^c(x'_{2 \leftarrow 1}) \\ L_{cyc}^{s_2} &= \mathbb{E}_{c_1, s_2}(\|s_2 - s'_{2 \leftarrow 1}\|_1), \quad s'_{2 \leftarrow 1} = E_2^s(x'_{2 \leftarrow 1}) \end{aligned} \quad (4)$$

Moreover, we add a GAN module to ensure the quality of generated samples. They should be indistinguishable from the real samples in the target domain  $\mathbb{X}_i$ . GAN loss is computed between  $x_j$  and  $x'_{i \leftarrow j}$  to represent the distance between two distributions  $p(x_j), p(x'_{i \leftarrow j})$ .

$$L_{GAN}^{x_i} = \mathbb{E}_{c_j, s_i}[\log(1 - D_i(x'_{i \leftarrow j}))] + \mathbb{E}_{x_i}[\log D_i(x_i)] \quad (5)$$

The full loss is the weighted sum of  $L_{recon}, L_{cycle}, L_{GAN}$ .

$$\begin{aligned} &\min_{E_1^c, E_1^s, E_2^c, E_2^s, G_1, G_2} \max_{D_1, D_2} L(E_1^c, E_1^s, E_2^c, E_2^s, G_1, G_2, D_1, D_2) \\ &= \lambda_s(L_{cyc}^{s_1} + L_{cyc}^{s_2}) + \lambda_c(L_{cyc}^{c_1} + L_{cyc}^{c_2}) + \lambda_x(L_{rec}^{x_1} + L_{rec}^{x_2}) + \lambda_g(L_{GAN}^{x_1} + L_{GAN}^{x_2}) \end{aligned} \quad (6)$$

where  $\lambda_s, \lambda_c, \lambda_x, \lambda_g$  control the weights of the components.

#### 5.4.2 Multi-domain transformation

In multi-domain case, there are  $4n$  agents in the game. To reduce complexity, we replace the domain-specific models  $E_i^c, G_i, D_i$  with a shared content encoder  $E^c$ , a shared decoder  $G$ , and a single multiclass classifier  $D^{cls}$ . Thus, only  $n + 3$  agents left. (form coalition?) Figure 3 shows the multi-domain transformation model.

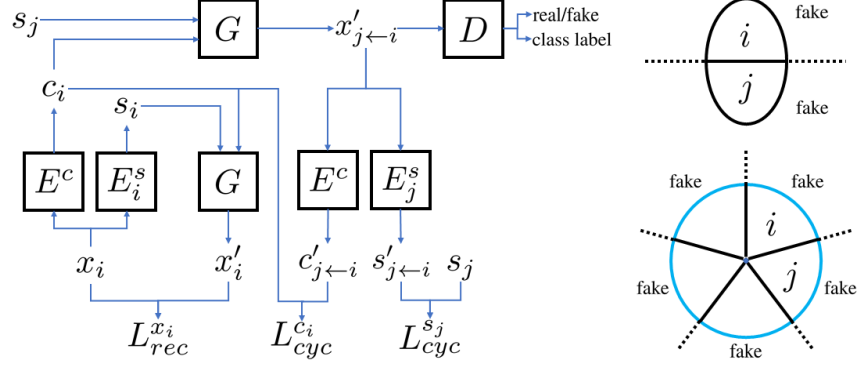


Figure 3: Learn multi-domain transformation. Left: conversion model with shared content encoder, decoder and classifier. Top-right: two domains. Bottom-right: multiple domains ( $n=5$ ).

When  $n = 2$ , there are 3 kinds of data: real in  $\mathbb{X}_1$ , real in  $\mathbb{X}_2$  and fake in  $\mathbb{X}_{fake}$ . Two real/fake discriminators  $D_1, D_2$  are enough for classification. If this idea is extended to multiple domains, there will be  $n$  binary discriminators and  $2^n$  outputs. Instead, we replace them with one binary real/fake discriminator  $D$  and one multiclass domain classifier  $D^{cls}$ . The two-step classification not only reduces complexity ( $2n$  outputs) but also makes the most of the training data. An example in domain  $i$  is also useful to train the generative model for other domains, as it is the common interest of all agents  $G_1, G_2, \dots, G_n$  to synthesize realistic data. Therefore, the multi-domain transformation model can be trained on different datasets with partially labeled data.

In the following sections, we will apply the proposed method on two real-world applications: image style transfer and emotinal speech conversion.

### 5.5 Case Study: Image Style Transfer

Image style transfer, or image-to-image translation is a hot topic in computer vision. The objective is to transfer the visual style of an image while keep its semantic content. Similar tasks includes texture synthesis, artistic style transfer, photorealistic image style transfer, etc.

Gatys et al. [7] introduced an algorithm to separate and recombine the content and style of natural images. They claimed that a Convolutional Neural Network (CNN) is the ideal representation to factorize semantic content and artistic style. High-level features are extracted by higher layers of the network, while low-level features are captured by the correlations between filter responses in various layers. However, the style is learnt from a single image, which limits the ability to capture the general theme of the target domain.

Instead of example-based style transfer, we will focus on learning the translation model between two image distributions. There are a broad range of researches on image synthesis and representation learning. We will expore the state-of-the-art neural network architecture and implement our model to learn the disentangled representations of image style and content. The proposed method will be tested on a typical image style transfer task.

### 5.6 Case Study: Emotional Voice Conversion

Voice transformation (VT) is a technique to modify some properties of human speech while preserving its linguistic information. VT can be applied to change the speaker identity, i.e., voice conversion (VC) [50], or to transform the speaking style of a speaker, such as emotion and accent conversion [65]. In this section, we will focus on emotion voice transformation. The goal is to change emotion-related characteristics of a speech signal while preserving its linguistic content and speaker identity. Emotion conversion techniques can be applied to various tasks, such as hiding negative emotions for customer service agents, helping film dubbing, and creating more expressive voice messages on social media.

Existing VC approaches cannot be applied directly because they change speaker identity by assuming pronunciation and intonation to be a part of the speaker-independent information. Since the speaker's emotion is mainly conveyed by prosodic aspects, some studies have focused on modelling prosodic

483 features such as pitch, tempo, and volume [66][67]. In [68], a rule-based emotional voice conversion  
 484 system was proposed. It modifies prosody-related acoustic features of neutral speech to generate  
 485 different types of emotions. A speech analysis-synthesis tool STRAIGHT [52] was used to extract  
 486 fundamental frequency ( $F_0$ ) and power envelope from raw audio. These features were parameterized  
 487 and modified based on Fujisaki model [69] and target prediction model [70]. The converted features  
 488 were then fed back into STRAIGHT to re-synthesize speech waveforms with desired emotions.  
 489 However, this method requires temporal aligned parallel data that is difficult to obtain in real  
 490 applications; and the accurate time alignment needs manual segmentation of the speech signal at  
 491 phoneme level, which is very time consuming.

492 To address these issues, we propose a nonparallel training method. Instead of learning example based  
 493 one-to-one mapping between paired emotional utterances  $(x_1, x_2)$ , we learn the conversion model  
 494 between two emotion domains  $(\mathbb{X}_1, \mathbb{X}_2)$ .

495 Inspired by the disentangled representation learning in image style transfer [31][9], we assume that  
 496 each speech signal  $x_i \in \mathbb{X}_i$  can be decomposed into a content code  $c \in \mathcal{C}$  that represents emotion-  
 497 invariant information and a style code  $s_i \in \mathcal{S}_i$  that represents emotion-dependent information.  $\mathcal{C}$   
 498 is shared across domains and contains the information we want to preserve.  $\mathcal{S}_i$  is domain-specific  
 499 and contains the information we want to change. In conversion stage, we extract content code of  
 500 the source speech and recombine it with style code of the target emotion. A generative adversarial  
 501 network (GAN) [15] is added to improve the quality of converted speech. Our approach is nonparallel,  
 502 text-independent, and does not rely on any manual operation.

503 For implementation, we use deep neural networks to learn the latent representations of speech.  
 504 Specifically, the encoders and decoders are implemented with one-dimensional CNNs to capture the  
 505 temporal dependences. The GAN discriminators are implemented with two-dimensional CNNs to  
 506 capture the spectra-temporal patterns. All networks are equipped with gated linear units (GLU) [73]  
 507 as activation functions.

508 We plan to test our approach on IEMOCAP [71] to learn the conversion models for four emotions:  
 509 angry, happy, neutral, sad. IEMOCAP is a nonparallel dataset widely used in emotional speech  
 510 recognition and analysis. It contains scripted and improvised dialogs in five sessions; each has labeled  
 511 emotional sentences pronounced by two professional English speakers. The emotions in scripted  
 512 dialogs have strong correlation with the lingual content. Since our task is to change emotion but keep  
 513 the speaker identity and linguistic content, we only use the improvised dialogs of the same speaker.

514 There are three metrics for performance evaluation: emotion correctness, voice quality and the ability  
 515 to retain speaker identity. For subjective evaluation, we will conduct listening tests on Amazon  
 516 MTurk to evaluate the converted speech. Each example is evaluated by a group of random listeners.  
 517 They will be asked to manually classify the emotion, and give 1-to-5 opinion scores on voice quality  
 518 and the similarity with the original speaker. The classification result and mean opinion score (MOS)  
 519 are two major measurements. For objective evaluation, we plan to use the state-of-the-art speech  
 520 emotion classifier [72] to check the emotion category of generated speech. It indicates a success if  
 521 our model can increase the proportion of the target emotions and reduce the original emotions. To our  
 522 knowledge, this is the first work for nonparallel emotion conversion. If there's time, we will develop  
 523 multidomain emotion conversion models for unseen speakers.

## 524 6 Dissertation Outline

525 The research will be split into the following four stages:

### 526 6.1 Introduction

### 527 6.2 Related Work

### 528 6.3 Distributionally Robust Games

529 In this part we introduce distributionally robust games and develop new filtering and learning  
530 architectures under this framework. The system may contain several competing neural networks: the  
531 attackers learn to generate synthetic samples that are supposed to have the same distribution as the  
532 original ones, while the defenders try to find counter-examples and create difficulties for the other  
533 side. Each player tries to perform better and beat the others, which forms a multi-agent zero-sum  
534 game with uncertain payoffs. The players use a robust optimization approach to contend with the  
535 worst-case scenario payoff. The attacker network is constructed based on the outcome of defender  
536 networks, and vice versa. The competing networks are trained together iteratively until achieving the  
537 distributional robust Nash equilibrium.

### 538 6.4 Wasserstein Metric

539 The loss function is designed to measure the similarity of two probability distributions. Unsupervised  
540 learning is conducted by minimizing the loss. We plan to study the properties of several widely used  
541 loss metrics:

- 542 • Compare L1, L2-loss, KL-divergence, f-divergence, and Wasserstein distance
- 543 • Study the time-dependent formulation of the optimal transportation cost
- 544 • Test the effect of translation and perturbation for a certain loss metric

### 545 6.5 Learning Algorithms for Robust Optimization

- 546 • Develop a specific learning algorithm to find robust Nash equilibria, which should be stable  
547 and efficient
- 548 • Compare with existing numerical optimization approaches in large-scale machine learning:  
549 SGD, Adam, Momentum, Ishikawa-Nesterov, Newton's method, conjugate gradient, natural  
550 gradient, etc.
- 551 • Compare with existing deep generative models: RBM, VAE, GAN, WGAN, etc.

### 552 6.6 Generative Modeling for Vehicle Tracking

### 553 6.7 Generative Modeling for Image Synthesis

### 554 6.8 Generative Modeling for Voice Conversion

### 555 6.9 Experiments on Large-scale Machine Learning datasets

- 556 • Maryland Traffic Surveillance Dataset (Vehicle tracking)
- 557 • Large-scale CelebFaces Attributes Dataset (CelebA, image synthesis)
- 558 • Large-scale Scene Understanding Challenge (LSUN, image synthesis)
- 559 • Interactive Emotional Dyadic Motion Capture (IEMOCAP, emotional voice conversion)

560 **6.10 Discussion**

561 **6.11 Conclusion and Future Work**

562 **7 Research Plan**

563 **7.1 Research Progress**

564 Literature review, planning  
565 Theory part on distributional robust games, Bregman learning and convex optimization  
566 Theoretical analysis and comparison for L2 distance, f-divergence and Wasserstein metric  
567 Algorithm design, overall integration, simulations, specific implementations on real problems  
568  
569 Application part on large-scale machine learning: experiments, evaluation and revision  
570 Documentation and Defence

571 **7.2 Application on Image and Audio Synthesis**

- 572 • Test on large-scale image dataset MNIST, CelebA and LSUN
- 573 • Literature review on emotional speech classification and audio synthesis
- 574 • Compare two sound representations in generative learning: waveform and spectrogram
- 575 • Design deep generative models for emotional speech generation
- 576 • Test on voice conversion or music style transfer if possible

577 **7.3 Timeline**

Fall 2018	Study on Generative Models for Voice Conversion
Spring 2019	Write Thesis
August 2019	Thesis Defense

578 **8 Conclusion**

579 Tackling the aforementioned problems would take us much closer to real intelligent systems, and  
580 defines three core pillars of Artificial Intelligence. However, there are many other problems which  
581 need to be solved and integrated to achieve a fully intelligent system, e.g. navigation, learning by  
582 imitation, cooperation, and many others.

583 **9 List of Publications**

584 **9.1 Thesis Related Publications**

- 585 Jian Gao and Hamidou Tembine, Distributionally Robust Games for Deep Generative  
586 Learning, July 2018. DOI: 10.13140/RG.2.2.15305.44644
- 587 Jian Gao, Yida Xu, Julian Barreiro-Gomez, Massa Ndong, Michalis Smyrnakis and Hamidou  
588 Tembine (September 5th 2018) Distributionally Robust Optimization. In Jan Valdmann,  
589 Optimization Algorithms, IntechOpen. DOI: 10.5772/intechopen.76686. ISBN: 978-1-  
590 78923-677-4
- 591 Jian Gao and Hamidou Tembine, Distributionally Robust Games: Wasserstein Metric,  
592 International Joint Conference on Neural Networks (IJCNN), Rio de Janeiro, Brazil, July  
593 2018
- 594 Jian Gao and Hamidou Tembine, Bregman Learning for Generative Adversarial Networks,  
595 Chinese Control and Decision Conference (CCDC), Shenyang, China, June 2018 (*Best  
596 Paper Finalist Award*)

597 Jian Gao and Hamidou Tembine, Distributed Mean-Field-Type Filter for Vehicle Tracking,  
 598 in American Control Conference (ACC), Seattle, USA, May 2017 (*Student Travel Award*)  
 599 Dario Bauso, Jian Gao and Hamidou Tembine, Distributionally Robust Games: f-Divergence  
 600 and Learning, 11th EAI International Conference on Performance Evaluation Methodologies  
 601 and Tools (VALUETOOLS), Venice, Italy, Dec 2017

## 602 9.2 Other Publications

603 J. Gao and H. Tembine, "Distributed Mean-Field-Type Filters for Traffic Networks," in  
 604 IEEE Transactions on Intelligent Transportation Systems. doi: 10.1109/TITS.2018.2816811  
 605 J. Gao and H. Tembine, "Empathy and berge equilibria in the forwarding dilemma in  
 606 relay-enabled networks," 2017 International Conference on Wireless Networks and Mobile  
 607 Communications (WINCOM), Rabat, 2017, pp. 1-8. doi: 10.1109/WINCOM.2017.8238199  
 608 (*Best paper Award*)  
 609 J. Gao and H. Tembine, "Correlative mean-field filter for sequential and spatial data process-  
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 611 2017, pp. 243-248. doi: 10.1109/EUROCON.2017.8011113  
 612 Fanhuai Shi, Jian Gao, Xixia Huang, An affine invariant approach for dense wide baseline  
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 615 J. Gao and H. Tembine, "Distributed Mean-Field-Type Filters for Big Data Assimilation,"  
 616 2016 IEEE 18th International Conference on High Performance Computing and Com-  
 617 munications; IEEE 14th International Conference on Smart City; IEEE 2nd International  
 618 Conference on Data Science and Systems (HPCC/SmartCity/DSS), Sydney, NSW, 2016, pp.  
 619 1446-1453. doi: 10.1109/HPCC-SmartCity-DSS.2016.0206

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 623 Aaron Courville, and Yoshua Bengio. Generative adversarial nets. In Z. Ghahramani, M. Welling,  
 624 C. Cortes, N. D. Lawrence, and K. Q. Weinberger, editors, *Advances in Neural Information*  
 625 *Processing Systems* 27, pages 2672–2680. Curran Associates, Inc., 2014.