123. Best Time to Buy and Sell Stock III

Say you have an array for which the i^{th} element is the price of a given stock on day *i*.

Design an algorithm to find the maximum profit. You may complete at most *two* transactions.

Note: You may not engage in multiple transactions at the same time (i.e., you must sell the stock before you buy again).

Example 1:

```
Input: [3,3,5,0,0,3,1,4]

Output: 6

Explanation:

Buy on day 4 (price = 0) and sell on day 6 (price = 3), profit = 3-0 = 3.

Then buy on day 7 (price = 1) and sell on day 8 (price = 4), profit = 4-1 = 3.
```

Example 2:

```
Input: [1,2,3,4,5]

Output: 4

Explanation:

Buy on day 1 (price = 1) and sell on day 5 (price = 5), profit = 5-1 = 4.

Note that you cannot buy on day 1, buy on day 2 and sell them later, as you are

engaging multiple transactions at the same time. You must sell before buying again.
```

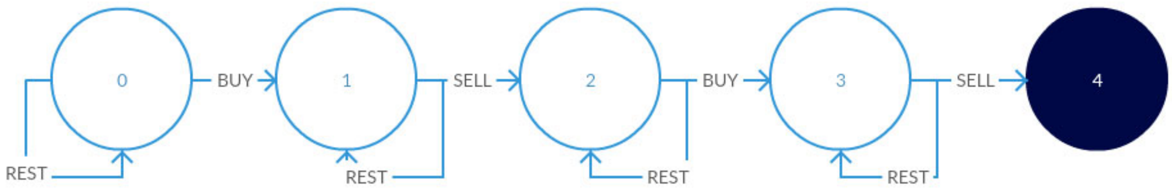
Example 3:

```
Input: [7,6,4,3,1]

Output: 0

Explanation: In this case, no transaction is done, i.e. max profit = 0.
```

Easy DP solution using state machine, O(n) time complexity, O(1) space complexity

This approach can be used for all the problems based on stock prices.

The idea is to design a state machine that correctly describes the problem statement.



Intuition behind the state diagram:
We begin at state `0`, where we can either rest (i.e. do nothing) or buy stock at a given price.

- If we choose to rest, we remain in state `0`
- If we buy, we spend some money (price of the stock on that day) and go to state `1`

From state `1`, we can once again choose to do nothing, or we can sell our stock.

- If we choose to rest, we remain in state `1`
- If we sell, we earn some money (price of the stock on that day) and go to state `2`

This completes one transaction for us. Remember, we can only do *at most* 2 transactions.

From state `2`, we can choose to do nothing or buy more stock.

- If we choose to rest, we remain in state `2`
- If we buy, we go to state `3`

From state `3`, we can once again choose to do nothing, or we can sell our stock for the last time.

- If we choose to rest, we remain in state `3`
- If we sell, we have utilized our allowed transactions and reach the final state `4`

Going from the state diagram to code

```
// Assume we are in state S

// If we buy, we are spending money but we can also choose to do nothing

// Doing nothing means going from S->S

// Buying means going from some state X->S, losing some money in the process
```

```
S = max(S, X-prices[i])

// Similarly, for selling a stock

S = max(S, X+prices[i])
```

Code:

```
int maxProfit(vector<int>& prices) {

        if(prices.empty()) return 0;

        int s1=-prices[0],s2=INT_MIN,s3=INT_MIN,s4=INT_MIN;

        for(int i=1;i<prices.size();++i) {

                s1 = max(s1, -prices[i]);

                s2 = max(s2, s1+prices[i]);

                s3 = max(s3, s2-prices[i]);

                s4 = max(s4, s3+prices[i]);

        }

        return max(0,s4);

}
```

We can create 4 variables, one for each state excluding the initial state since that's always 0, initializing `s1` to `-prices[0]` and the rest to `INT_MIN` since they will get overwritten later.

To reach `s1`, we either stay in `s1` or we buy stock for the first time.
To reach `s2`, we either stay in `s2` or we sell from `s1` and come to `s2`
Similarly for `s3` and `s4`.

In the end, we return `s4` or more accurately, `max(0,s4)` since we initialize `s4` to `INT_MIN`.

This idea works for all problems on stocks, as long as our state diagram is correct, we can code it up like this.

Side Note: Technically, this is a dynammic programming approach and we should actually be doing `s2[i] = max(s2[i-1], s1[i-1]+prices[i])` but we can be rest assured that the overwritten value of `s1` will always be better than the previous one and hence we do not need temporary variables.