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A wavelet neural network conjunction model for groundwater level forecasting

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SUMMARY

Accurate and reliable groundwater level forecasting models can help ensure the sustainable use of a watershed's aquifers for urban and rural water supply. In this paper, a new method based on coupling discrete wavelet transforms (WA) and artificial neural networks (ANN) for groundwater level forecasting applications is proposed. The relative performance of the proposed coupled wavelet–neural network models (WA–ANN) was compared to regular artificial neural network (ANN) models and autoregressive integrated moving average (ARIMA) models for monthly groundwater level forecasting. The variables used to develop and validate the models were monthly total precipitation, average temperature and average groundwater level data recorded from November 2002 to October 2009 at two sites in the Chateauguay watershed in Quebec, Canada. The WA–ANN models were found to provide more accurate monthly average groundwater level forecasts compared to the ANN and ARIMA models. The results of the study indicate the potential of WA–ANN models in forecasting groundwater levels. It is recommended that additional studies explore this proposed method, which can be used in turn to facilitate the development and implementation of more effective and sustainable groundwater management strategies.

1. Introduction

In many watersheds, groundwater is often one of the major sources of water supply for domestic, agricultural and industrial users. However, groundwater supplies for agricultural, industrial, and municipal purposes have been overexploited in many parts of the world (Konikow and Kendy, 2005). Various consequences of unsustainable groundwater use and management are becoming a serious issue globally, especially in developing countries (Konikow and Kendy, 2005). In many regions, groundwater has been withdrawn at rates far in excess of recharge, which leads to harmful environmental side effects such as major water-level declines, drying up of wells, reduction of water in streams and lakes, water-quality degradation, increased pumping costs, land subsidence, and decreased well yields (USGS, 2010). As a result, many watersheds are experiencing severe environmental, social and financial problems (Tsanis et al., 2008). As water demand will likely increase in the short and long term, there will be increasing pressures on groundwater resources (Sethi et al., 2010). Furthermore, climate change has a significant impact on the quantity and quality of groundwater resources. The sustainable management of groundwater resources in conjunction with surface waters in a watershed is very important to ensure the sustainability of a watershed's surface and groundwater resources (Mohanty et al., 2010). A critical component of planning and implementing integrated management of groundwater and surface water resources in a watershed is accurate and reliable forecasting of groundwater levels (Mohanty et al., 2010).

Accurate assessments of groundwater levels allow water managers, engineers, and stakeholders to: (i) develop better strategies to avoid or reduce adverse effects such as loss of pumpage in residential water supply wells, land surface subsidence, and aquifer compaction (Prinos et al., 2002); (ii) develop a better understanding of the dynamics and underlying factors that affect groundwater levels; and (iii) balance the needs of urban, agricultural, industrial and other demands and analyze the benefits and costs of water conservation. An important component of this is accurate groundwater level forecasts. The aim of this study is to develop a new data-based method of highly accurate groundwater level forecasting that can be used to help water managers, engineers, and stakeholders manage groundwater in a more effective and sustainable manner to help address some of the issues described above.

Conceptual or physically based models are often the main type of model used to depict hydrological variables and to understand physical processes occurring in a particular system. However, they have a number of practical limitations, including the need for large amounts of hydrogeological data. In the last decade, a number of studies have investigated the advantages and disadvantages of process based models for groundwater level forecasting and compared their forecasting performance to new data-based methods such as artificial neural network (ANN) models (e.g., Maskey et al., 2000; Daliakopoulos, 2005; Mohammadi, 2008). For

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Nomenclature

CWT	continuous wavelet transform	y_i	observed peak monthly groundwater level
N	number of data points used	$\hat{y}_i$	forecasted peak monthly groundwater level
s	scale parameter	τ	translation parameter
$x(t)$	signal	$*$	complex conjugate
$\bar{y}_i$	mean value taken over N	$\psi(t)$	mother wavelet

example, Mohammadi, 2008 found that ANN models were more accurate than process based models (in this case MODFLOW) for groundwater level forecasting, and found that the main disadvantage of process based models are the large number of input parameters as well as the computation time.

In watersheds where data is limited and obtaining accurate forecasts is more important than understanding underlying mechanisms, data based models are a suitable alternative. These methods are able to make generalizations of the process being studied. In data-based forecasting, statistical models have traditionally been used. Multiple linear regression (MLR) and autoregressive moving average (ARMA) models are probably the most common methods for hydrological forecasting (Raman and Sunilkumar, 1995; Young, 1999; Adamowski, 2007). In the area of groundwater level forecasting, Hodgson (1978) simulated groundwater levels through linear regression. Bierkens (1998) simulated water-table fluctuations with a stochastic differential equation. Bidwell (2005) developed an ARMA equation to forecast groundwater levels in New Zealand. Chenini and Khemiri (2009) evaluated groundwater quality using multiple linear regression and structural equation modeling in Tunisia. Lee et al. (2009) developed an ARMA model to forecast groundwater level data in Changwon, Korea. Jasmin et al. (2010) conducted a multiple linear correlation analysis to study the influence of rainfall, antecedent rainfall and antecedent groundwater table depth on groundwater depth in the upper Swarnamukhi River basin.

In recent years, artificial neural networks (ANN) have been used for groundwater level prediction (e.g., Daliakopoulos et al., 2005; Nayak et al., 2006; Uddameri, 2007; Krishna et al., 2008; Tsanis et al., 2008; Banerjee et al., 2009; Sreekanth et al., 2009; Sethi et al., 2010), aquifer parameter determination (Samani et al., 2007; Karahan and Ayvaz, 2008), and groundwater quality monitoring (Milot et al., 2002). ANN models are 'black box' models that are well suited to dynamic nonlinear system modeling. An important feature of ANN models is their ability to detect patterns in a complex system.

However, ANNs, ARIMA and other linear and non-linear methods frequently have limitations with non-stationary data (Cannas et al., 2006). ANN and ARIMA methods cannot handle non-stationary data without input data pre-processing (Tiwari and Chatterjee, 2010). The methods for dealing with non-stationary data are not as advanced as those for stationary data and additional research is needed to investigate methods that are better able to handle non-stationary data effectively. An example of such a method is wavelet analysis, which has received very little attention to date in the groundwater literature.

Wavelets are mathematical functions that give a time-scale representation of a time series and their relationships to analyze time series that contain non-stationarities. Preliminary studies have indicated that wavelet analysis appears to be a more effective tool than the Fourier Transform in analyzing non-stationary time series (Partal and Kisi, 2007; Adamowski, 2007). Wavelet analysis can be used to decompose an observed time series (such as groundwater levels) into various components so that the new time series can be used as inputs for an ANN model.

Wavelet analysis is a very new method in the areas of hydrology and water resources research. However, over the course of the last 5–6 years, it has begun to be investigated in a variety of hydrological applications. Cannas et al. (2006) developed a hybrid model for monthly rainfall–runoff forecasting in Italy. Adamowski (2007, 2008a,b) developed a completely new method of wavelet and cross wavelet based forecasting of floods. Kisi (2008) and Partal (2009) developed a hybrid model for monthly flow forecasting in Turkey. Kisi (2009) explored the use of WA–ANN models for daily flow forecasting of intermittent rivers. Wang et al. (2009) developed a wavelet neural network model to forecast the inflow at the Three Gorges Dam in Yangtze River. Adamowski and Sun (2010) developed WA–ANN models for flow forecasting at lead times of one and three days for three different rivers in Cyprus. These studies all found that the WA–ANN models outperformed the other models (such as multiple linear regression and regular artificial neural networks) that were studied for hydrological forecasting applications.

To date, no research has been published that explores coupling wavelet analysis with artificial neural networks for groundwater level forecasting with multiple hydrological input parameters. The objective of this research was to explore the use of the coupled WA–ANN method for monthly groundwater level forecasting and to compare it with two commonly used groundwater level forecasting methods. In this research, WA–ANN models were developed and compared with ANN models and ARIMA models for 1-month-ahead forecasting of groundwater levels at two sites in the Chateauguay watershed in Quebec, Canada.

2. Methods

2.1. ARIMA

The autoregressive integrated moving average (ARIMA) method has the ability to identify complex patterns in data and generate forecasts (Box and Jenkins, 1976). ARIMA models can be used to analyze and forecast univariate time series data. The ARIMA model function is represented by (p, d, q), with p representing the number of autoregressive terms, d the number of non seasonal differences, and q the number of lagged forecast errors in the prediction equation. The three steps to develop ARIMA models are identification, estimation and forecasting. ARIMA models are defined as follows (Box and Jenkins, 1976):

$$\Delta_1 Z_t = \Phi_1 Z_{t-1} + \cdots + \Phi_p Z_{t-p} + a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q}, \tag{1}$$

where $\Delta_1\ z_t$ is a differenced series (i.e., $z_t - z_t - 1$), z_t is the set of possible observations on the time-sequenced random variable, a_t is the random shock term at time t, $\Phi_1 \ldots \Phi_P$ are the autoregressive parameters of order p and $\theta_1 \ldots \theta_q$ are the moving average parameters of order q. As an example, a model described as $(0, 1, 3)$ signifies that it contains 0 autoregressive (p) parameters and 3 moving average (q) parameters which were computed for the series after it was differenced once (d).

2.2. Artificial neural networks

Artificial neural networks are inspired by the learning processes that take place in biological systems. An artificial neural network is composed of many artificial neurons that are linked together according to a specific network architecture. A neural network can be used to predict future values of possibly noisy multivariate time-series based on past histories, and it can be described as a network of simple processing nodes or neurons, interconnected to each other in a specific order, performing simple numerical manipulations. The objective of the neural network is to transform the inputs into meaningful outputs.

Hydrological variable forecasting in watershed systems is often a difficult task due to the complexity of the physical processes involved, as well as the variability of rainfall and temperature in space and time. ANNs have become popular in the last decade for hydrological forecasting such as rainfall runoff forecasting, groundwater and precipitation forecasting, and investigating water quality issues (e.g., Kisi, 2004, 2007; Adamowski, 2007, 2008a; Banerjee et al., 2009; Pramanik and Panda, 2009; Sreekanth et al., 2009; Adamowski and Sun, 2010; Sethi et al., 2010).

The most widely used neural network for hydrological modeling is the multilayer perceptron (MLP), which is also capable of nonlinear pattern recognition and memory association (Nayak et al., 2006). In the MLP, neurons are organized in layers, and each neuron is connected only with neurons in contiguous layers. A typical three-layer feedforward ANN is shown in Fig. 1. The input signal is transmitted through the network in a forward direction, layer by layer. The connections between neurons in different layers are supplied by adjusted weighting values. Each neuron is connected only with neurons in subsequent layers, and each neuron sums its inputs and later produces its output using an activation function.

The goal of an ANN model is to generalize a relationship of the form (Nayak et al., 2006):

$$Y^m = f(X^n), \tag{2}$$

where X^n is an n-dimensional input vector consisting of variables $x_1 \ldots x_i, \ldots, x_n$; while Y^m is an m-dimensional output vector consisting of the resulting variables of interest $y_1 \ldots y_i, \ldots, y_m$. In groundwater level forecasting, x_i may represent precipitation, temperature, and groundwater level values at different antecedent time lags and the value of y_i is generally the groundwater level for a subsequent period at a specific well (Nayak et al., 2006).

The Levenberg–Marquardt (LM) algorithm was used to train the ANN models in this study because it is fast, accurate, and reliable (Adamowski and Karapataki, 2010; Adamowski and Sun, 2010). In addition, previous studies have indicated that the LM is a very good algorithm to develop an ANN model for hydrological forecasting in terms of statistical significance as well as processing flexibility (Daliakopoulos et al., 2005; Sreekanth et al., 2009). The Levenberg–Marquardt algorithm is a modification of the classic Newton algorithm for finding an optimum solution to a minimization problem (Daliakopoulos et al., 2005). The LM algorithm is designed to approach second-order training speed and accuracy without having to compute the Hessian matrix. Second-order nonlinear optimization techniques are usually faster and more reliable.

2.3. Wavelet analysis

Wavelet transforms have recently begun to be explored as a tool for the analysis, de-noising and compression of signals and images. Wavelets are mathematical functions that give a time-scale representation of the time series and their relationships to analyze time series that contain non-stationarities. The data series is broken down by the transformation into its ‘wavelets’, a scaled and shifted version of the mother wavelet (Grossman and Morlet, 1984). Wavelet transform analysis, developed during the last two decades in the mathematics community, appears to be a more effective tool than the Fourier Transform (FT) in studying non-stationary time series (Partal and Kisi, 2007). The main advantage of wavelet transforms are their ability to simultaneously obtain information on the time, location and frequency of a signal, while the FT will only provide the frequency information of a signal.

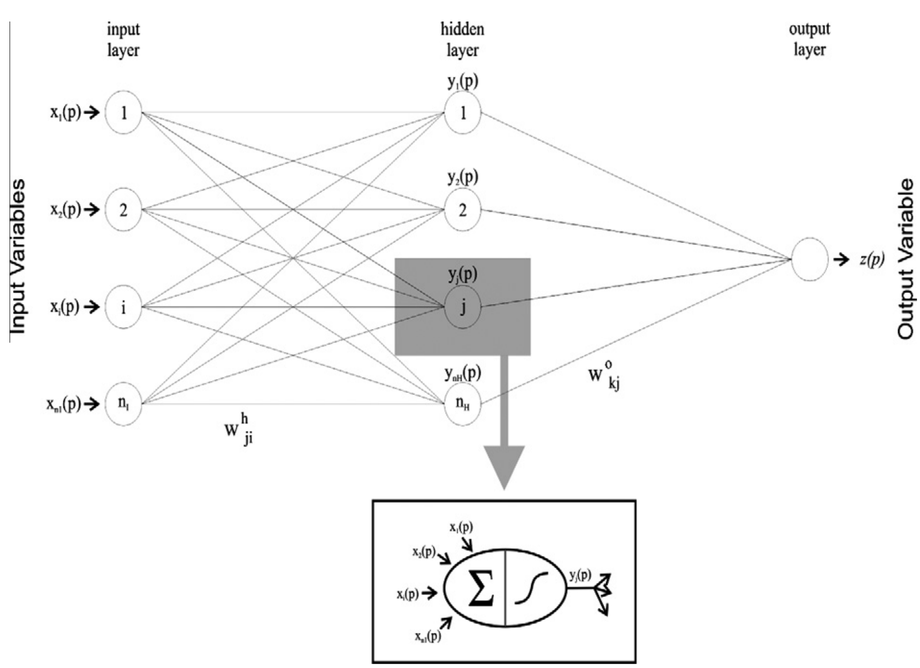

Fig. 1. ANN architecture with one hidden layer (Adamowski, 2007).

The continuous wavelet transform (CWT) of a signal $x(t)$ is defined as follows (Partal, 2009):

$$W(\tau, s) = s^{-1/2} \int_{-\infty}^{+\infty} x(t)\psi^* \left(\frac{t-\tau}{s}\right) dt, \tag{3}$$

where s is the wavelet scale, t is the time, τ is the translation parameter and '$*$' denotes the conjugate complex function. The translation parameter τ is the time step in which the window function is iterated. $W(\tau, s)$ presents a two-dimensional picture of wavelet power under a different scale. Scaling either dilates (expands) or compresses a signal. The mother wavelet ψ is the transforming function. Large scales (low frequencies) dilate the signal and provide detailed information hidden in the signal, while small scales (high frequencies) compress the signal and provide global information about the signal (Cannas et al., 2006).

The discrete wavelet transform (DWT) was used to decompose the time series data (i.e., groundwater level, temperature, and precipitation time series) for the WA–ANN models developed in this study. The classical continuous wavelet transform (CWT) requires a significant amount of computation time and data (Partal, 2009; Adamowski, 2007). In contrast, the DWT requires less computation time and is simpler to develop compared to the classical CWT (Christopoulou et al., 2002). The DWT scales and positions are usually based on powers of two (dyadic scales and positions) (Christopoulou et al., 2002). This is achieved by modifying the wavelet representation to (Grossman and Morlet, 1984):

$$\psi_{m,n}(t) = s_0^{-m/2} \psi\left(\frac{t - n\tau_0 s_0^m}{s_0^m}\right), \tag{4}$$

where s is the wavelet scale, t is the time, τ is the translation parameter, while m and n are integers that control, respectively, the scale and time; s_0 is a specified fixed dilation step greater than 1; and t_0 is the location parameter that must be greater than zero. The mother wavelet ψ is the transforming function.

In the DWT, a time-scale signal is obtained using digital filtering techniques. The original time series is passed through high-pass and low-pass filters, and detailed coefficients and approximation series are obtained with the wavelet algorithm (Zhang and Li, 2001). The filtering step is repeated every time some portion of the signal corresponding to some frequencies is eliminated, obtaining the approximation and one or more details.

2.4. Coupled wavelet and artificial neural networks (WA–ANN)

WA–ANN models are ANN models that use, as inputs, sub-series components (DWs), which are derived from the DWTs of the original time series data. As was already mentioned, the DWT was used in this study because it requires less computational effort than the CWT. One of the advantages of the WA–ANN method compared to the ANN method is its ability to identify data components in a time series such as irregular components with multi-level wavelet decomposition (Adamowski and Sun, 2010).

2.5. Model performance comparison

The performance of different forecasting models can be assessed in terms of goodness of fit once each of the model structures is calibrated using the training/validation data set and testing data set. The coefficient of determination (R^2), Nash–Sutcliffe model efficiency coefficient (E), and root-mean-squared error ($RMSE$) were used in this research.

R^2 measures the degree of correlation among the observed and predicted values. It measures the strength of the model by developing a relationship among input and output variables. R^2 values range from 0 to 1, with 1 indicating a perfect fit between the data and the line drawn through them, and 0 representing no statistical correlation between the data and a line. R^2 is given by (Sreekanth et al., 2009):

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i)^2 - \frac{\sum_{i=1}^{N}(\hat{y}_i)^2}{N}} \tag{5}$$

where $\bar{y}_i$ is the mean value taken over N, N is the number of data points used, y_i is the observed monthly groundwater level (in this study), and $\hat{y}_i$ is the forecasted groundwater level from the model.

The Nash–Sutcliffe model efficiency coefficient is used to assess the predictive power of hydrological models (Pulido-Calvo and Gutierrez-Estrada, 2009):

$$E = 1 - \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2} \tag{6}$$

An efficiency of one corresponds to a perfect match of forecasted data to the observed data. An efficiency of zero indicates that the model predictions are as accurate as the mean of the observed data, whereas an efficiency of less than zero ($E < 0$) occurs when the observed mean is a better predictor than the model (Pulido-Calvo and Gutierrez-Estrada, 2009).

The root mean square error ($RMSE$) evaluates the variance of errors independently of the sample size, and is given by (Sreekanth et al., 2009):

$$RMSE = \sqrt{\frac{SEE}{N}} \tag{7}$$

where SEE is the sum of squared errors, and N is the number of data points used. SEE is given by:

$$SEE = \sum_{i=1}^{N}(y_i - \hat{y}_i)^2 \tag{8}$$

with the variables having already been defined. $RMSE$ indicates the discrepancy between the observed and forecasted values. A perfect fit between observed and forecasted values would have an $RMSE$ of 0.

3. Study areas and data

3.1. Study watershed

The Chateauguay watershed is located on the Canadian–US border, where New York State and the Province of Quebec meet (Fig. 2). The watershed is home to around 100,000 people, of which around 73,000 live in Canada (SCABRIC-2, 2005). In the Canadian section of the watershed, there are 1057 farms that occupy a surface area of 98,633 ha, and a cultivation area of 73,235 ha. The livestock population is around 37,500 animals (SCABRIC-2, 2005). The Chateauguay watershed is under the influence of a moderated and subhumid climate. The annual mean temperature is around 6.3 °C and varies between −9.9 °C in January and 20.7 °C in July. The mean annual precipitations vary between 877 and 1039 mm/year depending on the location, while the total annual volume of precipitation is 2.25 billion m^3/year (Côté et al., 2006).

The source of the Chateauguay River is the Upper Chateauguay Lake in New York State. The 127 km river flows north into the Province of Quebec and drains into the St. Lawrence River (SCABRIC, 2005). The main tributaries of the Chateauguay River are the English River, the Trout River and the Outardes River, whose subwatersheds respectively drain 28%, 16% and 9% of the total area of the watershed. The watershed drains a territory of 2543 km^2 in which 62% (1444 km^2) is located in the Québec Region and 38% (1099 km^2) is in the United States (Fig. 2). The landscape of the wa-

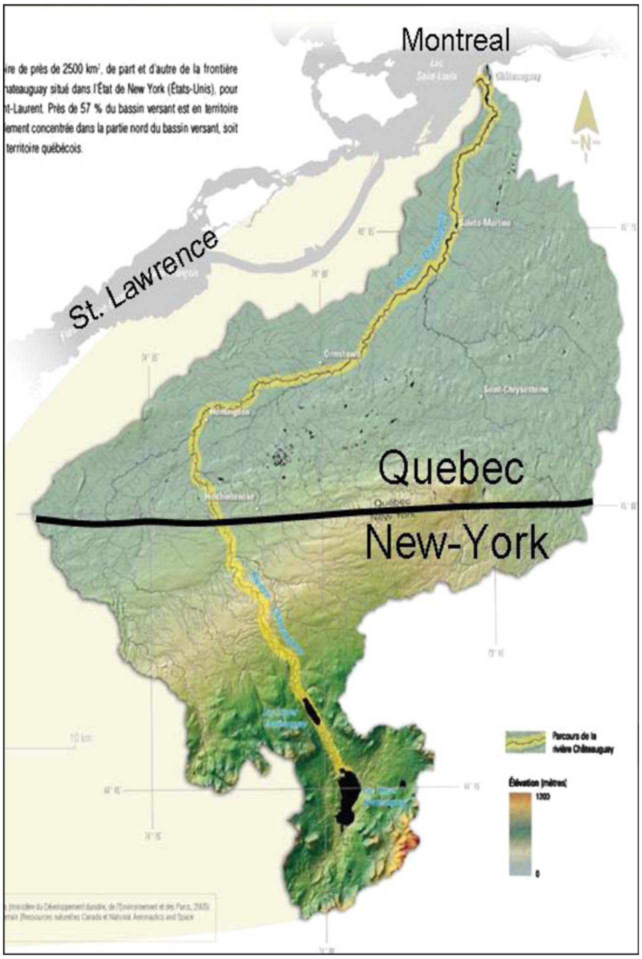

Fig. 2. Chateauguay watershed with national boundaries (Côté et al., 2006).

tershed is mainly composed of forested and cultivated land (Table 1), with forests dominating the southern parts of the watershed.

In the Chateauguay watershed in Quebec, Canada, groundwater has become a dependable source of water and it is used for many purposes and by many users (Côté et al., 2006). Only large users of groundwater are metered, and as such, the exact quantity of water withdrawn from the aquifer in the watershed is unknown, as is the case in most watersheds in Quebec. Nonetheless, the amount of groundwater extracted was estimated to be over 30 Mm^3/year (Table 2), which represents 48% of the total volume of water used with the remaining 52% coming from surface water. The local municipalities in the Chateauguay watershed have the highest level of groundwater extraction (38%), followed by the agricultural sector (27%), the industrial and commercial sector (24%), and finally private users (11%). In the agricultural sector, 75% of the water that is used is groundwater and 25% is surface water, indicating that agriculture depends heavily on groundwater in the Chateauguay watershed (Côté et al., 2006).

Table 1
Land use in the Chateauguay watershed (Côté et al., 2006).

Land use	Area (%)
Forest	38
Agriculture	34
Urban	9
Water	9
Cut, regeneration	7
Wetlands and peatlands	2
Not classified	2

Table 2
Estimated groundwater extraction in the Chateauguay watershed (Côté et al., 2006).

User	Total volume (Mm^3/year)	Proportion (%)
Municipalities	11.83	38
Private user	3.51	11
Agriculture	8.18	27
Commercial and industrial	7.51	24

Over the course of the last decade, groundwater level depletion and contamination have become an increasing concern in the Chateauguay watershed. Many abandoned wastes, including dangerous wastes, disposal facilities, and transit sites in the watershed are located in areas where they might enter the watershed's aquifer (Côté et al., 2006). These sites could leak and contaminate the groundwater in the future if they are not properly maintained and monitored. In addition, there is currently no uniform legislation to restrict land use at the watershed level (Côté et al., 2006). For example, a gravel quarry for storing industrial liquid waste was found at Mercier, a city in the Chateauguay watershed (that is also the location of one of the wells used in this study). Since the soils are extremely permeable at this site, liquid waste leaks very rapidly into the aquifer.

The problem is getting more acute as cities and populations in the watershed grow, and the demand for water increases in agriculture, industry and households. Poor management of groundwater resources leads to depletion of the aquifer storage, quality deterioration and declining groundwater levels (Banerjee et al., 2009). The accurate forecasting of groundwater levels is an important component of the sustainable management of groundwater resources. For example, accurate and reliable groundwater level forecasting in a watershed can help determine sustainable groundwater extraction policies, as well as facilitate environmental protection and the development of water price policies (Tsanis et al., 2008).

3.2. Data

The ANN and WA–ANN models in this study were developed using hydrological and meteorological variables. More specifically, the data used in this study consisted of monthly total precipitation (mm), monthly average temperature (°C), and monthly average groundwater level (mm). The average temperature and total precipitation data from November 2002 to October 2009 were obtained from the national climate data and information archive on the website of Environment Canada. Monthly average groundwater level data was obtained from November 2002 to October 2009 of two representative wells located in the cities of Mercier and St-Remi, both of which are within the Chateauguay watershed boundary. The groundwater level data was provided by the Ministry of Sustainable Development, Environment and Parks of Quebec.

4. Model development

4.1. ARIMA models

The ARIMA models for groundwater level forecasting for both study sites were developed using the Statistica software program (Statistica, 2006). Since the ARIMA method is a univariate time series analysis method, only one variable can be used (which in this case was the groundwater level data). The first step is to determine the stationarity of the input data series via the autocorrelation function (ACF). It was determined that both the Mercier and St-Remi groundwater level data series were not stationary. The ARIMA model requires the input data to have a constant mean, variance, and autocorrelation through time. Therefore, the input data series were transformed into a stationary model through a differencing process. In the models that were developed, the number of autoregressive terms (p) varied from 0 to 3, and the number of lagged forecast errors in the prediction equation (q) varied from 0 to 3. The number of nonseasonal differences (d) was set to 1 to ensure the stationarity of the data. Following this, parameter estimation in the ARIMA models was performed, and then groundwater levels were forecasted using the ARIMA models developed in the preceding steps. All ARIMA models were first trained using the data in the training set (November 2002 to February 2009), and then tested using the testing set (March 2009 to October 2009).

4.2. ANN models

The ANN models for groundwater level forecasting for both study sites were developed using the MATLAB R2010 software program (MATLAB, 2010). The regular ANN models with regular input data (i.e., those not using wavelet decomposed input data) consisted of an input layer, one single hidden layer, and one output layer consisting of one node denoting the targeted monthly average groundwater level. The ANN models were trained and tested based on different combinations of time series and numbers of neurons in the model's hidden layer. The input nodes consisted of various combinations of the following physical variables: the monthly average temperature, the monthly total precipitation, and the monthly average groundwater. Various combinations of these variables from the current month, from 1 month before, from 2 months before, from 3 months before, and from 4 months before were tested.

The number of neurons in the hidden layer was optimized using the available data through the use of a trial-and-error procedure (Jain et al., 2001). The number of neurons in the hidden layer is responsible for capturing the relationship among various input and output variables considered in developing an ANN. Each ANN model was tested on a trial-and-error basis for the optimum number of neurons in the hidden layer (found to be 2 for all models). The models were then compared using the statistical measures of goodness of fit described earlier.

For the ANN models, the data series were divided into a training set (November 2002 to June 2008), validation set (July 2008 to February 2009), and a testing set (March 2009 to October 2009).

4.3. WA–ANN models

The original data (monthly average groundwater level, monthly average temperature, and monthly total precipitation) was decomposed into a series of details (DWs) using a modified version of the a trous DWT (so that future data values are not used in the calculation). The DWs represented the detail frequency, time and location information of the original series. The decomposition process was iterated with successive approximation signals being decomposed in turn, so that the original time series was broken down into many lower resolution components. To select the number of decomposition levels or DWs, L = int[log(N)] was used (Nourani et al., 2009). L is the decomposition level while N is the number of time series data. In this study, N is 84 so L is approximately 2. As such, two wavelet decomposition levels were selected (DW1 and DW2).

In this research DW1 and DW2, as well as the approximate series, were summed and used as inputs to the ANN models. For the WA–ANN models, the ANN networks that were developed consisted of an input layer, a single hidden layer, and one output layer consisting of one node denoting the groundwater level. The input nodes consisted of various combinations of the following variables: the summed DW series (and the approximation series) of the average temperature, the total precipitation, and the average groundwater level (from the current month, from the previous month, from 2 months before, from 3 months before, and from 4 months before). As with the regular ANNs, each model was tested on a trial and error basis to determine the optimum number of neurons in the hidden layer based on different combinations of variables in the model's input layer and the number of neurons in the model's hidden layer. The optimum number of neurons was found to be 2 for all models.

For the WA–ANN models, the data series were divided into a training set (November 2002 to June 2008), a validation set (July 2008 to February 2009), and a testing set (March 2009 to October 2009).

5. Results and discussion

For both study sites (Mercier and St-Remi) the best WA–ANN models were found to provide more accurate groundwater level forecasts than both the best ANN models and the ARIMA models for 1 month lead time forecasting. For both study sites, the best WA–ANN models were a function of the total precipitation from the current month, the previous month and 2 months before; the average temperature from the current month, the previous month and 2 months before; and the average groundwater level from the current month and the previous month. The best WA–ANN models for both study sites had 2 neurons in the hidden layer.

For both study sites, the best regular ANN model had the same variables as the best WA–ANN model for both sites (i.e., the total precipitation from the current month, the previous month and 2 months before; the average temperature from the current month, the previous month and 2 months before; and the average groundwater level from the current month and the previous month). The best ANN models for both sites also had 2 neurons in the hidden layer. For both study sites, the best ARIMA model was a (1, 1, 1) ARIMA model, and so the number of autoregressive terms (p), the number of lagged forecast errors in the prediction equation (q), and the number of non seasonal differences (d) were each set at one.

The best WA–ANN models for the Mercier and St-Remi sites had a testing *RMSE* of 0.049 m and 0.553 m respectively (Tables 3 and 4), and were superior to the best ANN model and ARIMA model, which had a testing *RMSE* of 0.198 m and 0.246 m for the Mercier site and 0.814 m and 4.137 m for the St-Remi site. The lower *RMSE* values (with 0 being a perfect fit value) indicate that the best WA–ANN

Table 3
Comparison of the best WA–ANN model with the best ANN model and ARIMA model for groundwater-level forecasting at Mercier station during the testing period.

Model	Best ANN statistical results	Best WA–ANN statistical results	Best ARIMA statistical results
R^2	0.612	0.972	0.751
E	0.370	0.970	0.335
RMSE (m)	0.198	0.049	0.246

Table 4
Comparison of the best WA–ANN model with the best ANN model and ARIMA model for groundwater-level forecasting at St-Remi station during the testing period.

Model	Best ANN statistical results	Best WA–ANN statistical results	Best ARIMA statistical results
R^2	0.752	0.884	0.566
E	0.717	0.869	0.553
RMSE (m)	0.814	0.553	4.137

model had smaller differences and discrepancies between the forecasted groundwater level and the observed groundwater levels at both sites in the Chateauguay watershed.

The best WA–ANN models for the Mercier and St-Remi sites had a testing R^2 of 0.972 and 0.884 respectively (Tables 3 and 4), and were superior to the best ANN model and ARIMA model, which

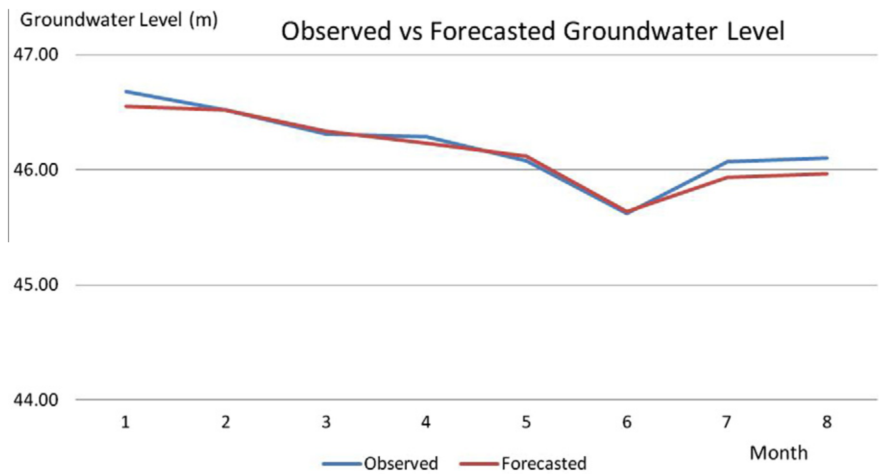

Fig. 3. Comparison of forecasted versus observed groundwater level at Mercier station using the best WA–ANN model for 1 month ahead forecasting during the testing period.

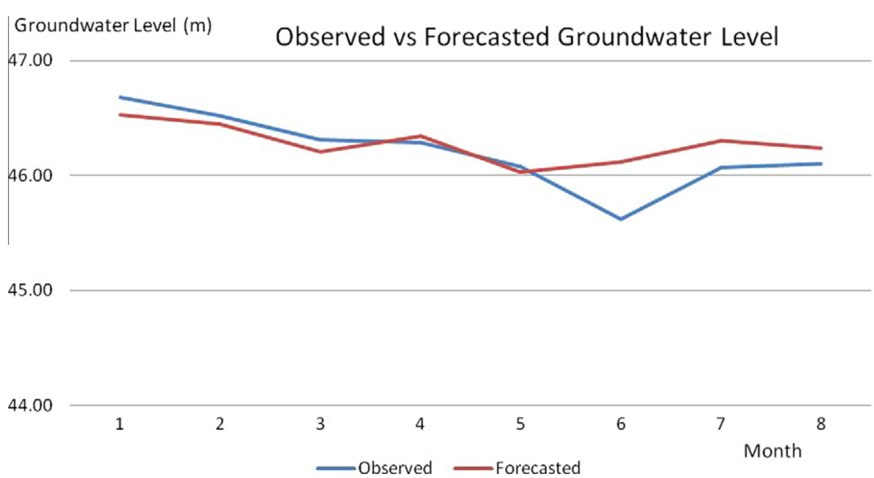

Fig. 4. Comparison of forecasted versus observed groundwater level at Mercier station using the best ANN model for 1-month ahead forecasting during the testing period.

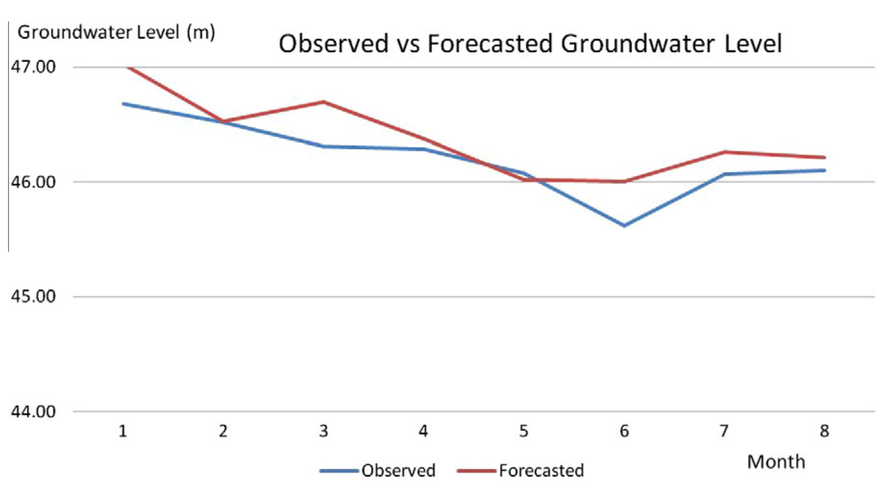

Fig. 5. Comparison of forecasted versus observed groundwater level at Mercier station using the best ARIMA model for 1-month ahead forecasting during the testing period.

had a testing R^2 of 0.612 and 0.751 for the Mercier site and 0.752 and 0.566 for the St-Remi site. The best WA–ANN models for the Mercier and St-Remi sites had a testing E of 0.970 and 0.869 respectively (Tables 3 and 4), and were superior to the best ANN model and ARIMA model, which had a testing E of 0.370 and 0.335 for the Mercier site and 0.717 and 0.553 for the St-Remi site.

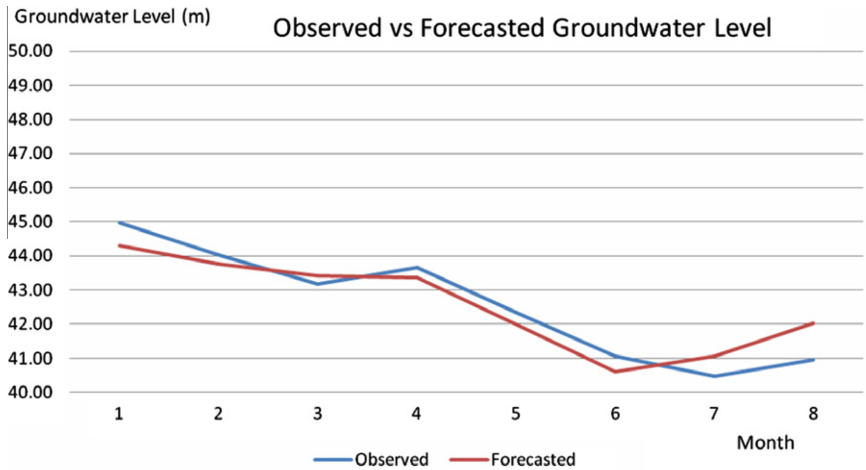

Fig. 6. Comparison of forecasted versus observed groundwater level at St-Remi station using the best WA–ANN model for 1-month-ahead forecasting during the testing period.

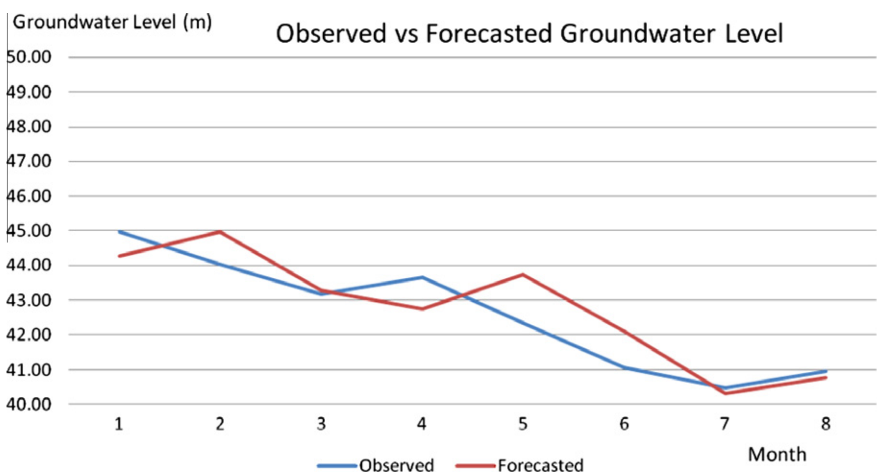

Fig. 7. Comparison of forecasted versus observed groundwater level at St-Remi station using the best ANN model for 1-month-ahead forecasting during the testing period.

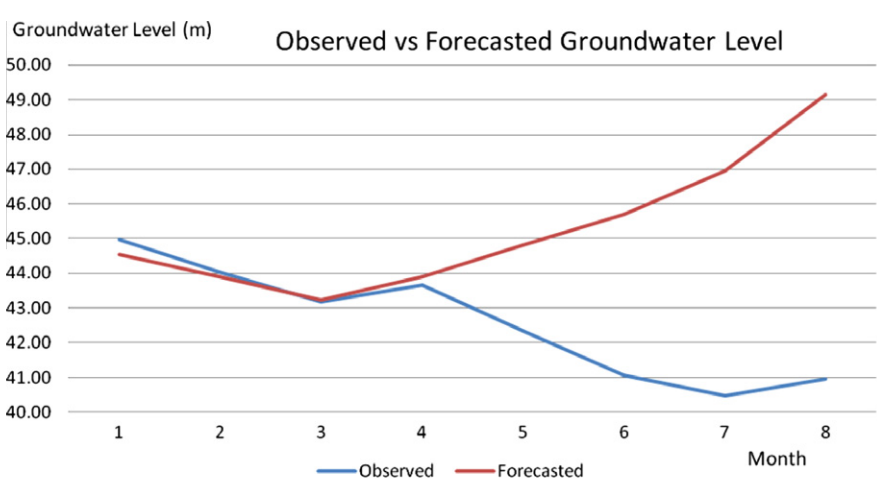

Fig. 8. Comparison of forecasted versus observed groundwater level at St-Remi station using the best ARIMA model for 1-month-ahead forecasting during the testing period.

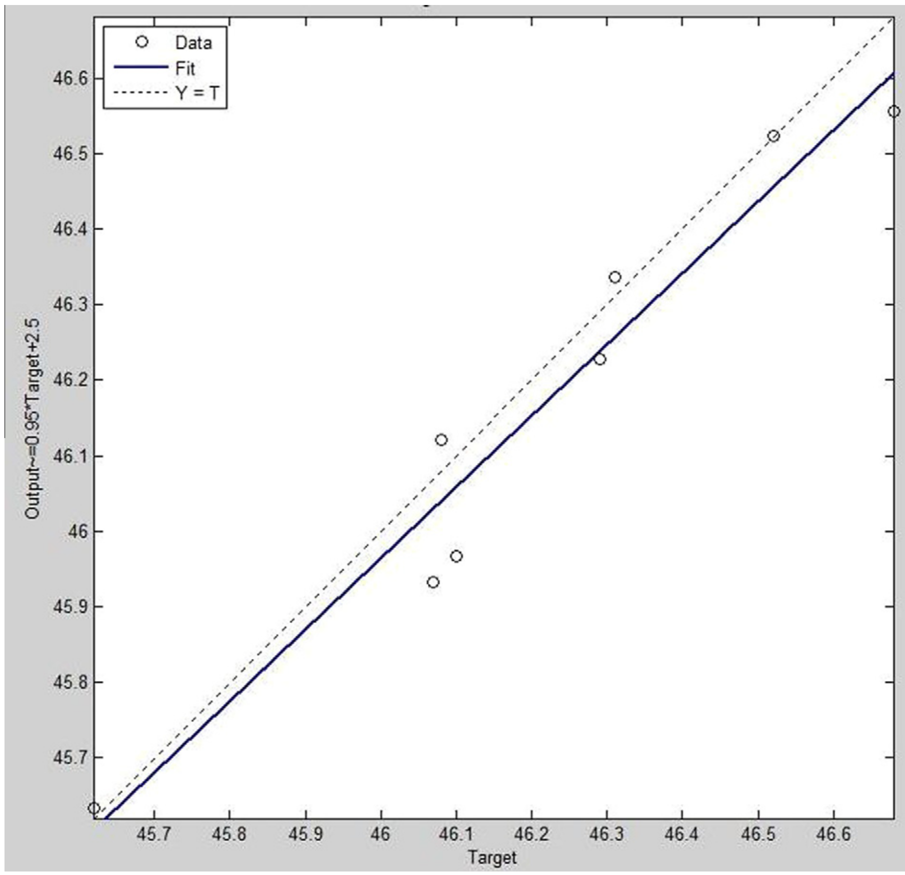

Fig. 9. Scatterplot comparing observed and forecasted groundwater levels using the best WA–ANN model for 1 month ahead forecasting during the testing period at Mercier station.

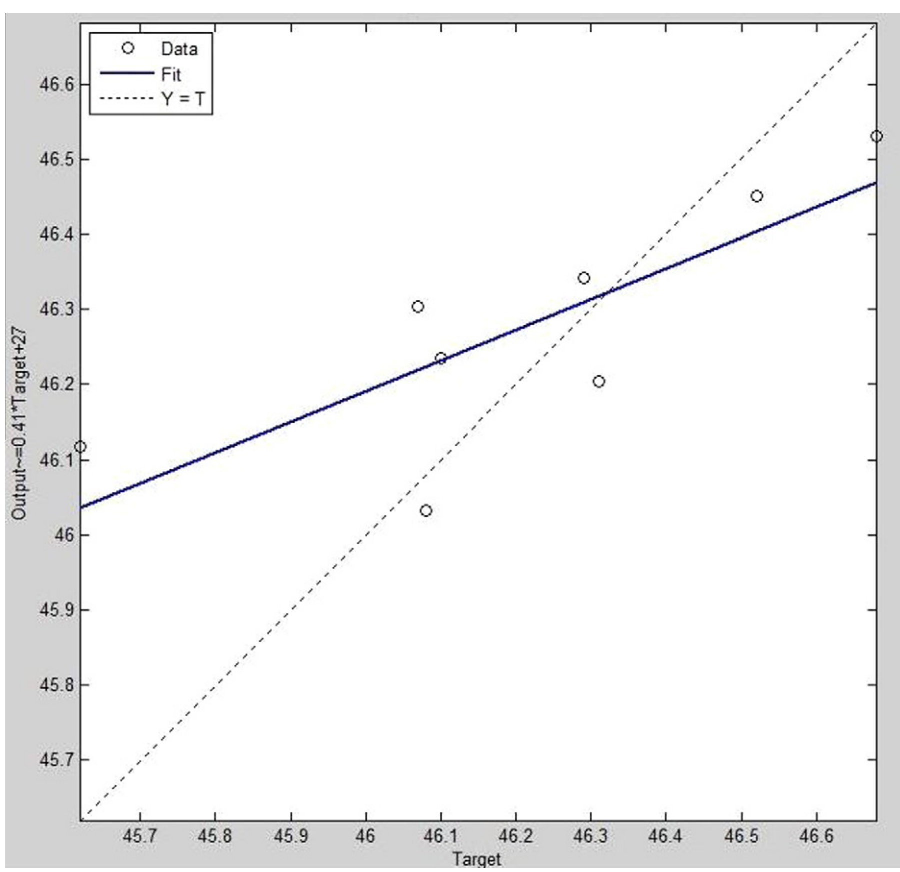

Fig. 10. Scatterplot comparing observed and forecasted groundwater levels using the best ANN model for 1 month ahead forecasting during the testing period at Mercier station.

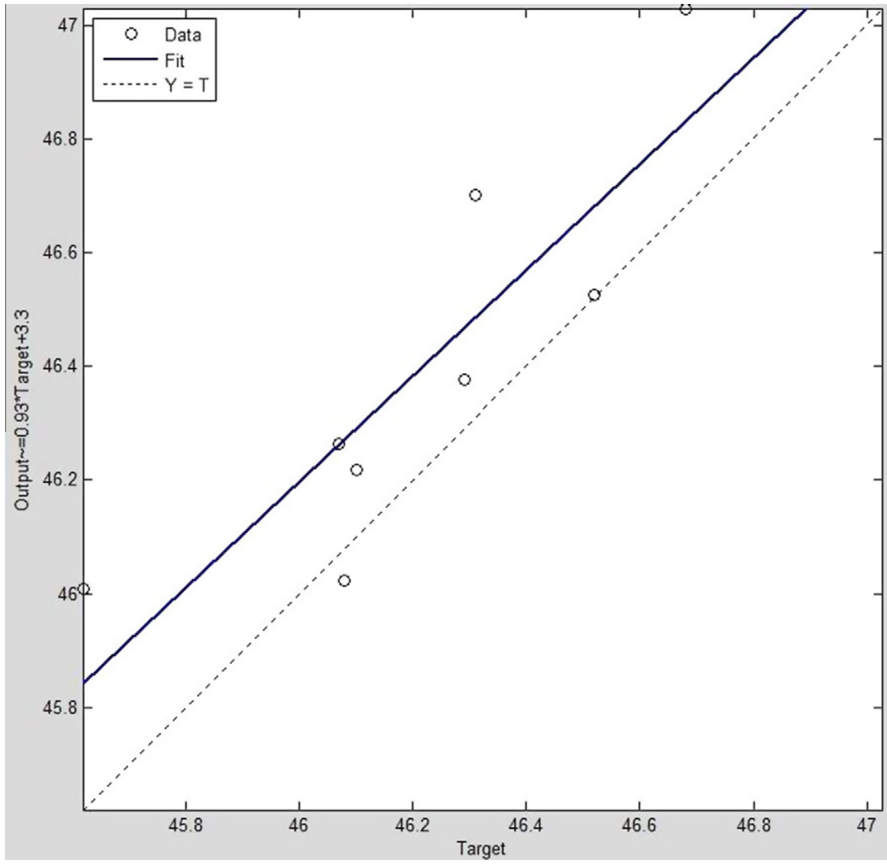

Fig. 11. Scatterplot comparing observed and forecasted groundwater levels using the best ARIMA model for 1 month ahead forecasting during the testing period at Mercier station.

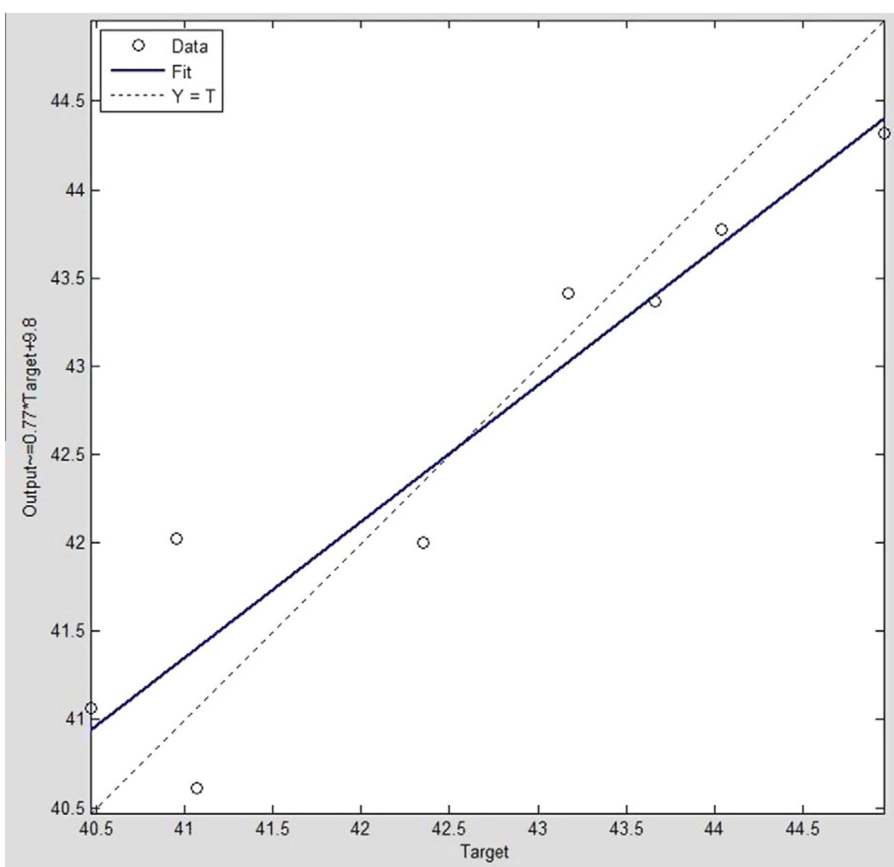

Fig. 12. Scatterplot comparing observed and forecasted groundwater levels using the best WA–ANN model for 1 month ahead forecasting during the testing period at St-Remi station.

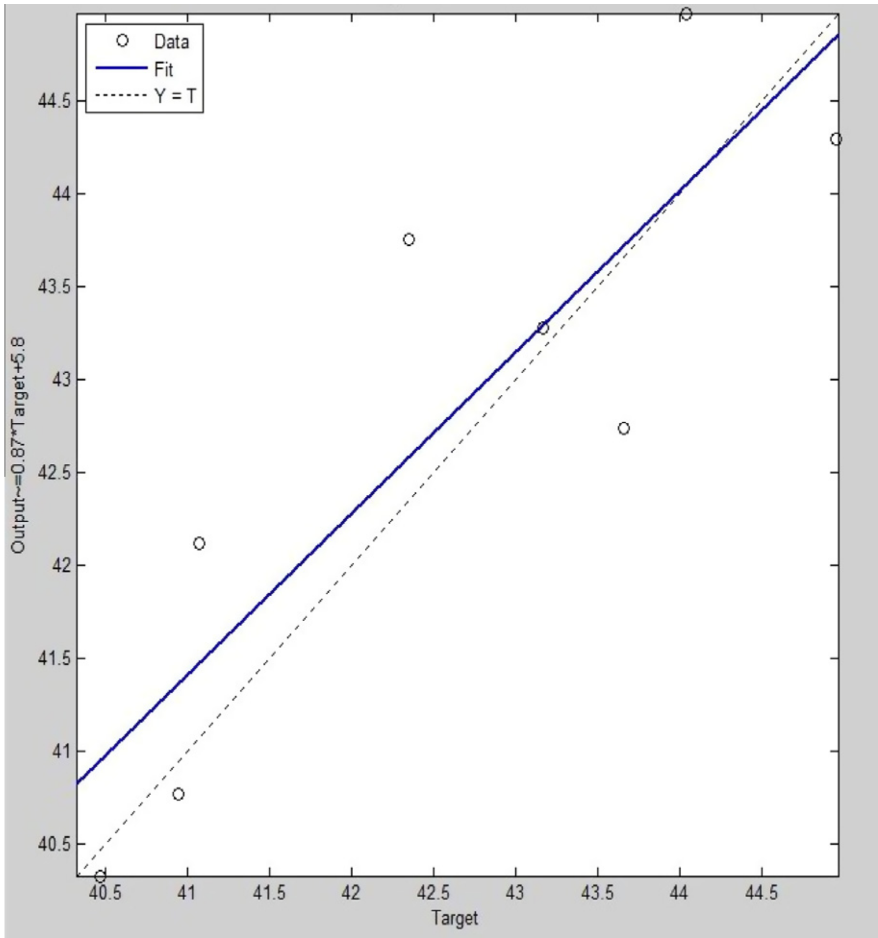

Fig. 13. Scatterplot comparing observed and forecasted groundwater levels using the best ANN model for 1 month ahead forecasting during the testing period at St-Remi station.

The higher the R^2 and E values (with 1 being a perfect fit value) indicate that the WA–ANN model is more accurate.

Figs. 3–5 compare the observed groundwater level with the forecasted groundwater level during the testing period at the Mercier site for the best WA–ANN, ANN, and ARIMA models, respectively. Figs. 6–8 compare the observed groundwater level with the forecasted groundwater level during the testing period at the St-Remi site for the best WA–ANN, ANN, and ARIMA models, respectively. It can be seen that the best ANN model and the best ARIMA model tend to over-forecast the groundwater level for both stations, while the WA–ANN model provides closer estimates to the corresponding observed groundwater level.

Figs. 9–14 are scatterplots comparing the observed and forecasted groundwater levels using the best WA–ANN model, the best ANN model and the best ARIMA model for 1 month lead time forecasting during the testing period at the Mercier and St-Remi sites. It can be seen that the WA–ANN model has less scattered estimates and that the values are denser in the neighborhood of the straight line compared to the ANN model and ARIMA model. Overall, it can be concluded the best WA–ANN model at both study sites provided more accurate forecasting results than the best ANN model and the best ARIMA model for groundwater level forecasting with a lead time of 1 month.

6. Conclusions

In this research, a new method based on coupling discrete wavelet transforms (WA) and artificial neural networks (ANN) for groundwater level forecasting applications was proposed to help watershed managers plan and manage groundwater supplies in a more effective and sustainable manner. The WA–ANN models were compared to regular ANN models and ARIMA models for average groundwater level forecasting with a 1 month lead time at two sites in the Chateauguay watershed in Quebec. The coupled wavelet–neural network models were developed by combining two methods, namely the discrete wavelet transform and artificial neural networks. Using the discrete wavelet transform, each of the original data series was decomposed into component series that carried most of the information, which were then used in forecasting via artificial neural networks. The discrete wavelet transform allowed most of the 'noisy' data to be removed and it facilitated the extraction of quasi-periodic and periodic signals in the original data time series.

This study found that the best WA–ANN model was substantially more accurate than the best ANN model and the best ARIMA model. It is hypothesized that the WA–ANN models are more accurate because wavelet transforms provide useful decompositions of the original time series, and the wavelet-transformed data improves the performance of the ANN forecasting model by analyzing useful information on various decomposition levels. The accurate forecasting results for both the St-Remi and Mercier sites in the Chateauguay watershed indicate that the WA–ANN method is a potentially very useful new method for groundwater level forecasting.

Highly accurate groundwater level forecasting models such as the WA–ANN model that was developed in this study are a useful

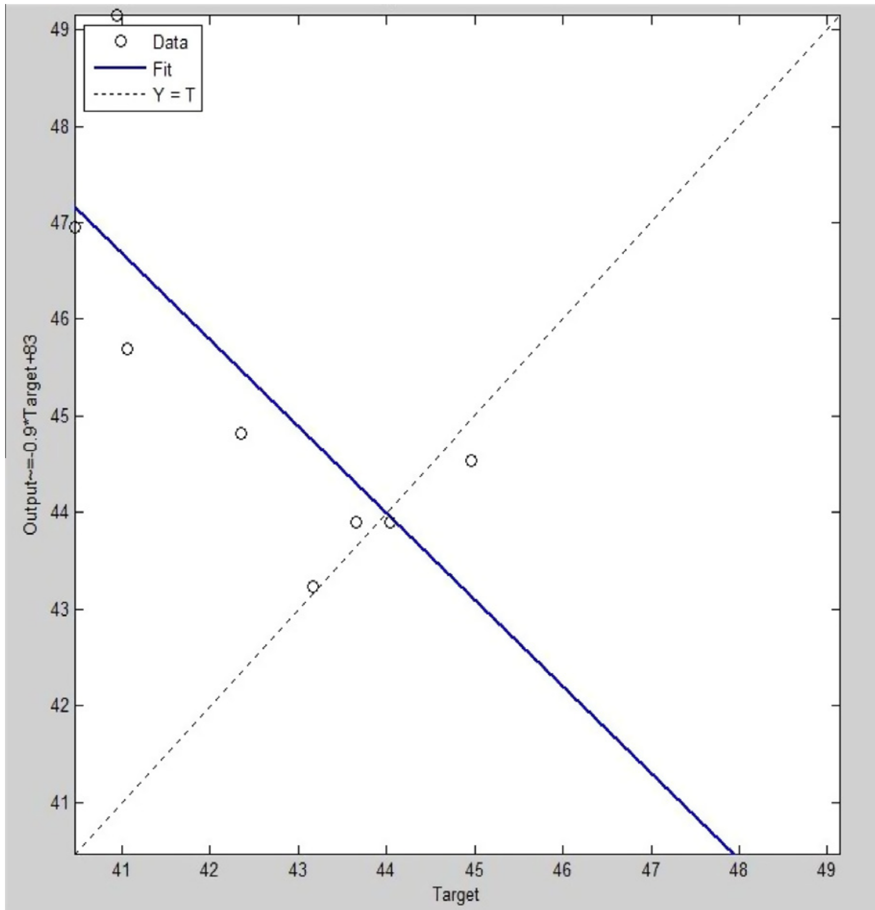

Fig. 14. Scatterplot comparing observed and forecasted groundwater levels using the best ARIMA model for 1 month ahead forecasting during the testing period at St-Remi station.

tool in sustainable groundwater extraction and optimized management in a watershed. It is recommended that future studies should explore the use of the WA–ANN method in groundwater level forecasting for: other watersheds in different geographical regions; other lead times (such as daily, weekly, or yearly forecasting); comparing the forecasting performance of the wavelet based noise removal method to other filtering methods; comparing the use of different types of continuous (Morlet and Mexican Hat) and discrete (Daubechies) mother wavelets in the wavelet decomposition phase of the wavelet neural network forecasting method; comparing the wavelet neural network method with other new methods such as support vector regression with localized multiple kernel learning; and ensemble forecasting via the use of the bootstrap method to develop wavelet–bootstrap–neural network models.

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