

# Empirical Macroeconomics

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April 1, 2019

# Growth and Fluctuations

## Supply and Demand

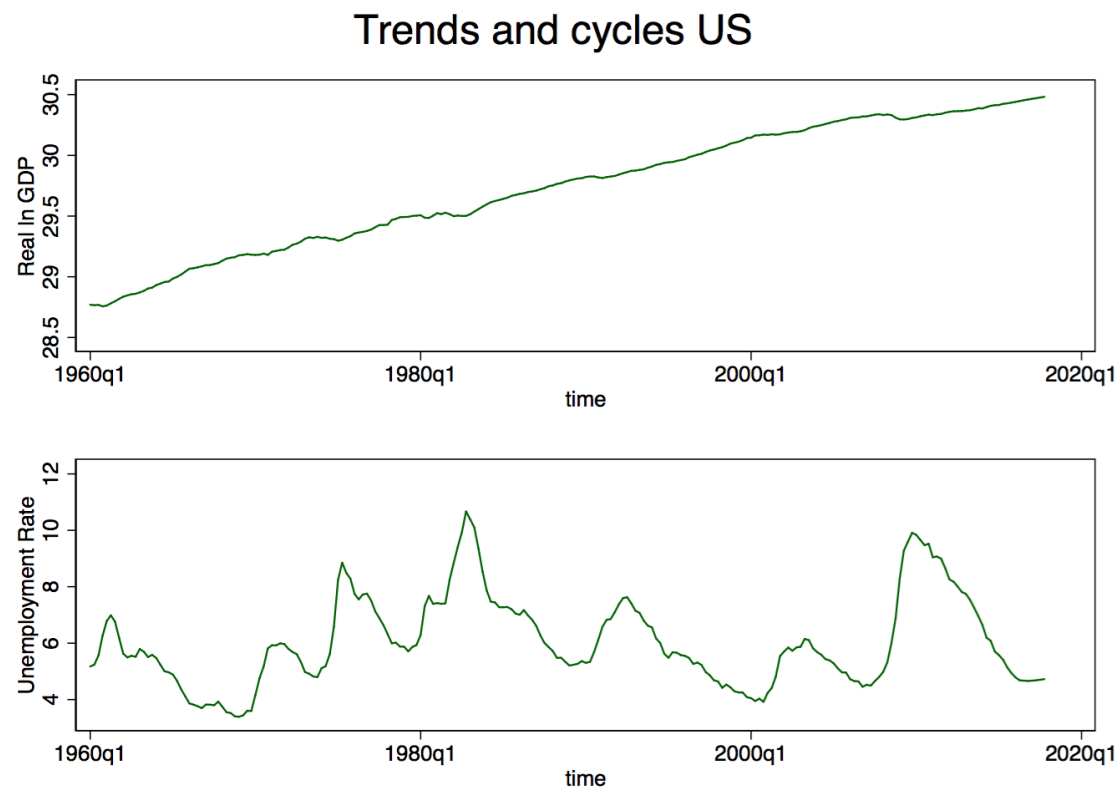


Figure : US dynamics

# Growth and Fluctuations

## Supply and Demand

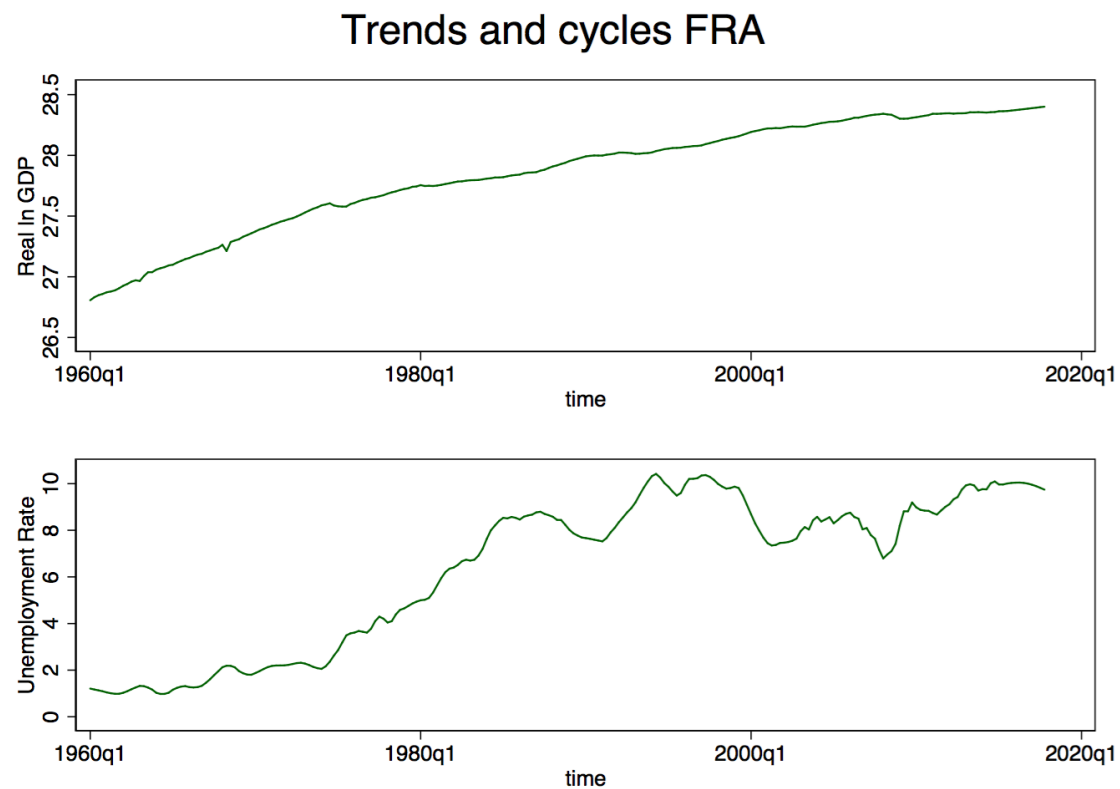


Figure : France dynamics

# Growth and Fluctuations

## Supply and Demand

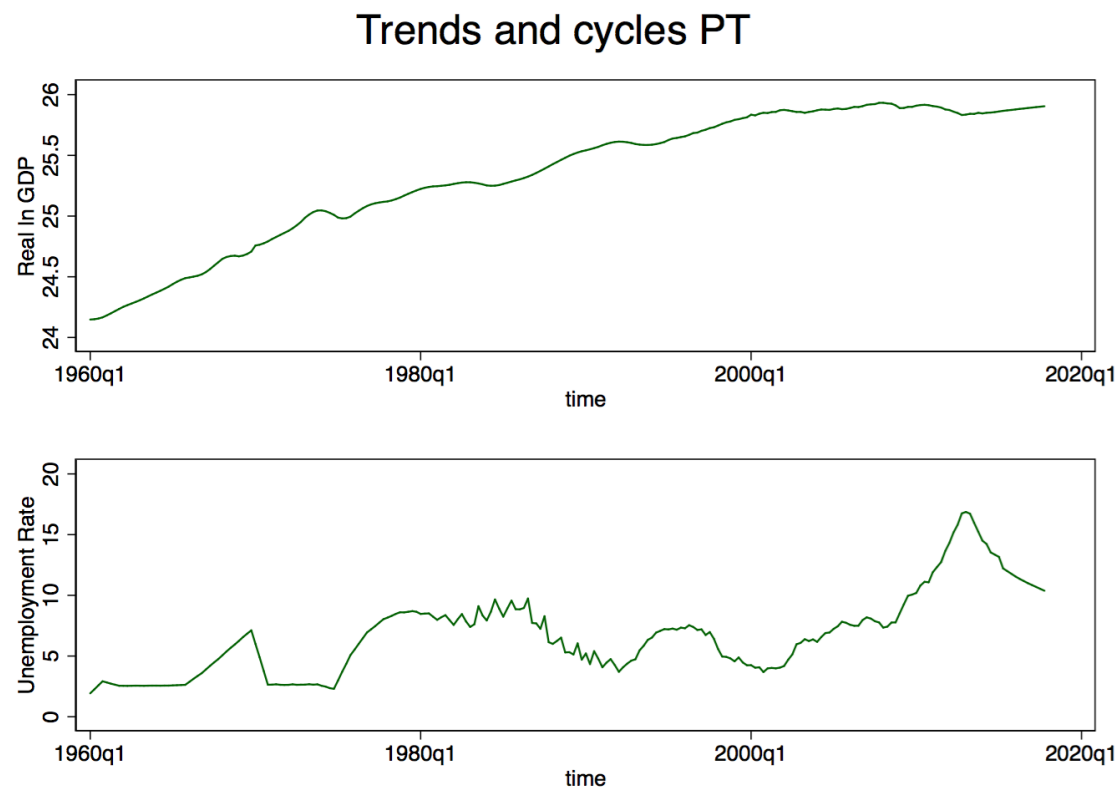


Figure : France dynamics

# SVAR

## Recap: What for

- ① study the expected response of the model variables to a given one-time structural shock: in our case permanent versus temporary.
- ② allow the construction of forecast error variance decompositions that quantify the average contribution of a given structural shock to the variability of the data.
- ③ used to provide historical decompositions that measure the cumulative contribution of each structural shock to the evolution of each variable over time.
- ④ allow the construction of forecast scenarios conditional on hypothetical sequences of future structural shocks.

# Var to SVAR

Recap: What for

- There is a large body of literature on the specification and estimation of reduced-form VAR models
- The success of such VAR models as descriptive tools and to some extent as forecasting tools is well established.
- Svar: after decomposing forecast errors into structural shocks that are mutually uncorrelated and have an economic interpretation we can assess the causal effects of these shocks on the model variables.

# The Economic Model

## MA

In the paper we find an example of a model that is a MA. This is fairly general. Let  $Y$  and  $U$  denote the logarithm of GNP and the level of the unemployment rate,  $e_d$  and  $e_s$  be the two disturbances.  $X = (\Delta Y, U)$ , and let  $\epsilon = (e_d, e_s)$ . The vector moving average (VMA) representation of the model :

$$X_t = A(0)\epsilon_t + A(1)\epsilon_{t-1} + \dots$$

- with the LR: identifying restriction  $\sum_{j=1}^{\infty} a_{11}(j) = 0$
- $Var(\epsilon) = I$ , orthogonality is identifying restriction, unit variance is normalization

# Wold representation

## Fundamental

Since  $X$  is stationary, it has a Wold-moving average representation

$$X_t = v_t + C(1)v_{t-1} + \dots + C(k-1)v_{t-k} + \dots$$

with  $Var(v) = \Omega$



# From Wold to Model

## Identification

Notice model is

$$X_t = A(0)\varepsilon_t + A(1)\varepsilon_{t-1} + \dots$$

Wold estimated from data is

$$X_t = v_t + C(1)v_{t-1} + \dots + C(k-1)v_{t-k} + \dots$$

now for every  $t$  :

$$v_t = A(0)\varepsilon_t$$

which means  $C(1)v_{t-1} = A(1)\varepsilon_{t-1} = C(1)A(0)\varepsilon_{t-1}$ . So finding  $A_0$  allows you to find all the original  $A_j$  (remember you estimate all the  $C_k$ )

# Identification

Expand

$$v = A(0)\epsilon$$
$$\begin{pmatrix} v_t^{\Delta Y} \\ v_t^U \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix}$$

first row:  $v_t^{\Delta Y} = a_{11}\epsilon_t^d + a_{12}\epsilon_t^s$

second row:  $v_t^U = a_{21}\epsilon_t^d + a_{22}\epsilon_t^s$

# Identification

Information comes from the variance of the data

$$\Omega = A(0)A(0)'$$

three restrictions

$$\begin{pmatrix} v_t^{\Delta Y} \\ v_t^U \end{pmatrix} \begin{pmatrix} v_t^{\Delta Y} \\ v_t^U \end{pmatrix}' = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix} \begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix}' \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}'$$

where now LHS is 3 distincts elements  $\omega_{11}, \omega_{12}, \omega_{22}$  and RHS is 4 distincts elements (remember to use  $\begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix} \begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix}' = I$ ). With one more restriction we can solve the system of 4 equation in 4 unknowns.

# Identification

In levels, we have for the first row of the Wold we have:

$$Y_t = v_{1t} + (I_1 + C_1(1))v_{1t-1} + .. + (I_1 + C_1(1) + .. + C_1(k))v_{t-k} + ...$$

which means that the Long Run effects are:

$$LR = (I + C(1) + C(2) + ...)A(0) = (I - C)^{-1}A(0)$$

and restriction is  $LR_{11} = 0$ .

# Issues

## Limitations:

- require an accurate estimate of the impulse responses at the infinite horizon. Similar issues that testing unit-root.
- require structural shocks to be fundamental (true for any identification strategy)

# Estimation

To recover the Wold we first estimating and then inverting the vector autoregressive representation of X (Not Always feasible...).

Estimate:

$$X_t = B(1)X_{t-1} + B(L)X_{t-2} + \dots + v_t$$

then invert

$$X_t = C(L)^{-1}v_t$$

$$X_t = v_t + C(1)v_{t-1} + \dots$$

# Estimation

The estimation of the VAR where  $T$  is sample,  $K$  is number of variables and  $p$  is number of lags,  $\mathcal{L}$  is log likelihood and  $t_p$  is the number of parameters.

- pre estimation number of lags selected through FPE, Akaike, Hannan and Quinn

$$FPE = |\Omega| \left( \frac{T + Kp + 1}{T + Kp - 1} \right)^K$$

$$AIC = -2 \left( \frac{\mathcal{L}}{T} \right) + 2 \frac{t_p}{T}$$

$$SBIC = -2 \left( \frac{\mathcal{L}}{T} \right) + 2 \frac{\ln(T)}{T} t_p$$

$$HQIC = -2 \left( \frac{\mathcal{L}}{T} \right) + \frac{2 \ln(\ln(T))}{T} t_p$$

# Estimation

The VAR residuals must be well-behaved: Normal: non serially correlated.



# Unit root

## Non stationarity

Widely disputed are the hypothesis that log GNP is reasonably characterized as a unit root process or a trend stationary process.

$$y_t = y_{t-1} + \epsilon_t$$

or

$$y_t = \beta t + \epsilon_t$$

*Our results make clear that uncritical repetition of the "we don't know, and we don't care" man- trais just as scientifically irresponsible as blind adoption of the view that "all macroeconomic series are difference-stationary," or the view that "all macroeconomic series are trend-stationary." There is simply no substitute for serious, case-by-case, analysis. Debiold and Senhadji. 1996*

# Unit root

## Non stationarity

In the OLS estimation of an AR(1)

$$y_t = \rho y_{t-1} + \epsilon_t$$

with  $\epsilon_t$  iid  $N(0, \sigma^2)$  and  $y_0 = 0$  the OLS estimate of  $\rho$  is given by

$$\hat{\rho} = \frac{\sum_{t=1}^n y_{t-1} y_t}{\sum_{t=1}^n y_{t-1}^2}$$

# Unit root

## Non stationarity

If  $|\rho| < 1$ , then

$$\sqrt{n}(\hat{\rho}_n - \rho) \rightarrow N(0, 1 - \rho^2)$$

But if this result was valid when  $\rho = 1$  then the distribution would have variance zero. (Theory in Hamilton chap 17). We need to find a suitable non degenerate distribution to test hypothesis  $H_0 : \rho = 1$

# Unit root

## Augmented Dickey-Fuller

we fit

$$\Delta y_t = \alpha + \beta y_{t-1} + \delta t + \sum_{j=1}^k \zeta_j \Delta y_{t-j} + e_t$$

via OLS. the test statistic for  $H_0 : \beta = 0$  is  $Z_t = \hat{\beta} / \hat{\sigma}_\beta$ . Critical values.

# Autocorrelation

## Test

### Definition

$\varepsilon_t$  is not iid since it is correlated with some  $\varepsilon_{t-s}$ .

Anything that causes correlation between the residuals and the regressor will make LS inconsistent. For instance, a model with a lagged dependent variable as regressor and autocorrelated shocks.

- 1 Estimate  $\rho = \text{corr}(\hat{\varepsilon}_t, \hat{\varepsilon}_{t-1})$  and use a t-test on  $\rho$
- 2 Durbin-Watson:  $DW \approx 2 - 2\rho$ . Reject  $H_0$  if  $DW \leq 1.5$ .

# Heteroskedasticity

Phillips Perron

PP correct both autocorrelation and heteroskedasticity with Newy-West but fits:  $y_t = \alpha + \rho y_{t-1} + \delta t + \epsilon_t$ . See code.