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# The Dynamic Effects of Aggregate Demand and Supply Disturbances: Comment

By Marco Lippi and Lucrezia Reichlin\*

In a recent paper published in this Review, Olivier J. Blanchard and Danny Quah (1989) propose a method to identify the dynamic effects of supply and demand disturbances on real GNP. Such a method is based on estimation of a bivariate VAR system, on which a set of structural justidentifying restrictions is imposed. From the results of their estimation and identification, Blanchard and Quah conclude that output fluctuations are mainly driven by demand shocks. Moreover, in an assessment of different studies, based on the same approach to identification, Blanchard (1989) concludes that the above result on demand shocks is robust to the particular identification restrictions imposed on the model.

In this note we make the point that Blanchard and Quah's results, as well as many other results derived from VAR estimation. are based on an arbitrary assumption about the moving-average representation, namely, that no zero of the determinant of the moving-average matrix polynomial is of modulus less than one. We show that a very simple and natural dynamic modification of the macroeconomic model underlying Blanchard and Quah's econometric work may lead to a moving-average representation with zeros on the "wrong" side of the unit circle. This has motivated us to explore alternative moving-average representations corresponding to a given estimated VAR. Some empirical results, based on Blanchard and Quah's data set, are commented upon in Section V. As we shall show, nonstandard moving-average representations may

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provide economically sensible alternatives to standard impulse-response functions, while implying a considerable reduction in the relative importance of demand as compared with Blanchard and Quah's results.

## I. Structural VAR Models

Let us summarize Blanchard and Quah's method to identify the supply and the demand component. They start with the (assumed stationary) vector:

$$\mathbf{X}(t) = \begin{bmatrix} (1-L)Y(t) \\ U(t) \end{bmatrix}$$

where Y(t) is the log of real output and U(t) is the unemployment rate. The first step consists in estimating a VAR model:

(1) 
$$\mathbf{B}(L)\mathbf{X}(t) = \mathbf{u}(t)$$

where  $\mathbf{B}(L)$  is a polynomial matrix,  $\mathbf{B}(0) = \mathbf{I}$ . Stationarity of  $\mathbf{X}(t)$  implies that all the zeros of  $\det \mathbf{B}(L)$  are of modulus greater than one

The second step consists in inverting (1):

(2) 
$$\mathbf{X}(t) = \mathbf{C}(L)\mathbf{u}(t)$$

where

$$\mathbf{C}(L) = \mathbf{B}(L)^{-1} = \mathbf{B}_{\mathrm{ad}}(L)/\det\mathbf{B}(L).$$

Obviously C(0) = I, while all the zeros of det C(L) are of modulus greater than one.

Consider now the following class of models:

(3) 
$$\mathbf{X}(t) = \mathbf{A}(L)\mathbf{e}(t)$$

where A(L) = C(L)P,  $e(t) = P^{-1}u(t)$ . The third step consists in identifying the matrix

P on the basis of structural relationships. This is the point of departure of structural VAR models from the traditional practice of imposing a recursive causal ordering between innovations. The structural restrictions can be imposed on contemporaneous short-run effects of the shocks, as in Blanchard (1989) and Ben Bernanke (1986), or on the long-effects, as in Blanchard and Quah and in Matthew Shapiro and Mark Watson (1988).<sup>1</sup>

As for the restrictions imposed by Blanchard and Quah, the matrix **P** is identified by requiring: (i) orthogonality of the components of  $\mathbf{e}(t)$ ; (ii) one of the components of  $\mathbf{e}(t)$  has no long-run effect on output; (iii) unit variance of the components of  $\mathbf{e}(t)$ . The component of  $\mathbf{e}(t)$  which has no long-run effect on output is then interpreted as a demand shock,  $e_{\mathbf{d}}(t)$ ; the other is interpreted as a supply shock,  $e_{\mathbf{s}}(t)$ . Representation (3) becomes:

$$(1-L)Y(t) = a_{11}(L)e_{d}(t) + a_{12}(L)e_{s}(t)$$
$$U(t) = a_{21}(L)e_{d}(t) + a_{22}(L)e_{s}(t)$$

where assumption (ii) above implies that  $a_{11}(1) = 0$ .

## II. A Simple Structural Model

The restrictions imposed by Blanchard and Quah are based on a very simple modification of a model by Stanley Fischer (1977):

$$(4) Y(t) = M(t) - P(t) + a\theta(t)$$

(5) 
$$Y(t) = N(t) + \theta(t)$$

(6) 
$$P(t) = W(t) - \theta(t)$$

(7) 
$$W(t) = W | \{E_{t-1}N(t) = \overline{N}\}$$

where Y, N, and  $\theta$  denote the logs of out-

put, employment, and productivity, respectively.  $\overline{N}$  is full employment; P, W, and M are the log of the price level, the nominal wage, and the money supply, respectively. The term  $a\theta(t)$  in (4) represents investment demand. In equation (7) nominal wages at t are set so that the expectation of the employment at t, formed at t-1, equals full employment. The evolution of M(t) and  $\theta(t)$  is given by

(8) 
$$M(t) = M(t-1) + e_d(t)$$

(9) 
$$\theta(t) = \theta(t-1) + e_s(t).$$

Solving the model for (1-L)Y(t) and U(t) leads to

(10) 
$$(1-L)Y(t) = (1-L)e_{d}(t)$$

$$+[1+(1-L)a]e_{s}(t)$$

$$U(t) = -e_{d}(t) - ae_{s}(t)$$

in which requirements (i), (ii), and (iii) are met, provided the shocks are orthogonal and have unit variance. The moving-average matrix is

$$\mathbf{A}(L) = \begin{bmatrix} (1-L) & 1+(1-L)a \\ -1 & -a \end{bmatrix}$$

which has determinant equal to unity.

## III. Learning-by-Doing as a Source of Dynamics

Model (10), though satisfying the required restrictions, has very poor dynamics as compared with estimated models. For that matter, Blanchard and Quah do not claim that (10) may be more than illustrative and suggest that "...more complex wage and price dynamics, such as in John B. Taylor (1980), will also satisfy the long-run properties..." (p. 658) of model (10).

Here we shall propose a simpler dynamic development of model (10), which is sufficient to motivate the empirical analysis reported in Section V. Let us write the modi-

<sup>&</sup>lt;sup>1</sup>George W. Evans (1989), like Blanchard and Quah, analyzes a bivariate VAR on output and unemployment but employs the traditional identification scheme based on a recursive structure.

fied equations:

$$(5') Y(t) = N(t) + \pi(t)$$

(6') 
$$P(t) = W(t) - \pi(t)$$

while (4), (7), (8), and (9) remain unchanged; and we add the equation

(11) 
$$\pi(t) = \pi(t-1) + d(L)e_s(t)$$

where d(1) = 1.

In the modified model,  $\theta(t) - \theta(t-1) = e_s(t)$  is that rate of increase of productivity which is eventually reached if the technical innovation becoming available at time t is installed. Equation (11) describes a learning-by-doing process: the rate of increase of productivity at time t+k is  $d_k e_s(t)$ . As the coefficients  $d_k$  sum to 1, the eventual increase in productivity is  $e_s(t)$ . The installation of technical innovation causes an immediate increase in investment demand equal to  $ae_s(t)$ , so that equation (4) is left unmodified. The solution of the model is immediate, and the matrix of the moving-average representation is:

(12) 
$$\begin{bmatrix} (1-L) & (1-L)a+d(L) \\ -1 & -a \end{bmatrix}$$

which has determinant equal to d(L).

The introduction of a learning-by-doing process has produced less-trivial dynamics in the model. Moreover, the determinant of (12) is identical to the learning-by-doing polynomial d(L) and therefore has the zeros of d(L).

The key point here is that, without further qualifications on d(L), representation (12) cannot be assumed to be fundamental, as in Blanchard and Quah's example. In that case, nonfundamentalness is ruled out by the implicit assumption d(L)=1. However, economic theory does not provide sufficient restrictions on the dynamics of  $\pi(t)$  to exclude polynomials d(L) with zeros within the unit circle. (Lars Peter Hansen and Thomas J. Sargent [1980, 1991] have made the same point in a different framework.)

Interesting examples can be obtained by assuming a bell-shaped pattern for the co-

efficients  $d_k$ , which implies the familiar S-shape for the sums  $\sum_{h=0}^{k} d_h$ . For instance, the polynomial

$$d(L) = d_0(1 + 2L + 4L^2 + 4L^3 + L^4 + 0.5L^5)$$

has five zeros whose moduli are 2.53, 0.86, and 0.60 (four of the zeros are complex).

This issue does not deserve further elaboration here. The productivity process has been introduced only to illustrate two points: (i) the long-run properties required by Blanchard and Quah can be obtained even in models whose dynamics are more complex than those of the prototype; (ii) it is possible to produce economically sensible examples in which the standard assumption on the zeros of the moving-average representation of a stationary vector is violated.

Point (ii) above implies that once a VAR model is estimated, we have to face two identification problems. First, we must give a criterion to determine the matrix **P**. Second, we must establish the position of the moving-average zeros.

The second problem, although mentioned in the literature as an important issue,<sup>3</sup> is never considered in applied work, and the rule that no zero has modulus smaller than unity is the basis of standard practice.

In the next section we recall some standard and less standard facts on representation theory for stationary stochastic vectors. In Section V we report results obtained by exploring nonfundamental moving-average representations based on the data set employed by Blanchard and Quah and based on the same identification criterion for P.

## IV. Nonfundamental Moving-Average Representations

Under the assumption of rational spectral density, the stochastic process X(t) admits

<sup>&</sup>lt;sup>2</sup>For an economic interpretation of the S-shaped pattern of the cumulated sum of d(L), see Lippi and Reichlin (1990).

<sup>&</sup>lt;sup>3</sup>Besides the already mentioned works by Hansen and Sargent, see, for instance, Quah (1990, 1992).

moving-average representations:

(13) 
$$\mathbf{X}(t) = \mathbf{D}(L)\mathbf{v}(t)$$

in which (i)  $\mathbf{v}(t)$  is a white-noise vector; (ii)  $\mathbf{D}(L)$  is a matrix of rational functions with no poles of modulus less or equal to unity [i.e., the denominators of the elements of  $\mathbf{D}(L)$  do not vanish inside or on the unit circle]; (iii)  $\det \mathbf{D}(L)$  has no zeros of modulus less than or equal to unity. Notice that representation (2), which is the inverse of (1), fulfills the conditions above. The white-noise  $\mathbf{v}(t)$ , which is unique up to multiplication by an orthogonal matrix, lies in the space  $\mathbf{H}_t$  spanned by current and past values of  $\mathbf{X}(t)$  (i.e., the econometrician's information set).

Both representation (13) and the whitenoise  $\mathbf{v}(t)$  are called *fundamental* for  $\mathbf{X}(t)$ . Nonfundamental representations are moving-average representations fulfilling (i) and (ii) but not (iii). From representation theory of stationary stochastic processes it follows that if

$$\mathbf{X}(t) = \mathbf{F}(L)\mathbf{w}(t)$$

is nonfundamental, then the white noise  $\mathbf{w}(t)$  is a linear combination of current, past, and future values of  $\mathbf{X}(t)$ , so that it does not belong to  $\mathbf{H}_t$  (see e.g., Yu A. Rozanov, 1967).

As shown in Lippi and Reichlin (1991), all possible  $\mathbf{F}(L)$  can be obtained by repeatedly multiplying  $\mathbf{D}(L)$  by matrices like  $\mathbf{KB}(L,\gamma)$ , where  $\mathbf{K}$  is an orthogonal matrix, while the definition of  $\mathbf{B}(L,\gamma)$  is

$$\mathbf{B}(L,\gamma) = \begin{bmatrix} \frac{L-\overline{\gamma}}{1-\gamma L} & 0\\ 0 & 1 \end{bmatrix}$$

where  $\bar{\gamma}$  is the complex conjugate of  $\gamma$ , and  $|\gamma| < 1$  (naturally, if  $\gamma$  is not real, also the matrix  $\mathbf{B}(L,\bar{\gamma})$  must be employed). Thus, all nonfundamental moving-average representations may be obtained as

(14) 
$$\mathbf{D}(L)\mathbf{K}_1\mathbf{B}(L,\gamma_1)\mathbf{K}_2\mathbf{B}(L,\gamma_2)\cdots\mathbf{K}_m\mathbf{B}(L,\gamma_m)$$
.

An interesting result, which is proved in the Appendix, is the following proposition.

PROPOSITION 1: Let Q be the set of all moving-average representations of X(t), fundamental and nonfundamental, fulfilling Blanchard and Quah's identifying restrictions. Let Z be the ratio of the demand component variance to the supply component variance. There exists a positive b, such that for any positive  $\beta$ ,  $\beta < b$ , a representation can be found in Q for which  $Z = \beta$ .

Such a result is possible because the manifold of nonfundamental representations is infinite and infinite-dimensional, as (14) clearly shows. Notice that the opposite result is already available in Quah (1982 [and previous versions]) and Lippi and Reichlin (1990).<sup>4</sup>

These results raise a difficult issue. How should one choose the numbers  $\gamma_i$  and the matrices  $\mathbf{K}_i$  in order to obtain economically relevant alternative representations? A thorough discussion of this problem would go far beyond the limits of this note. We will only mention here the argument developed in Lippi and Reichlin (1991). Suppose we start with the VAR approximation (1), which is rewritten here for convenience:

$$\mathbf{B}(L)\mathbf{X}(t) = \mathbf{u}(t)$$

with

$$\mathbf{B}(L) = \mathbf{B}_0 + \mathbf{B}_1 L + \cdots + \mathbf{B}_m L^m.$$

From (1) we obtain:

(15) 
$$\det \mathbf{B}(L)\mathbf{X}(t) = \mathbf{B}_{\mathrm{ad}}(L)\mathbf{u}_{t}.$$

Then, consider the zeros of  $\det \mathbf{B}(L)_{ad}$ :5

$$r_1, r_2, \ldots r_{2m}$$
.

Since they come from the inversion of  $\mathbf{B}(L)$ ,  $|r_i| > 1$ , i = 1, 2m. In Lippi and Reichlin

<sup>5</sup>These are identical to the zeros of det  $\mathbf{B}(L)$ .

<sup>&</sup>lt;sup>4</sup>The results in Quah (1992) and Lippi and Reichlin (1990) imply the existence of a decomposition with an arbitrarily small variance of the supply component.

(1991) we argue that nonfundamental representations obtained by using matrices  $\mathbf{B}(L,\gamma)$ , in which  $|\gamma|^{-1}$  is not among the moduli of the zeros  $r_i$ , should be ruled out as uninteresting from an economic point of view. This criterion considerably restricts the area of the complex plane in which to look for the coefficients  $\gamma$ . In the present paper the experiments have been conducted by using the zeros  $r_i$  themselves.<sup>6</sup>

## V. Empirical Results

Before reporting the results of our experiments, let us give some further details on the procedure used:

Step 1.—Let us go back to (15). Consider, for instance, the zero  $r_1$ . Let  $\mathbf{K}_1$  be an orthogonal matrix such that the polynomials on the first column of  $\mathbf{B}_{ad}(L)\mathbf{K}_1$  vanish for  $L = r_1$  [existence of  $\mathbf{K}_1$  is trivial since  $\det \mathbf{B}_{ad}(r_1) = 0$ ].

Step 2.—Multiply by  $\mathbf{K}_1\mathbf{B}(L, r_1^{-1})$ .

Step 3.—Impose Blanchard and Quah's restrictions. The resulting matrix,  $C_r(L)$ , has no poles, while its determinant has  $r_1^{-1}$ , but not  $r_1$ , among its zeros. The new representation will be

$$\det \mathbf{B}(L)\mathbf{X}(t) = \mathbf{C}_{r_1}(L)\mathbf{u}_1(t)$$

where the right-hand side is still an MA(m) whose determinant vanishes for

$$r_1^{-1}, r_2, \ldots, r_{2m}.$$

The impulse-response functions can be immediately obtained from

$$\frac{\mathbf{C}_{r_{\mathbf{I}}}(L)}{\det \mathbf{B}(L)}$$

Evidently, repeated applications of the above procedure lead to representations in

Table 1—Moduli of the Zeros of  $\det \mathbf{B}(L)$ 

Zeros moduli	E1	E2
1.2290	X	X
1.4357	X	X
1.4357	X	X
1.2971		
1.2971		
1.2825		
1.2825		
1.2828		
1.2828		
1.1908		
1.1908		
1.2140		
1.2140		
1.1876		X
1.1876		X
6.3442	X	X

*Note:* In columns 2 and 3, an X denotes that the reciprocal of the corresponding zero of column 1 was substituted in E1 or E2, respectively.

which the elements of any subset of  $\{r_i\}$  are replaced by their reciprocals.<sup>7</sup>

We performed many experiments by inverting some of the zeros in  $\{r_i\}$ . First, we discarded the cases producing impulseresponse functions with an "implausible" shape. Then we observed that the remaining cases could be grouped into a few classes containing impulse-response functions very close to one another. Lastly, we selected the two experiments (the best in their classes) with impulse-response shapes closest to the fundamental case: let us call such experiments E1 and E2.

In Table 1 we report the moduli of the zeros of  $\det \mathbf{B}(L)$  (repeated figures refer to complex conjugate zeros): in columns 2 and 3, an X indicates that the reciprocal of the zero of column 1 was substituted in E1 or E2, respectively.

Figures 1–4 contain the impulse-response functions for Blanchard and Quah's model (solid lines), experiment E1 (dashed lines) and experiment E2 (dotted lines). Figures 5 and 6 show output and unemployment fluc-

<sup>&</sup>lt;sup>6</sup>In Lippi and Reichlin (1991) we go further and provide ideas for distinguishing between zeros belonging to the AR and MA matrix polynomials of the underlying vector ARMA structure.

<sup>&</sup>lt;sup>7</sup>The procedure has been implemented using PC-MATLAB.

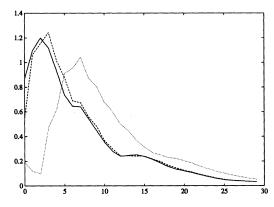


FIGURE 1. OUTPUT RESPONSE TO DEMAND

Key: Solid line = Blanchard and Quah (1989), dashed line = E1; dotted line = E2.

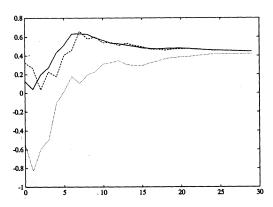


FIGURE 2. OUTPUT RESPONSE TO SUPPLY

Key: Solid line = Blanchard and Quah (1989); dashed line = E1; dotted line = E2.

tuations due to the demand shocks corresponding to the same cases.

Let us first comment on experiment E1. The variance of differenced output accounted for by the demand component is 90 percent in Blanchard and Quah's model, while it is 70 percent in E1. Such a substantial decrease is mainly due to first-impact (zero-lag) values of the responses to the demand shock (Fig. 1). The corresponding increase in the variance of the supply component of output in E1 with respect to Blanchard and Quah's case, emerges clearly from Figure 2, which represents the re-

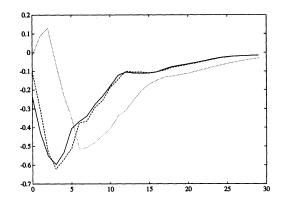


FIGURE 3. UNEMPLOYMENT RESPONSE TO DEMAND

*Key:* Solid line = Blanchard and Quah (1989); dashed line = E1; dotted line = E2.

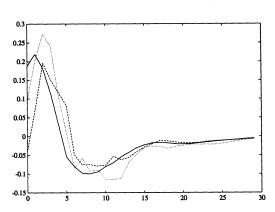


FIGURE 4. UNEMPLOYMENT RESPONSE TO SUPPLY

Key: Solid line = Blanchard and Quah (1989); dashed line = E1; dotted line = E2.

sponse of output to a supply shock.<sup>8</sup> First-impact responses are also responsible for the main differences in the shapes of the impulse-response functions (Figs. 1–4). In Figure 1, for example we can see that the first impact has the effect of somewhat accentuating the hump-shaped pattern of output response to demand.

<sup>&</sup>lt;sup>8</sup>Note that the variance of  $\Delta Y(t)$  is the sum of the squared increments of the functions graphed in Figures 1 and 2.

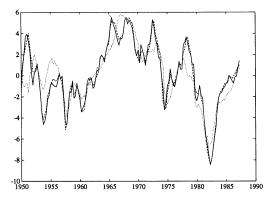


FIGURE 5. OUTPUT FLUCTUATIONS DUE TO DEMAND

Key: Solid line = Blanchard and Quah (1989); dashed line = E1; dotted line = E2.

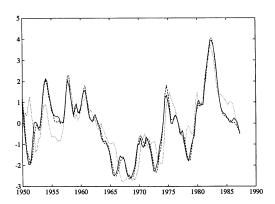


FIGURE 6. UNEMPLOYMENT FLUCTUATIONS
DUE TO DEMAND

*Key:* Solid line = Blanchard and Quah (1989); dashed line = E1; dotted line = E2.

Furthermore, as shown by Figures 5 and 6, the fluctuations of output and unemployment due to the E1 demand shock are remarkably similar to those due to the fundamental shock considered by Blanchard and Quah. In sum, from experiment E1 we obtain a rather different result for the relative importance of the demand component (70 percent instead of 90 percent), without altering too much either the shape of the impulse-responses or that of the fluctuations due to demand.

Let us now consider experiment E2, which is nested within E1, since it is obtained by taking some more zeros inside the unit circle (see Table 1). The variance of the demand component accounts for 36 percent of the total variance of differenced output. The shape of the responses to demand shocks is very interesting, showing a substantial shift in the lag structure with respect to Blanchard and Ouah's results. Overall the impulse-response functions are not difficult to interpret. However, we could not find a convincing explanation for the dramatic fall in output which occurs in Figure 2 as the first-impact effect of a positive productivity shock. As expected from the shape of the impulse-response functions, Figures 5 and 6 show that the fluctuations of output and unemployment due to the E2 shocks are more divergent from Blanchard and Ouah's fluctuations than is the case in our E1, but they still have a plausible shape. In summary, experiment E2 gives a picture of output and unemployment dynamics that differs substantially from that obtained in the fundamental case. The variances of the components, the impulse-response functions, and the fluctuations due to demand all differ considerably across the two experiments. It is worthwhile noting here that experiments performed with the VAR model estimated in Evans (1989) led to very similar results.<sup>9</sup>

#### VI. Concluding Remarks

The main point of this paper is that economic theory does not in general provide sufficient structure to choose among the many moving-average representations associated with an estimated VAR. Although this issue is relevant for VAR-based econometric research in general, here we have presented only an application to the bivariate output-unemployment model recently estimated by Blanchard and Quah (1989) and other authors.

<sup>&</sup>lt;sup>9</sup>Thanks are due to Danny Quah and George Evans for providing data and estimated coefficients.

In that case our exploration of nonstandard moving-average representations for a given estimated VAR shows economically interesting alternatives to fundamental representations. In particular, we can find valid representations in which the importance of demand is not as overwhelming as in Blanchard and Ouah's model.

#### APPENDIX

## PROOF OF PROPOSITION 1:

Consider the case in which X(t) is a finite MA (the ARMA case does not lead to any additional difficulties) and let its spectral density matrix be

$$\Gamma(z) = \begin{bmatrix} \Gamma_{11}(z) & \Gamma_{12}(z) \\ \Gamma_{12}(z^{-1}) & \Gamma_{22}(z) \end{bmatrix}$$

where  $z = e^{-i\lambda}$ . By assumption, the functions  $\Gamma_{ij}$  are finite polynomials in z and  $z^{-1}$ .

First, decompose  $\Gamma_{11}(z)$  and det  $\Gamma(z)$ :

$$\Gamma_{11}(z) = V(z)V(z^{-1})$$

$$\det \Gamma(z) = W(z)W(z^{-1})$$

 $\det \Gamma(z) = W(z)W(z^{-1})$ 

assuming that neither V(z) nor W(z) has zeros of modulus less than or equal to unity. Secondly, decompose the matrix  $\Gamma(z)$ :<sup>10</sup>

$$\Gamma(z) = \begin{bmatrix} z^{s}V(z^{-1}) & 0 \\ z^{s}\frac{\Gamma_{12}(z^{-1})}{V(z)} & \frac{W(z)}{V(z)} \end{bmatrix}$$

$$\times \begin{bmatrix} z^{-s}V(z) & z^{-s}\frac{\Gamma_{12}(z)}{V(z^{-1})} \\ 0 & \frac{W(z^{-1})}{V(z^{-1})} \end{bmatrix}$$

<sup>10</sup>These are the first steps in the procedure employed in Yu A. Rozanov (1967 pp. 44-7) to obtain the Wold representation from the spectral density in the rational case.

where s is the minimum integer such that neither  $z^s \mathbf{V}(z^{-1})$  nor  $z^s \Gamma_{12}(z^{-1})$  contains any negative powers of z. The above spectral decomposition corresponds to the representation

(16) 
$$\mathbf{X}(t) = \mathbf{M}(L)\mathbf{w}(t)$$

$$= \begin{bmatrix} L^{s}V(L^{-1}) & 0 \\ L^{s}\frac{\Gamma_{12}(L^{-1})}{V(L)} & \frac{W(L)}{V(L)} \end{bmatrix} \mathbf{w}(t).$$

Notice that M(L) has no poles of modulus less than or equal to unity, but

$$\det \mathbf{M}(L) = \frac{L^{s}V(L^{-1})W(L)}{V(L)}$$

has zeros inside the unit circle, so that (16) is a nonfundamental representation. Notice also that (16) trivially meets Blanchard and Ouah's restrictions.

Now let

$$N(L) = KB(L, \alpha)K$$

where  $\alpha$  is real,  $|\alpha| < 1$ , while **K** is the orthogonal matrix proportional to

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

and consider the representation

$$\mathbf{X}(t) = \mathbf{M}(L)\mathbf{N}(L)\tilde{\mathbf{w}}(t).$$

The first row of M(L)N(L), apart from a constant term, is

$$\left[ \frac{-(1+\alpha)(1-L)}{1-\alpha L} M_{11}(L) \quad \frac{(1-\alpha)(1+L)}{1-\alpha L} M_{11}(L) \right]$$

Blanchard and Quah's conditions are met. Moreover elementary calculations show that the supply-component variance (on the right) tends to zero as  $\alpha$  tends to 1, while the demand-component variance tends to zero as  $\alpha$  tends to -1.

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