Serie 1 OEL

Exercice 1

1°) a) Libres : absence d'excitateur (absence d'intervension exterieure)

* Amorties : Diminution des valeurs maximales des oscillations.

b) graphiquement : $1,75\,T = 1,05\ ms$

$$\boxed{T} = \frac{1,05}{1,75} = \boxed{0,6\ ms}$$

c) periode propre : $T_0 = 2\pi\sqrt{LC}$ $\boxed{\omega_0 = \frac{1}{\sqrt{LC}} = \frac{2\pi}{T_0} = 2\pi N_0}$

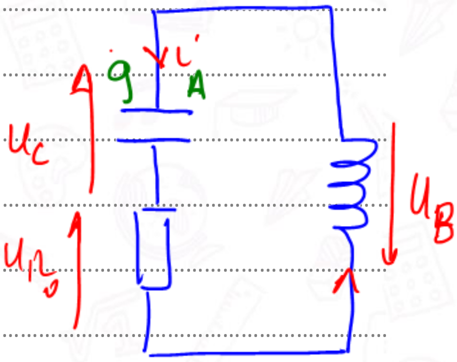
$$T \simeq T_0 = 2\pi\sqrt{LC} \Rightarrow T^2 = 4\pi^2 LC$$

$$\Rightarrow \boxed{L = \frac{T^2}{4\pi^2 C}} = \frac{0,36.10^{-6}}{4\pi^2\times 114.10^{-9}} = \boxed{0,08\,H}$$

2) loi des mailles :

$$u_C + u_{R_0} + u_B = 0$$

$$u_C + R_0 i + L\frac{di}{dt} + ri = 0$$



$$i = C\frac{du_C}{dt}$$

$$\Rightarrow \boxed{LC\frac{d^2u_C}{dt^2} + (R_0+r)C\frac{du_C}{dt} + u_C = 0}$$

3°) a) $\boxed{E = E_C + E_L = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2}$

b) $$\frac{dE}{dt} = \frac{1}{C}\underbrace{\frac{dq}{dt}}_{i}q + L\frac{di}{dt}.i = i\left[\underbrace{\frac{1}{C}q + L\frac{di}{dt}}_{-(R_0+r)i}\right] = i\left(u_C + L\frac{di}{dt}\right)$$

$$\frac{dE}{dt} = -(R_0+r)i^2 < 0.$$

$\frac{dE}{dt} < 0 \Rightarrow$ E diminue au cours du temps $\Rightarrow$ oscillateur non conservatif

4°) a) $u_b(t) = L\frac{di}{dt} + ri$

à $t = t_1$: * $\boxed{i(t_1) = -10^{-2}\,A}$

* $\frac{di}{dt}$ = pente de la tg à la courbe (p)

$\frac{di}{dt} = \frac{(-1-2)10^{-2}}{T/2} = \frac{-3\,10^{-2}}{0{,}3\,10^{-3}} = \boxed{-100\,A.s^{-1}}$

$\Rightarrow u_b(t_1) = -0{,}08 \times 100 - 5 \times 10^{-2} = -8{,}05\,V$.

b) loi des mailles à la date t_1 :

$u_C(t_1) + u_B(t_1) + u_{R_0}(t_1) = 0$

$\boxed{u_C(t_1)} = -[u_B(t_1) + R_0 i] = -[-8{,}05 - 20.10^{-2}] = \boxed{8{,}25\,V}$

$E(t_1) = E_C(t_1) + E_L(t_1) = \frac{1}{2}Cu_C(t_1)^2 + \frac{1}{2}Li(t_1)^2$

$\boxed{E(t_1)} = \frac{1}{2} \times 114\,10^{-9} \times 8{,}25^2 + \frac{1}{2} \times 0{,}08 \times 10^{-4} = \boxed{7{,}88\,10^{-6}\,J}$

c) $\boxed{E_{th} = E_{dis} = |\Delta E| = E(0) - E(t_1)}$

à $t = 0$: $\begin{cases} \text{Condensateur initialement chargé} : u_C(0) = U_0 . \\ i(0) = 0 \text{ (bobine s'oppose à l'augmentation instantanée du courant)} \Rightarrow E_L(0) = 0 \end{cases}$

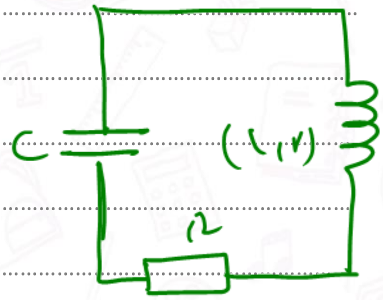
$\Rightarrow E(0) = \frac{1}{2}CU_0^2$

$$E_{th} = \frac{1}{2} C U_0^2 - E(t_1) \Rightarrow \frac{1}{2} C U_0^2 = E_{th} + E(t_1)$$

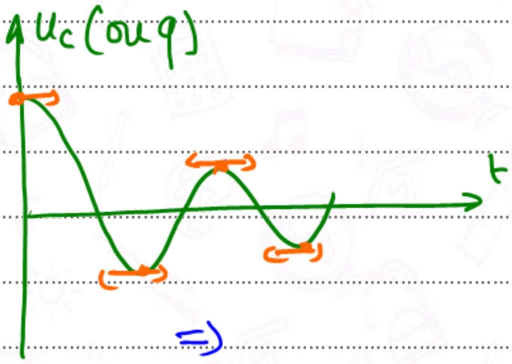
$$\boxed{U_0} = \sqrt{\frac{2(E_{th}+E(t_1))}{C}} = \sqrt{\frac{2[4{,}96\,10^{-6} + 7{,}88\,10^{-6}]}{114\,10^{-9}}} = \boxed{15V}$$

Rque

$\ast$ $u_C + (R+r)i + L\frac{di}{dt} = 0$

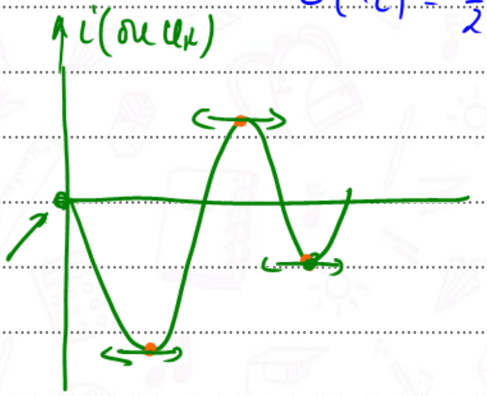


$\ast$



p.ou $\boxed{u_C = \pm U_{Cm}} \Rightarrow \frac{du_C}{dt} = 0 \Rightarrow \boxed{i = 0}$

$\Rightarrow$ $E(t_i) = \frac{1}{2} C U_i^2$



$E = E_{C0}$

$\frac{1}{2} C U_0^2$

$i = \pm I_m \Rightarrow \frac{di}{dt} = 0$

Loi des mailles

$u_C + (R+r)i + L\cancel{\frac{di}{dt}}^{\,0} = 0$

$\Rightarrow \boxed{u_C = -(R+r)i \neq 0}$

• oscillations libres $\boxed{\text{non amorties}}$ $\begin{cases} u_C = \pm U_{Cm} \Rightarrow i = 0 \\ i = \pm I_m \Rightarrow u_C = 0 \end{cases}$

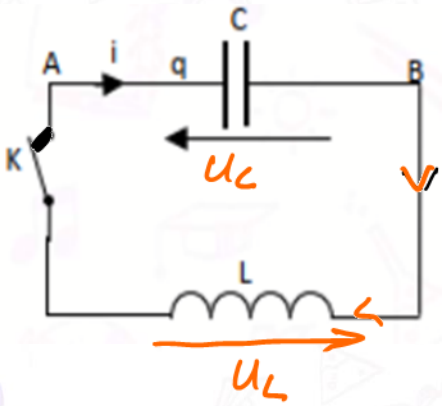
Partie B

1) Loi des mailles :

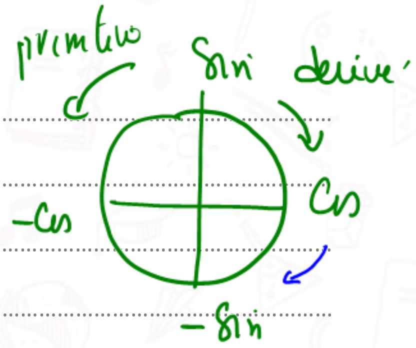
$u_C + u_L = 0$

$\frac{1}{C} q + L\frac{di}{dt} = 0$ or $i = \frac{dq}{dt}$

$\Rightarrow \underbrace{L\frac{d^2q}{dt^2} + \frac{1}{C} q = 0}\big) \frac{1}{L}$



$$\boxed{\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0}$$



b) $q(t) = Q_m \sin(\omega_0 t + \varphi)$

$$\frac{d^2q}{dt^2} = -\omega_0^2 \underbrace{Q_m \sin(\omega_0 t + \varphi)}_{q} = -\omega_0^2 q$$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC}q = -\omega_0^2 q + \frac{1}{LC}q$$

$$= q\left(-\omega_0^2 + \frac{1}{LC}\right)$$

$q(t)$ est une solution de l'éq. diff que si : $q\left[-\omega_0^2 + \frac{1}{LC}\right] = 0 \ \forall t$

$$\Rightarrow -\omega_0^2 + \frac{1}{LC} = 0 \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

c) $\boxed{T_0} = \frac{2\pi}{\omega_0} = \boxed{2\pi\sqrt{LC}}$

d) $q(t) = Q_m \sin(\omega_0 t + \varphi)$

$q(0) = Q_m \sin\varphi = \underline{\underline{C.U_0}} > 0$ ① ($q = C.U_c$ et $U_c(0) = U_0$
$Q_0 = CU_0$)

$i(0) = \frac{dq}{dt}(0) = \omega_0 Q_m \cos(\varphi) = 0$ ②

($q(0)$ ou $u_c(0) \longrightarrow i(0)$

① $\begin{cases} \sin\varphi > 0 \\ \cos\varphi = 0 \end{cases}$ ② $\Rightarrow \boxed{\varphi = \pi/2 \text{ rad.}}$

$$\Rightarrow Q_m = \frac{CU_0}{\sin \pi/2} = CU_0$$

$$\Rightarrow \boxed{q(t) = CU_0 \sin(\omega_0 t + \pi/2)}$$

$i(t) = \frac{dq}{dt} = CU_0\omega_0 \cos(\omega_0 t + \pi/2) =$ ($\cos\alpha = \sin(\alpha + \pi/2)$

$$\boxed{i(t) = C\omega_0 U_0 \sin(\omega_0 t + \pi)}$$

2°) a) $E = E_C + E_L = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2$

b) $\frac{dE}{dt} = \frac{1}{C}\underbrace{\frac{dq}{dt}}_{i}\cdot q + L\cdot\frac{di}{dt}\cdot i = i\Big[\underbrace{\frac{1}{C}q + L\frac{di}{dt}}_{=0\text{ (loi des mailles)}}\Big]$

$\frac{dE}{dt} = 0 \Rightarrow$ oscillateur conservatif

à $t=0$ $\begin{cases} u_C = U_0 \\ i = 0 \end{cases} \Rightarrow \boxed{E} = \frac{1}{2}Cu_C^2 = \boxed{\frac{1}{2}CU_0^2}$

3°) a) $E = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2$ or $E = \frac{1}{2}CU_0^2$

$\Rightarrow \cancel{\frac{1}{2}}\frac{q^2}{C} + \cancel{\frac{1}{2}}Li^2 = \cancel{\frac{1}{2}}CU_0^2 \Rightarrow \frac{q^2}{C} = -Li^2 + CU_0^2$

$\boxed{q^2 = -LCi^2 + (CU_0)^2} = Ai^2 + B$.

b) * $-LC = p$: pente $\Rightarrow p = \frac{(0-1).10^{-10}}{(1-0)\,10^{-4}} = -10^{-6}\ C^2A^{-2}$

$\Rightarrow L = -\frac{p}{C} = \frac{10^{-6}}{10^{-6}} \Rightarrow \boxed{L = 1\,H}$.

* $(CU_0)^2 = 10^{-10} \Rightarrow CU_0 = 10^{-5} \Rightarrow \boxed{U_0} = \frac{10^{-5}}{10^{-6}} = \boxed{10V}$.

$\Rightarrow \boxed{E} = \frac{1}{2}CU_0^2 = \frac{1}{2}\times 10^{-6}\times 10^2 = \boxed{5.10^{-5}\,J}$.