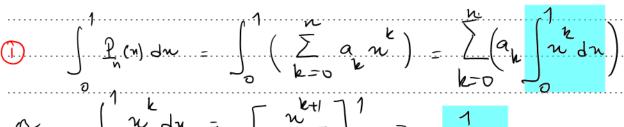


Exercice 1

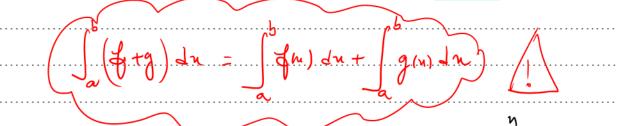
15 min



2 pts



or $\int_0^1 u \, du = \left[\frac{n^{k+1}}{k+1} \right]_0^1 = \frac{1}{k+1}$



Alors: $\int_{0}^{1} \frac{1}{n} (n) dn = \sum_{k=0}^{n} \left(\frac{2}{n} \frac{1}{k+1} \right) = \sum_{k=0}^{n} \left(\frac{2}{k+1} \frac{1}{n} \right)$

 $= \sum_{k=0}^{n} (k+1) = (1+(n+1))(n+1)$

 $\int_{0}^{1} \frac{1}{2} (n) dn = \frac{(n+2)(n+1)}{2}$

.....

\[\langle (\alpha k + \beta) = \frac{\longle \longle k}{\longle \tan \text{thme tight}} \\ \longle \longle \longle \text{Taism } \delta \\ \longle \delta = \frac{\longle \longle \text{tomme}}{\longle \text{tomme}} \longle \text{Summe}. \(\text{Summe} \)

.....





(2) * Lim
$$\int_{D}^{1} \frac{D_{n}(n) dn}{n-3+\infty} = \lim_{n\to+\infty} \frac{(n+2)(n+1)}{2} = +\infty$$

Lim
$$\int_{0}^{1} \int_{0}^{1} (n) du = \lim_{n \to +\infty} \int_{0}^{1} \frac{(n+1)(n+1)}{n}$$

$$- \lim_{\eta \to +\infty} \frac{1}{\chi^2} \frac{\chi^2 \left(1 + \frac{1}{\eta}\right) \left(1 + \frac{2}{\eta}\right)}{2} = \frac{1}{2}$$

Exercice 2





4 pts



f et Continue sur 12 et o∈12

donc F et la primitive de f sur 12

qui s'annule en o



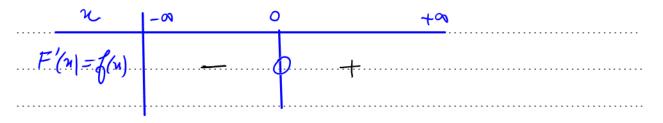
Si & or Continue sur I

er a E I alors

F st Léniable sur I et tre-I

F'(n) = f(n)

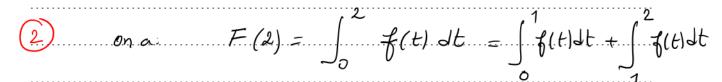
$$\forall n \in \mathbb{R}$$
 ; $F'(n) = \mathcal{J}(n)$







F & Shickman déconssante sur J-0; 0) er Stick + anissaute sur [o, + 0 [



pour $0 \le t \le 1$ ona $0 \le f(t) \le 2$ $0 \le \int_{0}^{1} f(t) dt \le 2 \qquad 0$

powr $1 \le t \le 2$ on a. $1 \le f(t) \le 2$ $\Rightarrow 1 \le \int_{1}^{2} f(t) dt \le 2$

1 (F(2) (4

 $F \in Y \quad Conh^n \text{nue} \quad Levivable \quad Sur \quad [1;2]$ $\forall \quad \chi \in [1,2] \qquad \qquad 1 \leq F'(n) = f(n) \leq 2$

dapie de T. I. A. F (2-1). $1 \subseteq F(2) - F(1) \subseteq 2(2-1)$

 $\Rightarrow 1 \leq F(2) - F(1) \leq 2$

= 0 $1+F(1) \subseteq F(2) \subseteq 2+F(1)$ Comme F(1) > F(0) donc F(1) > 0





$\Rightarrow 1+F(1) \leq F(2) \leq 2+F(1)$
$= 0 \qquad 1 < F(2) < 2+F(1) < 2+2=4$
Gar $F(1) = \int_{0}^{1} f(t) dt$ $0 \le t \le 1$ on $0 \le f(t) \le 1$ $0 \le \int_{0}^{1} f(t) dt \le 2$
(3) (2) $n_3 1$ on a 1 $\leq f(t)$ Lefontions Surv Continue $\int_1^n dt \leq \int_1^n f(t) dt = \int_1^n \int_1^n dt \leq \int_1^n f(t) dt = \int_1^n \int_1^n dt = \int_1^n dt = \int_1^n dt = \int_1^n \int_1^n dt =$
Just & Still Jt L Suv [1:2
$= n-1 \leq \int_{1}^{\infty} f(t) dt + \int_{0}^{\infty} f(t) dt$
⇒ n-1 ≤ - ∫ 1 f(t) dt + (F(n))
-D η-1 + F(1) ∠ F(η)
$\Rightarrow \qquad n-1 \leq F(n) \qquad \text{Cav} \qquad F(1) > F(0) = 0$
(b) ona: n, 1 F(n) > n-1
el Lim n-1 = +∞ n sto
donc $\lim_{n\to+\infty} F(n) = +\infty$







n u n² et dériable our [0,+0] $\mathcal{L}([0,+\infty[)=[0,+\infty[$ of gr Continue dur [0, +0 [et ontenne sur u(1) er 0 € u(t) due g gr dé vable sur alors F et dévirable our I

	ત્ (પ્ર)
F(α)=	(t) dt (n EI)
siu	et dérivable our I

Ju = I; F'(x)= u'(n) of (um)

4 n>,0 g'(n)=2n f(x) >0

n	0	+
81(n)	+	
		+00
···§(n)······		

	g (21)=	F(n2)	_ F	u (n)	 	
Lim u(4)	= li n'	= +0			 	
η-3+Φ	+ &		1			

 $donc \qquad \int_{t}^{\infty} dt \leq \int_{t}^{\infty} \xi(t) dt \leq \int_{t}^{\infty} 2 dt$





	n^2	
	$\Rightarrow n^2-1 \leq \int_{1}^{\infty} f(t) dt + \int_{0}^{\infty} f(t) dt \leq 2(n^2-1)$	
	$= 0 n^2 - 1 \leq - \int_0^1 f(t)dt + g(n) \leq 2(n^2 - 1)$	
1 -	$= 0 \qquad n^{2} - 1 + F(1) \leq g(n) \leq 2(n^{2} - 1) + F(1)$	
4 1/2	$\frac{1}{n} = \frac{1}{n} + \frac{F(1)}{n} \leq \frac{g(n)}{n} \leq 2\left(n - \frac{1}{n}\right) + \frac{F(1)}{n}$	
,	$\lim_{n\to+\infty} n - \frac{1}{n} + \frac{F(1)}{n} = +\infty$	
	Lone line $\frac{g(n)}{n} = +a$ Lone & admet	
	une branche parabolique de direction celle de (9))
	au voisinage de (+∞)	





Exercice 3

Q 25 min



4 pts

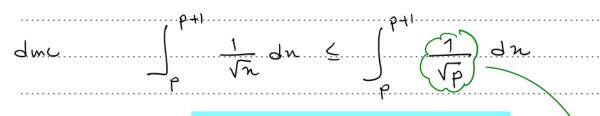
$$u_n = \sum_{p=1}^{n} \frac{1}{\sqrt{p}}$$

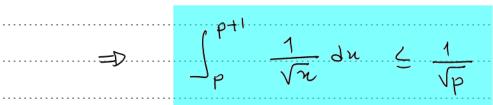
$$\sqrt{n} = \frac{U_n}{\sqrt{n}}$$

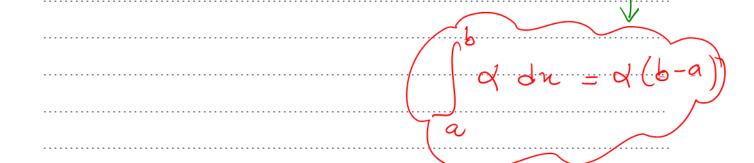
(1) @ P G IN*

$$\forall n \in [p, p+1]$$
 ona $\sqrt{p} \leq \sqrt{x} \leq \sqrt{p+1}$

$$\frac{1}{\sqrt{p+1}} \leq \frac{1}{\sqrt{x}} \leq \frac{1}{\sqrt{p}}$$
 Les fond



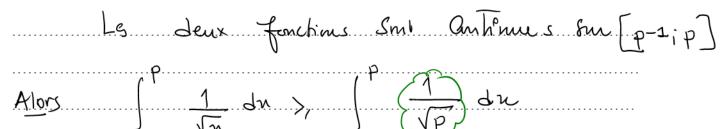




$$\forall n \in [p-1; p]$$
 on $a \in [p-1; p]$







$$= D \qquad \int_{P-1}^{P} \frac{1}{\sqrt{n}} dn > \frac{1}{\sqrt{p}}$$

$$\int_{\rho}^{\rho+1} \frac{du}{\sqrt{n}} \geq \frac{1}{\sqrt{\rho}}$$

$$= D \sum_{p=1}^{\infty} \left(\int_{p}^{p+1} \frac{dn}{\sqrt{n}} \right) \leq \sum_{p=1}^{\infty} \frac{1}{\sqrt{p}}$$

$$\Rightarrow \int \frac{dn}{\sqrt{n}} + \int \frac{dn}{\sqrt{n}} + \cdots + \int \frac{dn}{\sqrt{n}} \leq U_n$$

$$= \int_{1}^{n+1} \frac{du}{\sqrt{n}} \leq u_{n} \quad (1)$$

Pour tout
$$p \in \{2, 3, ..., n\}$$

$$\int_{\rho-1}^{\rho} \frac{dn}{\sqrt{n}} > \frac{1}{\sqrt{\rho}}$$





$$= \int_{1}^{2} \frac{dn}{\sqrt{n}} + \int_{2}^{3} \frac{dn}{\sqrt{n}} + \cdots + \int_{n-1}^{n} \frac{dn}{\sqrt{n}} > \sum_{p=1}^{n} \frac{1}{\sqrt{p}} - 1$$

$$=D \qquad \int_{1}^{n} \frac{dn}{\sqrt{n}} > U_{n} - 2 \qquad (11)$$

Comme
$$\int_{1}^{n} \frac{dn}{\sqrt{n}} = \left[2\sqrt{n} \right]_{1}^{n} = 2\sqrt{n} - 2\sqrt{n}$$

$$\int_{1}^{n+1} \frac{du}{\sqrt{n}} = \left[2\sqrt{n+1} - 2 \right]_{1}^{n+1}$$

$$2\sqrt{n+1} - 2 \leq 4 \ln \leq 2\sqrt{n} - 1$$

(2) on a:
$$\forall n > 1$$
, $U_n > 2\sqrt{n+1} - 2$

er
$$\lim_{n\to+\infty} 2\sqrt{n+1} - 2 = +\infty$$
 $\lim_{n\to+\infty} U_n = +\infty$

ona
$$\forall n > 1$$

 $-2 + 2 \sqrt{n+1} \leq U_n \leq -1 + 2 \sqrt{n}$

$$= \frac{2}{\sqrt{n}} + 2\sqrt{1+\frac{1}{n}} \leq \frac{U_n}{\sqrt{n}} \leq -\frac{1}{\sqrt{n}} + 2$$





Lim -	$-\frac{2}{\sqrt{n}}+2$	$\int 1 + \frac{1}{9} =$	2		
	$\frac{-1}{\sqrt{n}}$ + 2				
n→+∞	\sqrt{n}				
	done	1.m 4-0+00	12 =	2	
	• • • • • • • • • • • • • • • • • • • •				
	• • • • • • • • • • • • • • • • • • • •				

