Mathematics for Political Science

Day 3: Calculus I

Exercises

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1. (Gill 5.1 [adapted]) Find the following finite limits:
   1. $\lim_{x \to 4}[x^2 - 6x + 4]$
   2. $\lim_{x \to 0}[\frac{x - 25}{x + 5}]$
   3. $\lim_{x \to 4}[\frac{x^2}{3x - 2}]$
   4. $\lim_{x \to 1}[\frac{x^2 - 1}{x - 1}]$

2. (Gill 5.3 [adapted]) Find the following infinite limits and graph:
   1. $\lim_{x \to \infty}[\frac{9x^2}{x^2 + 3}]$
   2. $\lim_{x \to \infty}[\frac{3x - 4}{x + 3}]$
   3. $\lim_{x \to \infty}[\frac{2^x - 3}{2^x + 1}]$

3. (Gill 5.5 [adapted]) Calculate the following derivatives:

   a. $\frac{d}{dx} 3x^{\frac{1}{3}}$ b. $\frac{d}{dt}(14t - 7)$ c. $\frac{d}{dy}(y^3 + 3y^2 - 12)$

   d. $\frac{d}{dx}(x^2 + 1)(x^3 - 1)$ e. $\frac{d}{dy}(y^3 - 7)(1 + \frac{1}{y^2})$ f. $\frac{d}{dy}(y - y^{-1})(y - y^{-2})$

   g. $\frac{d}{dx} \frac{4x - 12x^2}{x^3 - 4x^2}$ h. $\frac{d}{dy} e^{y^2 - 3y + 2}$ i. $\frac{d}{dx} \ln(2\pi x^2)$

4. Consider the function $k(x) = 2(8(x^4 + 2) - 1)^2$. Find the derivative by:
   1. Expanding the polynomial and calculating the derivative using the power rule.
   2. Expressing $k(x)$ as the result of three nested functions $f(g(h(x)))$ and applying the chain rule.

   Show that these approaches yield the same answer.