1. Assume a year has 360 days and is composed of 12 months each of 30 days. How many people do you need in a room to **guarantee** that at least two people share the same *birthday month*?

A 12   B 13   C 14   D $11 \times 30 + 1 = 331$ E 360

2. Shirts come in 4 colors. You need to assign shirts to 5 students. In how many ways can you do this?

A 0   B 4^5 C 5^4 D 4!   E 5!

3. Shirts come in 4 colors. You need to assign shirts to 5 students, and no two students can get the same color shirt. In how many ways can you do this?

A 0   B 4^5 C 5^4 D 4!   E 5!

4. Shirts come in 4 colors. 5 students are in a row. You need to assign shirts to the students, and two students standing next to each other cannot get the same color shirt. In how many ways can you do this?

A 0   B $9\times8\times7\times6\times5$ C $\binom{9}{5}$ D $\binom{5}{4}$ E 4×3^4

5. x_1, x_2, x_3, x_4 are *natural numbers* $(1, 2, \ldots)$. In how many different ways can you choose x_1, x_2, x_3, x_4 so that $x_1 + x_2 + x_3 + x_4 = 10$? For example, two *different* solutions are (1,2,3,4) and (2,1,3,4).

A 54   B 64   C 74   D 84   E 98

6. You roll a pair of fair dice. What is the probability that the sum is even?

A $\frac{1}{6}$ B $\frac{12}{36}$ C $\frac{16}{36}$ D $\frac{2}{5}$ E $\frac{1}{2}$

7. You roll a pair of fair dice. What is the probability that the sum is even *given* that the two values rolled are different?

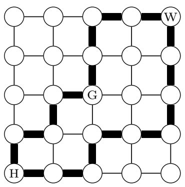
A $\frac{1}{6}$ B $\frac{12}{36}$ C $\frac{16}{36}$ D $\frac{2}{5}$ E $\frac{1}{2}$

8. You independently generate the 4 bits of a binary sequence $b_1b_2b_3b_4$ with $\mathbb{P}[b_i = 0] = \frac{1}{2}$. Compute the probability that $\sum_{i=1}^{4} b_i = 2$

A $\frac{1}{16}$ B $\frac{2}{16}$ C $\frac{4}{16}$ D $\frac{6}{16}$ E $\frac{8}{16}$

9. Problems 9 and 10 refer to the grid graph on the right.

There are three special nodes in this graph: H is home; W is work; G is the grocery store. Two shortest paths from Work to Home are highlighted in the graph, one goes through the grocery store and one does not. All shortest paths from W to H have length 8.
How many *different* shortest paths from W to H are there (two paths are different if there is an edge in one path that is not used in the other)?



A 40
B 2^8
C 50
D 60
E 70

10. If you randomly choose one of the shortest paths from W to H, with each shortest path being equally likely, what is the probability that you will be able to pick up groceries on your way home from work.

A $\frac{18}{35}$
B $\frac{1}{2}$
C $\frac{11}{30}$
D $\frac{12}{25}$
E $\frac{13}{20}$

1. How many guests do you need at a party to <u>*guarantee*</u> that two will be born on the same day of the week?

A 3

B 5

C 7

D 8

E Not possible

2. How many guests do you need at a party to <u>*guarantee*</u> that two will be born on a Monday?

A 3

B 5

C 7

D 8

E Not possible

3. How many numbers in the set $\{1, 2, 3, \ldots, 1000\}$ are divisible by 2 <u>*or*</u> 3.

A 657

B 660

C 667

D 830

E 833

4. How many different words can you get by rearranging the letters of the word BOOKKEEPER?

A 10!

B $\frac{10!}{2!\times 2!\times 3!}$

C $\binom{10}{6}$

D 6^{10}

E 10^6

5. You have 11 players and must form two teams of 5 for a practice match. How many different practice matches are possible. (Be careful! **TINKER**: for example, try 3 players forming two teams of 1)?

A 1386

B 1388

C 1390

D 2772

E 2774

6. You randomly pick an 8-bit sequence (independent bits and each bit is 1 with probability $\frac{1}{2}$). What is the probability that the sequence starts and ends in 1?

A $\frac{1}{2}$

B $\frac{1}{4}$

C $\frac{1}{8}$

D $\frac{1}{16}$

E $\frac{1}{32}$

7. A box contains 10 coins. 9 are *fair* and 1 has *two heads*. You pick a coin at random. You toss it three times. What is the probability of tossing three heads (HHH)?

A $\frac{1}{8}$

B $\frac{15}{80}$

C $\frac{16}{80}$

D $\frac{17}{80}$

E $\frac{18}{80}$

8. A box contains 10 coins. 9 are *fair* and 1 has *two heads*. You pick a coin at random. You toss your coin three times and get HHH. What is the probability that the coin you picked is fair?

A $\frac{9}{10}$

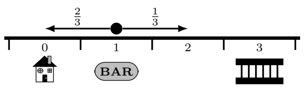
B $\frac{8}{17}$

C $\frac{9}{17}$

D $\frac{10}{17}$

E $\frac{11}{17}$

9. A drunk leaves the bar (at position 1), and takes independent steps: left (L) with probability $\frac{2}{3}$ or right (R) with probability $\frac{1}{3}$. What is the probability the drunk reaches home (at position 0) before reaching the lockup (at position 3)?



A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{4}{5}$

D $\frac{5}{6}$

E $\frac{6}{7}$

10. You roll 4 independent fair dice. What is the probability that you roll exactly *one* 2 and *one* 4?

A $\frac{3}{27}$

B $\frac{4}{27}$

C $\frac{5}{27}$

D $\frac{6}{27}$

E $\frac{7}{27}$

1. A drawer has 10 red and 10 blue socks. What is the minimum number of socks must you pull out to *guarantee* getting a pair of the same color?

A 2.

B 3.

C 11.

D 12.

E None of the above.

2. A drawer has 10 red and 10 blue socks. What is the minimum number of socks you must pull out to *guarantee* getting a blue pair of socks?

A 2.

B 3.

C 11.

D 12.

E None of the above.

3. How many numbers in the set $\{1, 2, 3, \ldots, 1000\}$ are divisible by 4 *or* 6.

A 250.

B 330.

C 375.

D 416.

E None of the above.

4. 100 runners finish a race. We are interested in the order of the first 10 runners. How many possible top-10 finishes are there?

A $\dfrac{100!}{90!}$.

B $\dfrac{100!}{10! \times 90!}$.

C 100^{10}.

D 10^{100}.

E None of the above.

5. 100 runners finish a race. The top-10 will form the State-team for the Nationals. How many possible state teams are there?

A $\frac{100!}{90!}$.

B $\frac{100!}{10! \times 90!}$.

C 100^{10}.

D 10^{100}.

E None of the above.

6. 10 end-of-season awards (sportsmanship, most improved, fittest, leadership, etc.) are given to 100 runners (the same runner may get more than one award). In how many ways can the 10 prizes be awarded?

A $\frac{100!}{90!}$.

B $\frac{100!}{10! \times 90!}$.

C 100^{10}.

D 10^{100}.

E None of the above.

7. You have 3 red, 3 green and 3 blue flags. In how many ways can you arrange the flags along the street for the parade?

A $9!$.

B 3^9.

C $9!/(3!)^3$.

D 9^3.

E None of the above.

8. In how many different ways can you place 10 identical rings on your 10 fingers? (All 10 rings can fit on one finger.)

A $10!$.

B 10^{10}.

C $\binom{20}{10}$.

D $\binom{19}{9}$.

E None of the above.

9. A probability space has four outcomes $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ with probabilities $P_1 = x$, $P_2 = 2x$, $P_3 = 3x$, $P_4 = 4x$. What is x?

A 0.1.

B 0.2.

C 0.3.

D 0.4.

E None of the above.

10. Randomly pick a 10-bit sequence (independent bits and each bit is 1 with probability $\frac{1}{2}$). What is the probability that the sequence starts with 111?

A 2^{-3}.

B 2^{-5}.

C 2^{-7}.

D 2^{-10}.

E None of the above.

11. A box has 6 fair coins and 4 two-headed coins ($\mathbb{P}[\mathrm{H}] = 1$ for two-headed coins). You picked a coin randomly, tossed it and got H. What is the probability you picked the fair coin, $\mathbb{P}[\text{fair coin} \mid \text{you got H}]$?

A $\frac{2}{5}$.

B $\frac{3}{7}$.

C $\frac{5}{10}$.

D $\frac{7}{10}$.

E None of the above.

12. Toss 20 fair coins independently. All coins that came up heads are *re-flipped* (the coins that came up tails are *not* re-flipped). At the end, what is the probability that 10 heads in total are flipped?

A 0.5.

B $\binom{20}{10} \times \left(\frac{1}{2}\right)^{10} \times \left(\frac{1}{2}\right)^{10}$.

C $\binom{20}{10} \times \left(\frac{1}{3}\right)^{10} \times \left(\frac{2}{3}\right)^{10}$.

D $\binom{20}{10} \times \left(\frac{1}{4}\right)^{10} \times \left(\frac{3}{4}\right)^{10}$.

E None of the above.

13. You roll a *loaded* 6-sided die (values 1,...,6). The probability of an even roll is twice the probability of an odd roll. What is the expected value of a single roll of this loaded die?

A $3\frac{1}{3}$.

B $3\frac{1}{2}$.

C $3\frac{2}{3}$.

D 4.

E None of the above.

14. A box has 6 fair coins and 4 two-headed coins. You pick a coin randomly and make 10 independent flips. Let $\mathbf{X}$ be the number of heads you get. What is $\mathbb{E}[\mathbf{X}]$? [Hint: Total Expectation]

A 5.

B 6.

C 7.

D 8.

E None of the above.

15. **[Hard]** A box has 1024 fair coins and 1 two-headed coin. You picked a random coin, flipped it 10 times and all 10 flips were H. You now keep flipping the *same* coin you picked until you flip a H. Let $\mathbf{X}$ be the number of additional flips you make. What is $\mathbb{E}[\mathbf{X}]$, the expected value of $\mathbf{X}$?

A 1.

B $1\frac{1}{2}$.

C 2.

D $2\frac{1}{2}$.

E None of the above.

1. A drawer has 5 red and 5 blue socks. It's dark and you can't see. What is the *minimum* number of socks must you pull out to *guarantee* getting at least one red sock and at least one blue sock?

A 3.

B 4.

C 5.

D 6.

E None of the above.

2. What is the *minimum* number of children needed to *guarantee* two are born on the same day of the week?

A 5.

B 6.

C 7.

D 8.

E None of the above.

3. What is the *minimum* number of children needed to *guarantee* two are born on a Monday?

A 6.

B 7.

C 8.

D 367.

E None of the above.

4. In how many ways can you pick a debate team of 3 students from 6 students?

A 20.

B 120.

C 6^3.

D 3^6.

E None of the above.

5. Which number could be a probability of some event?

A 3/2

B $\sqrt{2}$.

C $\sqrt{2}-1$.

D $\sqrt{2}-2$.

E π.

6. You randomly flip two independent fair coins. What is the probability of at least one flip being heads?

$\boxed{A}$ 0.

$\boxed{B}$ 1/4.

$\boxed{C}$ 1/4.

$\boxed{D}$ 3/4.

$\boxed{E}$ 1.

7. You randomly roll a pair of fair 6-sided dice. What is the most likely sum of the dice?

$\boxed{A}$ 5.

$\boxed{B}$ 6.

$\boxed{C}$ 7.

$\boxed{D}$ 8.

$\boxed{E}$ 9.

8. Random variable $\mathbf{X}$ has a uniform distribution on the ten values $\{1, 2, \ldots, 10\}$. What is $\mathbb{P}[\mathbf{X} \text{ is prime}]$?

$\boxed{A}$ 0.1.

$\boxed{B}$ 0.2.

$\boxed{C}$ 0.3.

$\boxed{D}$ 0.4.

$\boxed{E}$ None of the above.

9. Random variable $\mathbf{X}$ has values $\{1, 2, \ldots, 10\}$ with probabilities $\{x, 2x, \ldots, 10x\}$. What is $\mathbb{P}[\mathbf{X} \text{ is prime}]$?

$\boxed{A}$ 4/55.

$\boxed{B}$ 17/55.

$\boxed{C}$ 19/55.

$\boxed{D}$ 21/55.

$\boxed{E}$ It cannot be determined without knowing the value of x.

10. Randomly pick a 5-bit sequence (independent bits and each bit is 1 with probability $\frac{1}{2}$). What is the probability that the sequence starts and ends with the same bit?

$\boxed{A}$ 1/4.

$\boxed{B}$ 1/2.

$\boxed{C}$ 3/4.

$\boxed{D}$ 2/32.

$\boxed{E}$ None of the above.

11. Randomly pick a 5-bit sequence (independent bits and each bit is 1 with probability $\frac{1}{2}$). What is the probability that the sequence has at least one 1?

$\boxed{A}$ 1/32.

$\boxed{B}$ 9/32.

$\boxed{C}$ 27/32.

$\boxed{D}$ 31/32.

$\boxed{E}$ None of the above.

12. Which inequality for the AND of two events A and B is always correct? Assume $\mathbb{P}[A] > 0$ and $\mathbb{P}[B] > 0$.

$\boxed{A}$ $\mathbb{P}[A \cap B] \leq \mathbb{P}[A] \times \mathbb{P}[B]$.

$\boxed{B}$ $\mathbb{P}[A \cap B] \geq \mathbb{P}[A] \times \mathbb{P}[B]$.

$\boxed{C}$ $\mathbb{P}[A \cap B] \leq \min(\mathbb{P}[A], \mathbb{P}[B])$. ($\min(\cdot, \cdot)$ takes the minimum.)

$\boxed{D}$ $\mathbb{P}[A \cap B] \geq \min(\mathbb{P}[A], \mathbb{P}[B])$.

$\boxed{E}$ None of the above.

13. Which formula for the AND of two events A and B is always correct? Assume $\mathbb{P}[A] > 0$ and $\mathbb{P}[B] > 0$.

$\boxed{A}$ $\mathbb{P}[A \cap B] = \mathbb{P}[A] \times \mathbb{P}[B]$.

$\boxed{B}$ $\mathbb{P}[A \cap B] = \mathbb{P}[A] + \mathbb{P}[B]$.

$\boxed{C}$ $\mathbb{P}[A \cap B] = \mathbb{P}[A \mid B] + \mathbb{P}[B \mid A]$.

$\boxed{D}$ $\mathbb{P}[A \cap B] = \mathbb{P}[A \mid B] \times \mathbb{P}[B]$.

$\boxed{E}$ $\mathbb{P}[A \cap B] = \mathbb{P}[A \mid B] \times \mathbb{P}[B \mid A]$.

14. A box has 6 fair coins and 4 two-headed coins You pick a random coin and flip. What is $\mathbb{P}[H]$?

$\boxed{A}$ 4/10

$\boxed{B}$ 5/10

$\boxed{C}$ 6/10

$\boxed{D}$ 7/10

$\boxed{E}$ None of the above.

15. A box has two coins, one is fair and one is two-headed. You picked a coin randomly, flipped it twice and got HH. What are the chances you have the fair coin?

$\boxed{A}$ 1/2.

$\boxed{B}$ 1/3.

$\boxed{C}$ 1/4.

$\boxed{D}$ 1/5.

$\boxed{E}$ None of the above.

16. Which random variable $\mathbf{X}$ has a binomial distribution?

A Flip a fair coin until the second head appears. $\mathbf{X}$ is the number of flips made.

B Draw 10 cards from a randomly shuffled deck. $\mathbf{X}$ is the number of aces drawn.

C Hats of 100 men randomly land on the 100 heads. $\mathbf{X}$ is the number of men who get their hat back.

D Randomly answer 20 multiple-choice questions, each with 5 answers. $\mathbf{X}$ is the number correct.

E None of them have a binomial distribution.

17. Flip 5 fair coins independently. What is the probability to get exactly 3 heads.

A 3/16.

B 4/16.

C 5/16.

D 6/16.

E None of the above.

18. You flip a fair coin 3 times. What is the expected number of heads?

A 0.

B 1.

C 2.

D 3.

E None of the above.

19. A box has two fair coins and one two-headed coin. You randomly pick a coin and flip the coin you picked 3 times. What is the expected number of heads?

A 0.

B 1.

C 2.

D 3.

E None of the above.

20. Each sex is equally likely. A couple has kids until they have at least one boy and at least one girl. What is the expected number of kids the couple will have?

A 2.

B 3.

C 4.

D 5.

E None of the above.

1. How many students would guarantee two students with the same first initial. (First initials are $A, B, \dots, Z$.)

A 24.

B 25.

C 26.

D 27.

E None of the above.

2. How many numbers in $\{1, 2, \dots, 1000\}$ are divisible by 2 or 5.

A 200.

B 500.

C 600.

D 700.

E None of the above.

3. In how many ways can you pick a cooking team of 4 students from 7 student-chefs?

A 4×7.

B $7!/4!$.

C 7^4.

D $7!/(4! \times 3!)$.

E None of the above.

4. The only available majors at FOCS-University are CS and BIO. There are 70 students in total. There are 50 CS majors and 50 BIO majors. How many dual CS-BIO majors are there?

A 10.

B 20.

C 30.

D 40.

E None of the above.

5. The sum of the probabilities for all possible outcomes is always:

A 0

B 1/4.

C 1/2.

D 1.

E None of the above.

6. Randomly flip three independent fair coins. What are the chances of more H (heads)?

$\boxed{\text{A}}$ 0.

$\boxed{\text{B}}$ 2/8.

$\boxed{\text{C}}$ 4/8.

$\boxed{\text{D}}$ 6/8.

$\boxed{\text{E}}$ 1.

7. Randomly flip three independent fair coins. What are the chances of more H if the first flip is H?

$\boxed{\text{A}}$ 0.

$\boxed{\text{B}}$ 2/8.

$\boxed{\text{C}}$ 4/8.

$\boxed{\text{D}}$ 6/8.

$\boxed{\text{E}}$ 1.

8. Randomly flip three independent fair coins. What are the chances of more H if the first two flips are H?

$\boxed{\text{A}}$ 0.

$\boxed{\text{B}}$ 2/8.

$\boxed{\text{C}}$ 4/8.

$\boxed{\text{D}}$ 6/8.

$\boxed{\text{E}}$ 1.

9. Random variable $\mathbf{X}$ is a value in $\{1, 2, 3, 4\}$ with probabilities $\{x, 2x, 3x, 4x\}$ respectively. What is x?

$\boxed{\text{A}}$ 0.1.

$\boxed{\text{B}}$ 0.2.

$\boxed{\text{C}}$ 0.3.

$\boxed{\text{D}}$ 0.4.

$\boxed{\text{E}}$ It cannot be determined or none of the above.

10. For the random variable $\mathbf{X}$ in the previous problem, what is $\mathbb{E}[\mathbf{X}]$

$\boxed{\text{A}}$ 1.

$\boxed{\text{B}}$ 2.

$\boxed{\text{C}}$ 3.

$\boxed{\text{D}}$ 4.

$\boxed{\text{E}}$ It cannot be determined or none of the above.

11. What is the probability to get 4 or more heads in 5 independent flips of a biased coin with probability 1/3 of heads.

$\boxed{\text{A}}$ $10/3^5$.

$\boxed{\text{B}}$ $11/3^5$.

$\boxed{\text{C}}$ $12/3^5$.

$\boxed{\text{D}}$ $13/3^5$.

$\boxed{\text{E}}$ None of the above.

12. You flip a fair coin 10 times. What is the expected number of heads?

$\boxed{\text{A}}$ 3.

$\boxed{\text{B}}$ 4.

$\boxed{\text{C}}$ 5.

$\boxed{\text{D}}$ 6.

$\boxed{\text{E}}$ None of the above.

13. Boys are 4 times as likely as girls. You have kids till you get a girl. What is the expected number of kids you will have?

$\boxed{\text{A}}$ 2.

$\boxed{\text{B}}$ 3.

$\boxed{\text{C}}$ 4.

$\boxed{\text{D}}$ 5.

$\boxed{\text{E}}$ None of the above.

14. Random variables $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3$ have expected values $\mu_1 = 1, \mu_2 = 2, \mu_3 = 3$. What is $\mathbb{E}[2\mathbf{X}_1 + \mathbf{X}_2 + 3\mathbf{X}_3]$?

$\boxed{\text{A}}$ 11.

$\boxed{\text{B}}$ 12.

$\boxed{\text{C}}$ 13.

$\boxed{\text{D}}$ 14.

$\boxed{\text{E}}$ Can't be determined or none of the above.

15. 60% of students are men and 40% are women. Men have average hair length 5in. Women have average hair length 10in. What is the average hair length for all students?

$\boxed{\text{A}}$ 6in.

$\boxed{\text{B}}$ 7in.

$\boxed{\text{C}}$ 8in.

$\boxed{\text{D}}$ 9in.

$\boxed{\text{E}}$ None of the above.

16. A box has 5 fair and 5 two-headed coins Pick a random coin and flip 5 times. What are the chances of exactly 2 heads?

A 2/5

B 1/32

C 5/32

D 10/32

E None of the above.

17. A box has 5 fair and 5 two-headed coins Pick a random coin and flip 10 times. What is the expected number of heads?

A 5.

B 6.

C 7.

D 8.

E None of the above.

18. A test for COVID gives the right answer 90% of the time and 10% of people have COVID. You tested positive. What are the chances you have COVID.

A 9%.

B 10%.

C 50%.

D 90%.

E None of the above.

19. A test for COVID gives the right answer 90% of the time and 10% of people have COVID. You test 100 people. What is the expected number people that have a positive test.

A 9.

B 10.

C 18.

D 90.

E None of the above.

20. Starburst comes in two-packs and there are two equally likely colors. You buy two-packs until you have at least one starbust of each color. What is the expected number of two-packs you buy?

A 4/3.

B 5/3.

C 6/3.

D 7/3.

E None of the above.

1. How many subsets of $\{a, b, c, d, e\}$ contain the letter a?

A 8.

B 16.

C 32.

D 64.

E None of the above.

2. How many subsets of $\{a, b, c, d, e\}$ have exactly 3 letters?

A 8.

B 16.

C 32.

D 64.

E None of the above.

3. There are 40 young women and 40 gray women. There are 50 women who are young or gray. How many women are both young and gray?

A 10.

B 20.

C 30.

D 40.

E None of the above.

4. You flip a 2-sided coin and roll a 6-sided die. How many outcomes are in the probability space?

A 6.

B 8.

C 10.

D 12.

E None of the above.

5. An experiment has 3 possible outcomes A, B, C. $P(A) = 1/3$, $P(B) = 1/6$. What is $P(C)$?

A 0

B 1/4.

C 1/2.

D 1.

E None of the above.

6. Make 3 flips of a biased coin, with probability of heads 2/3. What are the chances of more heads than tails?

$\boxed{\text{A}}$ 1/2.

$\boxed{\text{B}}$ 8/9.

$\boxed{\text{C}}$ 16/27.

$\boxed{\text{D}}$ 20/27.

$\boxed{\text{E}}$ None of the above

7. In Problem 6, what are the chances of more heads than tails if all three flips match?

$\boxed{\text{A}}$ 1/2.

$\boxed{\text{B}}$ 8/9.

$\boxed{\text{C}}$ 16/27.

$\boxed{\text{D}}$ 20/27.

$\boxed{\text{E}}$ None of the above

8. 1-in-20 men are color blind and 1-in-400 women are color blind. There are an equal number of men and women. What are the chances a random person is color blind?

$\boxed{\text{A}}$ 1/20.

$\boxed{\text{B}}$ 20/800.

$\boxed{\text{C}}$ 21/800.

$\boxed{\text{D}}$ 22/800.

$\boxed{\text{E}}$ None of the above.

9. In Problem 8, what are the chances a random color blind person is a man?

$\boxed{\text{A}}$ 1/20.

$\boxed{\text{B}}$ 20/21.

$\boxed{\text{C}}$ 20/800.

$\boxed{\text{D}}$ 21/800.

$\boxed{\text{E}}$ None of the above

10. 1-in-1000 drivers are drunk. A breathalyzer test is correct at saying if you are drunk or not 99.9% of the time. The breathalyzer says a random person is drunk. What are the chances they are drunk?

$\boxed{\text{A}}$ 1/2.

$\boxed{\text{B}}$ 99/100.

$\boxed{\text{C}}$ 999/100.

$\boxed{\text{D}}$ 999/1000.

$\boxed{\text{E}}$ None of the above.

11. A box has 9 fair coins and 1 two-headed coin. Pick a random coin and flip. What are the chances of H?

$\boxed{\text{A}}$ 9/20.

$\boxed{\text{B}}$ 10/20.

$\boxed{\text{C}}$ 11/20.

$\boxed{\text{D}}$ 12/20.

$\boxed{\text{E}}$ None of the above

12. In problem 11, your flip was H. What are the chances you picked a fair coin?

$\boxed{\text{A}}$ 9/10.

$\boxed{\text{B}}$ 9/11.

$\boxed{\text{C}}$ 9/20.

$\boxed{\text{D}}$ 11/20.

$\boxed{\text{E}}$ None of the above

13. $\mathbf{X}$ is a uniform random variable taking a value in the set $\{1, 2, 3, 4\}$. What is $\mathbb{P}[\mathbf{X} = 2]$?

$\boxed{\text{A}}$ 0.1.

$\boxed{\text{B}}$ 0.2.

$\boxed{\text{C}}$ 0.3.

$\boxed{\text{D}}$ 0.4.

$\boxed{\text{E}}$ None of the above.

14. In Problem 13, what is $\mathbb{P}[\mathbf{X} \geq 2]$?

$\boxed{\text{A}}$ 0.

$\boxed{\text{B}}$ 0.25.

$\boxed{\text{C}}$ 0.5.

$\boxed{\text{D}}$ 0.75.

$\boxed{\text{E}}$ None of the above.

15. In Problem 13, what is $\mathbb{E}[\mathbf{X}]$?

$\boxed{\text{A}}$ 2.

$\boxed{\text{B}}$ 2.5.

$\boxed{\text{C}}$ 3.

$\boxed{\text{D}}$ 3.5.

$\boxed{\text{E}}$ None of the above.

16. Roll a fair 6-sided die 4 times. What are the chances to roll exactly 2 fours?

$\boxed{\text{A}}$ $140/6^4$.

$\boxed{\text{B}}$ $150/6^4$.

$\boxed{\text{C}}$ $160/6^4$.

$\boxed{\text{D}}$ $170/6^4$.

$\boxed{\text{E}}$ None of the above.

17. A biased coin has probability of heads $3/4$. What is the expected number of heads in 20 flips?

$\boxed{\text{A}}$ 5.

$\boxed{\text{B}}$ 10.

$\boxed{\text{C}}$ 15.

$\boxed{\text{D}}$ 20.

$\boxed{\text{E}}$ None of the above.

18. A box has 5 fair and 5 two-headed coins. Pick a random coin and make 10 flips. What is $\mathbb{E}[\text{number of heads}]$?

$\boxed{\text{A}}$ 6

$\boxed{\text{B}}$ 6.5

$\boxed{\text{C}}$ 7

$\boxed{\text{D}}$ 7.5

$\boxed{\text{E}}$ None of the above.

19. Boys are twice as likely as girls. What is the expected number of kids till you have a boy?

$\boxed{\text{A}}$ 1.

$\boxed{\text{B}}$ 2.

$\boxed{\text{C}}$ 3.

$\boxed{\text{D}}$ 4.

$\boxed{\text{E}}$ None of the above.

20. Ana and Amy have kids till a boy. For Ana, boys and girls are equally likely. For Amy, boys are twice as likely as girls. What is the expected number of kids Ana and Amy will have in total?

$\boxed{\text{A}}$ 3.

$\boxed{\text{B}}$ 3.5.

$\boxed{\text{C}}$ 4.

$\boxed{\text{D}}$ 4.5.

$\boxed{\text{E}}$ None of the above.