1. Which of the following would show that $p \to q$ is true?

$\boxed{\text{A}}$ Assume p is not true and show q is not true.

$\boxed{\text{B}}$ Show p is always true.

$\boxed{\text{C}}$ Show p is always false.

$\boxed{\text{D}}$ Assume q is true and show p is not true.

2. $p \to (q \wedge r)$ is equivalent to what other compound proposition:

$\boxed{\text{A}}$ $(p \to q) \wedge r$

$\boxed{\text{B}}$ $(p \to q) \wedge (p \to r)$

$\boxed{\text{C}}$ $(p \wedge q) \to r$

$\boxed{\text{D}}$ $p \vee (q \wedge r)$

3. Which reasoning is correct in the deductions below?

$\boxed{\text{A}}$ If it rains, then Kilam brings an umbrella. It did not rain. Therefore, Kilam did not bring an umbrella.

$\boxed{\text{B}}$ Everyone who eats apples is healthy. Malik is not healthy. Therefore, Malik does not eat apples.

$\boxed{\text{C}}$ At the party you can have cake or ice-cream. You had cake. Therefore, you did not have ice-cream.

$\boxed{\text{D}}$ Lights are turned on in the evenings. It is daytime. Therefore, the lights are turned off.

4. $P(n)$ is a predicate (n is an integer). $P(2)$ is true; and, $P(n) \to (P(n^2) \wedge P(n-2))$ is true for $n \geq 2$. For which n can we be **<u>sure</u>** $P(n)$ is true?

$\boxed{\text{A}}$ All $n \geq 2$.

$\boxed{\text{B}}$ All even $n \geq 0$.

$\boxed{\text{C}}$ All odd $n \geq 0$.

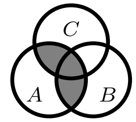
$\boxed{\text{D}}$ All n which are perfect squares.

5. You may take as known facts: $0 = 0$ and the standard operations of algebra from high-school math. Which of the following is a valid proof that $7 = 7$.

(I)	(II)	(III)
1. $7 = 7$	1. $7 \neq 7$	1. $0 = 0$
2. $7 - 7 = 7 - 7$	2. $7 - 7 \neq 7 - 7$	2. $0 + 7 = 0 + 7$
3. $0 = 0$ ✓	3. $0 \neq 0$ **!FISHY**	3. $7 = 7$ ✓
$\to$ $7 = 7$	$\to$ $7 = 7$	$\to$ $7 = 7$

$\boxed{\text{A}}$ I & II & III. $\boxed{\text{B}}$ I & II $\boxed{\text{C}}$ II & III. $\boxed{\text{D}}$ I & III.

6. Which expression represents the shaded region in the Venn diagram:



$\boxed{\text{A}}$ $A \cap B \cap C$ $\boxed{\text{B}}$ $A \cap (B \cup C)$ $\boxed{\text{C}}$ $A \cup (B \cap C)$ $\boxed{\text{D}}$ $A \cup B \cup C$

7. The domain of x, y is $\mathbb{R}$. True or false, $\exists x : (\forall y : xy = y)$?

$\boxed{\text{A}}$ True.

$\boxed{\text{B}}$ False.

$\boxed{\text{C}}$ Can't say because it depends on x.

$\boxed{\text{D}}$ Can't say because it depends on y.

8. T_n satisfies a recurrence $T_0 = 3$; $T_n = 2T_{n-1}$ for $n \geq 1$. Give a formula for T_n.

$\boxed{\text{A}}$ $T_n = 3(n+1) + \frac{3}{2}n(n-1)$

$\boxed{\text{B}}$ $T_n = 3 \cdot 2^{n+1} - 3(n+1)$

$\boxed{\text{C}}$ $T_n = 3 \cdot 2^n$

$\boxed{\text{D}}$ $T_n = 2^n$

9. The set $\mathcal{A}$ of arithmetic strings using characters in the set $\Sigma = \{1, +, \times, (,)\}$ has a recursive definition:

[Base Case:] $1 \in \mathcal{A}$;
[Constructor Rules:] $x, y, z \in \mathcal{A} \to (x + y + z) \in \mathcal{A}$
$x, y \in \mathcal{A} \to (x \times y) \in \mathcal{A}$.

Which string is in $\mathcal{A}$

$\boxed{\text{A}}$ $(1+1+1) \times (1+1)$

$\boxed{\text{B}}$ $(1+1+1) \times ((1+1+1)+1+1)$

$\boxed{\text{C}}$ $((1+1+1) \times ((1+1+1)+1+1))$

$\boxed{\text{D}}$ $((1 \times 1)+1+1+1)$

10. There are 5 rooted binary trees with 3 nodes. How many are there with 4 nodes?

$\boxed{\text{A}}$ 7

$\boxed{\text{B}}$ 12

$\boxed{\text{C}}$ 14

$\boxed{\text{D}}$ 16

1. We know that p is false. We do not know the truth value of q. Which of the following *must* be true?

(I) $\neg p \vee \neg q$ (II) $\neg p \wedge \neg q$ (III) $\neg(p \wedge q)$ (IV) $p \to q$

A I, II, III

B I, II, IV

C I, III, IV

D II, III, IV

2. For a set of horses $\mathcal{H}$, determine whether the following claim is true or false:

IF every subset of 10 horses has the same color, THEN every subset of 11 horses has the same color.

A Always true no matter what $\mathcal{H}$ is.

B Always false no matter what $\mathcal{H}$ is.

C Not enough information to determine whether it is true or false.

D False if $\mathcal{H}$ has fewer than 11 horses but true otherwise.

3. Which reasoning is correct in the deductions below?

A If it rains, then Kilam brings an umbrella. It did not rain. Therefore, Kilam did not bring an umbrella.

B Everyone who eats apples is healthy. Malik is healthy. Therefore, Malik eats apples.

C At the party you can have cake or ice-cream. You had cake. Therefore, you did not have ice-cream.

D Lights are turned on in the night. Lights are off. Therefore, it is day.

4. $P(n)$ is a predicate (n is an integer). $P(2)$ is true; and, $P(n) \to P(n+2)$ is true for $n \geq 0$. For which n can we be **sure** $P(n)$ is true?

A All $n \geq 2$.

B All even $n \geq 0$.

C All even $n \geq 2$.

D All n which are perfect squares.

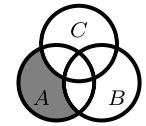
5. Which of the following, if any, is a valid way to prove $P(n) \to P(n+1)$.

(I) Let's see what happens if $P(n+1)$ is T.
$\vdots$ (valid derivations)
Look! $P(n)$ is T. ✓

(II) Let's see what happens if $P(n+1)$ is F.
$\vdots$ (valid derivations)
Look! $P(n)$ is F. ✓

A None B I C II D I and II

6. Which expression represents the shaded region in the Venn diagram:



A $A \cap B \cap C$ B $A \cap (B \cup C)$ C $A \cap \overline{B} \cap \overline{C}$ D $\overline{A} \cap B \cap C$

7. What is the more formal way to say: "There's a soul-mate for everyone"?

A $\exists x \in$ PEOPLE : ($\exists y \in$ PEOPLE : x is a soul-mate for y)

B $\exists x \in$ PEOPLE : ($\exists y \in$ PEOPLE : y is a soul-mate for x)

C $\forall x \in$ PEOPLE : ($\forall y \in$ PEOPLE : y is a soul-mate for x)

D $\forall x \in$ PEOPLE : ($\exists y \in$ PEOPLE : y is a soul-mate for x)

8. T_n satisfies a recurrence $T_0 = 2$; $T_n = T_{n-1} + 3n$ for $n \geq 1$. Compute T_{100}.

A 10,002

B 10,102

C 15,152

D 14,002

9. Determine the set $\mathcal{A}$ defined recursively by:

1 $1 \in \mathcal{A}$. **[basis]**

2 $x, y \in \mathcal{A} \to x + y \in \mathcal{A}$ **[constructors]**
$x, y \in \mathcal{A} \to x - y \in \mathcal{A}$.

3 Nothing else is in $\mathcal{A}$. **[minimality]**

A $\mathcal{A} = \{1, 2, 3, \ldots\}$

B $\mathcal{A} = \{0, 1, 2, 3, \ldots\}$

C $\mathcal{A} = \{\pm 1, \pm 2, \pm 3, \ldots\}$

D $\mathcal{A} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

10. 1 $1 \in \mathcal{S}$. **[basis]**

2 $x \in \mathcal{S} \to x + 1 \in \mathcal{S}$. **[constructor]**

This is a recursive definition of a set $\mathcal{S}$ *without* the minimality clause "Nothing else is in $\mathcal{S}$."

Which of the following *cannot* be the set $\mathcal{S}$

A $\mathbb{N}$

B $\mathbb{Z}$

C $\mathbb{N} \cup \{x \mid x = n + \frac{1}{2}, n \in \mathbb{N}\}$

D $\mathbb{N} \cup \{\frac{1}{2}\}$

1. $\sqrt{2}$ is what kind of number?

$\boxed{\text{A}}$ A natural number.

$\boxed{\text{B}}$ An integer.

$\boxed{\text{C}}$ A rational number.

$\boxed{\text{D}}$ A member of the set $\mathbb{Q}$.

$\boxed{\text{E}}$ None of the above.

2. What is the set $\mathbb{Z} \cap \overline{\mathbb{N}} \cap \mathcal{S}$, where $\mathcal{S}$ is the set of perfect square numbers. The universal set is $\mathbb{R}$.

$\boxed{\text{A}}$ $\varnothing$, the empty set.

$\boxed{\text{B}}$ $\{0\}$.

$\boxed{\text{C}}$ $\mathcal{S}$.

$\boxed{\text{D}}$ The non-positive integers.

$\boxed{\text{E}}$ The set is not well defined.

3. $A = \{2, 5\}$ and $B = \{3, 7\}$. What is the Cartesian Product $A \times B$?

$\boxed{\text{A}}$ $\{6, 14, 15, 35\}$.

$\boxed{\text{B}}$ $\{2, 3, 5, 7\}$.

$\boxed{\text{C}}$ $\{(2, 3), (2, 7), (5, 3), (5, 7)\}$.

$\boxed{\text{D}}$ $\{(2, 3), (3, 2), (2, 7), (7, 2)(5, 3), (3, 5), (5, 7), (7, 5)\}$.

$\boxed{\text{E}}$ None of the above.

4. How many rows in the truth table of $(p \to q) \vee p$ are T?

$\boxed{\text{A}}$ 0.

$\boxed{\text{B}}$ 1.

$\boxed{\text{C}}$ 2.

$\boxed{\text{D}}$ 3.

$\boxed{\text{E}}$ 4.

5. IF (you ace the final OR the quiz), THEN you get an A. You did get an A. *Did you ace the final*?

$\boxed{\text{A}}$ Yes, for sure.

$\boxed{\text{B}}$ No, for sure.

$\boxed{\text{C}}$ Yes, if and only if you did not ace the quiz.

$\boxed{\text{D}}$ Yes if you did not ace the quiz; otherwise we don't know.

$\boxed{\text{E}}$ None of the above.

6. Which mathematical claims are T. Note, $(a, b, c) \in \mathbb{R}^3$ stands for triples of real numbers (a, b, c).

(I) IF $\Big(\forall (a, b, c) \in \mathbb{R}^3 : ax^2 + bx + c = 0\Big)$, THEN $x = 0$

(II) $\forall (a, b, c) \in \mathbb{R}^3 : \Big($IF $ax^2 + bx + c = 0$, THEN x=0$\Big)$

$\boxed{\text{A}}$ I only.

$\boxed{\text{B}}$ II only.

$\boxed{\text{C}}$ Both I and II.

$\boxed{\text{D}}$ Neither I nor II.

7. For a natural number n, consider the implication: IF $n \geq n + 1$, THEN n+1$\geq n + 2$
Determine whether the *implication* is T or F?

$\boxed{\text{A}}$ Always T no matter what n is.

$\boxed{\text{B}}$ Always F no matter what n is.

$\boxed{\text{C}}$ T only for positive n.

$\boxed{\text{D}}$ T only for negative n.

$\boxed{\text{E}}$ None of the above.

8. What method of proof is used to prove that $\sqrt{2}$ is irrational?

$\boxed{\text{A}}$ Direct proof.

$\boxed{\text{B}}$ Contraposition proof.

$\boxed{\text{C}}$ Proof by contradiction.

$\boxed{\text{D}}$ Induction.

$\boxed{\text{E}}$ Strong induction.

9. Which gives a valid proof of the implication $(p \vee q) \to r$.

$\boxed{\text{A}}$ Assume p is T and show that r must be T.

$\boxed{\text{B}}$ Assume q is T and show that r must be T.

$\boxed{\text{C}}$ Assume r is F and show that p must be F.

$\boxed{\text{D}}$ Assume r is F and show that q must be F.

$\boxed{\text{E}}$ None of the above.

10. $P(n) =$ "n is even" and $Q(n) =$ "n is a sum of two primes". Translate "$\forall n \in \mathbb{N} : P(n) \to Q(n)$."

$\boxed{\text{A}}$ If n is a natural number then n is a sum of two primes.

$\boxed{\text{B}}$ Every prime number is a natural number.

$\boxed{\text{C}}$ There is a natural number which is a prime number.

$\boxed{\text{D}}$ Every positive even number is a sum of two primes.

$\boxed{\text{E}}$ Some positive even number is a sum of two primes.

11. $P(n)$ is a predicate (n is an integer). $P(1)$ is true; and, $P(n) \to P(2n-1) \wedge P(2n)$ is true for $n \geq 1$. Which set captures *all* n for which we can be **sure** $P(n)$ is T?

A All $n \geq 1$.

B All $n \geq 2$.

C All even $n \geq 1$.

D All even $n \geq 2$.

E None of the above.

12. Which of the following, if any, is a valid way to prove $P(n) \to P(n+1)$ in an induction proof.

(I) Let's see what happens if $P(n)$ is T.
⋮ (valid derivations)
Look! $P(n+1)$ is T. ✓

(II) Let's see what happens if $P(n+1)$ is F.
⋮ (valid derivations)
Look! $P(n)$ is F. ✓

A None. B I only. C II only. D Both I and II

13. We wish to break a group of n students into project-teams of 4 or 7 students.

A IF $n \geq 7$, THEN it can be done.

B IF $n \geq 11$, THEN it can be done.

C IF $n \geq 14$, THEN it can be done.

D IF $n \geq 19$, THEN it can be done.

E None of the above are T.

14. $A = \{x \,|\, x = 12m + 21n, \text{ for } m, n \in \mathbb{Z}\}$. T or F: $A = \mathbb{Z}$?

A T.

B F.

C Depends on m.

D Depends on n.

E None of the above.

15. What is the function defined recursively on the right for integer $n \geq 0$.

$$f(n) = \begin{cases} 1 & n = 0; \\ 2nf(n-1) & n \geq 1. \end{cases}$$

A $f(n) = n!$.

B $f(n) = 2^n$.

C $f(n) = 2^n \times n^n$.

D $f(n) = 2^n \times n!$.

E None of the above.

1. $\sqrt{2}$ is what kind of number?

A A natural number.

B A rational number.

C An irrational number.

D An integer.

E None of the above.

2. The set $S = \{n \mid n = (k-1)(-1)^k, \text{ where } k \in \mathbb{N}\}$. Which of these sets could be S?

A {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...}

B {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...}

C {0, 1, -2, 3, -4, 5, -6, 7, -8, 9, -10, ...}

D {1, -2, 3, -4, 5, -6, 7, -8, 9, -10, ...}

E {0, -1, 2, -3, 4, -5, 6, -7, 8, -9, 10, ...}

3. A and B are sets. Which answer is another way to represent $\overline{A \cap B}$.

A $A \cup B$.

B $A \cap B$.

C $\overline{A} \cup \overline{B}$.

D $\overline{A} \cap \overline{B}$.

E None of the above.

4. An integer $n \in \mathbb{Z}$ has an even square, that is n^2 is even. Which claim is true?

A n is odd.

B n is positive.

C n^2 is divisible by 4.

D n is divisible by 4.

E None of the above claims are true.

5. How many rows are there in the truth table of the compound proposition $\big((p \to q) \vee (p \to r)\big) \to (q \to r)$?

A 2.

B 4.

C 8.

D 12.

E 16.

6. On your car's back bumper is a sticker that says "Honk if you love FOCS." Joe was behind you and honked. Later, Sue was behind you and didn't honk. What would be a valid inference?

A Joe loves FOCS. We don't know about Sue.

B Sue loves FOCS. We don't know about Joe

C Joe does not love FOCS. We don't know about Sue.

D Sue does not love FOCS. We don't know about Joe

E Joe loves FOCS and Sue does not love FOCS.

7. For $x, y \in \mathbb{N} = \{1, 2, 3, \ldots\}$, determine T or F for the proposition $\forall y : (\exists x : x^2 = y)$.

A Can't be done because p is not a valid proposition which is either T or F.

B It depends on x.

C It depends on y.

D F.

E T.

8. What method of proof did we use to prove that $\sqrt{2} \notin \mathbb{Q}$?

A Direct proof

B Contraposition proof.

C Proof by induction.

D Proof by contradiction.

E None of the above.

9. What method would you use to *prove* that $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = (\frac{1}{2}n(n+1))^2$ for *all* $n \geq 1$?

A Direct proof

B Contraposition proof.

C Show that the formula is true for $n = 1$ up to $n = 1000$.

D Proof by induction.

E Proof by contradiction.

10. You must prove $P(n)$ for $n \geq 3$. You proved $P(n) \to P(n+3)$ for $n \geq 3$. What base cases do you need?

A $P(1)$

B $P(3)$

C $P(1)$, $P(2)$ and $P(3)$

D $P(3)$, $P(4)$ and $P(5)$

E None of the above.

11. For $x, y \in \mathbb{N}$, which statement is a contradiction (cannot possibly be true)?

A $x^2 < y$.

B $x^2 = y/2$

C $x^2 - y^2 \leq 1$

D $x^2 + y^2 \leq 1$

E None of the above. That is, each statement above can be true for specific choices of x, y.

12. Which gives a valid way to prove the implication $p \to q$.

A Assume p is F and show that q must be F.

B Assume q is T and show that p must be T.

C Assume p is T and show that q must be F.

D Assume p is T and q is F and derive a contradiction.

E None of the above.

13. What is the difference between using Induction versus Strong Induction to prove $P(n)$ for $n \geq 1$?

A The base cases are different.

B Induction is usually easier than Strong Induction.

C In Induction you prove $P(n+1)$. In Strong Induction you prove $P(n+2)$.

D In Induction you assume $P(n)$. In Strong Induction you assume $P(1) \wedge P(2) \wedge \cdots \wedge P(n)$.

E There is no difference between the two methods.

14. Compute the value of $(1 - \frac{1}{2}) \times (1 - \frac{1}{3}) \times (1 - \frac{1}{4}) \times (1 - \frac{1}{5}) \times \cdots \times (1 - \frac{1}{100})$.

A 1/5

B 1/10

C 1/50

D 1/100

E None of the above.

15. We wish to break a group of n students into project-teams. Each team must have either 4 or 6 students.

A IF $n \geq 4$, THEN it can be done.

B IF $n \geq 6$, THEN it can be done.

C IF $n \geq 10$, THEN it can be done.

D IF $n \geq 4$ and n is even, THEN it can be done.

E None of the above.

16. What are the first four terms A_0, A_1, A_2, A_3 in the the recurrence $A_n = \begin{cases} 1 & n = 0; \\ 2A_{n-1} + 1 & n \geq 1. \end{cases}$

$\boxed{A}$ $1, 2, 3, 4$.

$\boxed{B}$ $1, 2, 4, 8$.

$\boxed{C}$ $1, 3, 6, 12$.

$\boxed{D}$ $1, 3, 7, 15$.

$\boxed{E}$ None of the above.

17. For $n \geq 0$, what is a formula for A_n, where A_n satisfies the recurrence $A_n = \begin{cases} 1 & n = 0; \\ 2A_{n-1} + 1 & n \geq 1. \end{cases}$

$\boxed{A}$ $A_n = 1 + 2n$ for $n \geq 0$.

$\boxed{B}$ $A_n = 1 + n + n^2$ for $n \geq 0$.

$\boxed{C}$ $A_n = 1 + \frac{1}{3}(5n + n^3)$ for $n \geq 0$.

$\boxed{D}$ $A_n = 2^{n+1} - 1$ for $n \geq 0$.

$\boxed{E}$ None of the above.

18. String x is a palindrome, that is $x = x^{\mathrm{R}}$ where x^{R} is the reversal of x. Which statement about x is **false**?

$\boxed{A}$ x could be the string 1001.

$\boxed{B}$ The reversal of x must be a palindrome, that is x^{R} is a palindrome.

$\boxed{C}$ The concatenation of x with itself is a palindrome, that is $x \bullet x$ is a palindrome.

$\boxed{D}$ x must have even length.

$\boxed{E}$ The concatenation of x with its reversal is a palindrome, that is $x \bullet x^{\mathrm{R}}$ is a palindrome.

19. Rooted binary trees (RBTs) are recursively defined below. How many RBTs have 3 vertices?

$\boxed{A}$ 2

$\boxed{B}$ 3

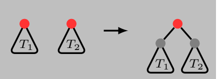
$\boxed{C}$ 4

$\boxed{D}$ 5

$\boxed{E}$ 6

Recursive Definition of RBT

1. The empty tree ε is an RBT.
2. If T_1, T_2 are disjoint RBTs with roots r_1 and r_2, then linking r_1 and r_2 to a *new* root r gives a new RBT with root r.
3. Nothing else is an RBT.



20. A rooted binary tree (RBT) has 8 vertices. How many links (edges) does the RBT have?

$\boxed{A}$ There is not enough information to determine the number of links.

$\boxed{B}$ 5

$\boxed{C}$ 6

$\boxed{D}$ 7

$\boxed{E}$ 8

1. Jodie asks John to solve $x^2 - a = 0$ and find x as a rational number. Which is true?

$\boxed{A}$ $\forall a \in \mathbb{N}$: John can find a rational solution x.

$\boxed{B}$ $\forall a \in \mathbb{N}$: John cannot find a rational solution x.

$\boxed{C}$ $\forall a \in \mathbb{Z}$: John can find a rational solution x.

$\boxed{D}$ $\forall a \in \mathbb{Z}$: John cannot find a rational solution x.

$\boxed{E}$ None of the above.

2. The set $S = \{4, 16, 64, 256, 1024, \ldots\}$. Which of these definitions using a variable could be S?

$\boxed{A}$ $S = \{n | n = 2^k, \text{ for } k \in \mathbb{N}\}$.

$\boxed{B}$ $S = \{n | n = 4^{1+k(k-1)/2}, \text{ for } k \in \mathbb{N}\}$.

$\boxed{C}$ $S = \{n | n = 2 \times 2^k, \text{ for } k \in \mathbb{N}\}$.

$\boxed{D}$ $S = \{x | x = 2^{2k}, \text{ for } k \in \mathbb{N}\}$.

$\boxed{E}$ None of the above.

3. $A = \{\text{positive multiples of } 2\}$ and $B = \{\text{positive multiples of } 3\}$. Which element is not in $\overline{A \cap B}$?

$\boxed{A}$ 4.

$\boxed{B}$ 8.

$\boxed{C}$ 12.

$\boxed{D}$ 16.

$\boxed{E}$ None of the above.

4. An integer $n \in \mathbb{Z}$ has a square that is divisible by 3, that is 3 divides n^2. Which claim *must be* true?

$\boxed{A}$ n is odd.

$\boxed{B}$ n is even.

$\boxed{C}$ n is positive.

$\boxed{D}$ n is divisible by 3.

$\boxed{E}$ None of the above claims must be true.

5. If it rains on a day, then it rains the next day. Today it didn't rain. Which is true?

$\boxed{A}$ It will rain tomorrow.

$\boxed{B}$ It will not rain tomorrow.

$\boxed{C}$ It did rain yesterday.

$\boxed{D}$ It did not rain yesterday.

$\boxed{E}$ None of the above.

6. Which method would succeed in *proving* $p \to (q \vee r)$?

$\boxed{\text{A}}$ You assumed p is true and showed q is true.

$\boxed{\text{B}}$ You assumed q is false and showed p is false.

$\boxed{\text{C}}$ You showed that p is true and that q is false.

$\boxed{\text{D}}$ You showed that p is true and that both q and r are false.

$\boxed{\text{E}}$ None of the above.

7. Which method would succeed in *disproving* $p \to (q \vee r)$?

$\boxed{\text{A}}$ You assumed p is true and showed q is true.

$\boxed{\text{B}}$ You assumed q is false and showed p is false.

$\boxed{\text{C}}$ You showed that p is true and that q is false.

$\boxed{\text{D}}$ You showed that p is true and that both q and r are false.

$\boxed{\text{E}}$ None of the above.

8. Determine true or false for the claim $\forall n \in \mathbb{Z} : (n > n+1) \to (n+1 > n+2)$.

$\boxed{\text{A}}$ This is not a valid proposition which is either true or false.

$\boxed{\text{B}}$ True for $n < 0$ and false otherwise.

$\boxed{\text{C}}$ True for $n = 0$ and false otherwise.

$\boxed{\text{D}}$ False.

$\boxed{\text{E}}$ True.

9. What method of proof would you use to *prove* that you cannot choose $a, b \in \mathbb{Z}$ so that $a^2 - 4b = 2$?

$\boxed{\text{A}}$ Direct proof.

$\boxed{\text{B}}$ Contraposition proof.

$\boxed{\text{C}}$ Proof by induction.

$\boxed{\text{D}}$ Proof by contradiction.

$\boxed{\text{E}}$ None of the above.

10. What method would you use to *prove* that $n^3 \le 2^n$ for *all* $n \ge 10$?

$\boxed{\text{A}}$ Direct proof

$\boxed{\text{B}}$ Contraposition proof.

$\boxed{\text{C}}$ Show that the formula is true for $n = 1$ up to $n = 1000$.

$\boxed{\text{D}}$ Proof by induction.

$\boxed{\text{E}}$ Proof by contradiction.

11. We wish to prove $P(n)$ for all $n \ge 10$. Which method accomplishes this?

$\boxed{\text{A}}$ Prove base case $P(1)$ and prove $P(n) \to P(n+2)$ for all $n \ge 10$.

$\boxed{\text{B}}$ Prove base cases $P(1), P(2)$ and prove $P(n) \to P(n+2)$ for all $n \ge 10$.

$\boxed{\text{C}}$ Prove base case $P(10)$ and prove $P(n) \to P(n+2)$ for all $n \ge 10$.

$\boxed{\text{D}}$ Prove base cases $P(10), P(11)$ and prove $P(n) \to P(n+2)$ for all $n \ge 10$.

$\boxed{\text{E}}$ None of the above methods works.

12. For $x, y \in \mathbb{Z}$, which statement is *not necassarily* a contradiction? (That is, which could be true?)

$\boxed{\text{A}}$ $x + 0 > x + 1$.

$\boxed{\text{B}}$ $x \ge y$ AND $x < y$.

$\boxed{\text{C}}$ $x^2 \ge y^2$ AND $|x| < |y|$.

$\boxed{\text{D}}$ $x^2 + y^2 \le 1$.

$\boxed{\text{E}}$ They are all contradictions.

13. Consider the predicate $P(n) : n^2 \le 2^n$. Which claim is true?

$\boxed{\text{A}}$ $P(n)$ is true for at most a finite number of $n \in \mathbb{N}$.

$\boxed{\text{B}}$ $P(n)$ is true for *all* $n \in \mathbb{N}$.

$\boxed{\text{C}}$ $P(n)$ is true for *all* even $n \in N$.

$\boxed{\text{D}}$ $P(n)$ is true for *all* odd $n \in N$.

$\boxed{\text{E}}$ None of the above claims is true.

14. Consider the predicate $P(n) : 8$ divides $n^2 - 1$. Which claim is true?

$\boxed{\text{A}}$ $P(n)$ is true for at most a finite number of $n \in \mathbb{N}$.

$\boxed{\text{B}}$ $P(n)$ is true for *all* $n \in \mathbb{N}$.

$\boxed{\text{C}}$ $P(n)$ is true for *all* even $n \in N$.

$\boxed{\text{D}}$ $P(n)$ is true for *all* odd $n \in N$.

$\boxed{\text{E}}$ None of the above claims is true.

15. Consider the predicate $P(n) : 1^2 + 2^2 + 3^2 + \cdots + n^2 > n^3/3$. Which claim is true?

$\boxed{\text{A}}$ $P(n)$ is true for at most a finite number of $n \in \mathbb{N}$.

$\boxed{\text{B}}$ $P(n)$ is true for *all* $n \in \mathbb{N}$.

$\boxed{\text{C}}$ $P(n)$ is true for *all* even $n \in N$.

$\boxed{\text{D}}$ $P(n)$ is true for *all* odd $n \in N$.

$\boxed{\text{E}}$ None of the above claims is true.

16. You wish to make postage n cents with 5-cent and 6-cent stamps. For which $n \in \mathbb{N}$ can you do it?

A All postages $n \geq 5$ cents.

B All postages $n \geq 10$ cents.

C All postages $n \geq 15$ cents.

D All postages $n \geq 20$ cents.

E None of the above.

17. $A_0 = 0$ and for $n > 0$, $A_n = n^2 + A_{n-2}$. What is A_6?

A It cannot be computed because this recurrence has only one base case.

B $A_6 = 12$.

C $A_6 = 52$.

D $A_6 = 56$.

E None of the above.

18. $f(1) = 1; f(2) = 1$ and for $n > 2$, $f(n) = n + f(n-3)$. For which $n \in \mathbb{N}$ can $f(n)$ be computed?

A *All* $n \in \mathbb{N}$.

B *All* $n \in \mathbb{N}$ which are even.

C *All* $n \in \mathbb{N}$ which are multiples of 3.

D *All* $n \in \mathbb{N}$ which are not multiples of 3.

E None of the above.

19. Rooted binary trees (RBTs) are recursively defined below. How many RBTs have 4 vertices and 2 links?

A 0.

B 5.

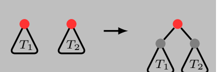
C 14.

D 42.

E 132.

Recursive Definition of RBT

1 The empty tree ε is an RBT.
2 If T_1, T_2 are disjoint RBTs with roots r_1 and r_2, then linking r_1 and r_2 to a *new* root r gives a new RBT with root r.
3 Nothing else is an RBT.



20. T_1 and T_2 are disjoint RBTs. RBT T_1 has 8 vertices and 7 links. RBT T_2 has 4 vertices and 3 links. Using the constructor for RBT, you get a child RBT T. How many vertices and links does T have?

A 12 vertices and 10 links.

B 12 vertices and 11 links.

C 13 vertices and 11 links.

D 13 vertices and 12 links.

E None of the above, or we can't say for sure.

1. Which set below is the set $S = \{2k \mid k \in \mathbb{N}\}$?

A All even numbers.

B All odd numbers.

C All non-negative even numbers.

D All non-negative odd numbers.

E None of the above.

2. Define sets $A = \{2k \mid k \in \mathbb{Z}\}$, $B = \{9k \mid k \in \mathbb{Z}\}$ and $C = \{6k \mid k \in \mathbb{Z}\}$. Which is true?

A $A \cap B = C$.

B $A \cap B \subseteq C$.

C $A \cap B = \overline{C}$.

D $A \cap B \subseteq \overline{C}$.

E None of the above.

3. How many rows are in the truth table of $p \to (p \vee q)$?

A 2.

B 4.

C 6.

D 8.

E None of the above.

4. True or false, $p \to (p \vee q)$?

A Can be true or false, depending on p.

B Can be true or false, depending on q.

C Always true.

D Always false.

E None of the above.

5. If you majored CS then you took FOCS. Joe took FOCS and Barb majored CS. What else do we know?

A Joe majored CS. We don't know anything more about Barb.

B We don't know anything more about Joe. Barb took FOCS.

C Joe majored CS. And, Barb took FOCS.

D Joe did not major CS. And, Barb took FOCS.

E None of the above.

6. What is the negation of the claim $\forall m, n \in \mathbb{N} : 3m + 6n \neq 10$?

$\boxed{\text{A}}$ $\forall m, n \in \mathbb{N} : 3m + 6n = 10$.

$\boxed{\text{B}}$ $\forall m, n \in \mathbb{N} : 3m + 6n \neq 10$.

$\boxed{\text{C}}$ $\exists m, n \in \mathbb{N} : 3m + 6n = 10$.

$\boxed{\text{D}}$ $\exists m, n \in \mathbb{N} : 3m + 6n \neq 10$.

$\boxed{\text{E}}$ None of the above.

7. Which proof-method is acceptable to prove the claim p?

$\boxed{\text{A}}$ Assume p is true and derive something known to be true, for example $0 = 0$.

$\boxed{\text{B}}$ Assume $\neg p$ is true and derive something known to be true, for example $0 = 0$.

$\boxed{\text{C}}$ Assume p is true and derive something known to be false, for example $1 > 2$.

$\boxed{\text{D}}$ Assume $\neg p$ is true and derive something known to be false, for example $1 > 2$.

$\boxed{\text{E}}$ None of the above.

8. Consider the claim $\exists m, n \in \mathbb{Z} : 9m + 21n = 7$. Is the claim true or false?

$\boxed{\text{A}}$ True.

$\boxed{\text{B}}$ False.

$\boxed{\text{C}}$ It depends on m.

$\boxed{\text{D}}$ It depends on n.

$\boxed{\text{E}}$ None of the above.

9. How do you *disprove* the claim $\forall n \in \mathbb{N} : \neg P(n) \to Q(n)$.

$\boxed{\text{A}}$ Show that for all $n \in \mathbb{N}$, $P(n)$ is true and $Q(n)$ is false.

$\boxed{\text{B}}$ Show that for all $n \in \mathbb{N}$, $P(n)$ is false and $Q(n)$ is false.

$\boxed{\text{C}}$ Show that for some $n \in \mathbb{N}$, $P(n)$ is true and $Q(n)$ is false.

$\boxed{\text{D}}$ Show that for some $n \in \mathbb{N}$, $P(n)$ is false and $Q(n)$ is false.

$\boxed{\text{E}}$ None of the above.

10. What is the first step in a proof by contradiction of the claim $\forall m, n \in \mathbb{N} : 3m + 6n \neq 10$.

$\boxed{\text{A}}$ Define the predicate $P(m, n) : 3m + 6n \neq 10$ and prove the base case $P(1, 1)$.

$\boxed{\text{B}}$ Assume $3m + 6n = 10$ for all $m, n \in \mathbb{N}$.

$\boxed{\text{C}}$ Assume $3m + 6n \neq 10$ for some $m, n \in \mathbb{N}$.

$\boxed{\text{D}}$ Assume $3m + 6n = 10$ for some $m, n \in \mathbb{N}$.

$\boxed{\text{E}}$ None of the above.

11. You decided to *prove* the claim $n^2 \leq 2^n$ for all $n \geq 4$. Which method of proof would you use?

$\boxed{\text{A}}$ Find a single value $n_* \in \mathbb{N}$ for which $n_*^2 > 2^{n_*}$.

$\boxed{\text{B}}$ Show that the formula $n^2 \leq 2^n$ is true for $n = 1$ up to $n = 1000$.

$\boxed{\text{C}}$ Proof by induction.

$\boxed{\text{D}}$ Contraposition proof.

$\boxed{\text{E}}$ Direct proof.

12. You decided to *disprove* the claim $n^2 \leq 2^n$ for all $n \geq 1$. Which method of proof would you use?

$\boxed{\text{A}}$ Find a single value $n_* \in \mathbb{N}$ for which $n_*^2 > 2^{n_*}$.

$\boxed{\text{B}}$ Show that the formula $n^2 \leq 2^n$ is true for $n = 1$ up to $n = 1000$.

$\boxed{\text{C}}$ Proof by induction.

$\boxed{\text{D}}$ Contraposition proof.

$\boxed{\text{E}}$ Direct proof.

13. How do you prove, by induction, the claim "5 divides $11^n - 6$" for all $n \geq 5$?

$\boxed{\text{A}}$ Show 5 divides $11^5 - 6$.

$\boxed{\text{B}}$ Show 5 divides $11^5 - 6, 11^6 - 6, 11^7 - 6$ all the way up to $11^{1{,}000{,}000} - 6$.

$\boxed{\text{C}}$ Show, for $n \geq 5$, if 5 divides $11^n - 6$ then 5 divides $11^{n+1} - 6$.

$\boxed{\text{D}}$ Show 5 divides $11^5 - 6$. And, show, for $n \geq 5$, if 5 divides $11^n - 6$ then 5 divides $11^{n+1} - 6$.

$\boxed{\text{E}}$ None of the above.

14. You wish to prove $n^4 \leq 2^n$ for $n \geq 16$. You showed that $n^4 \leq 2^n \to (n + 3)^4 \leq 2^{n+3}$ for $n \geq 16$. What base cases do you need to prove to complete the proof?

$\boxed{\text{A}}$ $n = 1$.

$\boxed{\text{B}}$ $n = 16$.

$\boxed{\text{C}}$ $n = 1$ and $n = 2$.

$\boxed{\text{D}}$ $n = 16$ and $n = 17$.

$\boxed{\text{E}}$ None of the above.

15. Define the predicate $P(n) : (2n - 1)^2 + 4$ is prime. For which n is $P(n)$ true?

$\boxed{\text{A}}$ $n \geq 1$.

$\boxed{\text{B}}$ $n \geq 2$.

$\boxed{\text{C}}$ $n \geq 3$.

$\boxed{\text{D}}$ $n \geq 4$.

$\boxed{\text{E}}$ None of the above.

16. Define the sum $S(n) = \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{(n-1) \times n}$ for $n \geq 2$. What is $S(100)$?

[A] 0.1.

[B] 0.01.

[C] 0.9.

[D] 0.99.

[E] None of the above.

17. $f(1) = 1$, $f(2) = 2$, and $f(n) = f(n-2) + 2$ for $n > 2$. What is $f(100)$?

[A] It cannot be computed because the recursion does not have enough base cases.

[B] 50.

[C] 100.

[D] 200.

[E] None of the above.

18. Define $\mathcal{A}$ recursively: (i) $1 \in \mathcal{A}$ (ii) $x \in \mathcal{A} \to x + 4 \in \mathcal{A}$ (iii) Nothing else is in $\mathcal{A}$. Which is true?

[A] Every number in $\mathcal{A}$ is even.

[B] Every even number is in $\mathcal{A}$.

[C] Every number in $\mathcal{A}$ is odd.

[D] Every odd number is in $\mathcal{A}$.

[E] None of the above.

19. A rooted binary tree (RBT) has 8 vertices. How many links does it have?

[A] 6.

[B] 7.

[C] 8.

[D] 9.

[E] None of the above.

20. There are 5 distinct rooted binary trees (RBT) with 3 vertices. How many have 4 vertices?

[A] 12.

[B] 13.

[C] 14.

[D] 15.

[E] None of the above.