QUIZ 1: 90 Minutes

Last Name: ______________________

First Name: ______________________

RIN: ______________________

Section: ______________________

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer

Total
100

1. Which of the following would show that $p \to q$ is true?

$\boxed{\text{A}}$ Assume p is not true and show q is not true.

$\boxed{\text{B}}$ Show p is always true.

$\boxed{\text{C}}$ Show p is always false.

$\boxed{\text{D}}$ Assume q is true and show p is not true.

2. $p \to (q \wedge r)$ is equivalent to what other compound proposition:

$\boxed{\text{A}}$ $(p \to q) \wedge r$

$\boxed{\text{B}}$ $(p \to q) \wedge (p \to r)$

$\boxed{\text{C}}$ $(p \wedge q) \to r$

$\boxed{\text{D}}$ $p \vee (q \wedge r)$

3. Which reasoning is correct in the deductions below?

$\boxed{\text{A}}$ If it rains, then Kilam brings an umbrella. It did not rain. Therefore, Kilam did not bring an umbrella.

$\boxed{\text{B}}$ Everyone who eats apples is healthy. Malik is not healthy. Therefore, Malik does not eat apples.

$\boxed{\text{C}}$ At the party you can have cake or ice-cream. You had cake. Therefore, you did not have ice-cream.

$\boxed{\text{D}}$ Lights are turned on in the evenings. It is daytime. Therefore, the lights are turned off.

4. $P(n)$ is a predicate (n is an integer). $P(2)$ is true; and, $P(n) \to (P(n^2) \wedge P(n-2))$ is true for $n \geq 2$. For which n can we be **<u>sure</u>** $P(n)$ is true?

$\boxed{\text{A}}$ All $n \geq 2$.

$\boxed{\text{B}}$ All even $n \geq 0$.

$\boxed{\text{C}}$ All odd $n \geq 0$.

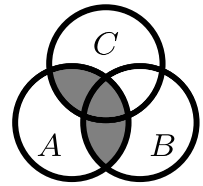
$\boxed{\text{D}}$ All n which are perfect squares.

5. You may take as known facts: $0 = 0$ and the standard operations of algebra from high-school math. Which of the following is a valid proof that $7 = 7$.

(I)			(II)			(III)		
1.	$7=7$		1.	$7\neq 7$		1.	$0=0$	
2.	$7-7=7-7$		2.	$7-7\neq 7-7$		2.	$0+7=0+7$	
3.	$0=0$	✓	3.	$0\neq 0$	**!FISHY**	3.	$7=7$	✓
$\to$	$7=7$		$\to$	$7=7$		$\to$	$7=7$	

$\boxed{\text{A}}$ I & II & III. $\boxed{\text{B}}$ I & II $\boxed{\text{C}}$ II & III. $\boxed{\text{D}}$ I & III.

6. Which expression represents the shaded region in the Venn diagram:



$\boxed{\text{A}}$ $A \cap B \cap C$ $\boxed{\text{B}}$ $A \cap (B \cup C)$ $\boxed{\text{C}}$ $A \cup (B \cap C)$ $\boxed{\text{D}}$ $A \cup B \cup C$

7. The domain of x, y is $\mathbb{R}$. True or false, $\exists x : (\forall y : xy = y)$?

$\boxed{\text{A}}$ True.

$\boxed{\text{B}}$ False.

$\boxed{\text{C}}$ Can't say because it depends on x.

$\boxed{\text{D}}$ Can't say because it depends on y.

8. T_n satisfies a recurrence $T_0 = 3$; $T_n = 2T_{n-1}$ for $n \geq 1$. Give a formula for T_n.

$\boxed{\text{A}}$ $T_n = 3(n+1) + \frac{3}{2}n(n-1)$

$\boxed{\text{B}}$ $T_n = 3 \cdot 2^{n+1} - 3(n+1)$

$\boxed{\text{C}}$ $T_n = 3 \cdot 2^n$

$\boxed{\text{D}}$ $T_n = 2^n$

9. The set $\mathcal{A}$ of arithmetic strings using characters in the set $\Sigma = \{1, +, \times, (,)\}$ has a recursive definition:

$$\begin{array}{rl} \textbf{[Base Case:]} & 1 \in \mathcal{A}; \\ \textbf{[Constructor Rules:]} & x, y, z \in \mathcal{A} \ \rightarrow (x + y + z) \in \mathcal{A} \\ & \quad x, y \in \mathcal{A} \ \rightarrow (x \times y) \in \mathcal{A}. \end{array}$$

Which string is in $\mathcal{A}$

$\boxed{\text{A}}$ $(1 + 1 + 1) \times (1 + 1)$

$\boxed{\text{B}}$ $(1 + 1 + 1) \times ((1 + 1 + 1) + 1 + 1)$

$\boxed{\text{C}}$ $((1 + 1 + 1) \times ((1 + 1 + 1) + 1 + 1))$

$\boxed{\text{D}}$ $((1 \times 1) + 1 + 1 + 1)$

10. There are 5 rooted binary trees with 3 nodes. How many are there with 4 nodes?

$\boxed{\text{A}}$ 7

$\boxed{\text{B}}$ 12

$\boxed{\text{C}}$ 14

$\boxed{\text{D}}$ 16

SCRATCH

QUIZ 1: 60 Minutes

Last Name: ____________________

First Name: ____________________

RIN: ____________________

Section: ____________________

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
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GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer

Total
100

1. We know that p is false. We do not know the truth value of q. Which of the following *must* be true?
(I) $\neg p \vee \neg q$ (II) $\neg p \wedge \neg q$ (III) $\neg(p \wedge q)$ (IV) $p \to q$

A I, II, III

B I, II, IV

C I, III, IV

D II, III, IV

2. For a set of horses $\mathcal{H}$, determine whether the following claim is true or false:
IF every subset of 10 horses has the same color, THEN every subset of 11 horses has the same color.

A Always true no matter what $\mathcal{H}$ is.

B Always false no matter what $\mathcal{H}$ is.

C Not enough information to determine whether it is true or false.

D False if $\mathcal{H}$ has fewer than 11 horses but true otherwise.

3. Which reasoning is correct in the deductions below?

A If it rains, then Kilam brings an umbrella. It did not rain. Therefore, Kilam did not bring an umbrella.

B Everyone who eats apples is healthy. Malik is healthy. Therefore, Malik eats apples.

C At the party you can have cake or ice-cream. You had cake. Therefore, you did not have ice-cream.

D Lights are turned on in the night. Lights are off. Therefore, it is day.

4. $P(n)$ is a predicate (n is an integer). $P(2)$ is true; and, $P(n) \to P(n+2)$ is true for $n \geq 0$. For which n can we be **sure** $P(n)$ is true?

A All $n \geq 2$.

B All even $n \geq 0$.

C All even $n \geq 2$.

D All n which are perfect squares.

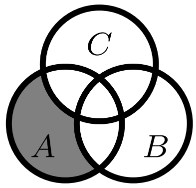
5. Which of the following, if any, is a valid way to prove $P(n) \to P(n+1)$.

(I) Let's see what happens if $P(n+1)$ is T.
$\vdots$ (valid derivations)
Look! $P(n)$ is T. ✓

(II) Let's see what happens if $P(n+1)$ is F.
$\vdots$ (valid derivations)
Look! $P(n)$ is F. ✓

A None B I C II D I and II

6. Which expression represents the shaded region in the Venn diagram:



$\boxed{\text{A}}$ $A \cap B \cap C$ $\boxed{\text{B}}$ $A \cap (B \cup C)$ $\boxed{\text{C}}$ $A \cap \overline{B} \cap \overline{C}$ $\boxed{\text{D}}$ $\overline{A} \cap B \cap C$

7. What is the more formal way to say: "There's a soul-mate for everyone"?

$\boxed{\text{A}}$ $\exists x \in \textsc{people} : (\exists y \in \textsc{people} : x \text{ is a soul-mate for } y)$

$\boxed{\text{B}}$ $\exists x \in \textsc{people} : (\exists y \in \textsc{people} : y \text{ is a soul-mate for } x)$

$\boxed{\text{C}}$ $\forall x \in \textsc{people} : (\forall y \in \textsc{people} : y \text{ is a soul-mate for } x)$

$\boxed{\text{D}}$ $\forall x \in \textsc{people} : (\exists y \in \textsc{people} : y \text{ is a soul-mate for } x)$

8. T_n satisfies a recurrence $T_0 = 2$; $T_n = T_{n-1} + 3n$ for $n \geq 1$. Compute T_{100}.

$\boxed{\text{A}}$ 10,002

$\boxed{\text{B}}$ 10,102

$\boxed{\text{C}}$ 15,152

$\boxed{\text{D}}$ 14,002

9. Determine the set $\mathcal{A}$ defined recursively by:

1	$1 \in \mathcal{A}$.	**[basis]**
2	$x, y \in \mathcal{A} \ \rightarrow x + y \in \mathcal{A}$ $x, y \in \mathcal{A} \ \rightarrow x - y \in \mathcal{A}$.	**[constructors]**
3	Nothing else is in $\mathcal{A}$.	**[minimality]**

$\boxed{\text{A}}$ $\mathcal{A} = \{1, 2, 3, \ldots\}$

$\boxed{\text{B}}$ $\mathcal{A} = \{0, 1, 2, 3, \ldots\}$

$\boxed{\text{C}}$ $\mathcal{A} = \{\pm 1, \pm 2, \pm 3, \ldots\}$

$\boxed{\text{D}}$ $\mathcal{A} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

10.

1	$1 \in \mathcal{S}$.	**[basis]**
2	$x \in \mathcal{S} \rightarrow x + 1 \in \mathcal{S}$.	**[constructor]**

This is a recursive definition of a set $\mathcal{S}$ *without* the minimality clause "Nothing else is in $\mathcal{S}$."

Which of the following *cannot* be the set $\mathcal{S}$

$\boxed{\text{A}}$ $\mathbb{N}$

$\boxed{\text{B}}$ $\mathbb{Z}$

$\boxed{\text{C}}$ $\mathbb{N} \cup \{x \mid x = n + \frac{1}{2}, n \in \mathbb{N}\}$

$\boxed{\text{D}}$ $\mathbb{N} \cup \{\frac{1}{2}\}$

SCRATCH

QUIZ 1: 60 Minutes

Last Name: ________________________

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Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
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GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show work to ensure getting full credit.

Total
150

1. $\sqrt{2}$ is what kind of number?

$\boxed{\text{A}}$ A natural number.

$\boxed{\text{B}}$ An integer.

$\boxed{\text{C}}$ A rational number.

$\boxed{\text{D}}$ A member of the set $\mathbb{Q}$.

$\boxed{\text{E}}$ None of the above.

2. What is the set $\mathbb{Z} \cap \overline{\mathbb{N}} \cap \mathcal{S}$, where $\mathcal{S}$ is the set of perfect square numbers. The universal set is $\mathbb{R}$.

$\boxed{\text{A}}$ $\varnothing$, the empty set.

$\boxed{\text{B}}$ $\{0\}$.

$\boxed{\text{C}}$ $\mathcal{S}$.

$\boxed{\text{D}}$ The non-positive integers.

$\boxed{\text{E}}$ The set is not well defined.

3. $A = \{2, 5\}$ and $B = \{3, 7\}$. What is the Cartesian Product $A \times B$?

$\boxed{\text{A}}$ $\{6, 14, 15, 35\}$.

$\boxed{\text{B}}$ $\{2, 3, 5, 7\}$.

$\boxed{\text{C}}$ $\{(2, 3), (2, 7), (5, 3), (5, 7)\}$.

$\boxed{\text{D}}$ $\{(2, 3), (3, 2), (2, 7), (7, 2)(5, 3), (3, 5), (5, 7), (7, 5)\}$.

$\boxed{\text{E}}$ None of the above.

4. How many rows in the truth table of $(p \to q) \vee p$ are T?

$\boxed{\text{A}}$ 0.

$\boxed{\text{B}}$ 1.

$\boxed{\text{C}}$ 2.

$\boxed{\text{D}}$ 3.

$\boxed{\text{E}}$ 4.

5. IF (you ace the final OR the quiz), THEN you get an A. You did get an A. *Did you ace the final*?

$\boxed{\text{A}}$ Yes, for sure.

$\boxed{\text{B}}$ No, for sure.

$\boxed{\text{C}}$ Yes, if and only if you did not ace the quiz.

$\boxed{\text{D}}$ Yes if you did not ace the quiz; otherwise we don't know.

$\boxed{\text{E}}$ None of the above.

6. Which mathematical claims are T. Note, $(a,b,c) \in \mathbb{R}^3$ stands for triples of real numbers (a,b,c).

(I) IF $\Big(\forall (a,b,c) \in \mathbb{R}^3 : ax^2 + bx + c = 0\Big)$, THEN $x = 0$

(II) $\forall (a,b,c) \in \mathbb{R}^3 : \Big($IF $ax^2 + bx + c = 0,$ THEN x=0$\Big)$

$\boxed{\text{A}}$ I only.

$\boxed{\text{B}}$ II only.

$\boxed{\text{C}}$ Both I and II.

$\boxed{\text{D}}$ Neither I nor II.

7. For a natural number n, consider the implication: IF $n \geq n+1,$ THEN n+1$\geq n+2$
Determine whether the *implication* is T or F?

$\boxed{\text{A}}$ Always T no matter what n is.

$\boxed{\text{B}}$ Always F no matter what n is.

$\boxed{\text{C}}$ T only for positive n.

$\boxed{\text{D}}$ T only for negative n.

$\boxed{\text{E}}$ None of the above.

8. What method of proof is used to prove that $\sqrt{2}$ is irrational?

$\boxed{\text{A}}$ Direct proof.

$\boxed{\text{B}}$ Contraposition proof.

$\boxed{\text{C}}$ Proof by contradiction.

$\boxed{\text{D}}$ Induction.

$\boxed{\text{E}}$ Strong induction.

9. Which gives a valid proof of the implication $(p \vee q) \to r$.

$\boxed{\text{A}}$ Assume p is T and show that r must be T.

$\boxed{\text{B}}$ Assume q is T and show that r must be T.

$\boxed{\text{C}}$ Assume r is F and show that p must be F.

$\boxed{\text{D}}$ Assume r is F and show that q must be F.

$\boxed{\text{E}}$ None of the above.

10. $P(n) =$ "n is even" and $Q(n) =$ "n is a sum of two primes". Translate "$\forall n \in \mathbb{N} : P(n) \to Q(n)$."

$\boxed{\text{A}}$ If n is a natural number then n is a sum of two primes.

$\boxed{\text{B}}$ Every prime number is a natural number.

$\boxed{\text{C}}$ There is a natural number which is a prime number.

$\boxed{\text{D}}$ Every positive even number is a sum of two primes.

$\boxed{\text{E}}$ Some positive even number is a sum of two primes.

11. $P(n)$ is a predicate (n is an integer). $P(1)$ is true; and, $P(n) \to P(2n-1) \wedge P(2n)$ is true for $n \geq 1$. Which set captures *all* n for which we can be **sure** $P(n)$ is T?

$\boxed{\text{A}}$ All $n \geq 1$.

$\boxed{\text{B}}$ All $n \geq 2$.

$\boxed{\text{C}}$ All even $n \geq 1$.

$\boxed{\text{D}}$ All even $n \geq 2$.

$\boxed{\text{E}}$ None of the above.

12. Which of the following, if any, is a valid way to prove $P(n) \to P(n+1)$ in an induction proof.

(I) Let's see what happens if $P(n)$ is T.
⋮ (valid derivations)
Look! $P(n+1)$ is T. ✓

(II) Let's see what happens if $P(n+1)$ is F.
⋮ (valid derivations)
Look! $P(n)$ is F. ✓

$\boxed{\text{A}}$ None. $\boxed{\text{B}}$ I only. $\boxed{\text{C}}$ II only. $\boxed{\text{D}}$ Both I and II

13. We wish to break a group of n students into project-teams of 4 or 7 students.

$\boxed{\text{A}}$ IF $n \geq 7$, THEN it can be done.

$\boxed{\text{B}}$ IF $n \geq 11$, THEN it can be done.

$\boxed{\text{C}}$ IF $n \geq 14$, THEN it can be done.

$\boxed{\text{D}}$ IF $n \geq 19$, THEN it can be done.

$\boxed{\text{E}}$ None of the above are T.

14. $A = \{x \,|\, x = 12m + 21n, \text{ for } m, n \in \mathbb{Z}\}$. T or F: $A = \mathbb{Z}$?

$\boxed{\text{A}}$ T.

$\boxed{\text{B}}$ F.

$\boxed{\text{C}}$ Depends on m.

$\boxed{\text{D}}$ Depends on n.

$\boxed{\text{E}}$ None of the above.

15. What is the function defined recursively on the right for integer $n \geq 0$.

$$f(n) = \begin{cases} 1 & n = 0; \\ 2nf(n-1) & n \geq 1. \end{cases}$$

$\boxed{\text{A}}$ $f(n) = n!$.

$\boxed{\text{B}}$ $f(n) = 2^n$.

$\boxed{\text{C}}$ $f(n) = 2^n \times n^n$.

$\boxed{\text{D}}$ $f(n) = 2^n \times n!$.

$\boxed{\text{E}}$ None of the above.

SCRATCH

QUIZ 1: 60 Minutes

Last Name: ____________________

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Answer **ALL** questions.
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GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

Total
200

1. $\sqrt{2}$ is what kind of number?

A A natural number.

B A rational number.

C An irrational number.

D An integer.

E None of the above.

2. The set $S = \{n \mid n = (k-1)(-1)^k, \text{ where } k \in \mathbb{N}\}$. Which of these sets could be S?

A $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$

B $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$

C $\{0, 1, -2, 3, -4, 5, -6, 7, -8, 9, -10, \ldots\}$

D $\{1, -2, 3, -4, 5, -6, 7, -8, 9, -10, \ldots\}$

E $\{0, -1, 2, -3, 4, -5, 6, -7, 8, -9, 10, \ldots\}$

3. A and B are sets. Which answer is another way to represent $\overline{A \cap B}$.

A $A \cup B$.

B $A \cap B$.

C $\overline{A} \cup \overline{B}$.

D $\overline{A} \cap \overline{B}$.

E None of the above.

4. An integer $n \in \mathbb{Z}$ has an even square, that is n^2 is even. Which claim is true?

A n is odd.

B n is positive.

C n^2 is divisible by 4.

D n is divisible by 4.

E None of the above claims are true.

5. How many rows are there in the truth table of the compound proposition $\big((p \to q) \vee (p \to r)\big) \to (q \to r)$?

A 2.

B 4.

C 8.

D 12.

E 16.

6. On your car's back bumper is a sticker that says "Honk if you love FOCS." Joe was behind you and honked. Later, Sue was behind you and didn't honk. What would be a valid inference?

A Joe loves FOCS. We don't know about Sue.

B Sue loves FOCS. We don't know about Joe

C Joe does not love FOCS. We don't know about Sue.

D Sue does not love FOCS. We don't know about Joe

E Joe loves FOCS and Sue does not love FOCS.

7. For $x, y \in \mathbb{N} = \{1, 2, 3, \ldots\}$, determine T or F for the proposition $\forall y : (\exists x : x^2 = y)$.

A Can't be done because p is not a valid proposition which is either T or F.

B It depends on x.

C It depends on y.

D F.

E T.

8. What method of proof did we use to prove that $\sqrt{2} \notin \mathbb{Q}$?

A Direct proof

B Contraposition proof.

C Proof by induction.

D Proof by contradiction.

E None of the above.

9. What method would you use to *prove* that $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = (\frac{1}{2}n(n+1))^2$ for *all* $n \geq 1$?

A Direct proof

B Contraposition proof.

C Show that the formula is true for $n = 1$ up to $n = 1000$.

D Proof by induction.

E Proof by contradiction.

10. You must prove $P(n)$ for $n \geq 3$. You proved $P(n) \to P(n+3)$ for $n \geq 3$. What base cases do you need?

A $P(1)$

B $P(3)$

C $P(1)$, $P(2)$ and $P(3)$

D $P(3)$, $P(4)$ and $P(5)$

E None of the above.

11. For $x, y \in \mathbb{N}$, which statement is a contradiction (cannot possibly be true)?

[A] $x^2 < y$.

[B] $x^2 = y/2$

[C] $x^2 - y^2 \leq 1$

[D] $x^2 + y^2 \leq 1$

[E] None of the above. That is, each statement above can be true for specific choices of x, y.

12. Which gives a valid way to prove the implication $p \to q$.

[A] Assume p is F and show that q must be F.

[B] Assume q is T and show that p must be T.

[C] Assume p is T and show that q must be F.

[D] Assume p is T and q is F and derive a contradiction.

[E] None of the above.

13. What is the difference between using Induction versus Strong Induction to prove $P(n)$ for $n \geq 1$?

[A] The base cases are different.

[B] Induction is usually easier than Strong Induction.

[C] In Induction you prove $P(n+1)$. In Strong Induction you prove $P(n+2)$.

[D] In Induction you assume $P(n)$. In Strong Induction you assume $P(1) \wedge P(2) \wedge \cdots \wedge P(n)$.

[E] There is no difference between the two methods.

14. Compute the value of $(1 - \frac{1}{2}) \times (1 - \frac{1}{3}) \times (1 - \frac{1}{4}) \times (1 - \frac{1}{5}) \times \cdots \times (1 - \frac{1}{100})$.

[A] 1/5

[B] 1/10

[C] 1/50

[D] 1/100

[E] None of the above.

15. We wish to break a group of n students into project-teams. Each team must have either 4 or 6 students.

[A] IF $n \geq 4$, THEN it can be done.

[B] IF $n \geq 6$, THEN it can be done.

[C] IF $n \geq 10$, THEN it can be done.

[D] IF $n \geq 4$ and n is even, THEN it can be done.

[E] None of the above.

16. What are the first four terms A_0, A_1, A_2, A_3 in the the recurrence $A_n = \begin{cases} 1 & n = 0; \\ 2A_{n-1} + 1 & n \geq 1. \end{cases}$

$\boxed{\text{A}}$ $1, 2, 3, 4.$

$\boxed{\text{B}}$ $1, 2, 4, 8.$

$\boxed{\text{C}}$ $1, 3, 6, 12.$

$\boxed{\text{D}}$ $1, 3, 7, 15.$

$\boxed{\text{E}}$ None of the above.

17. For $n \geq 0$, what is a formula for A_n, where A_n satisfies the recurrence $A_n = \begin{cases} 1 & n = 0; \\ 2A_{n-1} + 1 & n \geq 1. \end{cases}$

$\boxed{\text{A}}$ $A_n = 1 + 2n$ for $n \geq 0$.

$\boxed{\text{B}}$ $A_n = 1 + n + n^2$ for $n \geq 0$.

$\boxed{\text{C}}$ $A_n = 1 + \frac{1}{3}(5n + n^3)$ for $n \geq 0$.

$\boxed{\text{D}}$ $A_n = 2^{n+1} - 1$ for $n \geq 0$.

$\boxed{\text{E}}$ None of the above.

18. String x is a palindrome, that is $x = x^{\mathrm{R}}$ where x^{R} is the reversal of x. Which statement about x is **false**?

$\boxed{\text{A}}$ x could be the string 1001.

$\boxed{\text{B}}$ The reversal of x must be a palindrome, that is x^{R} is a palindrome.

$\boxed{\text{C}}$ The concatenation of x with itself is a palindrome, that is $x \bullet x$ is a palindrome.

$\boxed{\text{D}}$ x must have even length.

$\boxed{\text{E}}$ The concatenation of x with its reversal is a palindrome, that is $x \bullet x^{\mathrm{R}}$ is a palindrome.

19. Rooted binary trees (RBTs) are recursively defined below. How many RBTs have 3 vertices?

$\boxed{\text{A}}$ 2

$\boxed{\text{B}}$ 3

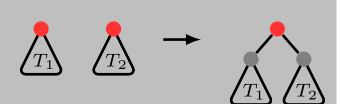
$\boxed{\text{C}}$ 4

$\boxed{\text{D}}$ 5

$\boxed{\text{E}}$ 6

Recursive Definition of RBT

1 The empty tree ε is an RBT.
2 If T_1, T_2 are disjoint RBTs with roots r_1 and r_2, then linking r_1 and r_2 to a *new* root r gives a new RBT with root r.
3 Nothing else is an RBT.



20. A rooted binary tree (RBT) has 8 vertices. How many links (edges) does the RBT have?

$\boxed{\text{A}}$ There is not enough information to determine the number of links.

$\boxed{\text{B}}$ 5

$\boxed{\text{C}}$ 6

$\boxed{\text{D}}$ 7

$\boxed{\text{E}}$ 8

SCRATCH

QUIZ 1: 60 Minutes

Last Name: ____________________

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NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

Total
200

INSTRUCTIONS

1. This is a **closed book** test. No electronics, books, notes, internet, etc.
2. The test will become available in Submitty at 8am on the test date.
3. Your PDF is due in Submitty by 2pm.
4. By submitting the test you attest that:
   – the work is entirely your own.
   – you obeyed the time limits of the exam.
5. Your submission *must* be typed and submitted as a PDF file.
6. The first page should list your twenty answers, something like:

(1)	A
(2)	B
(3)	C
(4)	D
⋮	
(20)	A

7. The *second* page onward *must* show your work for *every* answer, e.g.:

(1)	Because x is even
(2)	Because $\sqrt{2}$ is irrational.
(3)	Number of links is $$1 + 2 + \cdots + 10 = 55$$
⋮	
(20)	Because we proved in class that $\ell = n - 1$

   – Some problems may be "easy", so give a one line justification.
   – Some problems may require a detailed reasoning.

8. **If you don't show correct work, you won't get credit.**
9. Be careful. This is multiple choice.
   – Correct answers get 10 points.
   – Wrong answers or correct answers with no justification get 0.
10. Submit with plenty of time to spare. A late test won't be accepted.
    – We won't accept submissions that are even 1 second late.

1. Jodie asks John to solve $x^2 - a = 0$ and find x as a rational number. Which is true?

$\boxed{\text{A}}$ $\forall a \in \mathbb{N}$: John can find a rational solution x.

$\boxed{\text{B}}$ $\forall a \in \mathbb{N}$: John cannot find a rational solution x.

$\boxed{\text{C}}$ $\forall a \in \mathbb{Z}$: John can find a rational solution x.

$\boxed{\text{D}}$ $\forall a \in \mathbb{Z}$: John cannot find a rational solution x.

$\boxed{\text{E}}$ None of the above.

2. The set $S = \{4, 16, 64, 256, 1024, \ldots\}$. Which of these definitions using a variable could be S?

$\boxed{\text{A}}$ $S = \{n | n = 2^k, \text{ for } k \in \mathbb{N}\}$.

$\boxed{\text{B}}$ $S = \{n | n = 4^{1+k(k-1)/2}, \text{ for } k \in \mathbb{N}\}$.

$\boxed{\text{C}}$ $S = \{n | n = 2 \times 2^k, \text{ for } k \in \mathbb{N}\}$.

$\boxed{\text{D}}$ $S = \{x | x = 2^{2k}, \text{ for } k \in \mathbb{N}\}$.

$\boxed{\text{E}}$ None of the above.

3. $A = \{\text{positive multiples of } 2\}$ and $B = \{\text{positive multiples of } 3\}$. Which element is not in $\overline{A \cap B}$?

$\boxed{\text{A}}$ 4.

$\boxed{\text{B}}$ 8.

$\boxed{\text{C}}$ 12.

$\boxed{\text{D}}$ 16.

$\boxed{\text{E}}$ None of the above.

4. An integer $n \in \mathbb{Z}$ has a square that is divisible by 3, that is 3 divides n^2. Which claim *must be* true?

$\boxed{\text{A}}$ n is odd.

$\boxed{\text{B}}$ n is even.

$\boxed{\text{C}}$ n is positive.

$\boxed{\text{D}}$ n is divisible by 3.

$\boxed{\text{E}}$ None of the above claims must be true.

5. If it rains on a day, then it rains the next day. Today it didn't rain. Which is true?

$\boxed{\text{A}}$ It will rain tomorrow.

$\boxed{\text{B}}$ It will not rain tomorrow.

$\boxed{\text{C}}$ It did rain yesterday.

$\boxed{\text{D}}$ It did not rain yesterday.

$\boxed{\text{E}}$ None of the above.

6. Which method would succeed in *proving* $p \to (q \vee r)$?

$\boxed{\text{A}}$ You assumed p is true and showed q is true.

$\boxed{\text{B}}$ You assumed q is false and showed p is false.

$\boxed{\text{C}}$ You showed that p is true and that q is false.

$\boxed{\text{D}}$ You showed that p is true and that both q and r are false.

$\boxed{\text{E}}$ None of the above.

7. Which method would succeed in *disproving* $p \to (q \vee r)$?

$\boxed{\text{A}}$ You assumed p is true and showed q is true.

$\boxed{\text{B}}$ You assumed q is false and showed p is false.

$\boxed{\text{C}}$ You showed that p is true and that q is false.

$\boxed{\text{D}}$ You showed that p is true and that both q and r are false.

$\boxed{\text{E}}$ None of the above.

8. Determine true or false for the claim $\forall n \in \mathbb{Z} : (n > n+1) \to (n+1 > n+2)$.

$\boxed{\text{A}}$ This is not a valid proposition which is either true or false.

$\boxed{\text{B}}$ True for $n < 0$ and false otherwise.

$\boxed{\text{C}}$ True for $n = 0$ and false otherwise.

$\boxed{\text{D}}$ False.

$\boxed{\text{E}}$ True.

9. What method of proof would you use to *prove* that you cannot choose $a, b \in \mathbb{Z}$ so that $a^2 - 4b = 2$?

$\boxed{\text{A}}$ Direct proof.

$\boxed{\text{B}}$ Contraposition proof.

$\boxed{\text{C}}$ Proof by induction.

$\boxed{\text{D}}$ Proof by contradiction.

$\boxed{\text{E}}$ None of the above.

10. What method would you use to *prove* that $n^3 \leq 2^n$ for *all* $n \geq 10$?

$\boxed{\text{A}}$ Direct proof

$\boxed{\text{B}}$ Contraposition proof.

$\boxed{\text{C}}$ Show that the formula is true for $n = 1$ up to $n = 1000$.

$\boxed{\text{D}}$ Proof by induction.

$\boxed{\text{E}}$ Proof by contradiction.

11. We wish to prove $P(n)$ for all $n \geq 10$. Which method accomplishes this?

$\boxed{\text{A}}$ Prove base case $P(1)$ and prove $P(n) \to P(n+2)$ for all $n \geq 10$.

$\boxed{\text{B}}$ Prove base cases $P(1), P(2)$ and prove $P(n) \to P(n+2)$ for all $n \geq 10$.

$\boxed{\text{C}}$ Prove base case $P(10)$ and prove $P(n) \to P(n+2)$ for all $n \geq 10$.

$\boxed{\text{D}}$ Prove base cases $P(10), P(11)$ and prove $P(n) \to P(n+2)$ for all $n \geq 10$.

$\boxed{\text{E}}$ None of the above methods works.

12. For $x, y \in \mathbb{Z}$, which statement is *not necassarily* a contradiction? (That is, which could be true?)

$\boxed{\text{A}}$ $x + 0 > x + 1$.

$\boxed{\text{B}}$ $x \geq y$ AND $x < y$.

$\boxed{\text{C}}$ $x^2 \geq y^2$ AND $|x| < |y|$.

$\boxed{\text{D}}$ $x^2 + y^2 \leq 1$.

$\boxed{\text{E}}$ They are all contradictions.

13. Consider the predicate $P(n) : n^2 \leq 2^n$. Which claim is true?

$\boxed{\text{A}}$ $P(n)$ is true for at most a finite number of $n \in \mathbb{N}$.

$\boxed{\text{B}}$ $P(n)$ is true for *all* $n \in \mathbb{N}$.

$\boxed{\text{C}}$ $P(n)$ is true for *all* even $n \in N$.

$\boxed{\text{D}}$ $P(n)$ is true for *all* odd $n \in N$.

$\boxed{\text{E}}$ None of the above claims is true.

14. Consider the predicate $P(n) : 8$ divides $n^2 - 1$. Which claim is true?

$\boxed{\text{A}}$ $P(n)$ is true for at most a finite number of $n \in \mathbb{N}$.

$\boxed{\text{B}}$ $P(n)$ is true for *all* $n \in \mathbb{N}$.

$\boxed{\text{C}}$ $P(n)$ is true for *all* even $n \in N$.

$\boxed{\text{D}}$ $P(n)$ is true for *all* odd $n \in N$.

$\boxed{\text{E}}$ None of the above claims is true.

15. Consider the predicate $P(n) : 1^2 + 2^2 + 3^2 + \cdots + n^2 > n^3/3$. Which claim is true?

$\boxed{\text{A}}$ $P(n)$ is true for at most a finite number of $n \in \mathbb{N}$.

$\boxed{\text{B}}$ $P(n)$ is true for *all* $n \in \mathbb{N}$.

$\boxed{\text{C}}$ $P(n)$ is true for *all* even $n \in N$.

$\boxed{\text{D}}$ $P(n)$ is true for *all* odd $n \in N$.

$\boxed{\text{E}}$ None of the above claims is true.

16. You wish to make postage n cents with 5-cent and 6-cent stamps. For which $n \in \mathbb{N}$ can you do it?

$\boxed{A}$ All postages $n \geq 5$ cents.

$\boxed{B}$ All postages $n \geq 10$ cents.

$\boxed{C}$ All postages $n \geq 15$ cents.

$\boxed{D}$ All postages $n \geq 20$ cents.

$\boxed{E}$ None of the above.

17. $A_0 = 0$ and for $n > 0$, $A_n = n^2 + A_{n-2}$. What is A_6?

$\boxed{A}$ It cannot be computed because this recurrence has only one base case.

$\boxed{B}$ $A_6 = 12$.

$\boxed{C}$ $A_6 = 52$.

$\boxed{D}$ $A_6 = 56$.

$\boxed{E}$ None of the above.

18. $f(1) = 1; f(2) = 1$ and for $n > 2$, $f(n) = n + f(n-3)$. For which $n \in \mathbb{N}$ can $f(n)$ be computed?

$\boxed{A}$ *All* $n \in \mathbb{N}$.

$\boxed{B}$ *All* $n \in \mathbb{N}$ which are even.

$\boxed{C}$ *All* $n \in \mathbb{N}$ which are multiples of 3.

$\boxed{D}$ *All* $n \in \mathbb{N}$ which are not multiples of 3.

$\boxed{E}$ None of the above.

19. Rooted binary trees (RBTs) are recursively defined below. How many RBTs have 4 vertices and 2 links?

$\boxed{A}$ 0.

$\boxed{B}$ 5.

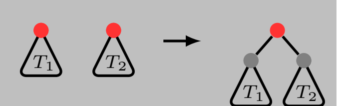
$\boxed{C}$ 14.

$\boxed{D}$ 42.

$\boxed{E}$ 132.

Recursive Definition of RBT

1. The empty tree ε is an RBT.
2. If T_1, T_2 are disjoint RBTs with roots r_1 and r_2, then linking r_1 and r_2 to a *new* root r gives a new RBT with root r.
3. Nothing else is an RBT.



20. T_1 and T_2 are disjoint RBTs. RBT T_1 has 8 vertices and 7 links. RBT T_2 has 4 vertices and 3 links. Using the constructor for RBT, you get a child RBT T. How many vertices and links does T have?

$\boxed{A}$ 12 vertices and 10 links.

$\boxed{B}$ 12 vertices and 11 links.

$\boxed{C}$ 13 vertices and 11 links.

$\boxed{D}$ 13 vertices and 12 links.

$\boxed{E}$ None of the above, or we can't say for sure.

SCRATCH

QUIZ 1: 60 Minutes

Last Name: ______________________

First Name: ______________________

RIN: ______________________

Section: ______________________

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show **CORRECT** work to get credit.

When in doubt, TINKER.

Total
200

1. Which set below is the set $S = \{2k \mid k \in \mathbb{N}\}$?

$\boxed{\text{A}}$ All even numbers.

$\boxed{\text{B}}$ All odd numbers.

$\boxed{\text{C}}$ All non-negative even numbers.

$\boxed{\text{D}}$ All non-negative odd numbers.

$\boxed{\text{E}}$ None of the above.

2. Define sets $A = \{2k \mid k \in \mathbb{Z}\}$, $B = \{9k \mid k \in \mathbb{Z}\}$ and $C = \{6k \mid k \in \mathbb{Z}\}$. Which is true?

$\boxed{\text{A}}$ $A \cap B = C$.

$\boxed{\text{B}}$ $A \cap B \subseteq C$.

$\boxed{\text{C}}$ $A \cap B = \overline{C}$.

$\boxed{\text{D}}$ $A \cap B \subseteq \overline{C}$.

$\boxed{\text{E}}$ None of the above.

3. How many rows are in the truth table of $p \rightarrow (p \vee q)$?

$\boxed{\text{A}}$ 2.

$\boxed{\text{B}}$ 4.

$\boxed{\text{C}}$ 6.

$\boxed{\text{D}}$ 8.

$\boxed{\text{E}}$ None of the above.

4. True or false, $p \rightarrow (p \vee q)$?

$\boxed{\text{A}}$ Can be true or false, depending on p.

$\boxed{\text{B}}$ Can be true or false, depending on q.

$\boxed{\text{C}}$ Always true.

$\boxed{\text{D}}$ Always false.

$\boxed{\text{E}}$ None of the above.

5. If you majored CS then you took FOCS. Joe took FOCS and Barb majored CS. What else do we know?

$\boxed{\text{A}}$ Joe majored CS. We don't know anything more about Barb.

$\boxed{\text{B}}$ We don't know anything more about Joe. Barb took FOCS.

$\boxed{\text{C}}$ Joe majored CS. And, Barb took FOCS.

$\boxed{\text{D}}$ Joe did not major CS. And, Barb took FOCS.

$\boxed{\text{E}}$ None of the above.

6. What is the negation of the claim $\forall m, n \in \mathbb{N} : 3m + 6n \neq 10$?

A $\forall m, n \in \mathbb{N} : 3m + 6n = 10$.

B $\forall m, n \in \mathbb{N} : 3m + 6n \neq 10$.

C $\exists m, n \in \mathbb{N} : 3m + 6n = 10$.

D $\exists m, n \in \mathbb{N} : 3m + 6n \neq 10$.

E None of the above.

7. Which proof-method is acceptable to prove the claim p?

A Assume p is true and derive something known to be true, for example $0 = 0$.

B Assume $\neg p$ is true and derive something known to be true, for example $0 = 0$.

C Assume p is true and derive something known to be false, for example $1 > 2$.

D Assume $\neg p$ is true and derive something known to be false, for example $1 > 2$.

E None of the above.

8. Consider the claim $\exists m, n \in \mathbb{Z} : 9m + 21n = 7$. Is the claim true or false?

A True.

B False.

C It depends on m.

D It depends on n.

E None of the above.

9. How do you *disprove* the claim $\forall n \in \mathbb{N} : \neg P(n) \to Q(n)$.

A Show that for all $n \in \mathbb{N}$, $P(n)$ is true and $Q(n)$ is false.

B Show that for all $n \in \mathbb{N}$, $P(n)$ is false and $Q(n)$ is false.

C Show that for some $n \in \mathbb{N}$, $P(n)$ is true and $Q(n)$ is false.

D Show that for some $n \in \mathbb{N}$, $P(n)$ is false and $Q(n)$ is false.

E None of the above.

10. What is the first step in a proof by contradiction of the claim $\forall m, n \in \mathbb{N} : 3m + 6n \neq 10$.

A Define the predicate $P(m, n) : 3m + 6n \neq 10$ and prove the base case $P(1, 1)$.

B Assume $3m + 6n = 10$ for all $m, n \in \mathbb{N}$.

C Assume $3m + 6n \neq 10$ for some $m, n \in \mathbb{N}$.

D Assume $3m + 6n = 10$ for some $m, n \in \mathbb{N}$.

E None of the above.

11. You decided to *prove* the claim $n^2 \leq 2^n$ for all $n \geq 4$. Which method of proof would you use?

$\boxed{\text{A}}$ Find a single value $n_* \in \mathbb{N}$ for which $n_*^2 > 2^{n_*}$.

$\boxed{\text{B}}$ Show that the formula $n^2 \leq 2^n$ is true for $n = 1$ up to $n = 1000$.

$\boxed{\text{C}}$ Proof by induction.

$\boxed{\text{D}}$ Contraposition proof.

$\boxed{\text{E}}$ Direct proof.

12. You decided to *disprove* the claim $n^2 \leq 2^n$ for all $n \geq 1$. Which method of proof would you use?

$\boxed{\text{A}}$ Find a single value $n_* \in \mathbb{N}$ for which $n_*^2 > 2^{n_*}$.

$\boxed{\text{B}}$ Show that the formula $n^2 \leq 2^n$ is true for $n = 1$ up to $n = 1000$.

$\boxed{\text{C}}$ Proof by induction.

$\boxed{\text{D}}$ Contraposition proof.

$\boxed{\text{E}}$ Direct proof.

13. How do you prove, by induction, the claim "5 divides $11^n - 6$" for all $n \geq 5$?

$\boxed{\text{A}}$ Show 5 divides $11^5 - 6$.

$\boxed{\text{B}}$ Show 5 divides $11^5 - 6, 11^6 - 6, 11^7 - 6$ all the way up to $11^{1{,}000{,}000} - 6$.

$\boxed{\text{C}}$ Show, for $n \geq 5$, if 5 divides $11^n - 6$ then 5 divides $11^{n+1} - 6$.

$\boxed{\text{D}}$ Show 5 divides $11^5 - 6$. And, show, for $n \geq 5$, if 5 divides $11^n - 6$ then 5 divides $11^{n+1} - 6$.

$\boxed{\text{E}}$ None of the above.

14. You wish to prove $n^4 \leq 2^n$ for $n \geq 16$. You showed that $n^4 \leq 2^n \rightarrow (n+3)^4 \leq 2^{n+3}$ for $n \geq 16$. What base cases do you need to prove to complete the proof?

$\boxed{\text{A}}$ $n = 1$.

$\boxed{\text{B}}$ $n = 16$.

$\boxed{\text{C}}$ $n = 1$ and $n = 2$.

$\boxed{\text{D}}$ $n = 16$ and $n = 17$.

$\boxed{\text{E}}$ None of the above.

15. Define the predicate $P(n) : (2n-1)^2 + 4$ is prime. For which n is $P(n)$ true?

$\boxed{\text{A}}$ $n \geq 1$.

$\boxed{\text{B}}$ $n \geq 2$.

$\boxed{\text{C}}$ $n \geq 3$.

$\boxed{\text{D}}$ $n \geq 4$.

$\boxed{\text{E}}$ None of the above.

16. Define the sum $S(n) = \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{(n-1) \times n}$ for $n \geq 2$. What is $S(100)$?

$\boxed{\text{A}}$ 0.1.

$\boxed{\text{B}}$ 0.01.

$\boxed{\text{C}}$ 0.9.

$\boxed{\text{D}}$ 0.99.

$\boxed{\text{E}}$ None of the above.

17. $f(1) = 1$, $f(2) = 2$, and $f(n) = f(n-2) + 2$ for $n > 2$. What is $f(100)$?

$\boxed{\text{A}}$ It cannot be computed because the recursion does not have enough base cases.

$\boxed{\text{B}}$ 50.

$\boxed{\text{C}}$ 100.

$\boxed{\text{D}}$ 200.

$\boxed{\text{E}}$ None of the above.

18. Define $\mathcal{A}$ recursively: (i) $1 \in \mathcal{A}$ (ii) $x \in \mathcal{A} \rightarrow x + 4 \in \mathcal{A}$ (iii) Nothing else is in $\mathcal{A}$. Which is true?

$\boxed{\text{A}}$ Every number in $\mathcal{A}$ is even.

$\boxed{\text{B}}$ Every even number is in $\mathcal{A}$.

$\boxed{\text{C}}$ Every number in $\mathcal{A}$ is odd.

$\boxed{\text{D}}$ Every odd number is in $\mathcal{A}$.

$\boxed{\text{E}}$ None of the above.

19. A rooted binary tree (RBT) has 8 vertices. How many links does it have?

$\boxed{\text{A}}$ 6.

$\boxed{\text{B}}$ 7.

$\boxed{\text{C}}$ 8.

$\boxed{\text{D}}$ 9.

$\boxed{\text{E}}$ None of the above.

20. There are 5 distinct rooted binary trees (RBT) with 3 vertices. How many have 4 vertices?

$\boxed{\text{A}}$ 12.

$\boxed{\text{B}}$ 13.

$\boxed{\text{C}}$ 14.

$\boxed{\text{D}}$ 15.

$\boxed{\text{E}}$ None of the above.

SCRATCH