1 30.10 THE CENTRAL LIMIT THEOREM

and its mean and variance are given by
$E[X] = a + b/2$, $V[X] = (b - a)^2/12$

30.10 The central limit theorem

In subsection 30.9.1 we discussed approximating the binomial and Poisson distributions by the Gaussian distribution when the number of trials is large. We now discuss why the Gaussian distribution is so common and therefore so important. The central limit theory may be stated as follows.

Central limit theorem. Suppose that Xi,i = 1,2,...,n, are independent random variables, each of which is described by a probability density function Fi(x) (these may all be different)with a mean u and variance. The random variable $Z = (sum * Xi)/n$, i.e. the 'mean' of the Xi, has the following properties:
(i) its expectations value is given by E[Z] = $(sum * Ui)$/n;
(ii) its variance is given by $V[Z] = (sum * sigma^2)/n^2$;
(iii) as n -¿ the probability function of Z tends to a Gaussian with corresponding mean and variance.

We note that for the theorem to hold, the probability density functions f(x) must posses formal means and variances. Thus, for example, if any of the X were described by a Cauchy distribution then the theorem would not apply. Properties (i) and (ii) of the theorem are easily proved, as follows. Firstly

a result which does not require that the X are independent random variables. If u = u for all i then this becomes
E[Z] = nu/n = u

Secondly, if the X are independent, it follows from an obvious extension of(30.68) that

Let us now consider property (iii), which is the reason for the ubiquity of the Gaussian distribution and is most easily proved by considering the moment generating function Mz(t) of Z. From (30.90), this MGF is given by
Mz(t) =
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