1. The Problem

The motivation of this work lies in the real-life version /show of Squid Game aired on Netflix, named "Squid Game: The Challenge". In this problem, there are 3 players in the semi final, so only the two of them can qualify to the final. The host brings three boxes with geometrical-shaped boxes inside. If a shape is pressed, a colored light appears and an associated action is taken. There are 3 shapes:

- Circle (C)
- Triangle (T)
- Rectangle (R)

When pressed, each shape is colored with a color that means something special. The green color means qualification to the final with the ability to choose the second finalist, the red one means the immediate elimination of the player and the gray color means that you have to wait for the next player - no tangible effect with the gray color.

Of course, each player considers their strategy and after some time to think, they start selecting shapes. Without violating the generality law, we can assign a shape to an action/color. From now on, let's map the winning shape (that will become green if pressed) to the Circle (**C**), the eliminating one (that will become red if pressed) to the Triangle (**T**) and the "idle" shape (that will become gray if pressed) to the Rectangle (**R**). So, we have done this custom mapping:

- C = 🟢 → **immediate qualification** to the final with the perk of selecting the second finalist
- T = 🔺 → **immediate elimination** of the player
- R = 🟫 → **waiting** for the next player's choice

The ultimate goal of each player is to reach the final. In the next section, we will clear out the game conditions in order to present the dynamics between the participating players.

2. Game Conditions

In this game, let's suppose that there is a lone player playing only for himself, without having any allies. Let's call this player "**M**". If "M" picks "C", he will immediately proceed to the final and inevitably select the second finalist to qualify. Let's also suppose that the remaining two players "**A**" and "**B**" are friends and allies in the whole game. This means that if one of them picks "C", they know that they will proceed to the final together. There is no chance of "A" or "B" selecting "M" as the second finalist in case they pick "C". At this point, it must be highlighted that picking a shape is a completely random action, since no player knows the actual meaning behind the shapes. So, to sum up:

- If "M" picks "C", he will qualify and select any of "A" and "B" with a 50%-50% chance.
- If "A" picks "C", he will qualify, selecting "B" as the second finalist, since "A" and "B" are allies
- If "B" picks "C", he will qualify, selecting "A" as the second finalist, since "B" and "A" are allies

At this point, it is clear that there is a huge advantage for the two cooperating players "A" and "B" over the lone one "M", but the question that arises is which is the ideal scenario (scenario = order of playing) for every one of these three players. A disclaimer here: we do not try to guess the order of playing (like an equilibrium state), we just try to identify the best-case strategy for each player, the strategy that guarantees the maximization of a player's probability to proceed to the final. In reality, each player cares about himself, but the cooperation of "A" and "B" shapes an environment in their favor, because they cooperate and they know that one of them picking "C" guarantees the qualification of the other ally. After these clarifications, the final question is: ***"What is the ideal scenario (order of playing) for every player in order to maximize his chances of proceeding to the final?"***

3. Game Analysis

Each player wishes to select the green-colored (if pressed) shape. In the current approach, we arbitrarily mapped this shape to "C" (arbitrary mapping). In the actual game, the players would pick a shape with a hidden color if pressed, but here we assume a mapping between shape and color and presume that the players pick a shape randomly without knowing its effect. Regarding the game process, an actual pre-agreement is not necessary. The player who feels that needs to start first, initially picks his shape. If the game does not end with this first choice, then the second player picks his color. When the second player chooses his shape (consider it as "second round"), the game will have finished, because in reality every game ends by the end of the "second round", so there is no need for a third player to pick his shape.

Why does this happen? In fact, there are four ways for such a game to end:

1. The **first** player picks **"C"**, so the game ends in the first round, since the winning player proceeds to the final alongside a finalist of his choice.
2. The **first** player picks **"T"**, so he is immediately eliminated, leaving the other 2 to proceed to the final. Again, the game ends in the first round.
3. The **first** player picks **"R"**, so nothing happens, until the second player chooses one of the remaining shapes. If the **second** player picks **"C"**, again two players qualify to the final and the game ends in the second round.
4. The **first** player picks **"R"**, so nothing happens, until the second player chooses one of the remaining shapes. If the **second** player picks **"T"**, the other two players qualify to the final and the game ends in the second round as well.

4. Scenarios

At this point, we can identify the 6 possible scenarios that define the order of the participating players. The number of scenarios is equal to the permutations of "M", "A" and "B", so it is computed as 3! = 6. Regarding the "mathematical" notation of the scenarios, we will use these 3 letters in the order the players pick shapes in this game.

1. **S1 = MAB** → First player = "M", second = "A" and third = "B"
2. **S2 = MBA** → First player = "M", second = "B" and third = "A"

3. **S3 = AMB** ⟶ First player = “A”, second = “M” and third = “B”
4. **S4 = ABM** ⟶ First player = “A”, second = “B” and third = “M”
5. **S5 = BMA** ⟶ First player = “B”, second = “M” and third = “A”
6. **S6 = BAM** ⟶ First player = “B”, second = “A” and third = “M”

In reality, the last player will not actually “play”, since the fate of the game will have been decided by the end of the first two rounds. Having identified the 6 possible scenarios (order of choosing shapes), we can conduct a probability analysis in order to reveal the best strategy for each player. In the next section, we will examine the winning events of players “M” and “A” and “B”. Obviously, the results of player “A” can be also applied for the case of player “B”, but in the opposite way.

5. Scenarios Analysis

For each scenario, the game can be finished through 4 ways as mentioned above. We will call them events. Each event is in favor of one or two players and they can be enumerated as shown below:

- Event 1: The game ends in the *first* round, because the **first** player picked **“C”**.
- Event 2: The game ends in the *first* round, because the **first** player picked **“T”**.
- Event 3: The game ends in the *second* round, because the **first** player picked **“R”** and the **second** player picked **“C”**.
- Event 4: The game ends in the *second* round, because the **first** player picked **“R”** and the **second** player picked **“T”**.

Regarding the corresponding probabilities of the identified events of each scenario:

$$Pr(ev_1) = 1/3$$

$$Pr(ev_2) = 1/3$$

$$Pr(ev_3) = 1/3 * 1/2 = 1/6$$

$$Pr(ev_4) = 1/3 * 1/2 = 1/6$$

$$Pr(P_i) = Pr(ev_1) + Pr(ev_2) + Pr(ev_3) + Pr(ev_4)$$

5.1 Scenario 1 - MAB

The identified events here are:

- Event 1: Player “M” picks “C” ⟶ Player “M” qualifies and chooses the second finalist between “A” and “B” in a random (**50% - 50%**) way.
- Event 2: Player “M” picks “T” ⟶ Player “M” is eliminated, so the remaining players, “**A**” and “**B**”, are the finalists.

- Event 3: Player “M” picks “R” and player “A” picks “C” → Player “**A**” proceeds alongside his ally “**B**”.
- Event 4: Player “M” picks “R” and player “A” picks “T” → Player “A” is eliminated, so the remaining players, “**M**” and “**B**”, qualify.

Now, we can calculate the probability of “M”, “A” and “B” proceeding to the final according to the probabilities of these events. For each player, we will only choose the events that guarantee them a qualification to the final of the competition.

For player M:

$$Pr(M_1) = Pr(ev_1) + Pr(ev_4) = 1/3 + 1/6 = 1/2$$

For player A:

$$Pr(A_1) = 1/2\, Pr(ev_1) + Pr(ev_2) + Pr(ev_3) = 1/2 * 1/3 + 1/3 + 1/6 = 2/3$$

For player B:

$$Pr(B_1) = 1/2\, Pr(ev_1) + Pr(ev_2) + Pr(ev_3) + Pr(ev_4) = 1/2 * 1/3 + 1/3 + 1/6 + 1/6 = 5/6$$

5.2 Scenario 2 - MBA

The identified events here are:

- Event 1: Player “M” picks “C” → Player “**M**” qualifies and chooses the second finalist between “A” and “B” in a random (**50%** - **50%**) way.
- Event 2: Player “M” picks “T” → Player “M” is eliminated, so the remaining players, “**A**” and “**B**”, are the finalists.
- Event 3: Player “M” picks “R” and player “B” picks “C” → Player “**B**” proceeds alongside his ally “**A**”.
- Event 4: Player “M” picks “R” and player “B” picks “T” → Player “B” is eliminated, so the remaining players, “**M**” and “**A**”, qualify.

Now, we can calculate the probability of “M”, “B” and “A” proceeding to the final according to the probabilities of these events. For each player, we will only choose the events that guarantee them a qualification to the final of the competition.

For player M:

$$Pr(M_2) = Pr(ev_1) + Pr(ev_4) = 1/3 + 1/6 = 1/2$$

For player B:

$$Pr(B_2) = 1/2\, Pr(ev_1) + Pr(ev_2) + Pr(ev_3) = 1/2 * 1/3 + 1/3 + 1/6 = 2/3$$

For player A:

$Pr(A_2) = 1/2\, Pr(ev_1) + Pr(ev_2) + Pr(ev_3) + Pr(ev_4) = 1/2 * 1/3 + 1/3 + 1/6 + 1/6 = 5/6$

5.3 Scenario 3 - AMB

The identified events here are:

- Event 1: Player “A” picks “C” ⟶ Player “**A**” proceeds alongside his ally “**B**”.
- Event 2: Player “A” picks “T” ⟶ Player “A” is eliminated, so the remaining players, “**M**” and “**B**”, are the finalists.
- Event 3: Player “A” picks “R” and player “M” picks “C” ⟶ Player “**M**” qualifies and chooses the second finalist between “A” and “B” in a random (**50%** - **50%**) way.
- Event 4: Player “A” picks “R” and player “M” picks “T” ⟶ Player “M” is eliminated, so the remaining players, “**A**” and “**B**”, qualify.

Now, we can calculate the probability of “A”, “M” and “B” proceeding to the final according to the probabilities of these events. For each player, we will only choose the events that guarantee them a qualification to the final of the competition.

For player A:

$Pr(A_3) = Pr(ev_1) + 1/2\, Pr(ev_3) + Pr(ev_4) = 1/3 + 1/2 * 1/6 + 1/6 = 7/12$

For player M:

$Pr(M_3) = Pr(ev_2) + Pr(ev_3) = 1/3 + 1/6 = 1/2$

For player B:

$Pr(B_3) = Pr(ev_1) + Pr(ev_2) + 1/2\, Pr(ev_3) + Pr(ev_4) = 1/3 + 1/3 + 1/2 * 1/6 + 1/6 = 11/12$

5.4 Scenario 4 - ABM

The identified events here are:

- Event 1: Player “A” picks “C” ⟶ Player “**A**” proceeds alongside his ally “**B**”.
- Event 2: Player “A” picks “T” ⟶ Player “A” is eliminated, so the remaining players, “**M**” and “**B**”, are the finalists.
- Event 3: Player “A” picks “R” and player “B” picks “C” ⟶ Player “**B**” qualifies and proceeds to the final alongside his ally “**A**”.
- Event 4: Player “A” picks “R” and player “B” picks “T” ⟶ Player “B” is eliminated, so the remaining players, “**A**” and “**M**”, qualify.

Now, we can calculate the probability of “A”, “B” and “M” proceeding to the final according to the probabilities of these events. For each player, we will only choose the events that guarantee them a qualification to the final of the competition.

For player A:

$Pr(A_4) = Pr(ev_1) + Pr(ev_3) + Pr(ev_4) = 1/3 + 1/6 + 1/6 = 2/3$

For player B:

$Pr(B_4) = Pr(ev_1) + Pr(ev_2) + Pr(ev_3) = 1/3 + 1/3 + 1/6 = 5/6$

For player M:

$Pr(M_4) = Pr(ev_2) + Pr(ev_4) = 1/3 + 1/6 = 1/2$

5.5 Scenario 5 - BMA

The identified events here are:

- Event 1: Player "B" picks "C" → Player "**B**" proceeds alongside his ally "**A**".
- Event 2: Player "B" picks "T" → Player "B" is eliminated, so the remaining players, "**M**" and "**A**", are the finalists.
- Event 3: Player "B" picks "R" and player "M" picks "C" → "**M**" qualifies and chooses the second finalist between "A" and "B" in a random (**50%** - **50%**) way.
- Event 4: Player "B" picks "R" and player "M" picks "T" → Player "M" is eliminated, so the remaining players, "**B**" and "**A**", qualify.

Now, we can calculate the probability of "B", "M" and "A" proceeding to the final according to the probabilities of these events. For each player, we will only choose the events that guarantee them a qualification to the final of the competition.

For player B:

$Pr(B_5) = Pr(ev_1) + 1/2 * Pr(ev_3) + Pr(ev_4) = 1/3 + 1/2 * 1/6 + 1/6 = 7/12$

For player M:

$Pr(M_5) = Pr(ev_2) + Pr(ev_3) = 1/3 + 1/6 = 1/2$

For player A:

$Pr(A_5) = Pr(ev_1) + Pr(ev_2) + 1/2 * Pr(ev_3) + Pr(ev_4) = 1/3 + 1/3 + 1/2 * 1/6 + 1/6 = 11/12$

5.6 Scenario 6 - BAM

The identified events here are:

- Event 1: Player "B" picks "C" → Player "**B**" proceeds alongside his ally "**A**".
- Event 2: Player "B" picks "T" → Player "B" is eliminated, so the remaining players, "**A**" and "**M**", are the finalists.
- Event 3: Player "B" picks "R" and player "A" picks "C" → "**A**" qualifies and proceeds alongside his ally "**B**".

- Event 4: Player "B" picks "R" and player "A" picks "T" → Player "A" is eliminated, so the remaining players, "**B**" and "**M**", qualify.

Now, we can calculate the probability of "B", "A" and "M" proceeding to the final according to the probabilities of these events. For each player, we will only choose the events that guarantee them a qualification to the final of the competition.

For player B:

$$Pr(B_6) = Pr(ev_1) + Pr(ev_3) + Pr(ev_4) = 1/3 + 1/6 + 1/6 = 2/3$$

For player A:

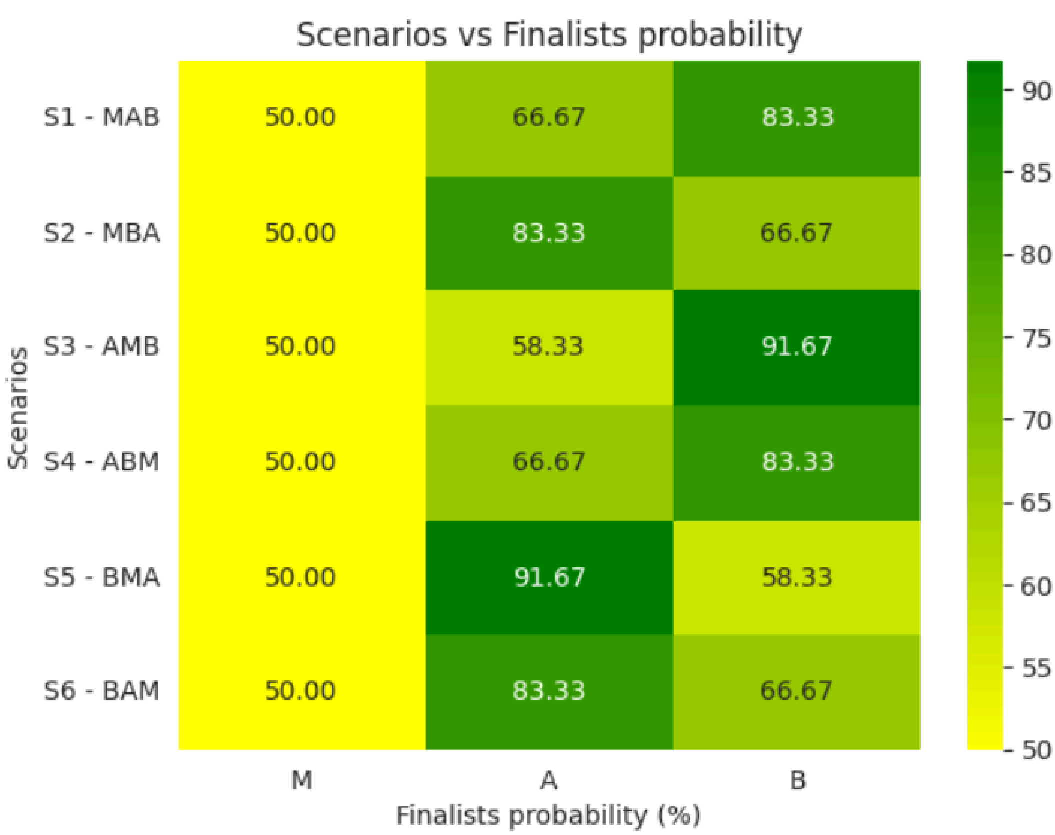
$$Pr(A_6) = Pr(ev_1) + Pr(ev_2) + Pr(ev_3) = 1/3 + 1/3 + 1/6 = 5/6$$

For player M:

$$Pr(M_6) = Pr(ev_2) + Pr(ev_4) = 1/3 + 1/6 = 1/2$$

6. Summary

To summarize, the results of the probability analysis can be visualized through a heatmap:



According to the heatmap:

- Player “**M**” has the same probability (**50.00%**) of proceeding to the final regardless of the scenario and the order of players.
- For player “**A**” the most fruitful strategy is scenario 5 - **BMA** that gives him a **91.67%** chance of qualifying. Generally speaking, the most efficient results for “A” are when he plays as the last player. On the contrary, playing as the first one in the game order, especially in scenario 3 - AMB, is the least convenient for “A”.
- What is true for “A” is also true for player “B”, but in the opposite way. So, for “**B**” the most fruitful strategy is scenario 3 - **AMB** that gives him a **91.67%** chance of qualifying. Note that scenario 3 is the least convenient for player “A”. The worst strategy for “B” is also playing first, especially in scenario 5, which is the best-case scenario for player “A”.

To conclude, the lone player “**M**” has the **same probability of proceeding to the fina**l, while the allies “**A**” and “**B**” are **benefitted when playing last**. This happens because when their ally plays before them, they can win if their ally chooses "C" (they qualify together) or "T" (the ally is eliminated/sacrificed, but they proceed themselves) or even "R", where they wait for the second player's move. Of course, their ideal scenario is to play last, while their ally plays first and “sacrifices” himself. They simply have more options to win - in some cases, all four game-ending events are in their favor - while the lone player ends up winning in two of the four game-ending events in every possible scenario and that is why the probability of qualification always culminates in the same sum that gives the 50% chance ($1/3 + 1/6 = 1/2$).