Two-way Analysis of Variance (ANOVA)

Between-subjects designs

Konstantinos I. Bougioukas, MSc, PhD

School of Psychology
Faculty of Philosophy
Aristotle University of Thessaloniki

Έλεγχος υποθέσεων-Βήματα

1. Καθορίζεται η **μηδενική υπόθεση Η0 (=)** και **εναλλακτική υπόθεση Η1 (≠) .**

2. Ορίζεται το **επίπεδο σημαντικότητας** α (συνήθως α=0.05).

3. Επιλέγεται μια κατάλληλη **στατιστική δοκιμασία** και υπολογίζεται η τιμή του στατιστικού με βάση τα δεδομένα του δείγματος.

4. Σύγκριση της **πιθανότητας p** να έχουμε την συγκεκριμένη τιμή του στατιστικού (ή κάτι πιο ακραίο) θεωρώντας ότι ισχύει η Ho, με το **επίπεδο σημαντικότητας α** (0.05). Στατιστικά σημαντικό αποτέλεσμα (p <0.05).

5. **Ερμηνεία** αποτελεσμάτων.

Two-way between-subjects ANOVA

ANOVA: Y = numeric variable → **Dependent**

X_1 = categorical (factor A) variable → **Independent**

X_2 = categorical (factor B) variable → **Independent**

EXAMPLE

Assume that researchers are interested in exploring the role of **analogical thinking** in the **problem-solving skills** of **children and adolescents.**

Y: Number of mistakes made while attempting to solve the problems

X_1: intervention control group, exp. group 1 (exposure to similar examples), and exp. group 2 (exposure + instructions)

X_2: school age (primary school, secondary school)

Two-way between-subjects ANOVA

Null and alternative hypotheses

Main effect: intervention

- H_0: There is no significant difference in number of mistakes between intervention groups ($\mu_{control} = \mu_{exper1} = \mu_{exper2}$)
- H_1: At least one intervention has a significantly different mean number of mistakes.

Main effect: school age

- H_0: There is no significant difference in number of mistakes between primary and secondary school. ($\mu_{primary} = \mu_{secondary}$)
- H_1: There is a significant difference in number of mistakes between primary and secondary school ($\mu_{primary} \neq \mu_{secondary}$).

Interaction effect between intervention and school age

- H_0: There is no interaction effect between intervention and school age on number of mistakes (i.e., the effect of intervention on number of mistakes does not depend on school age).
- H_1: There is a significant interaction effect between intervention and school age on number of mistakes (i.e., the effect of intervention on number of mistakes varies depending on school age).

Assumptions of two-way between-subjects ANOVA

- **Between-subjects design**

- **Normality:** The number of mistakes should be approximately normally distributed within each group (i.e., for each combination of intervention and school age).

  ➡ Check the residuals (Shapiro-Wilk test)

- **Homogeneity of Variance (Homoscedasticity):** The variance of number of mistakes should be approximately equal across all groups.

  ➡ Check for homogeneity of variances (Levene's test)

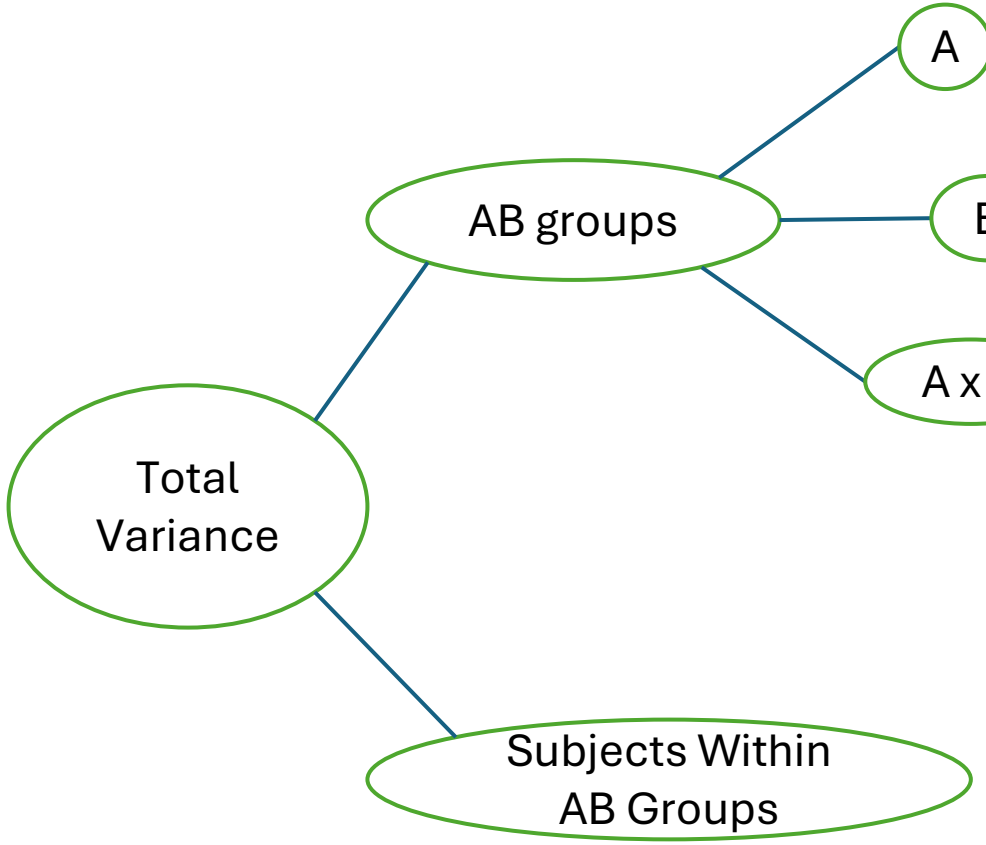
DATA (3x2)

How it works

		Factor A			MEAN
		control	exp1	exp2	
Factor B					
Secondary		38	20	10	
		47	20	8	
		42	22	12	
		37	24	5	
		33	32	6	
		31	19	0	
		33	36	0	
	Mean	37.3	24.7	5.9	22.6
Primary		34	29	37	
		39	47	45	
		37	28	48	
		26	43	29	
		46	41	34	
		40	44	32	
		27	31	51	
	Mean	35.6	37.6	39.4	37.5
	MEAN	36.4	31.1	22.6	30.1

Two-way between-subjects ANOVA

Partitioning the total variance into its sources



A — Main effect of factor A — $df_A=a-1=3-1=2$

B — Main effect of factor B — $df_B=b-1=2-1=1$

A x B — Interaction of two factors (moderation effect) — $df_{AxB}=df_A \cdot df_B = 2$

Subjects Within AB Groups — Residuals or Error — $df_{error}=a \cdot b \cdot (n-1) = 3 \cdot 2 \cdot (7-1) = 36$

Interaction

An interaction occurs when the effect of one factor changes according to the level of the other factor.

- E.g., the number of mistakes in the exposure-to-examples group is lower among secondary school students than among primary school students.

ANOVA TABLE (two-factors with interaction)

ANOVA - mistakes

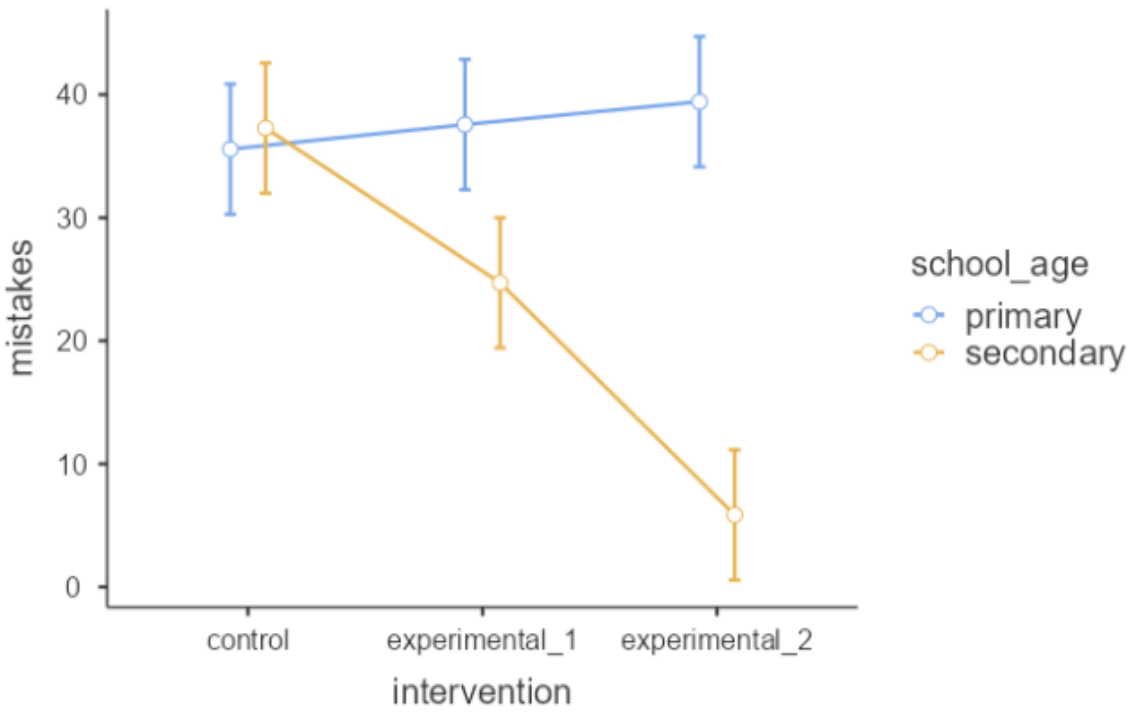
	Sum of Squares	df	Mean Square	F	p	η^2
Main effect A → intervention	1354.429	2	677.214	14.217	< .001	0.178
Main effect B → school_age	2332.595	1	2332.595	48.968	< .001	0.307
Interaction → intervention ✻ school_age	2200.905	2	1100.452	23.102	< .001	0.289
Residuals	1714.857	36	47.635			

A key principle in interpreting and reporting factorial analysis results is that **interactions** take precedence over main effects.

→ Conduct a **simple effects** analysis

Two-way between-subjects ANOVA

Simple effects analysis



ANOVA for Simple Effects of intervention

Moderator					
school_age	F	Num df	Den df	p	η^2p
primary	0.547	2	36	0.584	0.029
secondary	36.772	2	36	< .001	0.671

ANOVA for Simple Effects of school_age

Moderator					
intervention	F	Num df	Den df	p	η^2p
control	0.216	1	36	0.645	0.006
experimental_1	12.146	1	36	0.001	0.252
experimental_2	82.810	1	36	< .001	0.697

Post hoc Tests

- Bonferroni Procedure
- Tukey's Test
- Scheffe
- Holm