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Exercises Computational Physics

6 Extrem value statistics

Assumption: Variable x has probability density $p(x)$ and distribution function $P(x) = \int_{-\infty}^{x} dx' p(x')$. Let z the maximum of N independent values $x_1, \ldots, x_N$:

$$z = \max\{x_1, \ldots, x_N\}\,. \tag{1}$$

The probability density for z is

$$q(z) = Np(z)P(z)^{N-1} \tag{2}$$

(the distribution function for z is $Q(z) = G(z)^N$, then $q(z) = dQ(z)/dz$.)

Example: exponential distribution $p(x) = \lambda e^{-\lambda x}$ and $P(x) = 1 - e^{-\lambda x}$. Let $z_0 := \log(N)/\lambda$, hence $e^{\lambda z_0} = N \Rightarrow$

$$\begin{aligned} q(z) &= Np(z)P(z)^{N-1} = N\lambda e^{-\lambda z}\left(1 - e^{-\lambda z}\right)^{N-1} \\ &= N\lambda e^{-\lambda z}\left(1 - \frac{N}{N}e^{-\lambda z}\right)^{N-1} = \lambda e^{-\lambda(z-z_0)}\left(1 - \frac{e^{-\lambda(z-z_0)}}{N}\right)^{N-1} \\ &\to \lambda e^{-\lambda(z-z_0)} \exp\left(-e^{-\lambda(z-z_0)}\right) =: g(z) \end{aligned} \tag{3}$$

(because $(1 - k/N)^N \to \exp(-k)$ for $N \to \infty$).
The density $g(z)$, and the corresponding distribution function $G(z) = \exp\left(-e^{-\lambda(z-z_0)}\right)$ describe the so-called Gumbel distribution.

1. Download the program `exponential.c` from StudIp.
2. Change the rogramm such that in each iteration instead of one single now N exponentially distributed numbers are drawn and from these N numbers the maximum is obtained. The histogram of these maxima z shall be redirected from standard output into a file `max_<N>.out` (replace `<N>` by the current number N). (5 P)
3. Measure the (normalized) histograms for $\lambda = 1$ and several values of N (z.B. 1, 3, 5, 10,100) and good statistics ($> 10^5$). (1 P)
4. Plot (with `gnuplot`) the data together with the density function of the Gumbel distribution with parameters λ and z_0. For which value of N is the agreement sufficient? (1 P)

5. Perform similar computational experiments, when you replace the exponential distribution by the uniform distribution in $[0, \lambda)$ (with e.g. $\lambda = 1$). Does the Gumbel function macht the data? (2 P)

6. What is the exact density according the above formula? (1 P)