Prof. Dr. A.K. Hartmann WiSe 2023/24, 5.04.4521
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Exercises Computational Physics

5 Random sampling in trees

Download `tree_fragment.c` (or maybe `tree_fragment3.c` is provided) from StudIP.
Write a complete function (no frame is provided now!) which estimates the number of leaves of a tree by a random algorithm (Knuth's algorithm): (5 P)

algorithm knuth(tree)
begin
  $b := 1$
  while tree has successor **do**
    if tree has one successorr **then**
      tree := tree $->$ succesort (`left` or `right`)
    else
    begin
      $b := 2 * b$
      tree := left/right successor with same probability
    end
  return (b)
end

One can show (easily by recursion): the expectation value $\langle b \rangle$ is equal to the number of leaves.

Test some (in your program implemented) trees, how fast , i.e. as a function of the number t of measurements, the mean $\langle b \rangle$ converges to the correct number of leaves. Suitable tree sizes are in the range $N = 30..40$ nodes. Plot the curves $\langle b \rangle(t)$, e.g.. with `gnuplot`.

- Random tree: Generate N times a random number, generate a node with this number and insert the node into the sorted tree. (2 P)
- A tree for which the algorithm "converges" quickly. (1 P)
- A tree for which the algorithm converges very slowly. (2 P)