Machine Learning I, Exercise 2, Group3

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1. $P(x|\lambda) = \exp(-\lambda)\frac{\lambda^x}{x!}$, $x \in \mathbb{N}$

What is max. likelihood solution if N datapoints $x_1, \cdots x_N$ are given ($x_n \in \mathbb{N}$)?

■ Data likelihood function:

$$L(\theta) = P(x^{(1)}, \cdots x^{(n)}|\theta) = \prod_{n=1}^{N} P(x^{(n)}|\theta)$$

■ Log likelihood function:

$$\mathcal{L}(x^{(1)}, \cdots x^{(N)}; \theta) = \sum_{n=1}^{N} \ln(P(x^{(n)}|\theta))$$

max of likelihood = max. of log likelihood

$$\mathcal{L}(x_1, \cdots, x_N; \lambda) = \sum_{n=1}^{N} \ln(P(x_n|\lambda))$$

$$= \sum_{n=1}^{N} \ln\left(\exp(-\lambda)\frac{\lambda^{x_n}}{x_n!}\right)$$

$$= \sum_{n=1}^{N} \left[\ln(\exp(-\lambda)) + \ln(\lambda^{x_n}) - \ln(x_n!)\right]$$

$$= \sum_{n=1}^{N} \left[-\lambda + x_n \ln(\lambda) - \ln(x_n!)\right]$$

$$= -N\lambda + \ln(\lambda)\sum_{n=1}^{N} x_n - \sum_{n=1}^{N} \ln(x_n!)$$

$$\max \Rightarrow \frac{\partial}{\partial\lambda}\mathcal{L}(x_1, \cdots x_N; \lambda) \stackrel{!}{=} 0$$

$$\frac{\partial}{\partial\lambda}\left[-N\lambda + \ln(\lambda)\sum_{n=1}^{N} x_n - \sum_{n=1}^{N}\ln(x_n!)\right] = 0$$

$$= -N + \frac{1}{\lambda}\sum_{n=1}^{N} x_n = 0$$

$\div N$

$$-1 + \frac{1}{N\lambda}\sum_{n=1}^{N} x_n = 0$$

$\times \lambda$

$$-\lambda + \frac{1}{N}\sum_{n=1}^{N} x_n = 0$$

$$\therefore \hat{\lambda} = \frac{1}{N}\sum_{n=1}^{N} x_n$$

// Sample mean