Task 3 B) Maximum likelihood parameters μ^* and $(\sigma^2)^*$, where PDF = Gaussian

$$P(x\mid\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

max of likelihood = max of log likelihood

$$\mathcal{L}(x_1,\cdots x_N;\mu,\sigma^2) = \sum_{n=1}^{N} \ln\left(P(x\mid\mu,\sigma^2)\right)$$

$$= \sum_{n=1}^{N} \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x_n-\mu)^2}{2\sigma^2}\right)\right)$$

$$= \sum_{n=1}^{N}\left(-\frac{1}{2}\ln(2\pi\sigma^2) - \frac{(x_n-\mu)^2}{2\sigma^2}\right)$$

$$= -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma^2) - \frac{1}{2}\sum_{n=1}^{N}\frac{(x_n-\mu)^2}{\sigma^2}$$

$$\max \Rightarrow \frac{\partial}{\partial\mu}\mathcal{L}(x_1\cdots x_N;\mu,\sigma^2) \overset{!}{=} 0$$

$$\frac{\partial}{\partial\mu}\left(-\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma^2) - \frac{1}{2}\sum_{n=1}^{N}\frac{(x_n-\mu)^2}{\sigma^2}\right) = 0$$

$$-\frac{1}{2}\sum_{n=1}^{N}\frac{-2x_n+2\mu}{\sigma^2} = 0$$

$\cdot\sigma^2$

$$\sum_{n=1}^{N}(x_n-\mu) = 0$$

$$\therefore \underline{\mu = \frac{1}{N}\sum_{n=1}^{N}x_n} \quad \text{// sample mean}$$

$$\max \Rightarrow \frac{\partial}{\partial\tilde\sigma}\mathcal{L}(x_1,\cdots,x_N;\mu,\tilde\sigma) \overset{!}{=} 0 \quad \text{, where } \tilde\sigma = \sigma^2$$

$$\frac{\partial}{\partial\tilde\sigma}\left(-\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\tilde\sigma) - \frac{1}{2}\sum_{n=1}^{N}\frac{(x_n-\mu)^2}{\tilde\sigma}\right) \overset{!}{=} 0$$

$$= -\frac{N}{2\tilde\sigma} - \frac{1}{2}\sum_{n=1}^{N}\left(-\frac{1}{\tilde\sigma^2}(x_n-\mu)^2\right) = 0$$

$$= \sum_{n=1}^{N}\left(-\frac{1}{2}\right)\left(\frac{1}{\tilde\sigma} - \frac{(x_n-\mu)^2}{\tilde\sigma^2}\right) = 0$$

$\cdot -2\tilde\sigma$

$$\sum_{n=1}^{N}\left(1 - \frac{(x_n-\mu)^2}{\tilde\sigma}\right) = 0$$

$$N - \frac{1}{\tilde\sigma}\sum_{n=1}^{N}(x_n-\mu)^2 = 0 \qquad \therefore \underline{\tilde\sigma = \sigma^2 = \frac{1}{N}\sum_{n=1}^{N}(x_n-\mu)^2} \quad \text{// variance}$$