Script:

Mathematical description of F-chart and Utilizability Methods for solar thermal system sizing and analysis

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Note: This script is fully based on the thorough and detailed original description of the mentioned methods that can be found here: (Duffie und Beckman 2013). Thus, the script does not represent any original work from the author, but merely a condensed summary of the method as described by Duffie and Beckman (2013).

1. The F-chart Method: a general introduction

The F-chart Method is a design method for assessing the long term performance of solar thermal systems. It is based on correlations and data fitting functions performed on the basis of extensive measurements and results from dynamic simulations of solar thermal systems. Since it is not an analytical method based on equations describing the performance of each system component in detail, its validity is only guaranteed for systems of similar characteristics to those whose data were used as a basis for obtaining the correlation functions. The systems for which the F-chart is valid are:

- Solar thermal systems providing energy demands at temperature levels close to 20°C. Thus, it is fully applicable for space heating (SH) or domestic hot water (DHW) supply. But it cannot be applied for systems supplying process heat at temperatures higher than 50°C or solar cooling applications.
- Air heating systems providing SH and DHW demands
- Liquid based systems providing either only SH or SH and DHW demands simultaneously.

There method is also valid as long as the following system parameters are within the ranges mentioned in Table 1.

Table 1. Ranges for design parameters within the validity range of the F-Chart method.

Ranges for design parameters	Units
$0.6 \leq (\tau\alpha)_n \leq 0.9$	-
$30° \leq \beta \leq 90°$	°
$2.1 \leq U_L \leq 8.3$	W/m^2K
$5 \leq F'_R A_c \leq 120$	m^2
$83 \leq UA_h \leq 667$	W/m^2K

IMPORTANT NOTE: all angles to be used in the following equations of the F-chart and Utilization methods need to be entered in degrees °!! Using radians or other units will lead to wrong results!

2. F-Chart Method Equations

a. Core correlations

The solar fraction *f,* of the monthly heating load supplied by solar energy as a function of two dimensionless parameters, *X* and *Y*. Where *X* is the ratio of collector losses to heating loads and *Y* is the ratio of absorbed solar radiation to the heating loads are given in equations (1) and (2).

$$f = 1.029Y - 0.065X - 0.245Y^2 + 0.0018X^2 + 0.0215Y^3 \quad \text{(Eq. 1)}$$

$$X = F_R U_L \frac{F'_R}{F_R}(T_{ref} - \overline{T_a})\Delta t \frac{A_c}{L} \quad \text{(Eq. 2)}$$

$$Y = F_R(\tau\alpha)_n \frac{F'_R}{F_R}\frac{(\overline{\tau\alpha})}{(\tau\alpha)_n}\overline{H}_T N \frac{A_c}{L} \quad \text{(Eq. 3)}$$

Where:

f	[-]	Monthly solar fraction
Y	[-]	Ratio of collector losses to heating loads
X	[-]	Ratio of absorbed solar radiation to heating loads
$F_R U_L$	W/ m^2	Heat loss factor of the collectors
$\frac{F'_R}{F_R}$	[-]	Collector heat exchanger correction factor
T_{ref}	[°C]	Empirical reference temperature
$\overline{T_a}$	[°C]	Monthly average ambient temperature
Δt	[s/month]	Number of seconds in the month
A_c	[m^2]	Area of the collector field
$F_R(\tau\alpha)_n$	[-]	Absorption coefficient of the collectors
N	[Days]	Number of days of the month
$(\tau\alpha)_n$	[-]	Normal absorptance transmittance product
$(\overline{\tau\alpha})$	[-]	Monthly Average absorptance transmittance product
$\overline{H}_T$	[J/m^{2}day]	Monthly average daily incident radiation onto tilted collector plane
L	[J]	Monthly loads

b. Conversion of collector parameters

All collector parameters expressed in the correlations shown above are expressed in terms of **gross collector area, A_g.** Thus, before getting started with the calculations and corrections necessary for applying the method, you need to make sure that your collector parameters η_0, a_1 and a_2 are expressed related to the gross collector area. If they are related to the aperture area instead A_a, they should be corrected by multiplying them with the factor A_a / A_g.

From the collector database you have characterized your collectors by using the parameters η_0, a_1 and a_2. However the collectors in equations (2) and (3) are characterized in terms of $F_R U_L$ and $F_R(\tau\alpha)_n$. In order to convert the parameters from the collector database to the required coefficients for the F-Chart method the following equations can be used:

$$F_{av}(\tau\alpha)_n - F_{av}U_L\frac{(T_{av}-T_a)}{G_T} = \eta_0 - a_1\frac{(T_{av}-T_a)}{G_T} - a_2\left(\frac{(T_{av}-T_a)}{G_T}\right)^2 G_T \quad \text{(Eq. 4)}$$

Where:

T_{av}	[°C]	Average collector temperature
G_T	[W/ m^2]	Global radiation on the tilted collector plane
a_1	[W/ m^{2}K]	First order heat loss coefficient
a_2	[W/ m^{2}K^2]	Second order heat loss coefficient
T_a	[°C]	Ambient temperature

Assuming two different but reasonable values for $\frac{(T_{av}-T_a)}{G_T}$ of 0.12 and 0.05 Km^2/W, and an average value of the radiation on the tilted collector plane, G_T, of 800 W/m^2 you can obtain a system of two equations (one with each set of values) and two unknowns, $F_{av}(\tau\alpha)_n$ and $F_{av}U_L$. From these you can easily obtain the values for the collector performance $F_{av}(\tau\alpha)_n$ and $F_{av}U_L$.

The values of $F_{av}(\tau\alpha)_n$ and $F_{av}U_L$ can be used to obtain the parameters depicting the performance of your collector (field) required for equations (2) and (3), $F_R(\tau\alpha)_n$ (Eq. 5) and F_RU_L (Eq. 6).

$$F_R U_L = F_{av} U_L \left(1 + \frac{A_{c1} F_{av} U_L}{2\dot{m} c_p}\right)^{-1} \qquad \text{(Eq. 5)}$$

$$F_R (\tau\alpha)_n = F_{av} (\tau\alpha)_n \left(1 + \frac{A_{c1} F_{av} U_L}{2\dot{m} c_p}\right)^{-1} \qquad \text{(Eq. 6)}$$

Where:

A_{c1}	[m^2]	Area of one **single collector**
c_p	[J/kgK]	Specific heat capacity of collector fluid
$\dot{m}$	[kg/s]	Mass flow of one **single collector**

These values derived express the performance of one single collector in your collector field. In case all your collectors in the collector field are connected in parallel their operating conditions should be the same. Thus, those values can be directly applied in equations (2) and (3) respectively.

c. Correction factor for series or parallel correction

In case your collector field is a combination of collectors connected in series in paralell the operation of the collectors in series cannot be expressed solely as a function of $F_R(\tau\alpha)_n$ and F_RU_L obtained in the previous section. Thus, if any series connection is present on your collector field, the following corrections need to be applied to the factors $F_R(\tau\alpha)_n$ and F_RU_L obtained in the previous section. In the equations (7) and (8) the values of $F_{R1}(\tau\alpha)_{n1}$ and $F_{R1}U_{L1}$ represent the values for one single collector as obtained from equations (5) and (6) respectively. In turn, the values of $F_R(\tau\alpha)_n$ and F_RU_L are those representing the behavior of all your collector field, with the given series connections and should be used in equations (2) and (3) accordingly.

$$F_R (\tau\alpha)_n = F_{R1} (\tau\alpha)_{n1} \left(\frac{1-(1-K)^N}{NK}\right) \qquad \text{(Eq. 7)}$$

$$F_R U_L = F_{R1} U_{L1} \left(\frac{1-(1-K)^N}{NK}\right) \qquad \text{(Eq. 8)}$$

$$K = \frac{A_{c1} F_{R1} U_{L1}}{\dot{m} c_p} \qquad \text{(Eq. 9)}$$

Where:

N	[-]	Number of collectors connected in series (in one branch!)
K	[-]	Correction factor for assessing the different losses in series connection resulting of higher temperatures in the last collectors
$\dot{m}$	[kg/s]	Mass flow of one **single collector**

d. Collector heat exchanger factor

The collector heat exchanger factor is an indication of the penalty in collector performance due to the higher temperatures required from the collector field for compensating the temperature drop in the heat exchanger of the solar collector loop.

$$\frac{F'_R}{F_R} = \left[1 + \left(\frac{A_c F_{av} U_L}{(\dot{m}c_p)_c}\right)\left(\frac{(\dot{m}c_p)_c}{\varepsilon(\dot{m}c_p)_{min}} - 1\right)\right]^{-1} \quad \text{(Eq. 10)}$$

Where:

ε	[-]	Effectiveness of the heat exchanger in the solar collector loop
$(\dot{m}c_p)_c$	[W/K]	Fluid capacitance rate in the collector loop (product of mass flow – heat capacity of the collector fluid)
$(\dot{m}c_p)_{min}$	[W/K]	Minimum fluid capacitance rate of the fluids in the collector and storage loops

e. Absorptance-transmittance product

As a first simplified approach for estimating the absorptance-transmittance product a value of 0.94 can be assumed if the following conditions are given:

- Collectors with an azimuth of 0°
- Heating applications with significant collector use in the winter months
- Two-covers collectors

If any of these assumptions is not fulfilled a detailed calculation is required. For this the following procedure is required. The equation required to calculate the ratio between the monthly average transmittance-absorptance product and that for the normal incidence $\frac{(\overline{\tau\alpha})}{(\tau\alpha)_n}$ is given in the following equation:

$$\frac{(\overline{\tau\alpha})}{(\tau\alpha)_n} = \frac{(\overline{\tau\alpha})_b}{(\tau\alpha)_n}\left(\frac{\overline{H}_b}{\overline{H}_T}\right)\overline{R}_b + \frac{(\overline{\tau\alpha})_d}{(\tau\alpha)_n}\frac{\overline{H}_d}{\overline{H}_T}\left(\frac{1 + cos\beta}{2}\right) + \frac{\overline{H}\rho_g}{\overline{H}_T}\frac{(\overline{\tau\alpha})_g}{(\tau\alpha)_n}\left(\frac{1 - cos\beta}{2}\right)$$

The ratio of the transmittance-absorptance product for the beam and diffuse components of the incident radiation $\frac{(\overline{\tau\alpha})}{(\tau\alpha)_n}$ can be obtained from Figure 1 as a function of the beam angle in each month.

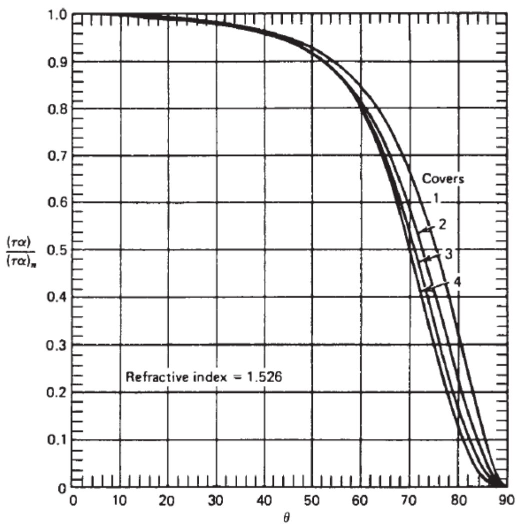


Figure 1. Typical (τα)/(τα)n curves for one to four covers. (Duffie, Beckmann, 2013, p.215)

The ratio of the transmittance-absorptance product for the beam component of the incident radiation $\frac{(\overline{\tau\alpha})_b}{(\tau\alpha)_n}$ can be obtained from the above figure, once the incidence angle for the beam radiation in each month is known. For estimating the incidence angle for the beam radiation for each month at different latitudes and azimuths (in the northern hemisphere) of the collector field Figures 2 or 3 can be used.

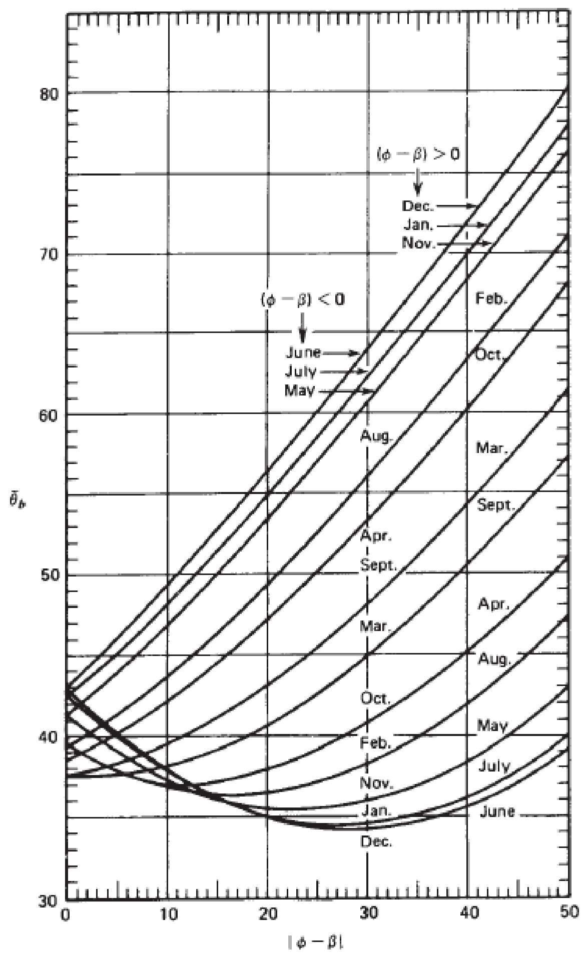
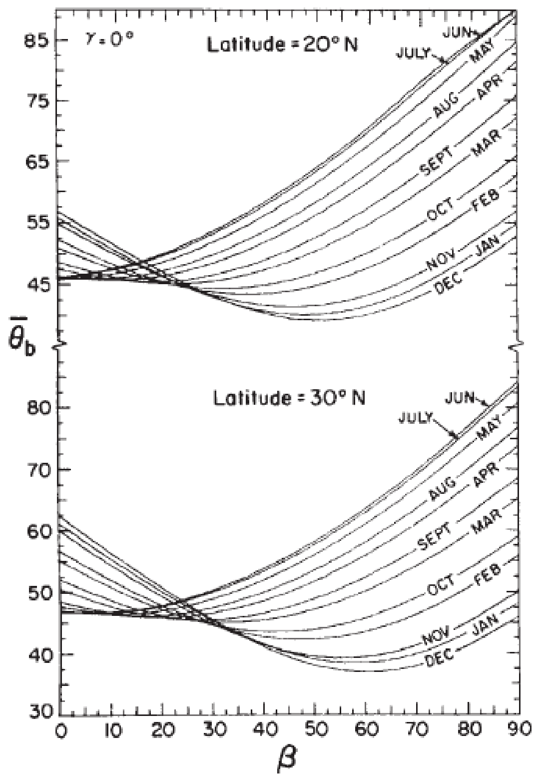
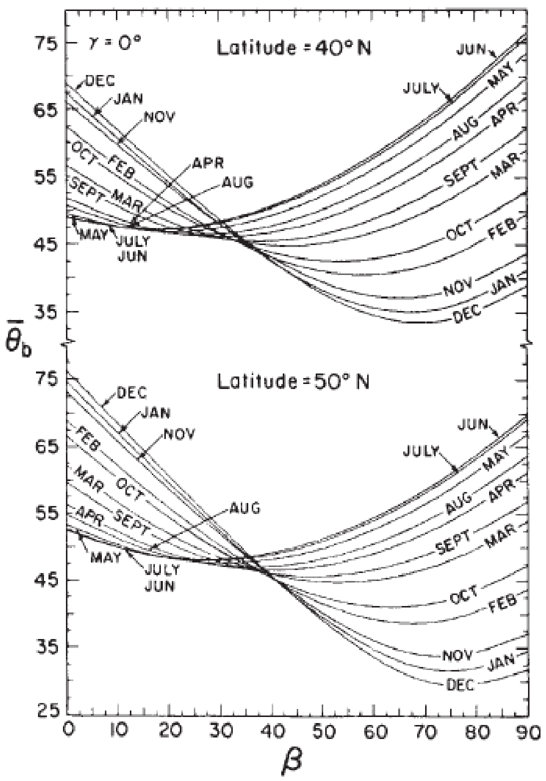


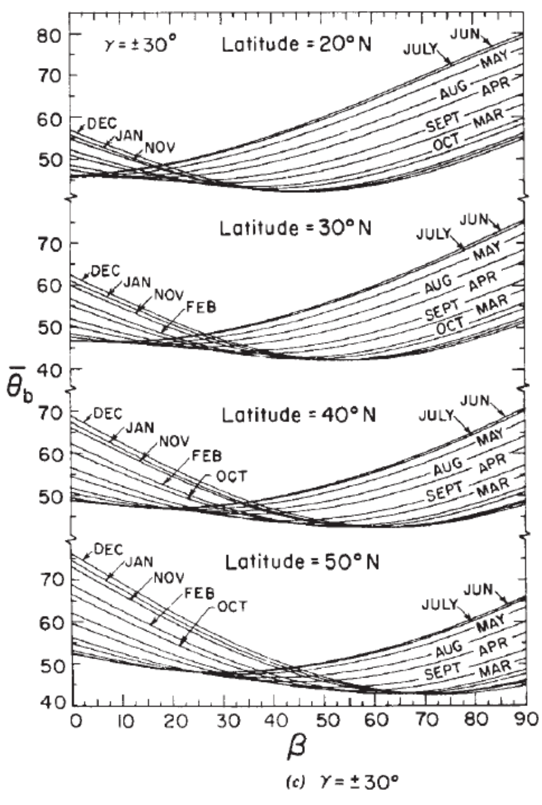
Figure 2. Monthly mean incidence angle for beam radiation for surfaces facing the equator in the northern hemisphere for space heating systems. For the southern hemisphere, interchange the two inequality signs (Duffie, Beckmann, 2013, p.228)



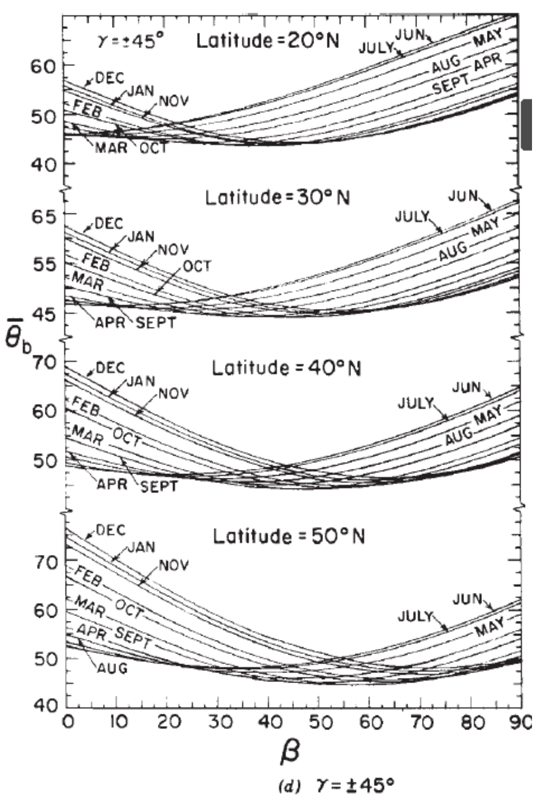
(a) $\gamma = 0°$



(b) $\gamma = 0°$



(c) $\gamma = \pm 30°$



(d) $\gamma = \pm 45°$

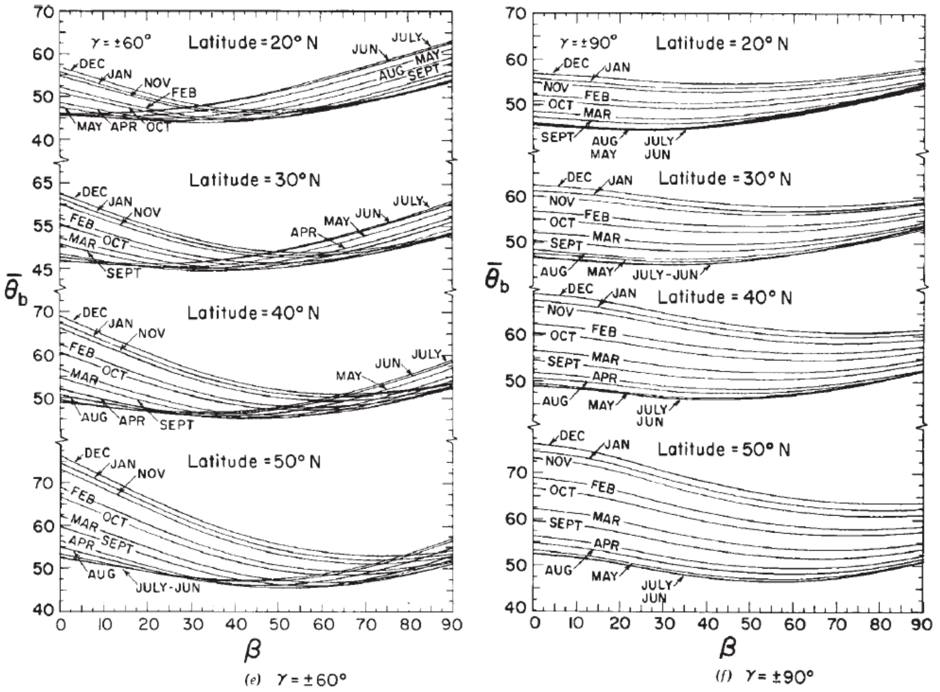


Figure 3. Monthly average beam incidence angle for various latitudes and orientations. For southern hemisphere interchange months as follows (north = south): June = Dec; July= Jan; May = Nov; Aug= Feb; Apr =Oct; Sept=March; March =Sept; Oct = Apr; Feb=Aug.

The ratio of the transmittance-absorptance product for the diffuse radiation components $\frac{(\overline{\tau\alpha})_d}{(\tau\alpha)_n}$ and $\frac{(\overline{\tau\alpha})_g}{(\tau\alpha)_n}$ can be obtained also from Figure 1 After the incidence angle of the diffuse radiation for the ground reflected and diffuse components have been obtained from Figure 4

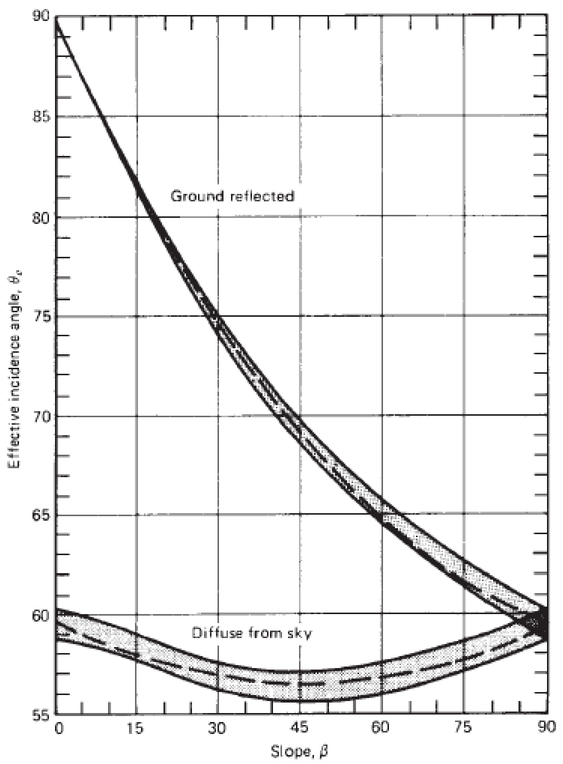


Figure 4. Effective incidence angle of isotropic diffuse radiation and isotropic ground-reflected radiation on sloped surfaces. (Duffie, Beckmann, 2013, p.212)

f. Monthly radiation onto collector plane (northern hemisphere)

The radiation data that can be more easily found for any site is the global horizontal radiation per month. From this, the monthly average daily global horizontal radiation can be obtained, $\overline{H}$. For designing solar thermal systems however the monthly average daily radiation incident on the collector tilted plane $\overline{H}_T$ needs to be calculated. Assuming that the diffuse and ground reflected components are isotropic the monthly total radiation on an unshaded tilted plane, $\overline{H}_T$, can be calculated as follows:

$$\overline{H}_T = \overline{H}\left(1-\frac{\overline{H}_d}{\overline{H}}\right)\overline{R}_b + \overline{H}\frac{\overline{H}_d}{\overline{H}}\left(\frac{1+cos\beta}{2}\right) + \overline{H}\rho_g\left(\frac{1-cos\beta}{2}\right) \qquad \text{(Eq. 11)}$$

Where:

β	[°]	Tilt angle of the collector plane, slope
$\overline{H}_d$	[J/m^{2}day]	Diffuse component of the horizontal global radiation
$\overline{R}_b$	[-]	Average monthly ratio of beam radiation on the tilted surface to that on a horizontal surface
ρ_g	[-]	Ground reflectivity

The monthly average daily ratio of beam radiation on the tilted surface to that on a horizontal surface can be calculated with the following equation for surfaces with an azimuth of zero degrees in the northern hemisphere, *ϒ=0°* (or 180° in the southern hemisphere):

$$\overline{R}_b = \frac{\cos(\phi-\beta)cos\delta\ sin\omega'_s + \left(\frac{\pi}{180}\right)\omega'_s sin(\phi-\beta)\ sin\delta}{cos\phi\ cos\delta\ sin\omega_s + \left(\frac{\pi}{180}\right)\omega_s\ sin\phi\ sin\delta} \qquad \text{(Eq. 12)}$$

Where:

ϕ	[°]	Latitude
δ	[°]	Declination
ω_s	[-]	Sunset hour angle for horizontal surface
ω'_s	[-]	Sunset hour angle for the tilted surface

The declination can be calculated following the equation (13). In that equation n is the average day of the month which can be determined using the table below.

$$\delta = 23.45 \sin 360\left(\frac{284+n}{365}\right) \qquad \text{(Eq. 13)}$$

Table 2. Average day of each month, *n*.

Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
17	47	75	105	135	162	198	228	258	288	318	344

The sunset hour angle for the tilted plane can be calculated as follows:

$$\omega'_s = min\begin{bmatrix}\cos^{-1}(-\tan\phi\tan\delta)\\ \cos^{-1}(-\tan(\phi-\beta)\tan\delta)\end{bmatrix} \qquad \text{(Eq. 14)}$$

And the sunset hour angle on the horizontal plane can be obtained by:

$$\omega_s = \cos^{-1}(-\tan\phi\tan\delta) \qquad \text{(Eq. 15)}$$

For calculating the fraction of diffuse radiation $\frac{\overline{H}_d}{\overline{H}}$ the clearness index $\overline{K}_T$ needs to be obtained first:

$$\overline{K_T} = \frac{\overline{H}}{\overline{H}_0} \qquad \text{(Eq. 16)}$$

In the previous equation $\overline{H}_0$ is the monthly average daily extraterrestrial radiation,in [J/m^{2}d]. For latitude between ±60° the value of $\overline{H}_0$can be estimated with the following relationship:

$$\overline{H}_0 = \frac{24\,.3600\;G_{sc}}{\pi}(1+0.033\cos\frac{360n}{365})(cos\,\phi\cos\delta\sin\omega_s + \frac{\pi\omega_s}{180}sin\delta\;sin\phi) \qquad \text{(Eq. 17)}$$

The fraction of diffuse radiation $\frac{\overline{H}_d}{\overline{H}}$ can be calculated as a function of the clearness index $\overline{K}_T$ and the hour angle on the horizontal plane, ω_s . For values of the clearness index among 0.3 and 0.8 the following relationships can be used:

$$0.3 \le \overline{K}_T \le 0.8 \qquad \text{(Eq. 18)}$$

$$\omega_s \le 81.4; \qquad \frac{\overline{H}_d}{\overline{H}} = 1.391 - 3.56\overline{K}_T + 4.189\overline{K}_T^{\;2} - 2.137\overline{K}_T^{\;3} \qquad \text{(Eq. 19)}$$

$$\omega_s > 81.4; \qquad \frac{\overline{H}_d}{\overline{H}} = 1.311 - 3.022\overline{K}_T + 3.427\overline{K}_T^{\;2} - 1.821\overline{K}_T^{\;3} \qquad \text{(Eq. 20)}$$

g. Correction for storage size

The F-chart method was developed for a standard storage capacity of 75 liters of stored water per square meter of collector area. The performance of systems with storage capacities in the range of 37.5 to 300 liters/m^2 can be determined by correcting the dimensionless correlation value for the heat losses, X to X_c, according to the following equation:

$$for\;\; 0.5 \le \left(\frac{actual\ storage\ capacity}{standard\ storage\ capacity}\right) \le\; 4.0\;\;, \quad \frac{X_c}{X} = \left(\frac{actual\ storage\ capacity}{standard\ storage\ capacity}\right)^{-0.25}$$

h. Correction for load heat exchanger

The size of the heat exchanger for supplying the loads (after the storage tank) influences the temperature required from the storage: smaller capacity (UA value) of the heat exchanger leads to higher temperatures required from the storage to be able to supply the required demand. This leads to higher temperatures in the collector loop and thus, reduced efficiency. This effect of different capacities or sizes of the load heat exchanger in the system performance can be assessed by the correction factor addressed in this section.

The F-Chart method was developed for load heat exchangers having a value $\frac{\varepsilon_L\, C_{min}}{(UA)_h}$ of 2. For any other heat exchanger size the following correction for the heat gain correlation parameter Y should be performed.

$$for\ \ 0.5 \leq \left(\frac{\varepsilon_L\, C_{min}}{(UA)_h}\right) \leq\ 50\ ,\quad \frac{Y_C}{Y}\ = 0.39 + 0.65\ e^{-\left(\frac{0.139\,(UA)_h}{\varepsilon_L\, C_{min}}\right)} \qquad \text{(Eq. 21)}$$

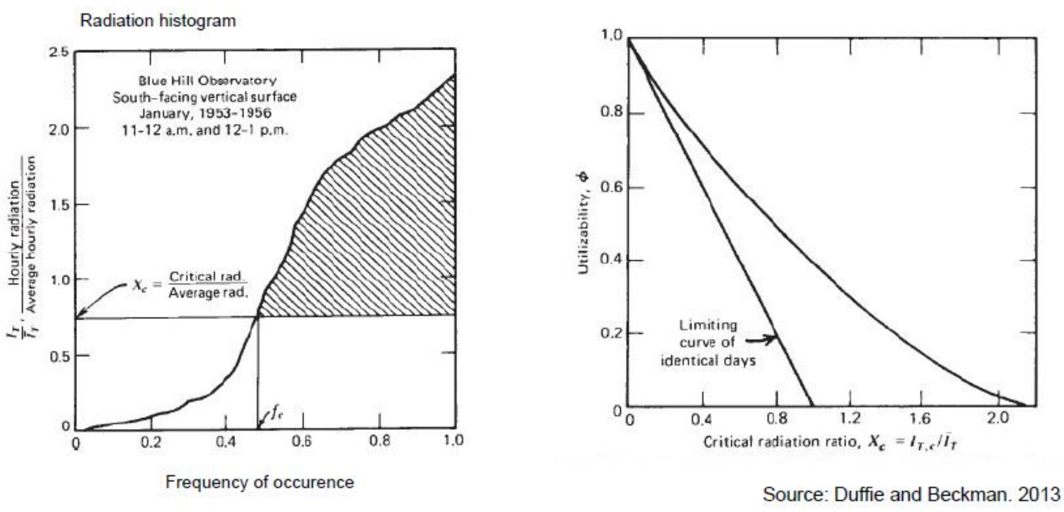
Where:

Y_c	[°]	Corrected Y parameter for the actual heat exchanger size
$(UA)_h$	[W/K]	Heat transfer coefficient of the heat exchanger
C_{min}	[W/K]	Minimum fluid capacitance rate
ε_L		Effectivenes of the load heat exchanger

3. The Utilizability Method: a general introduction

For solar thermal systems providing energy demands at temperature levels significantly different from those of DHW and SH demands (i.e. close to 20°C), the F-chart method cannot be used. Enhancing the F-chart method with the "Utilizability" concept can be of great help for avoiding that limitation.

The utilizability (ϕ) is the fraction of the incident solar radiation that can be converted into useful heat. The higher the temperature to be supplied by the collector field, the lower this fraction would be and thereby the lower the utilizability results.



(a) (b)

Figure 4. (a) Histogram showing the frequency of occurrence of different hourly radiations as compared to the average hourly radiation in a given time span, e.g. one month. The parameter Xc represents the ratio between the critical radiation level and the average one and is defined by the solar thermal system designer as a function of the required temperatures to be supplied by the system; (b) Dependency between the critical radiation level and the utilizability. (Duffie, Beckmann, 2013, p.116-117)

Figure 5 (a) shows the value of the critical radiation required as compared to the average hourly radiation level, X_c, for a given location and time span of e.g. one month. Higher temperatures to be supplied by the solar thermal system leads to higher critical radiation levels required (i.e. higher values for X_c which is represented in the Y-axis). The shadowed area in Figure 5 (a) gives therefore the cumulative value of the radiation over the represented time span than can be used by the solar thermal system for the supply temperatures required. Radiation values below X_c cannot be effectively used to supply the required temperatures. Figure 5 (b) shows that the utilizability decreases for increasing value of the critical radiation level.

The concept of utilizability allows therefore accounting for variable critical radiation levels resulting from variable collector inlet temperatures (due to storage temperature fluctuations, load patterns, seasonal variations….). It can be determined on an hourly or monthly basis. The last one is used when combined with the F-Chart method.

The utilizability can be determined for different solar thermal applications than SH and DHW supply as a function of the inlet temperatures in the collector field and the required outlet temperatures for the demand. This allows applying this simplified sizing method for applications such as solar cooling systems or process heat applications.

This sizing method allows obtaining the yearly performance of such systems. Since it is not a dynamic system calculation method it cannot determine the outlet temperature from the collector field. It merely estimates a minimum collector outlet temperature. Thereby, any devices present on the system whose efficiency is directly and strongly influenced by the collector outlet temperature will be penalized (e.g. thermally driven chillers, heat pumps or turbines). A detailed estimation of the efficiency of these system components cannot be assessed.

a. Core correlations

The ratio of the collector losses to the energy demands (loads) to be provided for the utilizability method *X′* can be calculated as shown in the equation below. As it can be seen, in this equation the temperature difference $\left(T_{ref} - \overline{T_a}\right)$ has been substituted by 100 as an empirical constant.

$$X = F_R U_L \frac{F'_R}{F_R} 100 \Delta t \frac{A_c}{L} \qquad \text{(Eq. 22)}$$

The ratio of the absorbed solar radiation to the energy demands to be supplied for the utilizability method is calculated by correcting the Y factor with the maximum daily average utilizability for each month, $\bar{\phi}_{max}$.

$$\bar{\phi}_{max} Y = \bar{\phi}_{max} F_R (\tau\alpha)_n \frac{F'_R}{F_R} \frac{(\overline{\tau\alpha})}{(\tau\alpha)_n} \overline{H}_T N \frac{A_c}{L} \qquad \text{(Eq. 23)}$$

$$f = \bar{\phi}_{max} Y - 0.015\left(e^{3.85f} - 1\right)\left(1 - e^{-0.15X}\right) {R_s}^{0.76} \qquad \text{(Eq. 24)}$$

a. Required additional variables

The **utilizability $\bar{\phi}_{max}$,** is a function of several parameters such as the ratio of the radiation on the tilted radiation to horizontal one *R*, that ratio at noon*, R_n*, or the dimensionless parameters *a, b* and *c*. The main design variable strongly determining the utilizability is, however, the critical radiation level X_c. Since the F-chart method is used for monthly averages, average monthly values of the utilizability $\bar{\phi}_{max}$ and critical radiation levels $\overline{X_c}$ are used in the equation.

$$\bar{\phi}_{max} = \exp\left\{\left[a + b\left(\frac{R_n}{\bar{R}}\right)\right]\left[\overline{X_c} + c\overline{X_c}^2\right]\right\} \qquad \text{(Eq. 25)}$$

The **dimensionless parameters *a, b* and *c*** are an function of the clearness index and can be obtained with the following equations:

$$a = 2.943 - 9.271\bar{K}_T + 4.031\bar{K}_T^{\ 2} \qquad \text{(Eq. 26)}$$

$$b = -4.345 + 8.853\bar{K}_T - 3.602\bar{K}_T^{\ 2} \qquad \text{(Eq. 27)}$$

$$c = -0.710 - 0.3061\bar{K}_T + 2.936\bar{K}_T^{\ 2} \qquad \text{(Eq. 28)}$$

The **values of the ratio of tilted radiation on the tilted plane to that on the horizontal surface *R*** and that ratio for noon conditions, R_n, can be obtained with the following equations:

$$\bar{R} = \frac{\bar{H}_T}{\bar{H}}$$

$$R_n = \left(1 - \frac{r_{d,n}}{r_{t,n}}\frac{\bar{H}_d}{\bar{H}}\right)R_{b,n} + \frac{r_{d,n}}{r_{t,n}}\frac{\bar{H}_d}{\bar{H}}\left(\frac{1+cos\beta}{2}\right) + \rho_g\left(\frac{1-cos\beta}{2}\right) \qquad \text{(Eq. 29)}$$

$$R_{b,n} = \frac{cos|\phi-\delta-\beta|}{cos|\phi-\delta|} \qquad \text{(Eq. 30)}$$

The **average noon to average daily diffuse radiation $r_{d,n}$** can be calculated as a function of the hour angle at noon ω_n and the sunset hour angle ω_s:

$$r_{d,n} = \frac{\pi}{24}\frac{cos\omega_n - \pi\omega_s}{\sin\omega_s - \frac{\pi\omega_s cos\omega_s}{180}} \qquad \text{(Eq. 31)}$$

The **average noon to average daily total radiation $r_{t,n}$** can be calculated as a function of the hour angle at noon ω_n and the sunset hour angle ω_s:

$$r_{t,n} = \frac{\pi}{24}(a' + b'\,cos\omega_n)\frac{cos\omega_n - \pi\omega_s}{\sin\omega_s - \frac{\pi\omega_s cos\omega_s}{180}} \qquad \text{(Eq. 32)}$$

The **parameters a' and b'** for calculating $r_{t,n}$ can be obtained from:

$$a' = 0.409 + 0.5061\,sin(\omega_s - 60) \qquad \text{(Eq. 33)}$$

$$b' = 0.6609 + 0.4767\,sin(\omega_s - 60) \qquad \text{(Eq. 34)}$$

Finally, the **minimum monthly average critical radiation** can be defined as a function of the minimum temperature to be supplied by the solar thermal system, T_{min} as follows:

$$\bar{X}_{c,min} = \bar{X}_c = \frac{F_R U_L (T_{min} - \bar{T}_a)}{F_R(\tau\alpha)_n \frac{(\overline{\tau\alpha})}{(\tau\alpha)_n} r_{t,n} R_n \bar{K}_T H_0} \quad \text{(Eq. 35)}$$

The last parameter missing for obtaining the solar fraction f is then ***R_s*, the correction factor for different storage capacities** than the standard one. The standard storage heat capacity per unit of collector area is defined as 350 kJ/m^{2}K. The value of R_s is then determined as a function of the actual storage capacity (*in kJ/m^{2}K) :

$$R_s = \frac{350}{actual\ storage\ capacity*} \quad \text{(Eq. 36)}$$

References

Duffie, John A.; Beckman, William A. (2013): Solar Engineering of Thermal Processes. Fourth Edition: Wiley Science.