Ising Model – Metropolis (1.5 LP) and Wolff (+ 0.5 LP) algorithms

The aim of the project is to simulate the 2D Ising model in equilibrium with use of Monte Carlo algorithms. The Ising model is considered on a square lattice $L \times L$ with periodic boundary conditions, ranging in size from $L = 20$ to $L = 100$. The coupling constant J and k_B are set to one.

Simulations follow the standard Metropolis and Wolff algorithms as described in the book *M. E. J. Newman and G. T. Barkema, Monte Carlo Methods in Statistical Physics, OUP, 1999.*

(1) Give examples of time evolution of the magnetization M and snapshots of the lattice state for different temperatures below and above the phase transition

(2) Calculate the autocorrelation function of $|M|$, for some L, as function of temperature; observe difference in the Metropolis and Wolff algorithms for this quantity

(3) Find temperature dependence of the average magnetization $\langle |M| \rangle (T)$ and of susceptibility $\chi(T) = \beta L^2 (\langle |M|^2 \rangle - (\langle |M| \rangle)^2)$ for several values of L. Calculate also errors (e.g. using blocking method). Compare CPU time for Metropolis and Wolff methods with similar errors.

(4) Check the finite-size scaling by plotting $\chi L^{-a/b}$ vs $L^{1/b} t$ where $t = \frac{T - T_c}{T_c}$. Adjust T_c, a, b so that the data points for different L collapse.

Basic Literature: Newman, Barkema, Monte Carlo Methods in Statistical Physics, Oxford University Press

Ising Model and its Monte-Carlo simulation is also described in almost all books on computational physics, e.g. *An Introduction to Computer Simulation Methods: Applications to Physical Systems, by Gould and Tobochnik*