Schrödinger equation (1.5 LP)

We consider one-dimensional Schrödinger equation in dimensionless form

$$i\frac{\partial\psi}{\partial t} = \hat{H}\psi = (-\frac{\partial^2}{\partial x^2} + V(x,t))\psi$$

Problem 1: Consider the eigenvalue problem for this equation and find first four eigenvalues and eigenfunctions for the double-well potential $V(x) = \frac{A}{2}(1-x^2)^2$.

Problem 2: Simulate time-dependent Schrödinger equation using the Crank-Nicholson dicretization scheme.

Problem 2a: Check accuracy of the total probability $\int_{-\infty}^{\infty} |\psi|^2 \, dx$ conservation in dependence on the space step.

Problem 2b: Use eigenfunctions found as initial conditions and check stationarity in the time-dependent runs

Problem 2c: Use combinations of two eigenfunctions as initial conditions and check if time evolution of the observable $p_{left} = \int_{-\infty}^{0} |\psi|^2 \, dx$ shows the expected periodicity in time.

Literature:
Scherer, Computational Physics, sec. 17.3.4 (Crank-Nicholson method), Ch. 19 (Schrödinger eq.)
Koonin, Computational Physics, sec. 3.5 (stationary eq.), Ch. 7 (nonstationary eq.)
Landau, Paez, Bordeianu, A Survey of Computational Physics, sec. 9.10-9.11 (stationary eq.); sec. 18.5-18.7 (nonstationary eq.)