6. Consider the differential equation

$$y^{-1}y' + p(x)\ln y = q(x), \tag{9}$$

where $p(x)$ and $q(x)$ are continuous functions on some interval (a, b).

2

a) Show that the change of variable $u = \ln y$ reduces the equation (9) to the linear differential equation

$$u' + p(x)u = q(x).$$

b) Show that the general solution to the equation (9) is

$$y(x) = \exp\left\{e^{-\int p(x)\,dx}\left[\int q(x)e^{\int p(x)\,dx}\,dx + c\right]\right\}.$$

c) Use the above technique to solve the initial value problem

$$y^{-1}y' - 2x^{-1}\ln y = x^{-1}(1 - 2\ln x), y(1) = e.$$