Sequences

- succession of numbers
  - ex odd numbers

    $a_i = 2i+1$ for $i = 0, 1, 2 \dots \infty$

    multiples of 3

    $a_i = 3i$ for $i = 1, 2, 3 \dots \infty$

→ explicit def.

n recursive definitions

odd numbers

$a_0 = 1$

$a_{n+1} = a_n + 2$

ex $a_0 = 1$

$a_1 = 1 + 2 = 3$

$a_2 = a_1 + 2 = 5$

$a_3 = a_2 + 2 = 7$

$\vdots$

geometric sequence

$a_0 = a$ $\quad$ a given

$a_{i+1} = a_i \cdot r$ $\quad$ r given

ex powers of 2 : $a = 1,\ r = 2$

$a_0 = 1$

$a_1 = 2a_0 = 2$

$a_2 = 2a_1 = 4$

$\vdots$

given a, r, N, compute a_N

Recursive approach

```
if N == 0:
    return a
else:
    return r · (a_{N-1})
```

Non-recursive approach

→ compute $a_0, a_1, a_2 \ldots a_N$

$i = \cancel{0}\ 1$

$a_{old} = a$

while $i \leq N$

\# $a_i = r\, a_{i-1}$

$a_{new} = 2 * a_{old}$

$a_{old} = a_{new}$

$i = i + 1$

i	a_{old}	a_{new}
0	a	/
1	a	ra
2	ra	$r^2 a$
3	$r^2 a$	$r^3 a$

if $a_0 = a$

$a_{i+1} = r\, a_i$

~~then~~ $a_i = a\, r^i$

$a_0 = a$

$a_1 = a r$

$a_2 = a r^2$

$a_3 = a r^3$

Proof that $a_i = a\, r^i$:

if $i = 0$ $a_0 = a$: statement is true if $i = 0$

suppose statement is true for i

$a_{i+1} = r\, a_i = r\, a r^i = a r^{i+1}$

then it is true for $i+1$

Geometric series.

$$s_n = \sum_{i=0}^{n} a^i$$

$i=1, \; a=2 \qquad s_n = 1 + 2 + 4 + 8 + \cdots$

if $|a|<1$, then $\lim_{n \to \infty} s_n$ exists