Computer Vision
ETH Zürich
Fall 2020

Lab Assignment 4 - Model Fitting

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1 Line Fitting

Using the RANSAC algorithm for line fitting gives relevant results to model the real line. As we can see in figure 1, the RANSAC line fitting (in red) is nearly superposed to the real line (in black), while the least square result (in green) is more affected by the outliner.

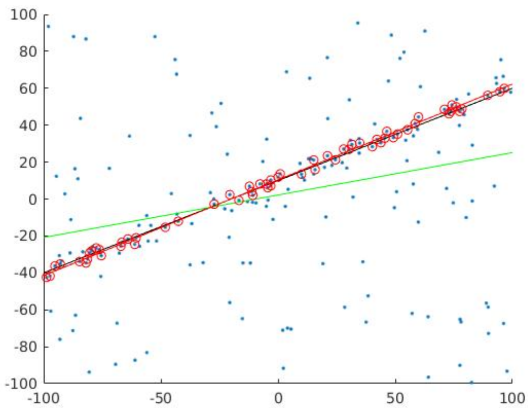


Figure 1: a nice plot

This can also be seen by comparing the loss of different algorithm to the real line loss in table 1. The RANSAC error is nearly the same as the real line error, but the least square model error is to high.

Model	Real	Least Square	RANSAC
Error	38.0261	181.2809	38.0509

Table 1: Table of errors measured on the true inliers.

2 Fundamental Matrix Estimation

Given n point matches $u'_i \leftrightarrow u_i$, I started by standardizing the points to get a zero mean and a unit standard deviations using the transformation matrices T and T', i.e., $\tilde{u} = Tu$

and $\tilde{u}' = T'u'$. Then, I solved the equation $\tilde{u}'^T F \tilde{u} = 0$ by introducing the equation matrix A and the column vector f such that $Af = 0$ and

$$A = \begin{pmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 & 1 \\ \vdots & & & & \ddots & & & & \vdots \\ u_nu'_n & u_nv'_n & u_n & v_nu'_n & v_nv'_n & v_n & u'_n & v'_n & 1 \end{pmatrix}$$

$$f = \begin{pmatrix} F_{11} & F_{21} & F_{31} & F_{12} & F_{22} & F_{32} & F_{13} & F_{23} & F_{33} \end{pmatrix}^T$$

Finally, I calculated F', the closest singular matrix to F under Frobenius norm using the Singular Value Decomposition. The epipolar lines are shown in figure 2.

```
[U,S,V] = svd(F);
S(3,3) = 0;
Fh = U*S*V';
```

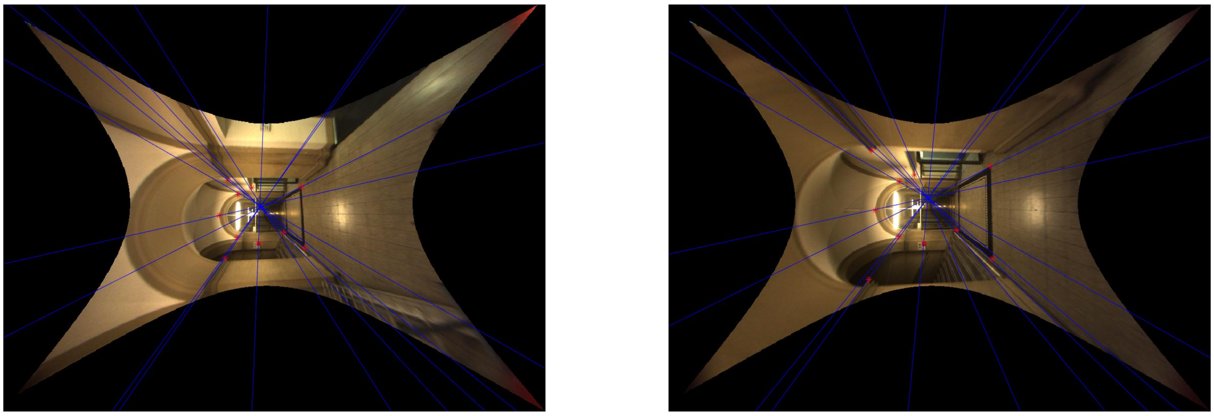
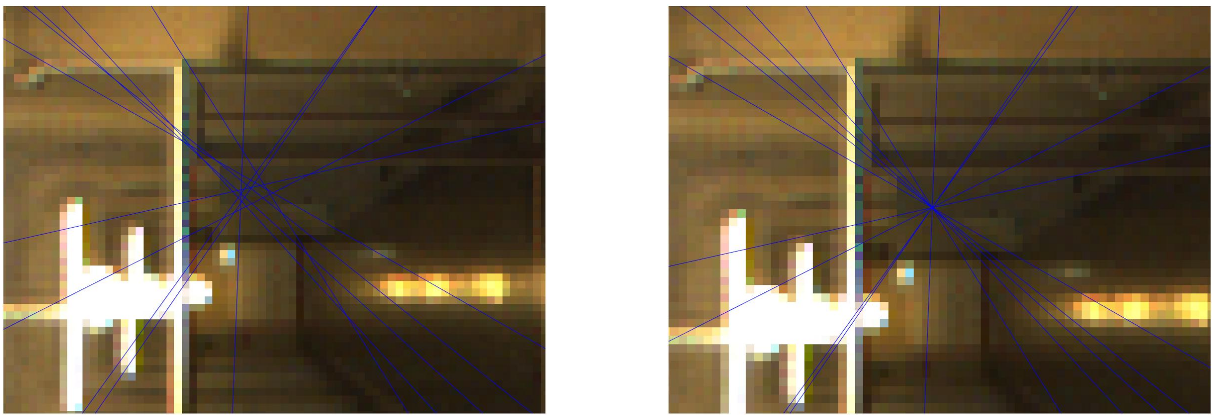


Figure 2: Epipolar lines for both images using singularity constraint.

We can clearly see in figure 3 the different results when using F with and without singularity constraint. In fact, when $det(F) \neq 0$, the fundamental matrix does not have epipoles, i.e., the epipolar lines do not intersect in a single point.



(a) Without singularity constraint. (b) With singularity constraint

Figure 3: Epipolar lines intersection.

3 Feature Extraction and Matching

For the same image pairs, SIFT is used to extract and match features. These matches are shown in figure 4.

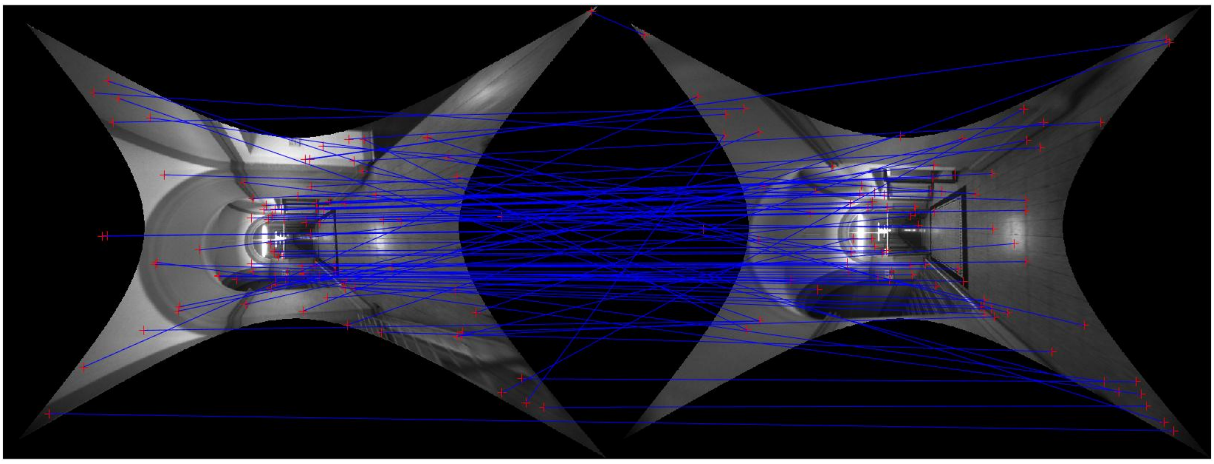


Figure 4: Feature extraction and matching.

4 Eight-Point RANSAC

4.1 Simple RANSAC

Given l and p, the homogeneous line and point respective equations, the distance between them is $d = l \cdot p$. So, I have performed an element-wise multiplication between the two $3 \times N$ matrices and then performed a sum for each column.

```
a = abs(lines .* points);
d = sum(a, 1);
```

At each iteration, 8 points are randomly selected to calculate the fundamental matrix. Then, an error is calculated between each point and the corresponding epipolar line where d is the distance introduced earlier.

$$e(x, x') = 0.5 \times (d(x', Fx) + d(x, F^T x'))$$

Only points with $e < t$ are kept, where t is a predefined threshold. Results for $t = 1$ are shown in figure 5.

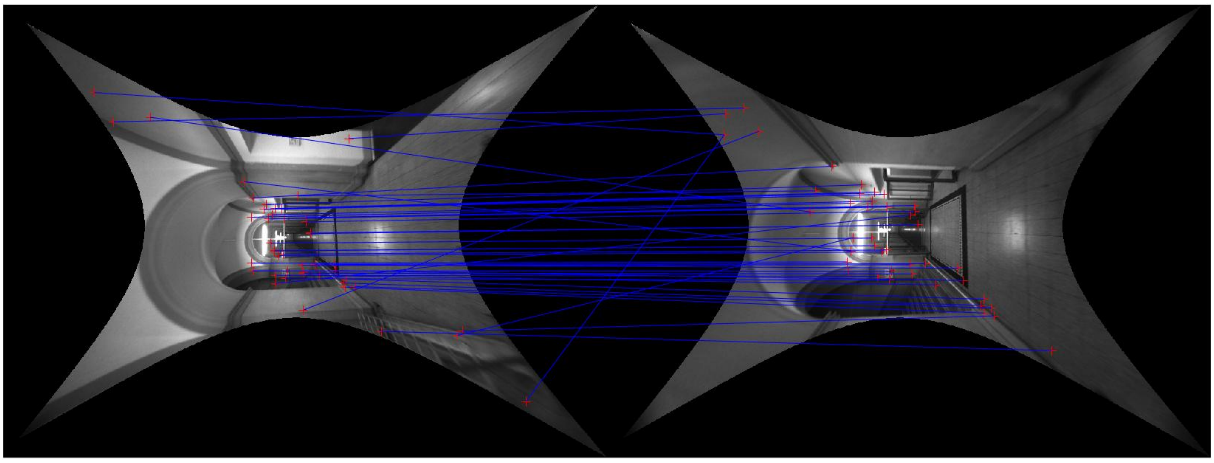


Figure 5: Feature extraction and matching using simple RANSAC.