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1. Exact solution of Differential equation

Given Initial Value Problem:

$$\begin{cases} y' = f(x, y) \\ y(x_0) = y_0 \\ x \in (x_0; X) \end{cases}$$

Given Parts:

$$f(x, y) = e^y - \frac{2}{x}$$

$$y_0 = -2$$

$$x_0 = 1$$

$$X = 7$$

1.1 Find General Solution

Let's write out equation in alternative way:

$$\frac{dy}{dx} = e^y - \frac{2}{x}$$

$$e^y - \frac{2}{x} - \frac{dy}{dx} = 0$$

$$\frac{e^y \cdot x - x \cdot \frac{dy}{dx} - 2}{x} = 0$$

$$e^y \cdot x - x \cdot \frac{dy}{dx} - 2 = 0$$

Let's make substiotution:

$$y = \ln\left(\frac{v}{x}\right)$$

$$\frac{dy}{dx} = \frac{x \left(\frac{\frac{dv}{dx}}{x} - \frac{v}{x^2} \right)}{v}$$

$$v - \frac{dy}{dx} \cdot x - 2 = 0$$

$$v - \frac{x^2 \left(\frac{\frac{dv}{dx}}{x} - \frac{v}{x^2} \right)}{v} - 2 = 0$$

Simplify:

$$v(x) - \frac{x \cdot \frac{dv}{dx}}{v} - 2 = 0$$

Let's put x's and v's on opposite sides

$$\frac{dv}{(v-1)(v)} = \frac{dx}{x}$$

Integrate both sides:

$$\int \frac{dv}{(v-1)(v)} = \int \frac{dx}{x}$$

Evaluate the integrals:

$$\ln(-v+1) - \ln(x) = \ln(x) + C_1$$

Let's expents of both sides:

$$e^{\ln(-v+1) - \ln(x)} = e^{\ln(x) + C_1}$$

$$v = \frac{1}{e^{C_1}x + 1} = \frac{1}{c_1 + 1}$$

Substitute back for $y = \ln\left(\frac{v}{x}\right)$:

$$v = x \cdot e^y$$

And we got our general solution:

$$y = -\ln(C_1 \cdot x^2 + x)$$

1.2 Let's solve initial value problem:

$$e^{-y_1} = (c_1 \cdot x_1^2 + x_1)$$

$$e^{-y_1} - x_1 = c_1 \cdot x_1^2$$

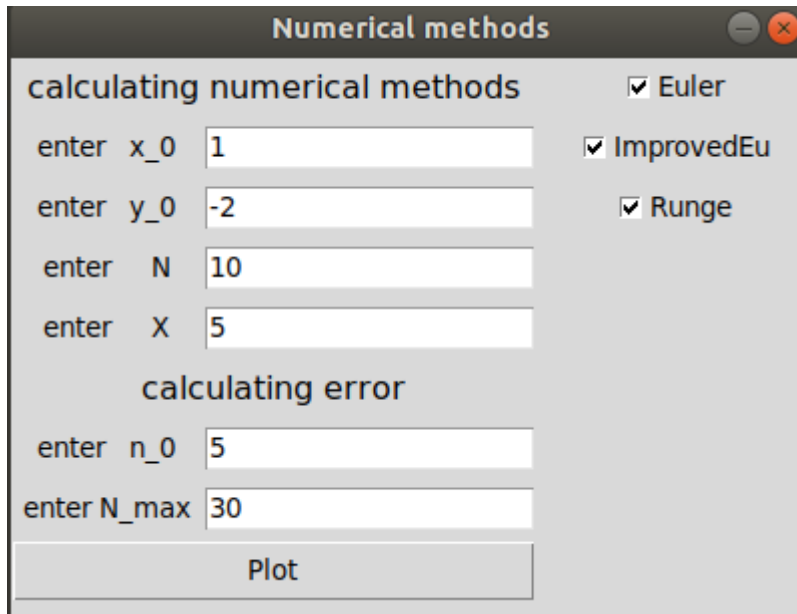
$$\frac{e^{-y_1} - x_1}{x_1^2} = c_1$$

Now let's look at what we obtained finally:

$$y = -\ln\left(\frac{e^{-y_1} - x_1}{x_1^2} \cdot x^2 + x\right)$$

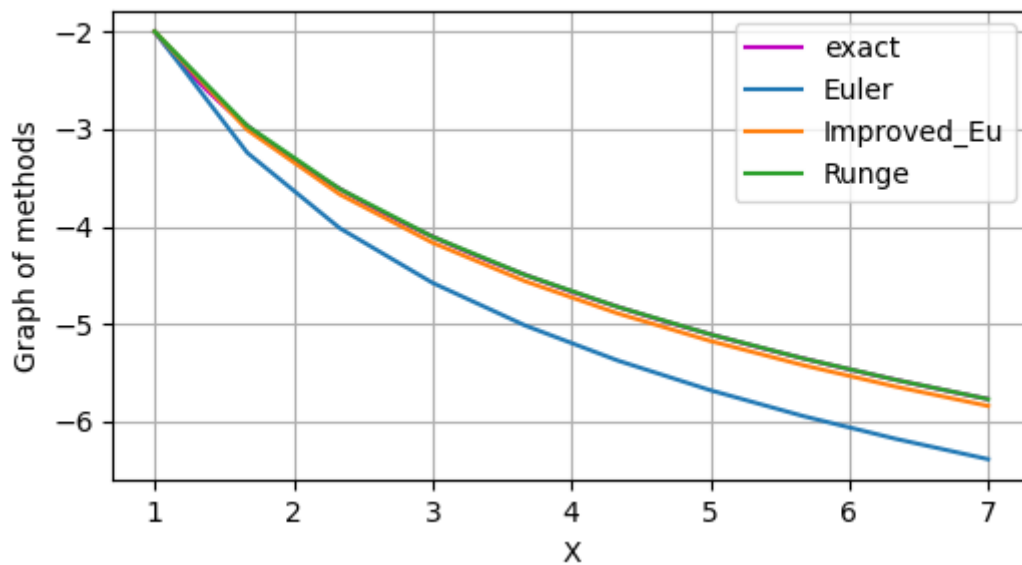
2. Computed Solution of Initial Value Problem

2.1 Plotting of exact function $y(x)$, and numerical methods



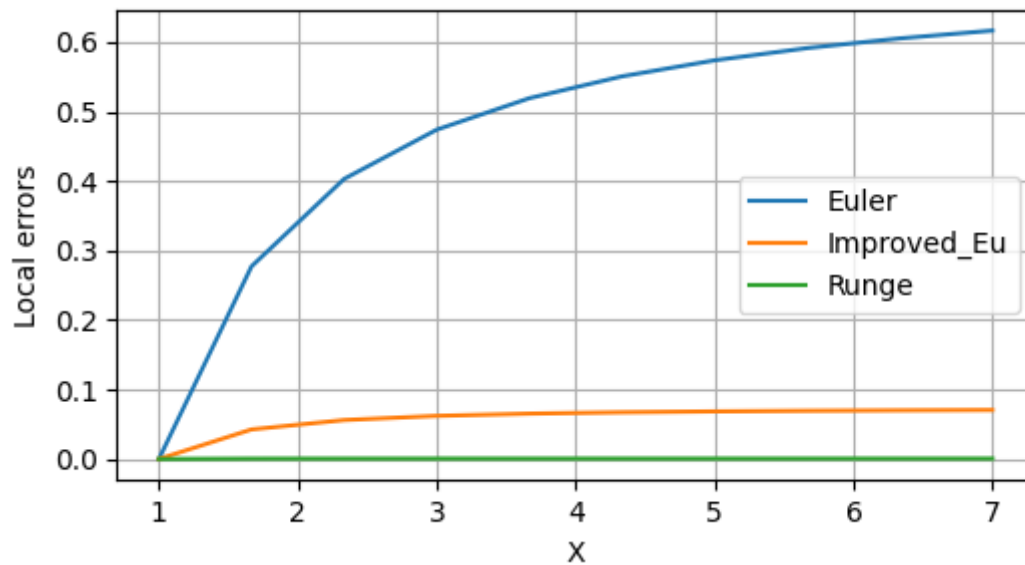
The screenshot shows a window titled "Numerical methods" with a standard Windows-style title bar (minimize, maximize, close buttons). The window is divided into two main sections. The top section, titled "calculating numerical methods", contains four input fields: "enter x_0" with value 1, "enter y_0" with value -2, "enter N" with value 10, and "enter X" with value 5. To the right of these fields are three checked checkboxes: "Euler", "ImprovedEu", and "Runge". The bottom section, titled "calculating error", contains two input fields: "enter n_0" with value 5 and "enter N_max" with value 30. At the bottom of the window is a large button labeled "Plot".

User interface.

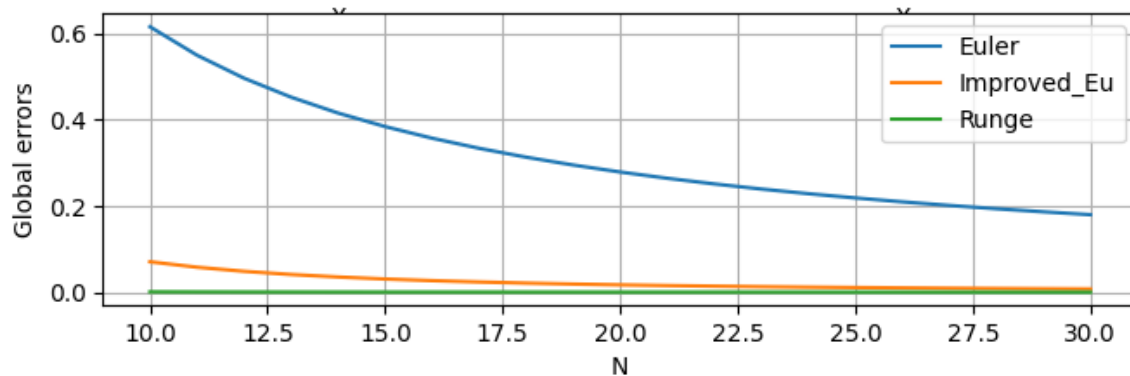


Graph of the implemented methods

2.2 Plotting of errors, on fixed steps and average of some section



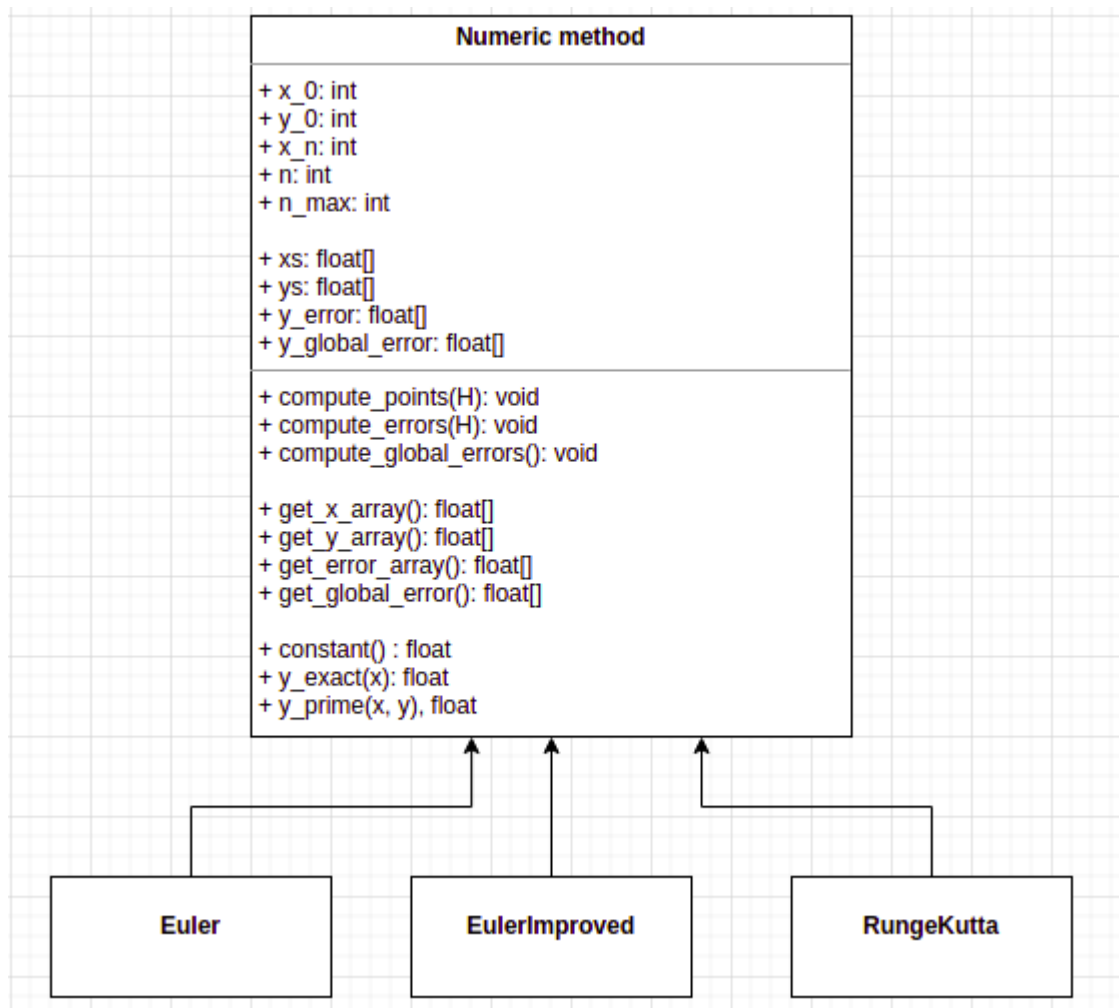
Graph of the errors with dividing the interval into 20 parts



Graph of the highest errors on interval $N \in \{10, 30\}$

3. Code comments

3.1 UML Diagrams



3.2 Comments

Most of the operations are done in `NumericMethod.py` and `Application.py`. Program is written in more simple way, but it can be used for solving other equation also, just needed to modify the solutions.

3.3 Source code

<https://github.com/bovvlet/Numeric-Methods>

4. Conclusion

As we can see, the Runge Kutta method is the most accurate. Also we can notice that if our N is big than method works better, which means that smaller steps guarantee a more accurate approximation.