Computational practicum

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1. Exact solution of Differential equation

Given Initial Value Problem:

$$\begin{cases} y' = f(x, y) \\ y(x_0) = y_0 \\ x \in (x_0; X) \end{cases}$$

Given Parts:

$$f(x, y) = e^y - \frac{2}{x}$$

$$y_0 = -2$$

$$x_0 = 1$$

$$X = 7$$

1.1 Find General Solution

Let’s write out equation in alternative way:

$$\frac{dy}{dx} = e^y - \frac{2}{x}$$

$$e^y - \frac{2}{x} - \frac{dy}{dx} = 0$$

$$\frac{e^y \cdot x - x \cdot \frac{dy}{dx} - 2}{x} = 0$$

$$e^y \cdot x - x \cdot \frac{dy}{dx} - 2 = 0$$

$$Let's\ make\ substiotution:$$

$$y = \ln\left(\frac{v}{x}\right)$$

$$\frac{dy}{dx} = \frac{x\left(\frac{\frac{dv}{dx}}{x} - \frac{v}{x^2}\right)}{v}$$

$$v - \frac{dy}{dx} \cdot x - 2 = 0$$

$$v - \frac{x^2\left(\frac{\frac{dv}{dx}}{x} - \frac{v}{x^2}\right)}{v} - 2 = 0$$

$$Simplify:$$

$$v(x) - \frac{x \cdot \frac{dv}{dx}}{v} - 2 = 0$$

$$Let's\ put\ x's\ and\ v's\ on\ opposite\ sides$$

$$\frac{dv}{(v-1)(v)} = \frac{dx}{x}$$

$$Integrate\ both\ sides:$$

$$\int \frac{dv}{(v-1)(v)} = \int \frac{dx}{x}$$

$$Evaluate\ the\ integrals:$$

$$\ln(-v + 1) - \ln(x) = \ln(x) + C_1$$

$$Let's\ expents\ of\ both\ sides:$$

$$e^{\ln(-v + 1) - \ln(x)} = e^{\ln(-v + 1) - \ln(x)}$$

$$v = \frac{1}{e^{C_1}x + 1} = \frac{1}{c_1 + 1}$$

$$Substitute\ back\ for\ y = \ln\left(\frac{v}{x}\right):$$

$$v = x \cdot e^y$$

And we got our general solution:

$$y = -\ln\left(C_1 \cdot x^2 + x \right)$$

1. 2 Let's solve initial value problem:

$$e^{-y_1} = \left(c_1 \cdot x_1^{\,2} + x_1 \right)$$

$$e^{-y_1} - x_1 = c_1 \cdot x_1^{\,2}$$

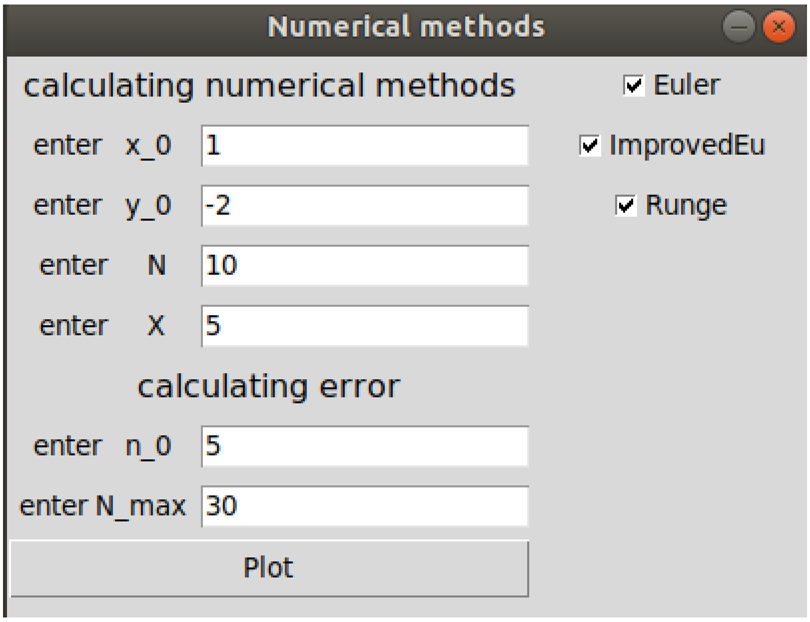
$$\frac{e_1^{-y} - x_1}{x_1^{\,2}} = c_1$$

Now let's look at what we obtained finally:

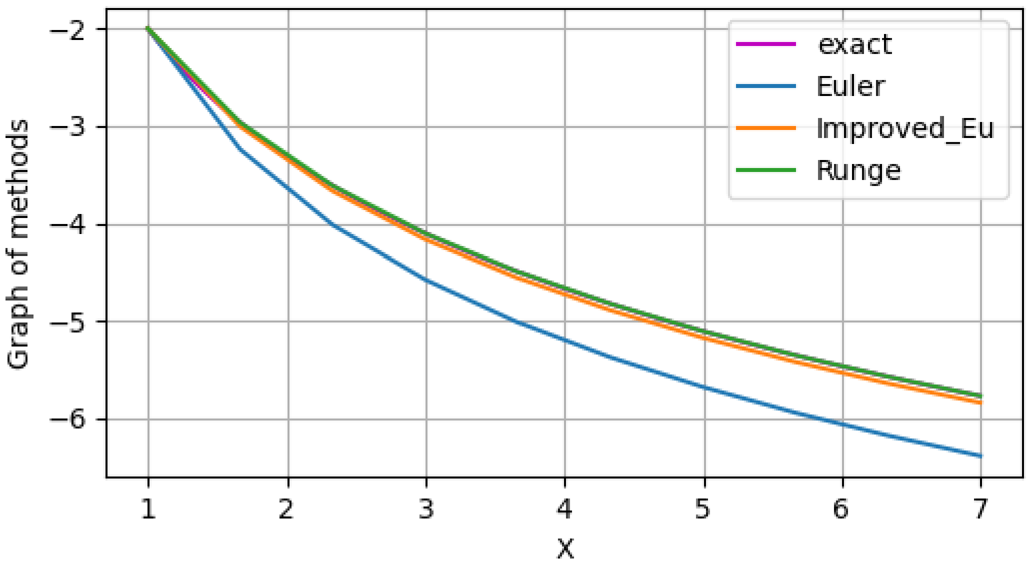
$$y = -\ln\left(\frac{e^{-y}{}_1 - x_1}{x_1^{\,2}} \cdot x^2 + x \right)$$

2. Computed Solution of Initial Value Problem

2.1 Plotting of exact function y(x), and numerical methods

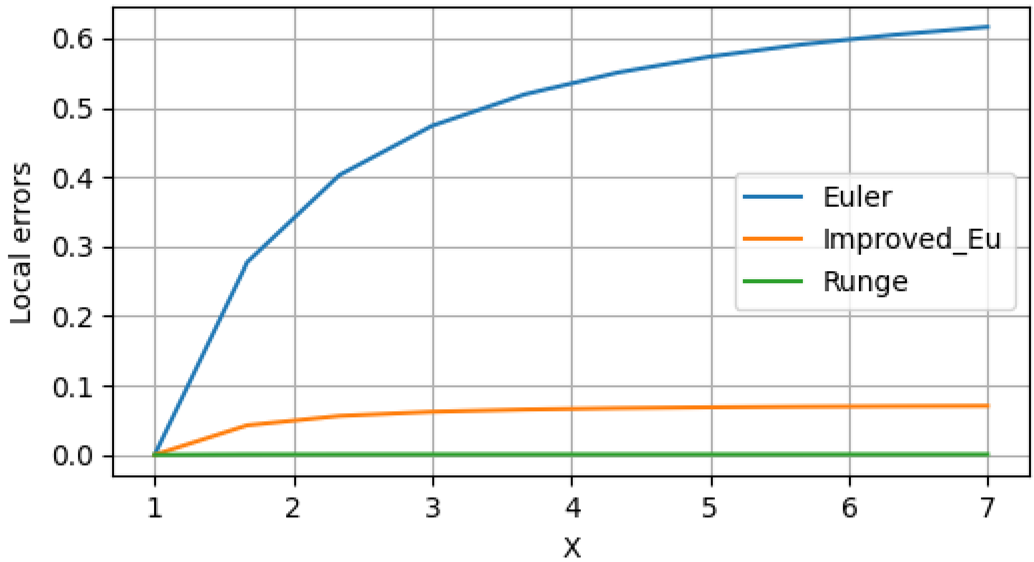


User interface.

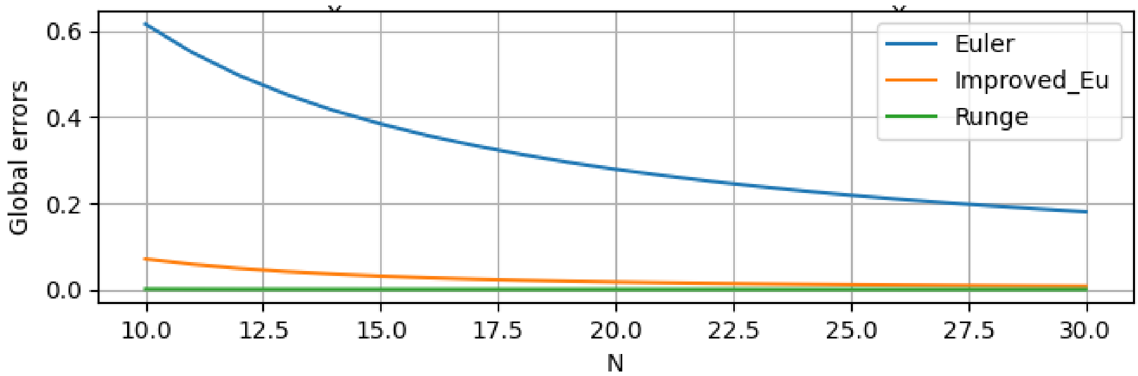


Graph of the implemented methods

2.2 Plotting of errors, on fixed steps and average of some section



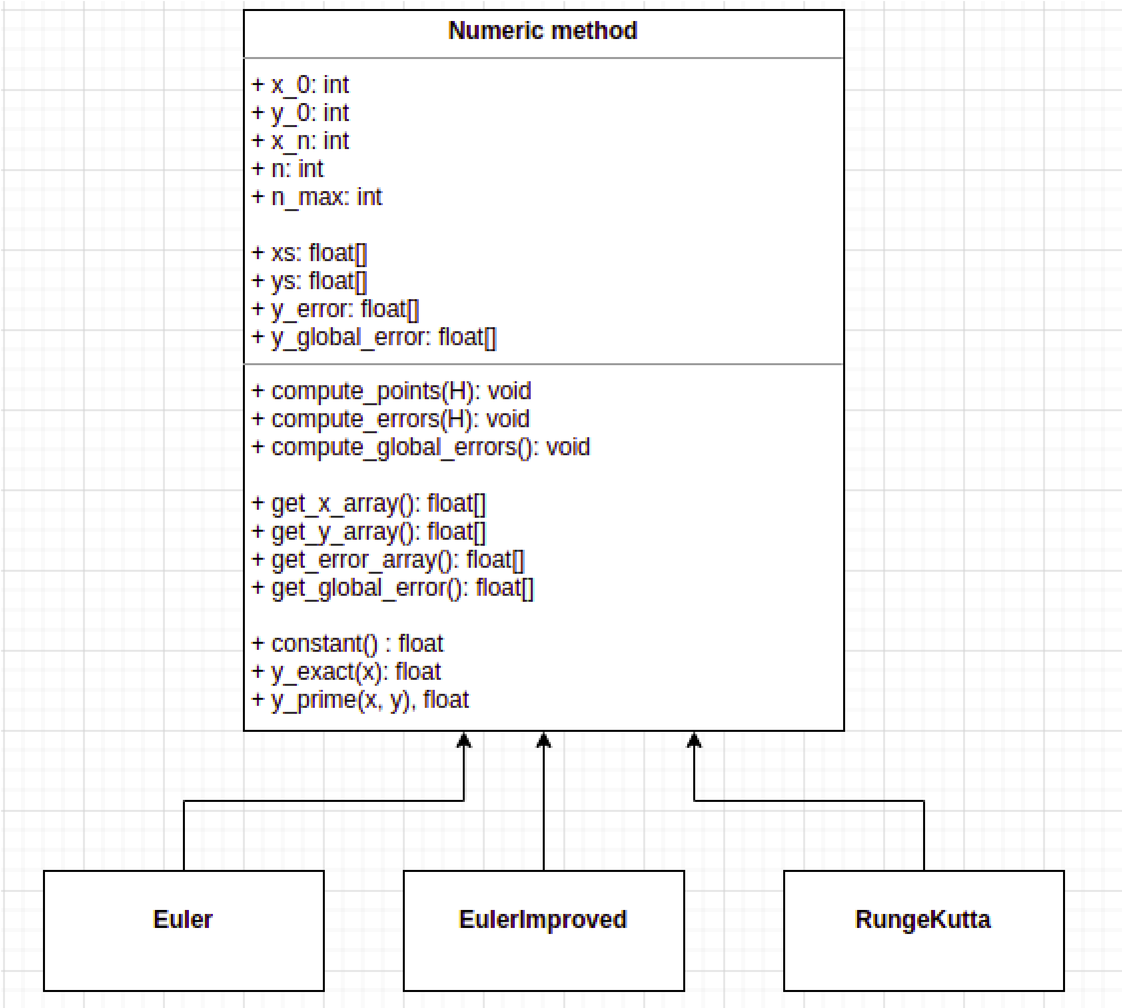
Graph of the errors with dividing the interval into 20 parts



Graph of the highest errors on interval N{10, 30}

3. Code comments

3.1 UML DIagrams



3.2 Comments

Most of the operations are done in NumericMethod.py and Application.py. Program is written in more simple way, but it can be used for solving other equation also, just needed to modify the solutions.

3.3 Source code

https://github.com/bovvlet/Numeric-Methods

4. Conclusion

As we can see, the Runge Kutta method is the most accurate. Also we can notice that if our N is big than method works better, which means that smaller steps guarantee a more accurate approximation.