MFDS – Make Up Answer Key

Q1 a) $L(S)$ contains S is quite obvious – 1 Mark.
To prove that it is smallest, consider any other subspace W of V that contains S. Now any linear combination of elements of S would be in W and hence $L(S)$ is contained in W, which shows that $L(S)$ is the smallest subspace of V containing S – 1 Mark.

Q1 b) Assume there is a linear transformation T from R^4 to R^3. Since Range(T) is a subset of R^3

$$\dim(\text{Range}(T)) \le 3 \longrightarrow 1 \text{ Mark}$$

Since $\dim(\text{Ker}(T)) = 0$, we have a contradiction to the Rank Nullity Theorem as $0 + \dim(\text{Range}(T))$ cannot be equal to 4. ⟶ 1 Mark.

Q1c) $e^{-i\theta}$ and $e^{i\theta}$ are EIGEN VALUES (1 Mark)
Evaluating two eigen vectors ⟶ ($0.5 \times 2 = 1$ Mark)

Q1d) 1 Mark each for pivot operations and 1 Mark for Back Substitution.

$$\left[\begin{array}{cccc|c} 0.02 & 0.01 & 0 & 0 & 0.02 \\ 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 100 & 200 & 800 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 0.02 & 0.01 & 0 & 0 & 0.02 \\ 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 100 & 200 & 800 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 1 \\ 0.02 & 0.01 & 0 & 0 & 0.02 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 100 & 200 & 800 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} \boxed{1} & \boxed{2} & \boxed{1} & \boxed{0} & \boxed{1} \\ 0.02 & 0.01 & 0 & 0 & 0.02 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 100 & 200 & 800 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 1 \\ 0 & -0.03 & -0.02 & 0 & 0 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 100 & 200 & 800 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & -0.03 & -0.02 & 0 & 0 \\ 0 & 0 & 100 & 200 & 800 \end{array}\right]$$

$\cancel{0} \quad \cancel{0} \quad \cancel{100}$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 0.04 & 0.03 & 0.12 \\ 0 & 0 & 100 & 200 & 800 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 100 & 200 & 800 \\ 0 & 0 & 0.04 & 0.03 & 0.12 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 100 & 200 & 800 \\ 0 & 0 & 0 & -0.05 & -0.20 \end{array}\right]$$

$\cancel{-0.20x}$ $-0.05x_4 = -0.20$

$\boxed{x_4 = 4}$

$100x_3 + 200x_4 = 800 \Rightarrow \boxed{x_3 = 0}$

$x_2 + 2x_3 + x_4 = 4. \Rightarrow \boxed{x_2 = 0}$

$x_1 + 2x_2 + x_3 = 1 \Rightarrow \boxed{x_1 = 1}$

Q2a) $\lfloor x \rfloor + \lfloor y \rfloor + \lfloor x + y \rfloor \leq \lfloor 2x \rfloor + \lfloor 2y \rfloor$ for all real numbers x and y.

Yes. (Can Prove)

If value of fractional part of either x/y is more than 0.5 $\rightarrow$ Right side becomes more than left hand side by 1.

If not, both LHS and RHS are same.

2b) Graph of function $f(x) = \left\lfloor 2\left\lceil \frac{x}{2} \right\rceil + \frac{1}{2} \right\rfloor$

$f(-6) = \left\lfloor 2\left\lceil -6/2 \right\rceil + \frac{1}{2} \right\rfloor = \lfloor 2(-3) + 1/2 \rfloor = \lfloor -5.5 \rfloor = -6$

$f(-4) = \left\lfloor 2\left\lceil \frac{-4}{2} \right\rceil + \frac{1}{2} \right\rfloor = \lfloor 2(-2) + 1/2 \rfloor = \lfloor -3.5 \rfloor = -4.$

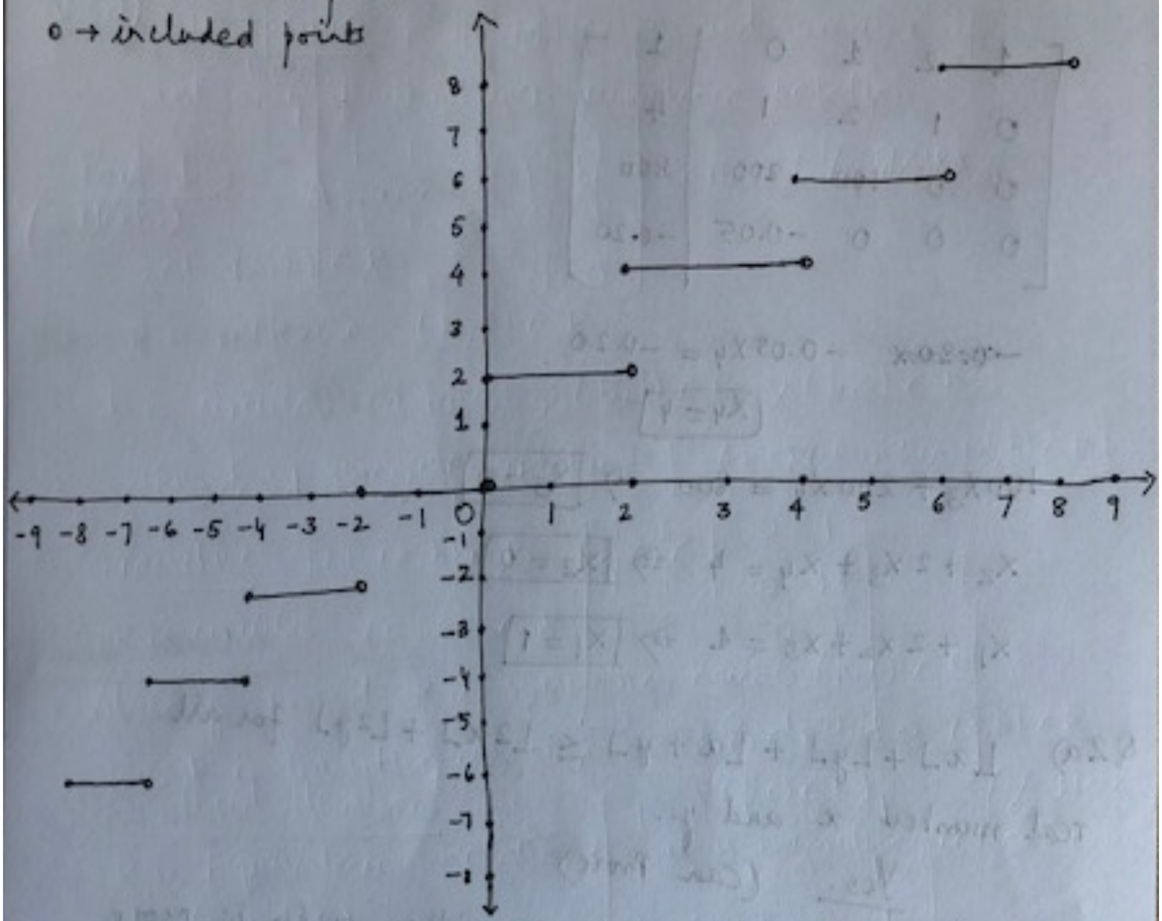
$f(-2) = -2$, $f(0) = 0$.

$f(2) = 2$.

$f(4) = 4$, $f(6) = 6$.

corresponding points of graph are $(-6,6)$, $(-4,4)$, $(-2,-2)$, $(0,0)$, $(2,2)$, $(4,4)$, $(6,6)$.

•→ Not included points
o → included points



2c) To find transitive closure on $\{a,b,c,d,e\}$ for relation $\{(a,b),(a,c),(a,e),(b,a),(b,c),(c,a),(c,b),(d,a),(e,d)\}$

$$W_0 = M_R = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

<u>First iteration</u> $\cancel{M_R} = \cancel{M_R \odot M_R}$.

W_1 has a 1 for (i,j)th element if W_0 has a 1 for (i,j)th element / if there is a path from a_i to a_j with $a_1 = a$ as only interior vertex.

$$W_1 = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \text{ — } (1/2 \text{ Mark})$$

Second iteration

$$W_2 = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

<u>Third iteration</u>

$$W_3 = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(1/2 Mark)

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \longrightarrow (1/2 \text{ Mark})$$

$$W_5 = M_R^* = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$\rightarrow$ (1/2 Mark)

Q2 d) To prove R is reflexive and circular if and only if R is an equivalence Relation.

Part 1 :- Assume R to be Reflexive & circular
(2 Marks)

Let $(a,b) \in R$

R is reflexive $(b,b) \in R$.

R is circular $(a,b) \in R$, $(b,b) \in R \Rightarrow (b,a) \in R$

Thus R is Symmetric relation $[\ (b,a) \in R$ when $(a,b) \in R]$

Transitive Let $(a,b) \in R$ $(b,c) \in R$.

R is circular $(c,a) \in R$.

Already proved R is symmetric hence $(a,c) \in R$

Thus R is transitive relation (as $(a,c) \in R$ when $(a,b) \in R$ & $(b,c) \in R$)

R is Equivalence Relation (as it is transitive, reflexive and symmetric)

Part 2 (2 Marks) Let R be equivalence Relation
↳ it is reflexive, symmetric, transitive.

Since R is transitive
$(a,c) \in R$.

R is symmetric $(c,a) \in R$.

Thus R is circular Relation $[\ (c,a) \in R$ whenever $(a,b) \in R$ and $(b,c) \in R]$

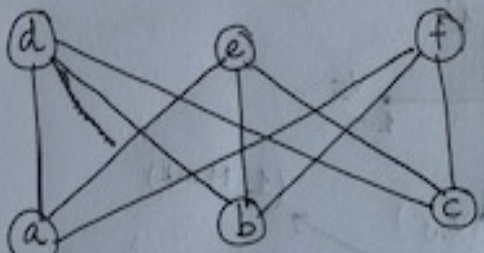
First Part → if R is reflexive & circular ⟹ R is equivalence Relation

Second Part → if R is equivalence Relation ⟹ R is reflexive & circular

Hence
R is reflexive & circular if & only if
R is equivalence Relation

Q3 a) Adjacency matrix of $K_{3,3}$ is

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \text{(1 Mark)}$$



$$A^2 = A \cdot A = \begin{bmatrix} 3 & 3 & 3 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 3 & 3 & 3 \end{bmatrix}$$

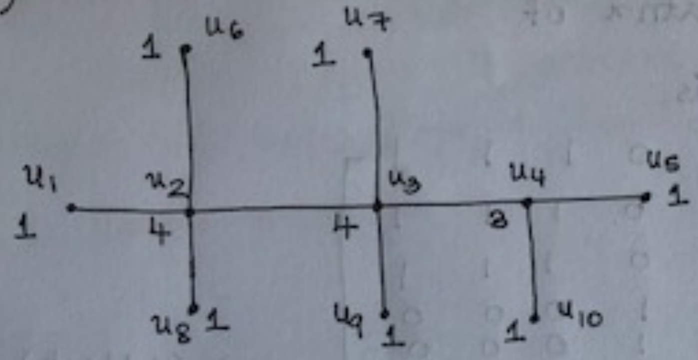
$\hookrightarrow$ (1 Mark)

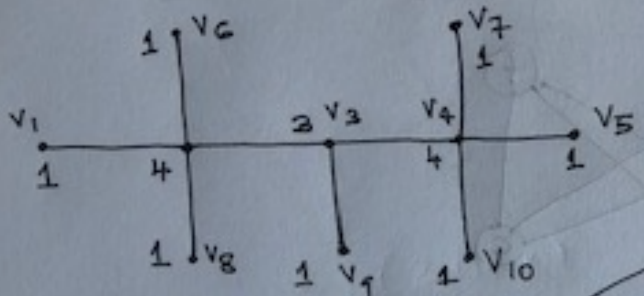
$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 0 & 0 & 0 & 9 & 9 & 9 \\ 0 & 0 & 0 & 9 & 9 & 9 \\ 0 & 0 & 0 & 9 & 9 & 9 \\ 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \end{bmatrix}$$

There are 9 paths of length 3 between adjacent vertices

3 b)





(1 Mark)

degree of vertices to be mentioned. Note in 1^{st} graph two adjacent vertices of degree 4 are there [1 Mark] but in 2^{nd} graph vertices of degree 4 are not adjacent

Hence NOT ISOMORPHIC [1 Mark]

3 c) $\deg(a) = 3$ $\quad \deg(d) = 3$ $\quad \deg(g) = 3$

$\deg(b) = 3$ $\quad \deg(e) = 3$ $\quad \deg(h) = 3$

$\deg(c) = 3$ $\quad \deg(f) = 3$ $\quad \deg(i) = 3$

$\deg(j) = 3$

↓

DIRAC's THEOREM is not satisfied

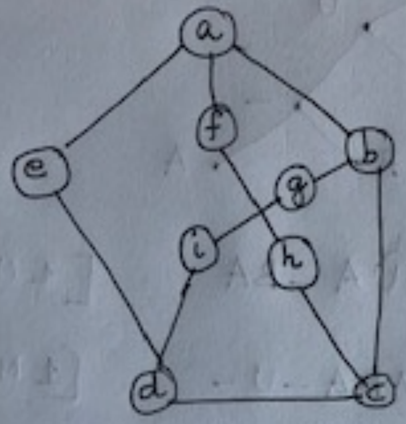
(degree below $n/2 = \frac{10}{2} = 5$)

[2 Marks]

Removing Vertex (2 Marks)

Original graph is SYMMETRIC and choosing any vertex will work / have a Hamilton circuit.

Remove vertex j



Results in Hamilton circuit.

Q4 a) Construct ordered rooted tree.

Given Preorder Traversal is a, b, f, c, g, h, i, d, e, j, k, l

a has 4 children

ROOT = a.

[Steps - 1 Mark]

a → children of a = b, c, d, e.

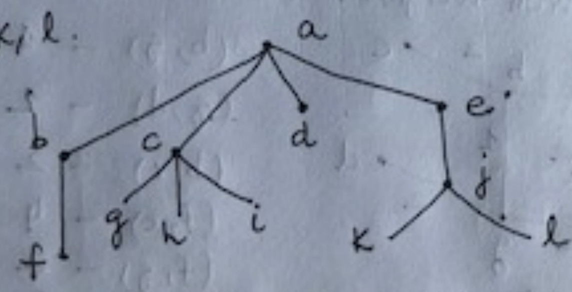
b → child f.

c → g, h. (children), i.

h does not have children.

i " " " "

j → k, l.

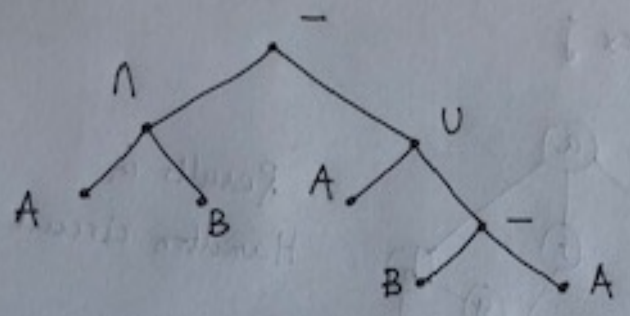


(2 Marks)

Ordered Rooted Tree

Q4b) $(A \cap B) - (A \cup (B - A))$

Ordered rooted tree



[½ Mark]

Prefix notation

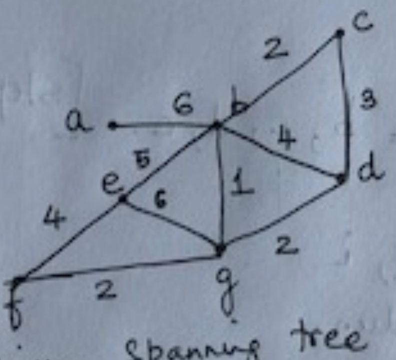
$- \cap A B \cup A - B A$ [1 Mark]

Postfix notation $A B \cap A B A - \cup -$ [1 Mark]

Infix notation $((A \cap B) - (A \cup (B - A)))$ [½ marks.]

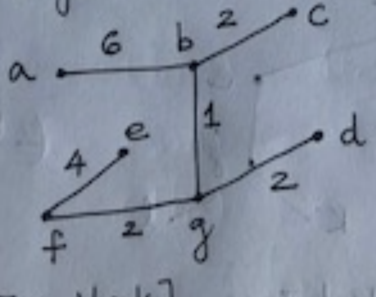
given expression fully parenthesized.

Q4c).



Find minimum spanning tree each method - 2 Marks.

choice of edges with weights — 1 mark. ← in that.



[1 Mark]

(b, g) − 1

(g, d) − 2

(b, c) − 2

(f, e) − 4

(f, g) − 2

(a, b) − 6

17

Table edges + Total weight (1 Mark)