Course Title   :   Data Structures and Algorithms
Course No   :   SS ZG 519
Total   :   50 marks
Nature of Exam   :   Open Book
Duration   :   3 hours
Date   :   07/11/2015 (AN)

No. of Pages = 2
No. of Questions = 5

Note:

1. Please follow all the Instructions to Candidates given on the cover page of the answer book.
2. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
3. Assumptions made if any, should be stated clearly at the beginning of your answer.

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1. (a) Suppose that each row of an $n \times n$ array A consists of 1's and 0's such that, in any row of A, all the 1's come before any 0's in that row. Assuming A is already in memory, describe a method running in $O(n)$ time for finding the row of A that contains the most 1's.   (4 *Marks*)

   (b) Take the following functions and arrange them in ascending order of growth rate.

   $$2^{\sqrt{\log n}} \qquad 2^n \qquad 2^{2^n} \qquad n^{4/3}$$
   $$n(\log n)^3 \qquad n^{\log n} \qquad 2^{n^2}$$

     (3 *Marks*)

   (c) The integers 1, 2, 3, 4 and 5 are to be inserted into an empty stack using the following sequence of push() operations:

   push(1) push(2) push(3) push(4) push(5)

   Assume that pop() removes an element from the stack and outputs the same. Which of the following output sequences can be generated by inserting suitable pop() operations into the above sequence of push() operations? Justify your answer.

   1. 5 4 3 2 1
   2. 1 2 3 4 5
   3. 3 2 1 4 5
   4. 5 4 1 2 3.

     (3 *Marks*)

2. (a) Illustrate the performance of selection sort algorithm on the following input sequence:
   2,5,16,4, 10, 23, 39, 18, 26, 15   (2 *Marks*)

   (b) Provide average case analysis for insertion sort.   (3 *Marks*)

   (c) Illustrate the performance of heap sort algorithm on the following input sequence:
   2,5,16,4, 10, 23, 39, 18, 26, 15   (3 *Marks*)

   (d) Give an $O(n \log k)$-time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists.   (4 *Marks*)

3. Draw the 11-item hash table resulting from hashing the keys 23, 55, 24, 99, 34, 105, 22, 50, 31, 27, 16

   (a) Using the hash function $h(i) = (2i + 5) \mod 11)$ and assuming collisions are handled by chaining.   (3 *Marks*)

(b) Using the same hash function but collisions are handled by linear probing. (*2Marks*)

(c) Using the same hash function but collisions are handled by quadratic probing. (*2Marks*)

(d) Using the same hash function but collisions are handled by double hashing with secondary hash function (*3Marks*)
$h'(k) = 7 - (k \mod 7)$.

4. (a) Insert items with the following keys (in the given order) into an initially empty binary search tree: 31, 41, 25, 59, 29, 27, 12, 14. Draw the tree after each insertion. (*3Marks*)

(b) Remove the root node from the binary search tree constructed in the previous part (*2Marks*)

5. (a) Write a pseudocode, which runs in $O(nlogn)$, to count the number of inversions in a given sequence of numbers $a_1, a_2, \ldots, a_n$. We call the pair (a_i, a_j) is an inversion if $a_i > a_j$ and $i < j$. (*5Marks*)

(b) Construct the adjacency matrix and adjacency list for the following graph.
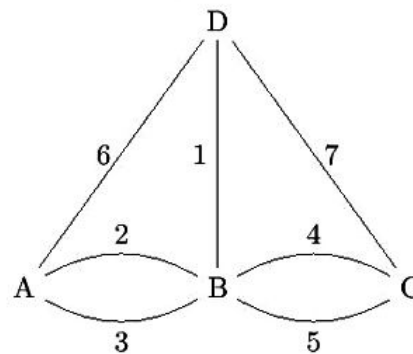
(*4Marks*)

(c) Construct a simple, connected, weighted graph with 8 vertices and 14 edges and each with unique edge weights. Identify one vertex as a start vertex and illustrate a running on Dijkstra's algorithm on this graph. (*4Marks*)