Bow Simulator v0.5

User Manual

Contents

1 Introduction

About this Manual

This is the user manual for Bow Simulator, a software tool for bow and arrow physics simulation. It shows how to use the program, explains the various input parameters and output results and also contains some additional background information.

For the latest version of the software and this manual visit the project's website at
`http://bow-simulator.org/`.

Support

If you need help, want to report a problem or give feedback you can either use the mailing lists on the website or contact the author at
`s-pfeifer@gmx.net`.

2 The Bow Editor

The bow editor is the main window of the application. Here you can load, edit and save bow models and start static and dynamic simulations.

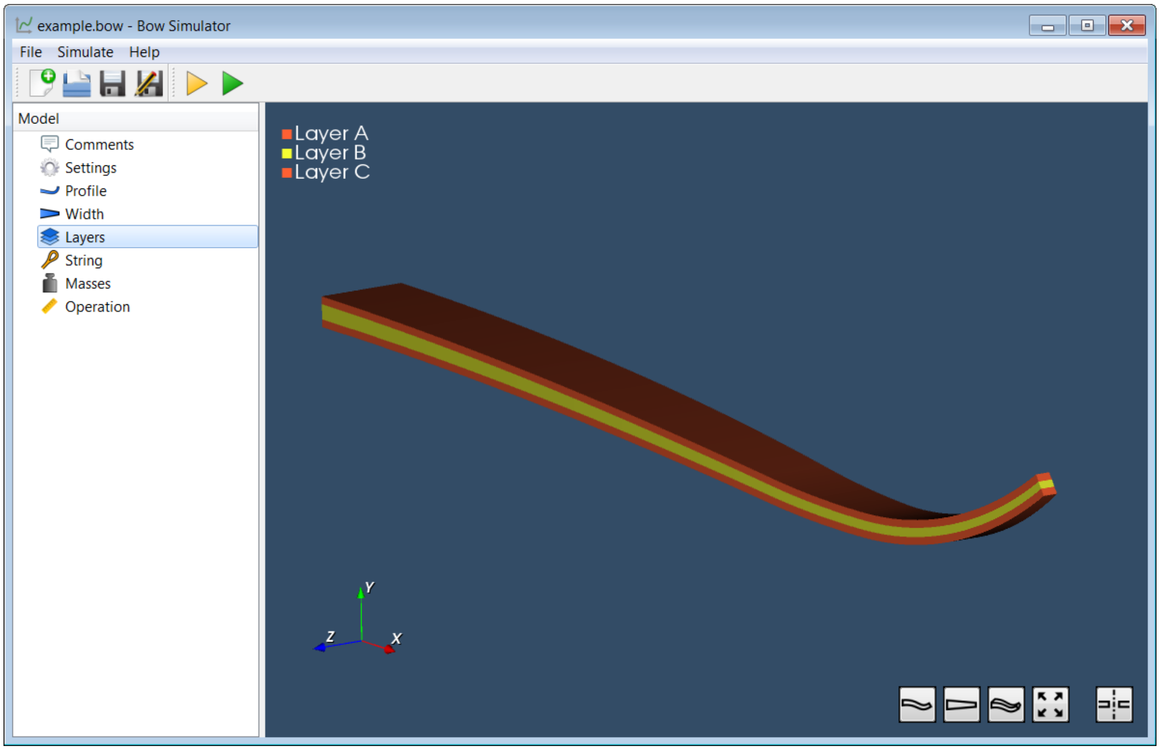


Figure 1: Bow Editor

Double-click any of the list items on the left to edit the respective model properties. Each of those will open an associated input dialog. Those will be explained in more detail in the following sections.

The 3D view on the right shows the current limb geometry. Use the mouse to rotate, zoom and shift the view. More view options are available through the buttons on the bottom-right corner.

2.1 Profile

The profile curve is the geometric centerline of the bow in unbraced state. Use the table on the left to edit the parameters, the resulting curve is shown on the plot on the right.

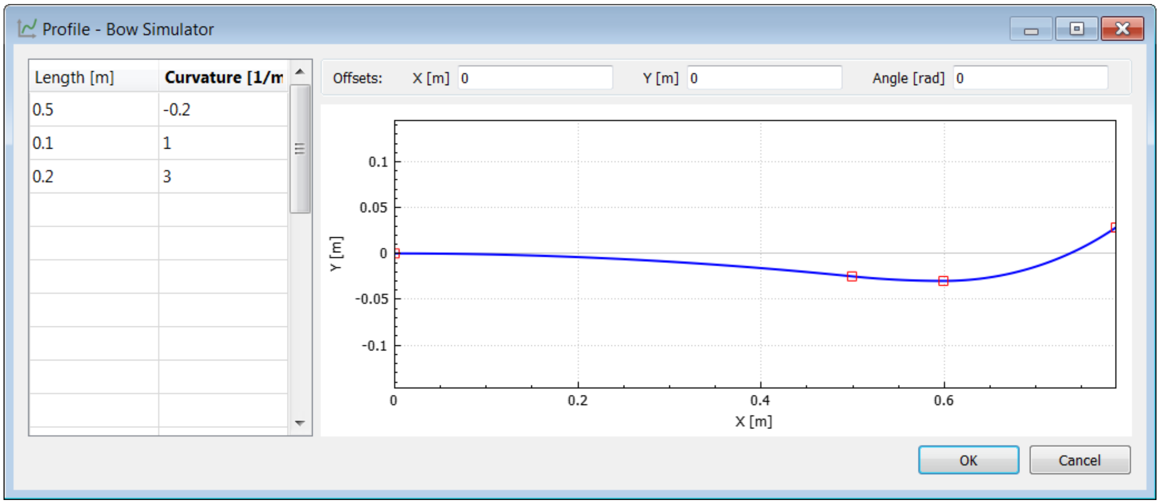


Figure 2: Profile dialog

The profile curve consists of a series of arc segments that each have a certain length and curvature. Each row in the table defines one such arc segment.

Note: Mathematically, the curvature κ of a segment is the reciprocal of its radius r, so you can calculate the curvature via $\kappa = \frac{1}{r}$ if you know the radius and vice versa.

With the text fields on the top right you can specify x-, y- and angular offsets that control the starting point and orientation of the profile curve. Those can be used to account for a stiff middle section/riser.

2.2 Width

The width dialog defines the limb's width along its profile curve. This width is the same for all layers of the bow.

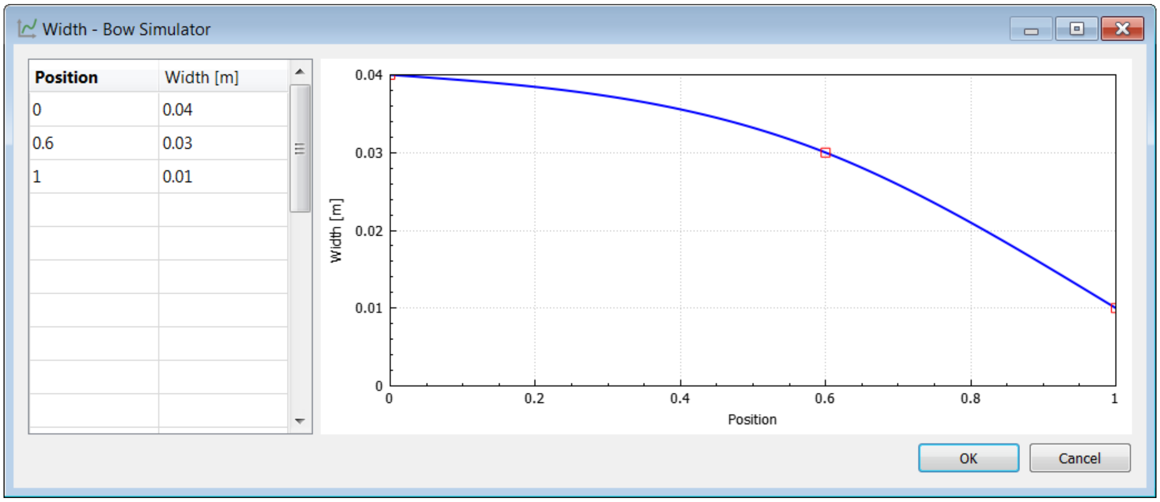


Figure 3: Width dialog

On the table on the left you can specify values for the width at certain relative positions (between 0 and 1) along the limb's profile curve. This definition of cross section properties relative to the total length of the profile curve makes it possible to change the profile without having to adjust any cross sections.

The actual width distribution of the limb is constructed as a smooth curve (cubic spline) passing through the supplied values as shown on the plot on the right.

2.3 Layers

With the layer dialog you can create any number of layers and specify their height/thickness and material properties.

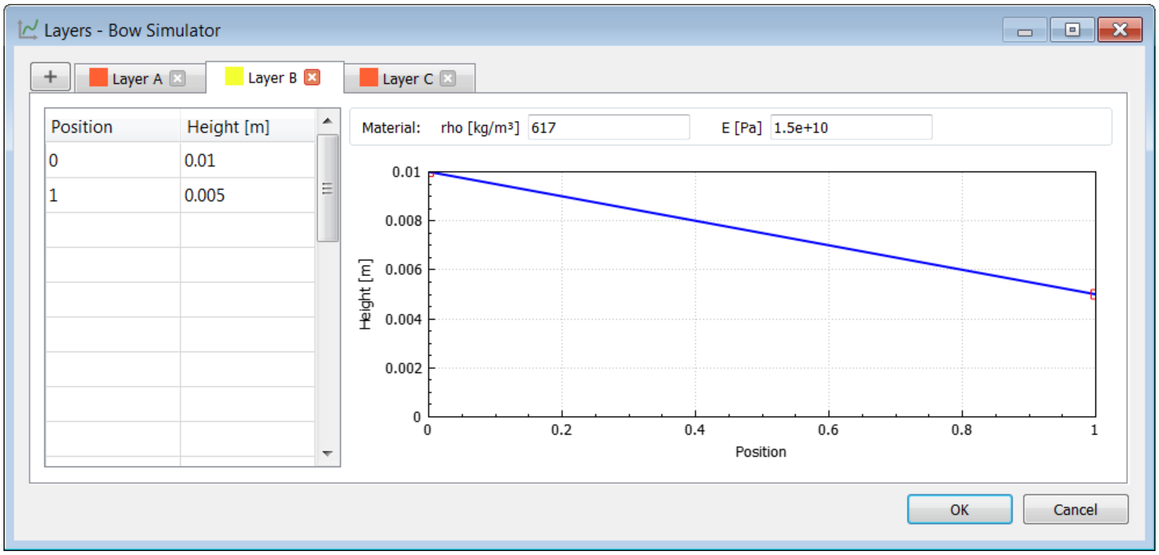


Figure 4: Layer dialog

Click the plus button on the top left to add layers. You can later rearrange them by dragging the tabs. Double-click on the tab to rename a layer.

The table on the left sets the height distribution of the layer. It works the same way as the limb's width: You specify a number of values at different relative positions and the program creates an interpolating curve.

The layer material is specified by the following two constants,

- **rho:** Density (Mass per unit volume)
- **E:** Elastic modulus (Measure for the stiffness of a solid material)

For manufactured materials like e.g. fiberglass or steel you might find those numbers in a datasheet provided by the manufacturer.

Wood is a bit more difficult, because the properties can vary quite a bit even within the same species of tree. You can find average numbers on the internet, for example at `http://www.wood-database.com`. Those are probably good enough as a first reference. However, in order to be really sure about a specific piece of wood there is no other way than to test it. One possibility is a bending test as shown in Appendix D.

2.4 String

Here you can define the mechanical properties of the string by providing data for the string material and the number of strands being used.

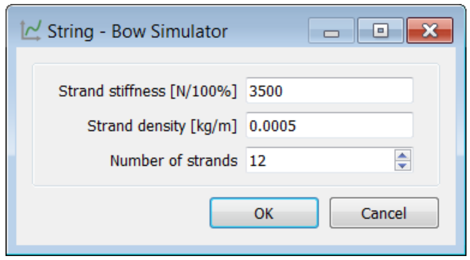


Figure 5: String dialog

- **Strand density:** Linear density of the strands (mass per unit length)
- **Strand stiffness:** Stiffness of the strands against elongation (force per unit strain)
- **Number of strands:** Total number of strands in the string

The linear density of a string material can be easily determined with a kitchen scale (weight divided by length), but the stiffness is much more difficult to obtain. Table 1 shows some reference values taken from the SuperTiller V6.6 Excel spreadsheet by Alan Case[1].

Note: The stiffness of the string material is an important parameter in dynamic analysis. The static results however aren't affected very much by it as long as it is high enough to prevent significant elongation.

Material	**Stiffness** [N/100%]	**Density** [kg/m]	**Breaking strength** [N]	**Source/Comment**
Dacron B50	3113.76	0.000333	217.96	BCY assuming 7% elongation at break (linearized)
Fast Flight	14086.04	0.000182	422.58	BCY assuming 3% elongation at break (linearized)
Dyneema	18860.46	0.000160	578.27	Calculated assuming linear stress-strain (to break)
Linen 40/3	1668.08	0.0642	49.38	Maurice Taylor, Archery The Technical Side, 1947
Silk	1026.51	0.0930	65.83	Maurice Taylor, Archery The Technical Side, 1947

Table 1: Properties of common bowstring materials according to SuperTiller V6.6

[1] `http://www.buildyourownbow.com/build-alongs/how-to-use-supertiller-build-along/`

2.5 Masses

These are used to account for the various dead weights that can be attached to a bow.

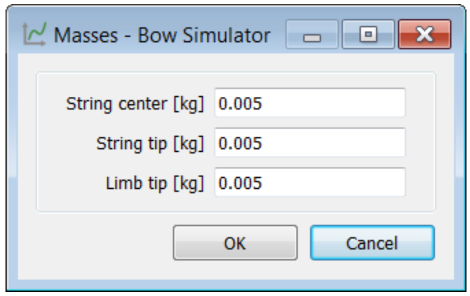


Figure 6: Masses dialog

- **String center:** Additional mass at the string center (serving, nocking point)
- **String tip:** Additional mass at the ends of the string (serving, silencers)
- **Limb tip:** Additional mass at the limb tips (overlays and the like)

2.6 Operation

Here you can find all parameters that define the conditions under which the bow operates.

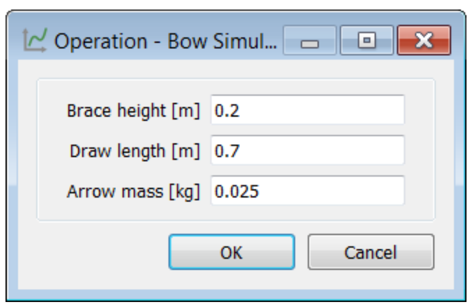


Figure 7: Operation dialog

- **Brace height:** Distance of the string center to the coordinate origin ($x = 0$, $y = 0$) in braced state
- **Draw length:** Distance of the string center to the coordinate origin in fully drawn state
- **Arrow mass:** Total mass of the arrow

2.7 Comments

The comments are meant for documenting the bow model. Any notes about the bow and the simulation results can be added here.

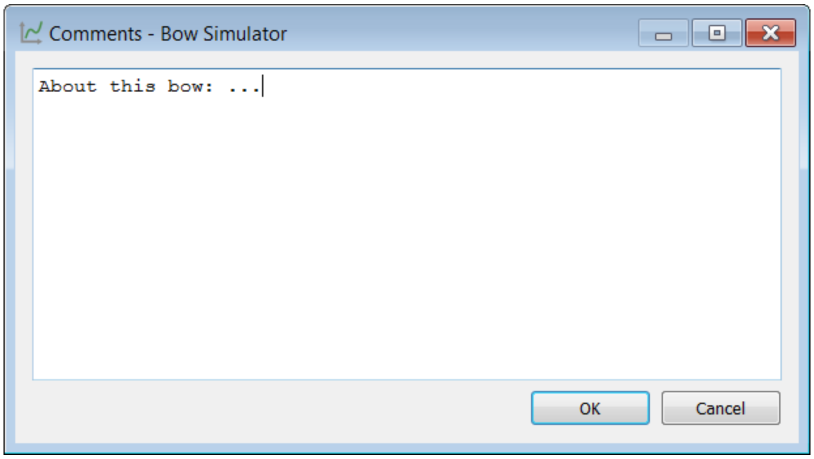


Figure 8: Comments dialog

2.8 Settings

These are numerical settings used by the simulation. You can do some fine-tuning here, but most of the time the default values should be fine. However, as the default values favor accuracy and realiability over performance there might be use cases where finding faster settings is worth it. Think about running a large number of simulations in batch mode for example.

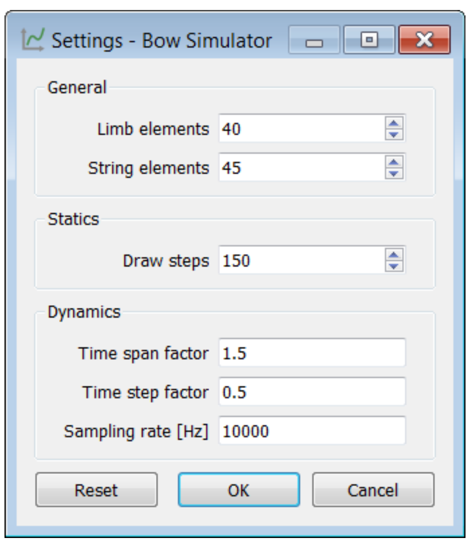


Figure 9: Settings dialog

These are the individual settings:

- **General**
  - **Limb elements:** Number of finite elements that are used to approximate the limb. More elements increase the accuracy but also the computing time.
  - **String elements:** Same as above.
- **Statics**
  - **Draw steps:** Number of steps that are performed by the static simulation from brace height to full draw. This determines the resolution of the static results. You can usually decrease this value to speed up the simulation.
- **Dynamics**
  - **Time span factor:** This controls the time period that is simulated. A value of 1 corresponds to the time at which the arrow reaches brace height. The default value is larger than that in order to capture some of the things that occur after the arrow left the bow (for example the maximum dynamic loads on limb and string).
  - **Time step factor:** When carrying out the dynamic simulation the program will repeatedly use the current state of the bow at time t to calculate the next state at time $t + \Delta t$ where Δt is some small timestep. This timestep has to be chosen small enough to get an accurate and stable solution but also as large as possible to keep the total number of steps low. The program can estimate this optimal timestep, but to be on the safe side the estimation is reduced by a safety factor between 0 and 1 that you can choose here.
  - **Sampling rate:** Limits the time resolution of the output data. This is done because the dynamic simulation usually produces much finer grained data than is actually useful. Not including all of that in the final output saves memory and computing time.

3 Simulation Results

You can start static or dynamic simulations by using the yellow and green toolbar buttons or the simulation menu. Once the simulation has finished, a new window with the results will open.

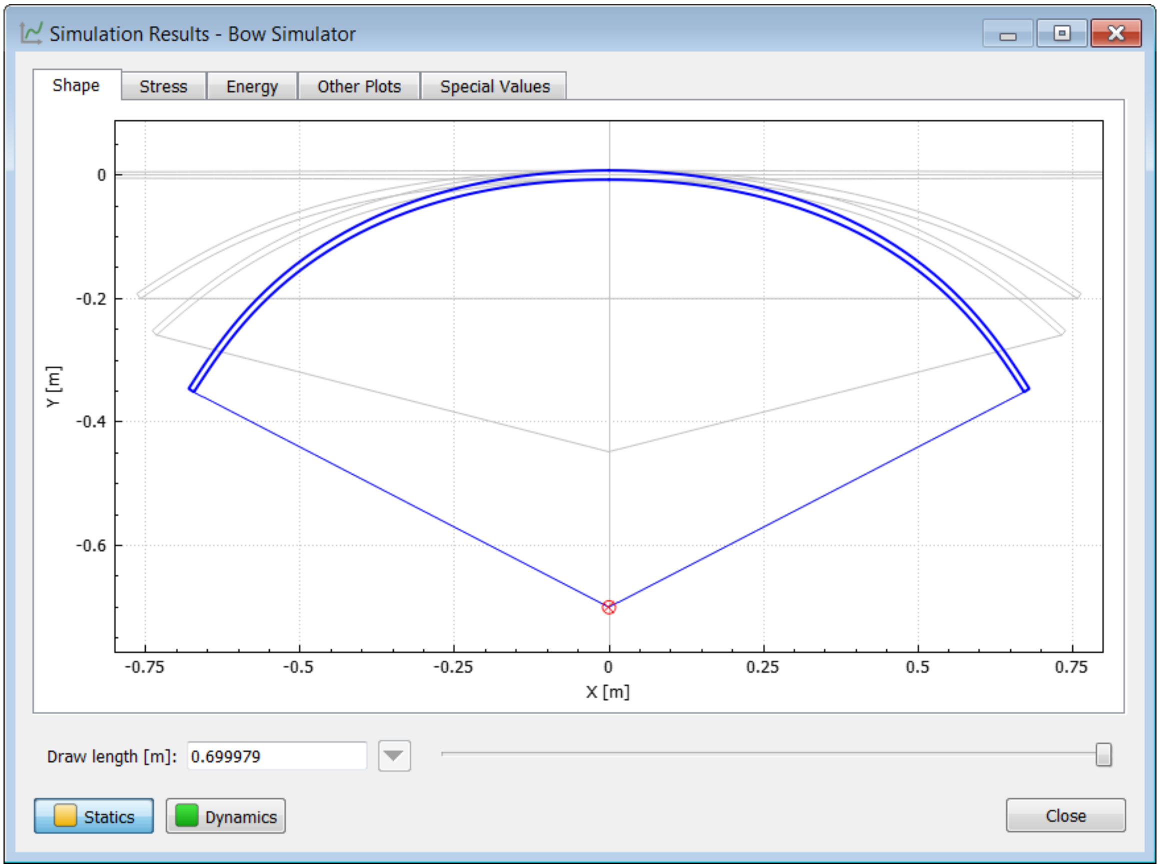


Figure 10: Simulation results

With the buttons on the bottom left you can switch between the static and dynamic (if available) results. The results themselves are organized in different tabs. At the bottom of the window there is a slider where you can change the current draw length (statics) or time (dynamics). This value applies to all of the result tabs.

These are the contents of the individual result tabs:

- **Shape:** Shows the shape of the limb and string as well as the position of the arrow at different stages of either the draw (statics) or the shot (dynamics)
- **Stress:** Shows the distribution of material stress along the length of the limb for the back and belly of each layer
- **Energy:** Shows how potential and kinetic energy of the different parts of the bow develop during the simulation
- **Other Plots:** Here you can combine arbitrary simulation results and plot them together, e.g. things like the draw curve of the bow or the velocity of the arrow.

- **Special Values (Statics):**
  - **String length:** Initial length of the string such that the bow meets the specified brace height
  - **Final draw force:** Draw force in fully drawn state
  - **Drawing work:** Total work done by drawing the bow.
  - **Storage ratio:** This is an indicator of the bow's capability to store energy and is defined (/made up by the author) as

    $$\text{storage\_ratio} = \frac{\text{drawing\_work}}{1/2 \cdot \text{draw\_force} \cdot (\text{draw\_length} - \text{brace\_height})}.$$

    It describes the amount of energy stored by the bow's draw curve in relation to a fictious linear draw curve with the same final draw force.

- **Special Values (Dynamics):**
  - **Arrow velocity:** Velocity of the arrow when leaving the bow
  - **Arrow energy:** Kinetic energy of the arrow when leaving the bow
  - **Efficiency:** Degree of efficiency of the bow. Useful energy output (kinetic energy of the arrow) divided by energy input (static drawing work).

4 Command Line Interface

The command line interface can be used to start simulations in batch mode, without opening the GUI. This way Bow Simulator can be called from other programs for performing more advanced computations like parameter studies and optimizations.

The command line parameters are as follows:

bow-simulator [input] [output] [options]

- **input:** Path to an input file (`.bow`)
- **output:** Path for the output file (`.dat`)
- **options:** Simulation options, `--static` or `--dynamic`

All of the arguments are optional. Calling Bow Simulator with either no arguments or only an input file will open the GUI. If in addition to that either an output file or simulation options are provided, the simulation is carried out silently and the results are written to disk. If not specified, a default output file named after the input file is created.

Note: To use the command line interface on Windows you have to either use the complete path to the `bow-simulator.exe` executable or add the installation directory to your `PATH` environment variable.

Input and Output Files

Bow Simulator's input files use the JSON[1] format, a human readable text format that stores different types of data in a hierarchical way. The output files use MessagePack[2], a more compact binary format that is otherwise very similar to JSON. Both are very common formats with implementations available in many programming languages. An example for using Bow Simulator with Python can be found in Appendix C.

The exact definition and layout of the data contained in the input and output files is documented in Appendix A and B, respectively. However please note that those aren't stable yet. Future releases are very likely to introduce breaking changes.

[1] `http://json.org/`
[2] `http://msgpack.org/`

5 Background Information

5.1 The Internal Bow Model

This section is intended to give interested users an overview of the mathematical bow model behind Bow Simulator, i.e. how the different components of the bow are modeled and what the assumptions and limitations are. This section will eventually be replaced by a separate technical documentation of the simulation model.

Limb: The limb is regarded as an Euler-Bernoulli beam. This means that all cross-sections of the beam are assumed to stay flat and perpendicular to the beam axis during deformation. The Euler-Bernoulli beam theory therefore only accounts for bending deformation and neglects shear deformation, which is usually a valid thing to do for long, slender beams.

The material of the limb is considered linear-elastic, so the relation between material stress σ and strain ϵ at any point in the limb is given by the linear equation $\sigma = E \cdot \epsilon$ with the elastic modulus E as a material constant. The overall behaviour of the limb however is nonlinear due to the nonlinear kinematics/geometry of large deformations.

String: Contrary to the limb, the string only transfers longitudinal forces and has no flexural rigidity. The material is considered linear-elastic as well. The string has a constant cross section and is internally implemented as a chain of point masses connected by springs. Additional point masses at the center and the tips represent things like servings and nocking point.

Arrow: The arrow is modeled as a point mass. Deformation and vibration of the arrow (known as *archers paradox*) is neglected/not captured by this model. That's because the scope of this program is only to evaluate overall bow performance, things like final arrow velocity, degree of efficiency, etc. For this purpose a point mass is sufficient.

Symmetry: The bow is assumed to be symmetric. This is often only an approximation as most bows besides crossbow prods are actually slightly asymmetric. The assumption of symmetry simplifies the definition of the parameters by the user (no need to define the limb twice). It also allows the program to simulate only one half of the bow, which reduces the computing time. (As a user you don't have to take this into account, all input and output data of the program corresponds to the complete bow.)

5.2 Validation of Simulation Results

An important task for the development of this software is to make sure the simulation results agree reasonably well with real world examples. This section shows the validation efforts made so far. It is still very sparse, so if you have used this program for a real world application, let me know about your results and they will be added here.

Statics of a straight steel bow

In this experiment the draw curve and limb shapes of a small steel bow with a constant cross section have been measured and compared to version 2014.4 of Bow Simulation Tool (now Bow Simulator). The bow is shown in figure 11 and has been made from an old saw blade. Steel is a good material for this kind of test because of its homogenous mechanical properties. The elastic modulus was assumed to be $E = 210\ \mathrm{GPa}$ which is a good estimate for most types of steel.

Figure 11: Steel bow. Cross section: 16.85 $\times$ 0.75mm. Length: 269mm. Brace heighth: 49.8mm

The experiment was carried out by hanging a plastic bag at the string center. The draw force was then gradually applied by counting steel balls with a known mass into the bag. After every load step the draw length was measured and a photo of the bow was taken.

Figure 12 shows a comparison between the measured and the simulated draw curve and figure 13 compares the pictures of the limb against the simulated limb shapes.

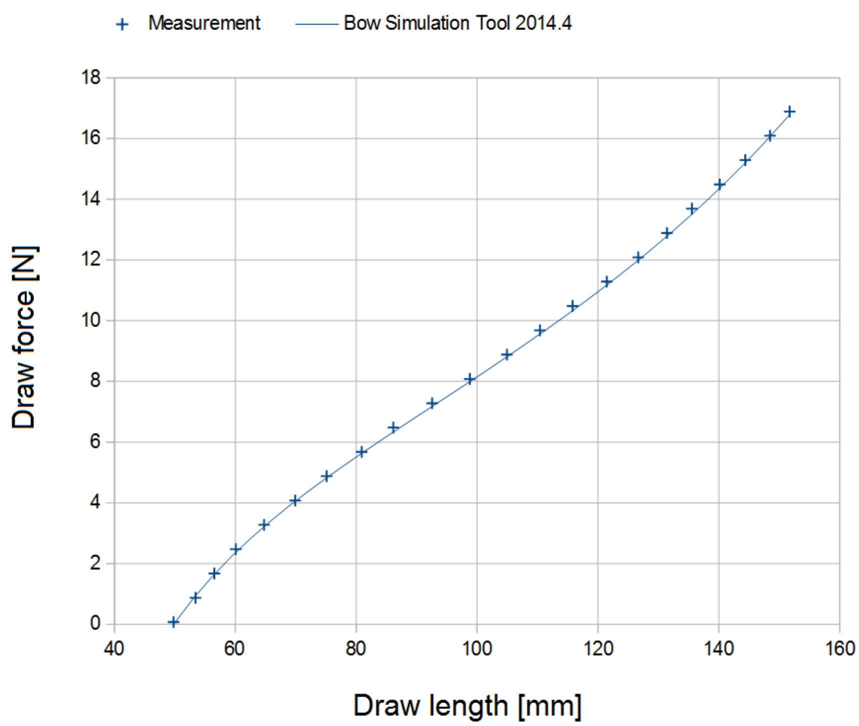


Figure 12: Experimental and simulated draw curves

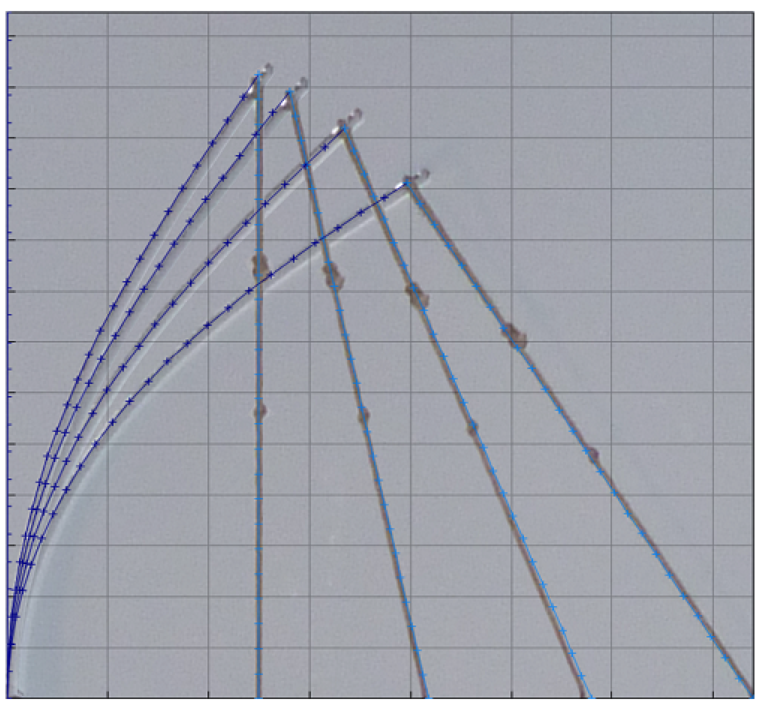


Figure 13: Experimental and simulated limb shapes

The agreement between experiment and simulation is surprisingly good here. It really shows the potential of such kinds of simulations, provided that the material properties are well known and a the bow can be built exactly as simulated, with low tolerances.

But this is still a very simple example. The next step would be to repeat this experiment with bows that have varying cross sections and non-straight profiles. Another open question are the dynamic simulation results. It's unclear if they can match experiments as good as the static results do, because there are much more uncertainties involved.

A Input File Structure

Field	Type	Units	Description
meta			
version	`string`	–	Internally used version string
comments	`string`	–	User comments
settings			
n_limb_elements	`integer`	–	Number of limb elements
n_string_elements	`integer`	–	Number of string elements
n_draw_steps	`integer`	–	Number of steps for the static simulation
time_span_factor	`double`	–	Factor for modifying total simulation time
time_step_factor	`double`	–	Factor for modifying simulation time steps
sampling_rate	`double`	Hz	Time resolution for the dynamic output
profile			
segments	`double[][]`	m, 1/m	Table with segment lengths and curvatures
angle	`double`	rad	Angular offset of the profile curve
x_pos	`double`	m	X offset of the profile curve
y_pos	`double`	m	Y offset of the profile curve
width	`double[][]`	–, m	Table with positions and widths
layers			
{			
name	`string`	–	Name of the layer
height	`double[][]`	–, m	Table with positions and heights
rho	`double`	kg/m^3	Density of the layer material
E	`double`	Pa	Elastic modulus of the layer material
}			
{ ... }			
string			
strand_stiffness	`double`	N	Stiffness of the string material
strand_density	`double`	kg/m	Density of the string material
n_strands	`integer`	–	Number of strands
masses			
string_center	`double`	kg	Additional mass at string center
string_tip	`double`	kg	Additional mass at string tips
limb_tip	`double`	kg	Additional mass at limb tips
operation			
brace_height	`double`	m	Brace height
draw_length	`double`	m	Draw length
arrow_mass	`double`	kg	Arrow mass

B Output File Structure

P: Number of limb nodes
Q: Number of string nodes
R: Number of layer nodes

Field	Type	Unit	Description
limb_properties			
length	`double[P]`	m	Arc lengths of the limb nodes (unbraced)
angle	`double[P]`	rad	Orientation angles of the limb nodes (unbraced)
x_pos	`double[P]`	m	X coordinates of the limb nodes (unbraced)
y_pos	`double[P]`	m	Y coordinates of the limb nodes (unbraced)
width	`double[P]`	m	Cross section width
height	`double[P]`	m	Cross section height (total)
rhoA	`double[P]`	kg/m	Linear density of the cross sections
Cee	`double[P]`	N	Longitudinal stiffness of the cross sections
Ckk	`double[P]`	Nm^2	Bending stiffness of the cross sections
Cek	`double[P]`	Nm	Coupling between bending and elongation
layers			
{			
length	`double`	m	Arc lengths of the layer nodes
He_back	`double[R][P]`	N/m^2	Stress evaluation matrix[1] (back)
Hk_back	`double[R][P]`	N/m	Stress evaluation matrix[1] (back)
He_belly	`double[R][P]`	N/m^2	Stress evaluation matrix[1] (belly)
Hk_belly	`double[R][P]`	N/m	Stress evaluation matrix[1] (belly)
}			
{ ... }			
statics			
states			
{ ... }			Sequence of bow states (see table below)
string_length	`double`	m	Initial length of the string
final_draw_force	`double`	N	Final draw force
drawing_work	`double`	J	Drawing work
storage_ratio	`double`	–	Storage ratio
dynamics			
states			
{ ... }			Sequence of bow states (see table below)
final_arrow_velocity	`double`	m/s	Final velocity of the arrow
final_arrow_energy	`double`	J	Final energy of the arrow
efficiency	`double`	–	Degree of efficiency

N: Number of simulation steps
P: Number of limb nodes
Q: Number of string nodes

Field	Type	Unit	Description
states			
time	double[N]	s	Time
draw_length	double[N]	m	Draw length
draw_force	double[N]	N	Draw force
string_force	double[N]	N	String force (total)
strand_force	double[N]	N	String force (strand)
grip_force	double[N]	N	Grip force
pos_arrow	double[N]	m	Arrow position
vel_arrow	double[N]	m/s	Arrow velocity
acc_arrow	double[N]	m/s^2	Arrow acceleration
x_pos_limb	double[N][P]	m	X coordinates of the limb nodes
y_pos_limb	double[N][P]	m	Y coordinates of the limb nodes
angle_limb	double[N][P]	rad	Rotation angles of the limb nodes
epsilon	double[N][P]	–	Longitudinal strain at the limb nodes
kappa	double[N][P]	m	Bending curvature at the limb nodes
x_pos_string	double[N][Q]	m	X coordinates of the string nodes
y_pos_string	double[N][Q]	m	Y coordinates of the string nodes
e_pot_limbs	double[N]	J	Potential energy of the limbs
e_kin_limbs	double[N]	J	Kinetic energy of the limbs
e_pot_string	double[N]	J	Potential energy of the string
e_kin_string	double[N]	J	Kinetic energy of the string
e_kin_arrow	double[N]	J	Kinetic energy of the arrow

[1]**Note:** For space efficiency reasons, the stresses for each layer aren't stored directly in the output data. Instead they can be calculated as needed by multiplying the layer's stress evaluation matrices with the strain and curvature of the limb at the given bow state,

$$\texttt{sigma\_back} = \texttt{He\_back} \cdot \texttt{epsilon} + \texttt{Hk\_back} \cdot \texttt{kappa}$$
$$\texttt{sigma\_belly} = \texttt{He\_belly} \cdot \texttt{epsilon} + \texttt{Hk\_belly} \cdot \texttt{kappa}$$

The result is a vector of stresses corresponding to the nodes of the layer.

C Python Scripting Example

The code example below shows how simulations can be automated with the Python programming language. It loads, modifies and saves an input file, runs a static simulation with it, loads the output results and calculates the maximum stress of the first layer.

Two external packages are needed: `msgpack` for reading the output files and `numpy` for evaluating stresses (matrix multiplication). They can be installed via

```
pip install msgpack
pip install numpy
```

Reading and writing input files is possible out of the box by using Python's `json` standard library module. Bow Simulator itself is called like any other command line tool either with `os.system` or `subprocess.call`.

```python
import json          # Loading and saving input files
import msgpack       # Loading output files
import numpy         # Evaluating stresses
import os            # Runnig the simulation command

# Load input file
with open("input.bow", "r") as file:
    input = json.load(file)

# Modify input
input["string"]["n_strands"] += 1

# Save modified input
with open("input.bow", "w") as file:
    json.dump(input, file)

# Run a static simulation
os.system("bow-simulator input.bow output.dat --static")

# Load the output file
with open("output.dat", "rb") as file:
    output = msgpack.unpackb(file.read())

# Calculate the maximum stress at the back of layer 0
He_back = numpy.array(output["limb_properties"]["layers"][0]["He_back"])
Hk_back = numpy.array(output["limb_properties"]["layers"][0]["Hk_back"])

epsilon = numpy.array(output["statics"]["states"]["epsilon"][-1])
kappa   = numpy.array(output["statics"]["states"]["kappa"][-1])

sigma_back = He_back.dot(epsilon) + Hk_back.dot(kappa)
print(sigma_back.max())
```

D A Simple Bending Test

A bending test is an easy way to determine the elastic modulus of a material. It can be done without any special equipment. Figure 14 shows the setup. A test piece with length l is clamped on one side and subjected to a vertical force F at its free end. The deflection s due to this load is measured.

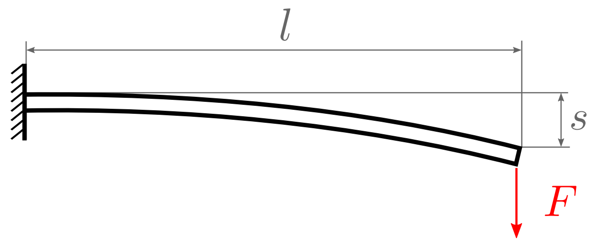


Figure 14: Experimental setup

The elastic modulus can then be calculated depending on the cross section geometry using the equations in Table 2.

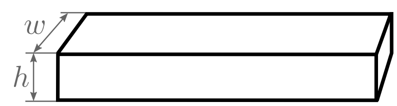
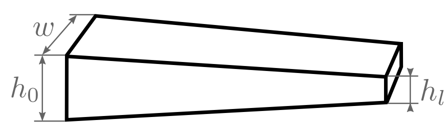
Test geometry	**Elastic Modulus**
	$E = \frac{4}{wh^3} \frac{Fl^3}{s}$
	$E = \frac{12 \ln(h_l\, l) + 6}{w\,(h_l - h_0)^3} \frac{Fl^3}{s}$

Table 2: Elastic modulus for different test geometries

Here are a few practical considerations:

- The precision of the cross sections is very important, especially the height.
- The equations above hold for slender beams and small deflections. The test setup should be chosen accordingly. As a rule of thumb: $h, s < l/15$.
- A simple way to apply a defined force is to hang a mass m onto the beam tip and use $F = m \cdot g$, with $g = 9.81\,\mathrm{m/s^2}$.
- If there is some small initial deflection due to gravity, then s is simply the difference in deflection after application of the force.