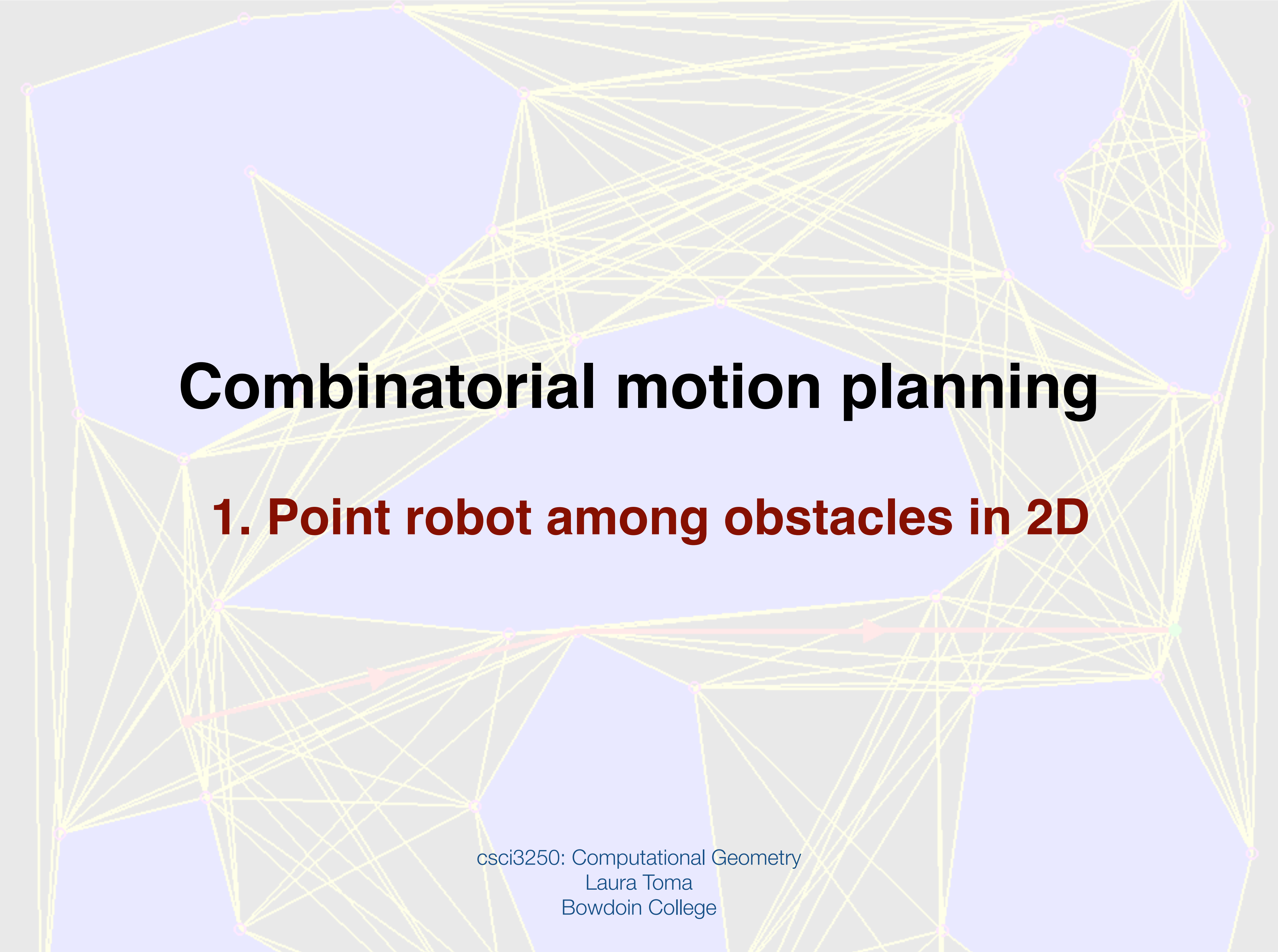
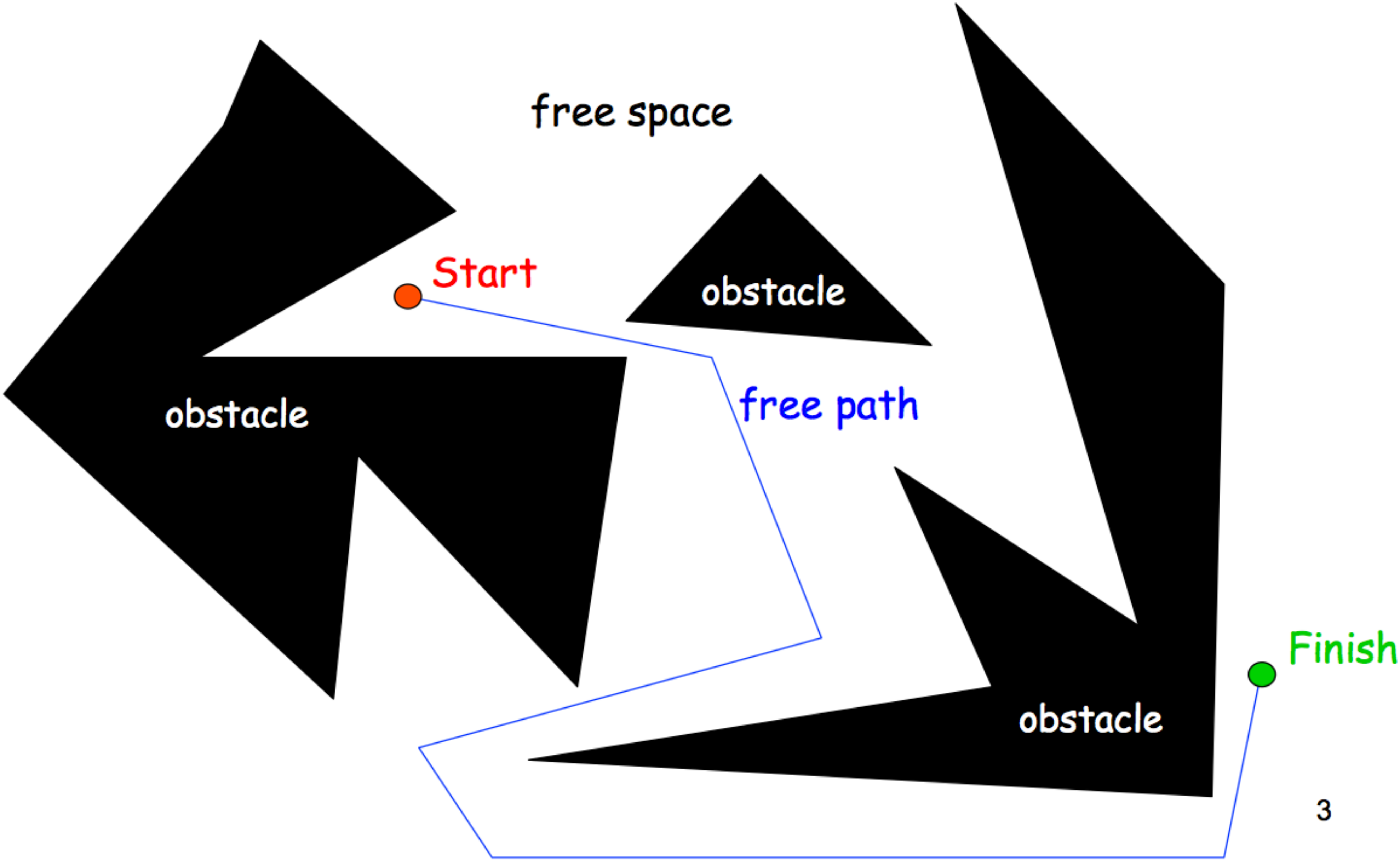


Combinatorial motion planning

1. Point robot among obstacles in 2D

csci3250: Computational Geometry
Laura Toma
Bowdoin College

Motion planning



screenshot from: ai.stanford.edu/~latombe/cs26n/2012/slides/**point-robot**-bug.ppt

Motion Planning

Input:

- a robot R
- start and end position
- a set of obstacles $S = \{O_1, O_2, \ldots\}$

Find a path from start to end (that optimizes some objective function).

Parameters:

- physical space (2d, 3d)
- geometry of obstacles (polygons, disks, convex, non-convex, etc)
- geometry of robot (point, polygon, disc)
- robot movement —how many degrees of freedom (dof); 2d, 3d
- objective function to minimize (euclidian distance, nb turns, etc)
- static vs dynamic environment
- exact vs approximate path planning
- known vs unknown map

Motion Planning

algorithm that finds a path

Ideally we want a **planner** to be complete and optimal.

- A planner is complete: it always finds a path when a path exists

- A planner is optimal: it finds an optimal path (wrt an objective function)

Path planning problems

we'll talk about these

- point robot moving among (arbitrary) polygons in 2D
- polygonal robot moving among (arbitrary) polygons in 2D
    - translation only
    - translation+rotation
- ….
- robot with arms and articulation moving in 3D

harder

Approaches

- Combinatorial (exact)
    - Used for path planning in 2D
    - Idea:
        - Compute an exact representation of free space as a graph
        - Find a path using the graph
- Approximate
    - Used for higher dimensional planning
    - Idea:
        - sample and approximate free space

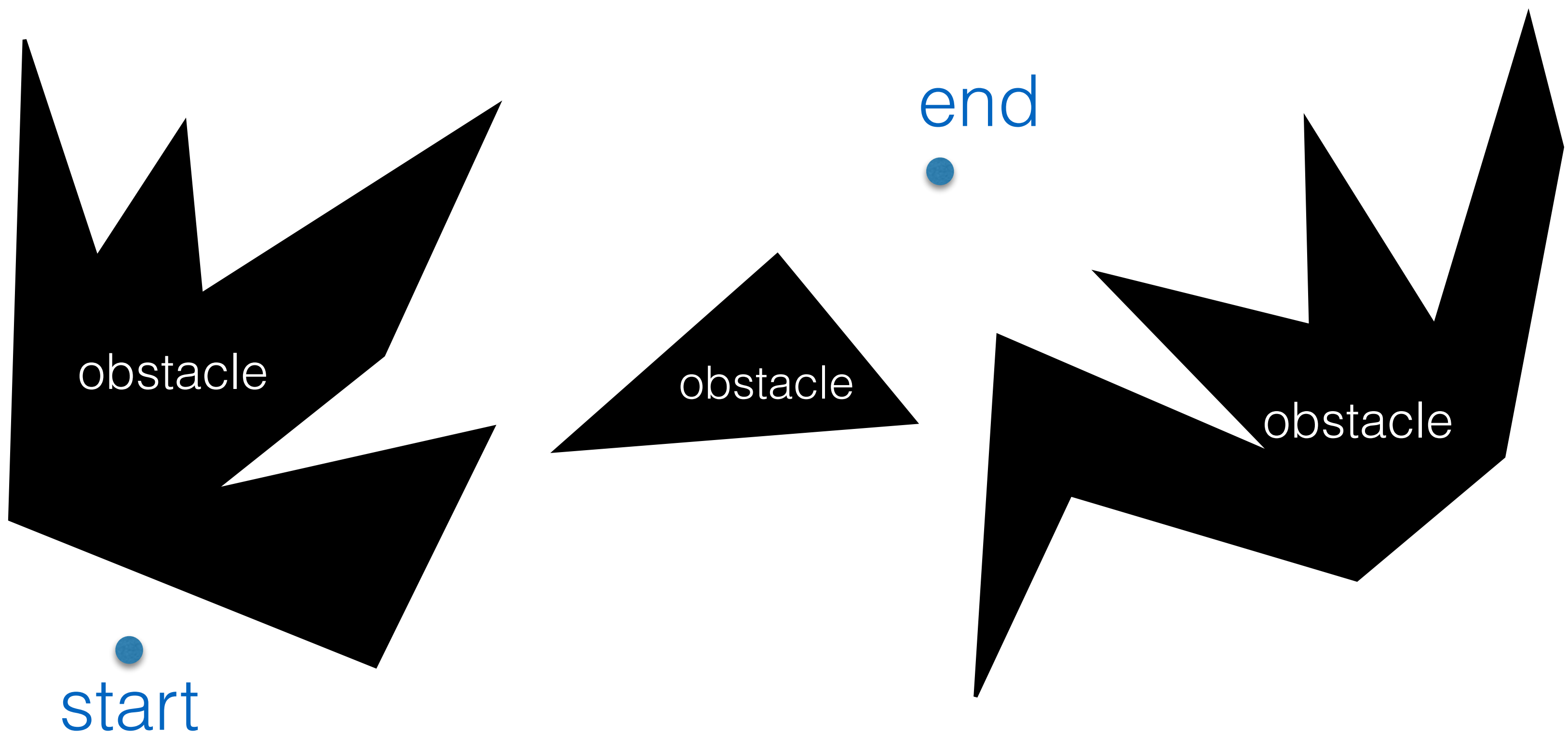
1. Point robot moving in 2D

Point robot in 2D

Input:

- start and end position
- a set of polygonal obstacles $S = \{O_1, O_2, \ldots\}$

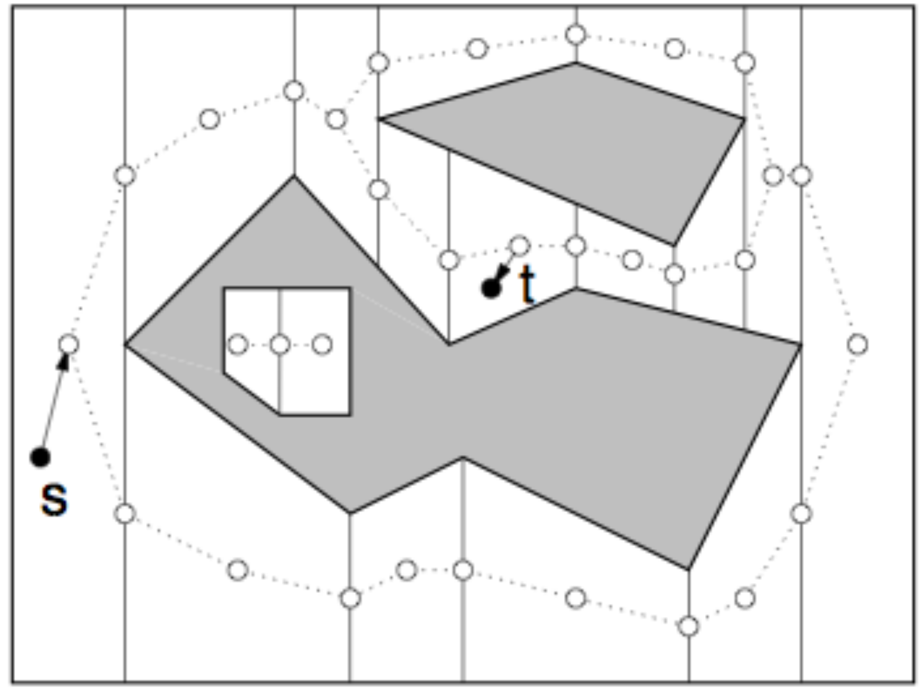
Find a path from start to end.



Point robot in 2D

General idea

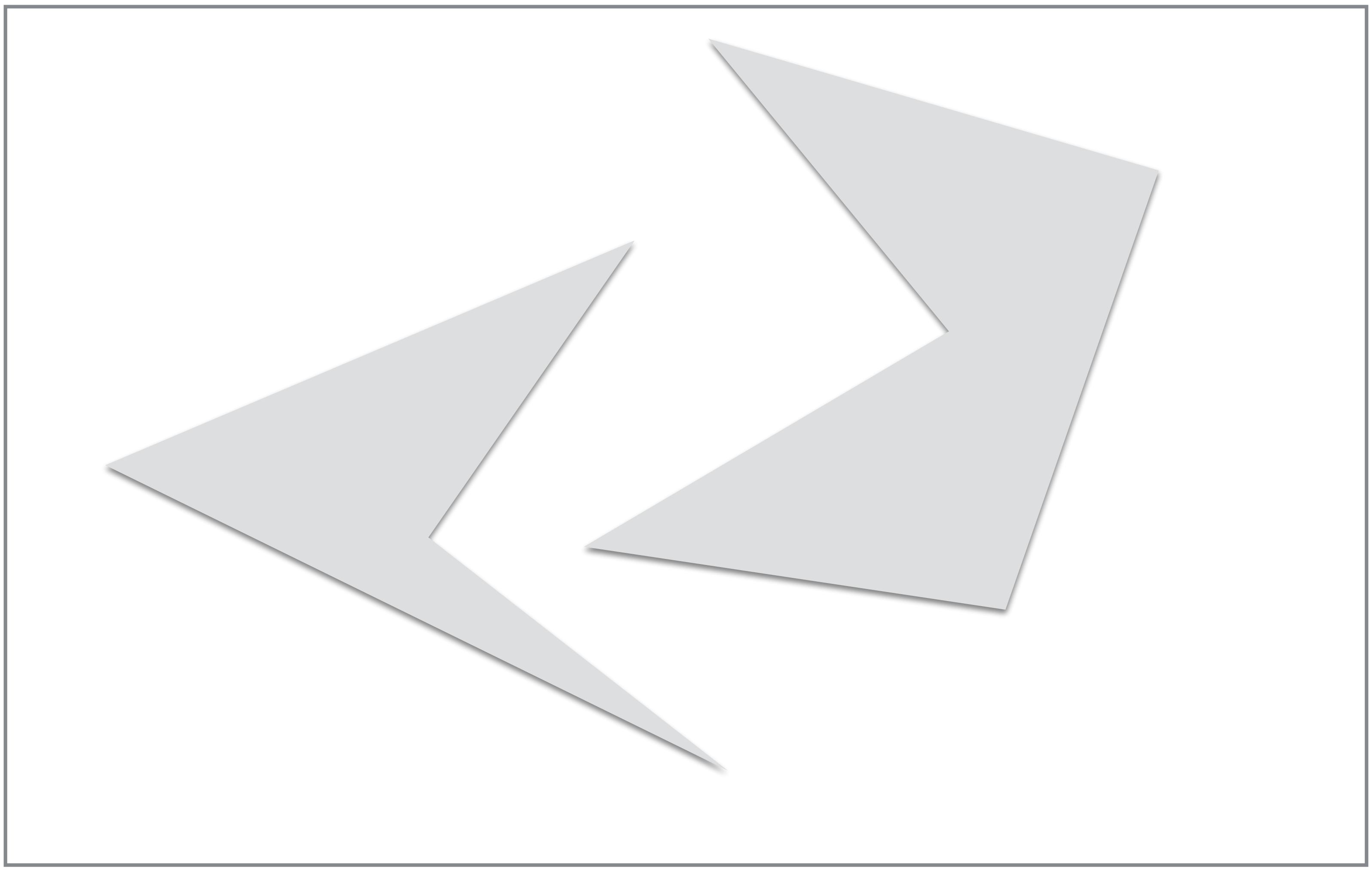
- Build a graph that represents movement through the free space
    - based on trapezoid decomposition of free space
- Search graph to find path



(screenshot from O'Rourke)

- Questions: How? How long? Size?

Let’s consider the following scene. Show a trapezoid decomposition of free space and the corresponding graph (“roadmap”).



Point robot in 2D

n = complexity of obstacles (total number of edges)

- Compute a trapezoid partition of free space ← Has size $O(n)$ and can be computed in $O(n \lg n)$ time
- Build graph of free space ← Has size $O(n)$ and can be computed in $O(n)$ time
- Search graph to find path ← BFS or DFS in $O(n)$ time

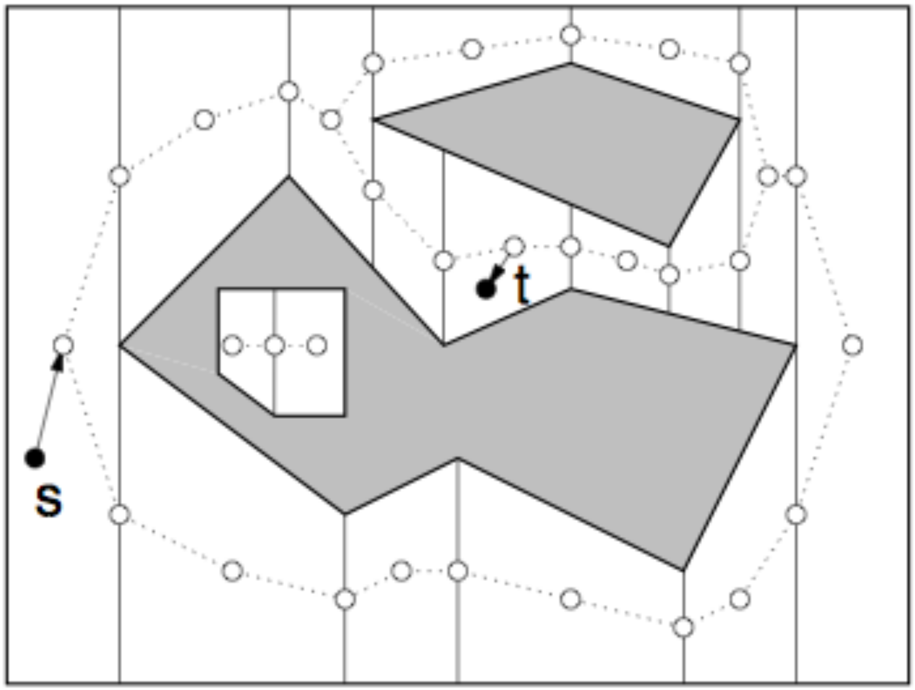
Result: Let R be a point robot moving among a set of polygonal obstacles in 2D with n edges in total. We can pre-process the scene in O(n lg n) expected time such that, between any start and goal position, a collision-free path for R can be computed in O(n) time, if it exists.

- Big idea: Path planning for point robot in 2D reduces to graph search in the "free space" graph

Point robot in 2D

Is this complete?

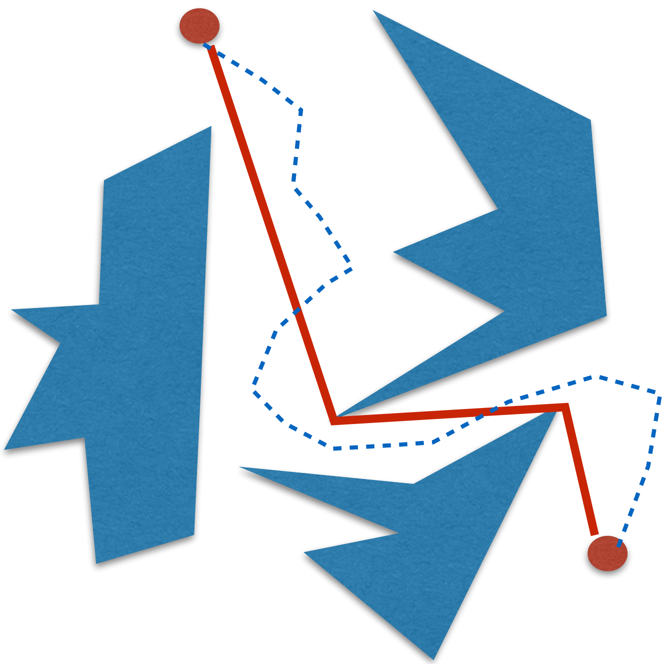
YES



Is this optimal?

No

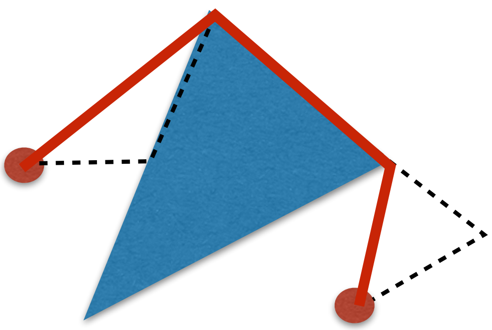
What if we wanted an **optimal** path?

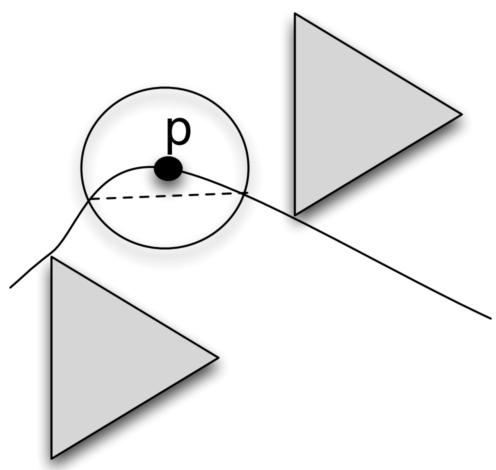


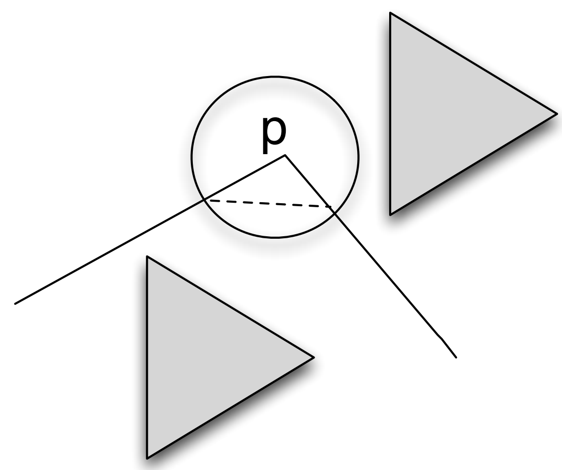
Theorem:

Any shortest path among a set S of disjoint polygonal obstacles:

1. is a polygonal path (that is, not curved)
2. its vertices are the vertices of S.

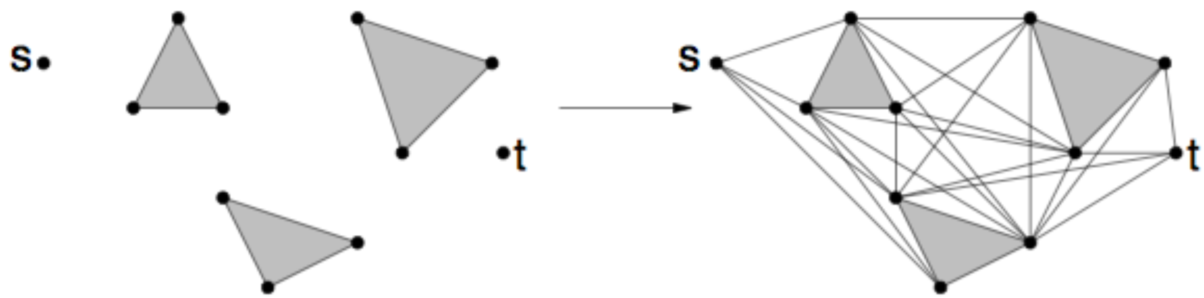






Visibility graph

- Idea: Since the vertices of any shortest path are the vertices of S, build a graph that represents all possible ways to travel between the vertices of the obstacles
    - V = {set of vertices of obstacles + p_{start} + p_{end}}
    - E = {all pairs of vertices (v_i, v_j) such that $v_i v_j$ are visible to each other (and not inside a polygon)
- Claim: any shortest path must be a path in the VG

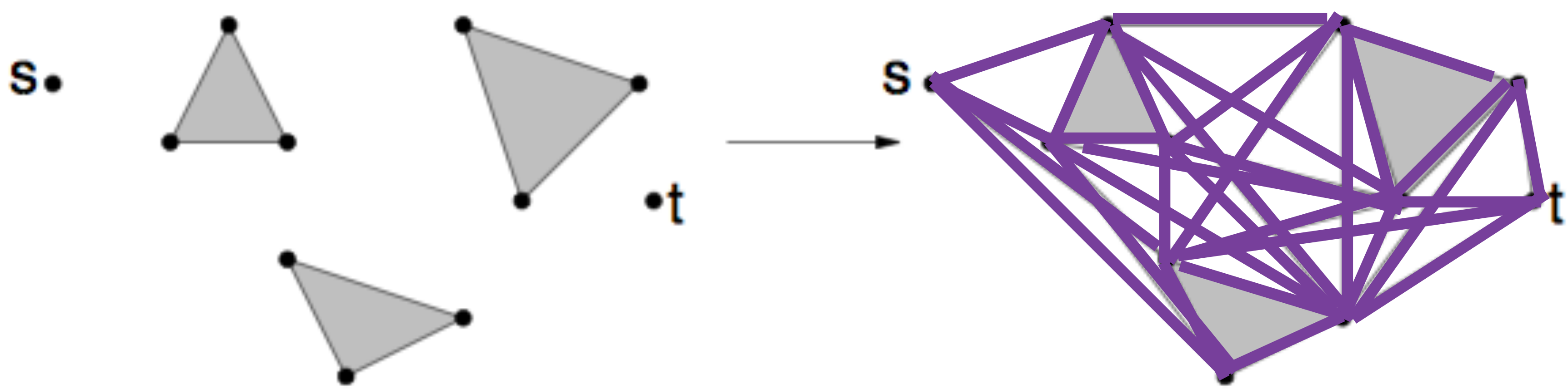


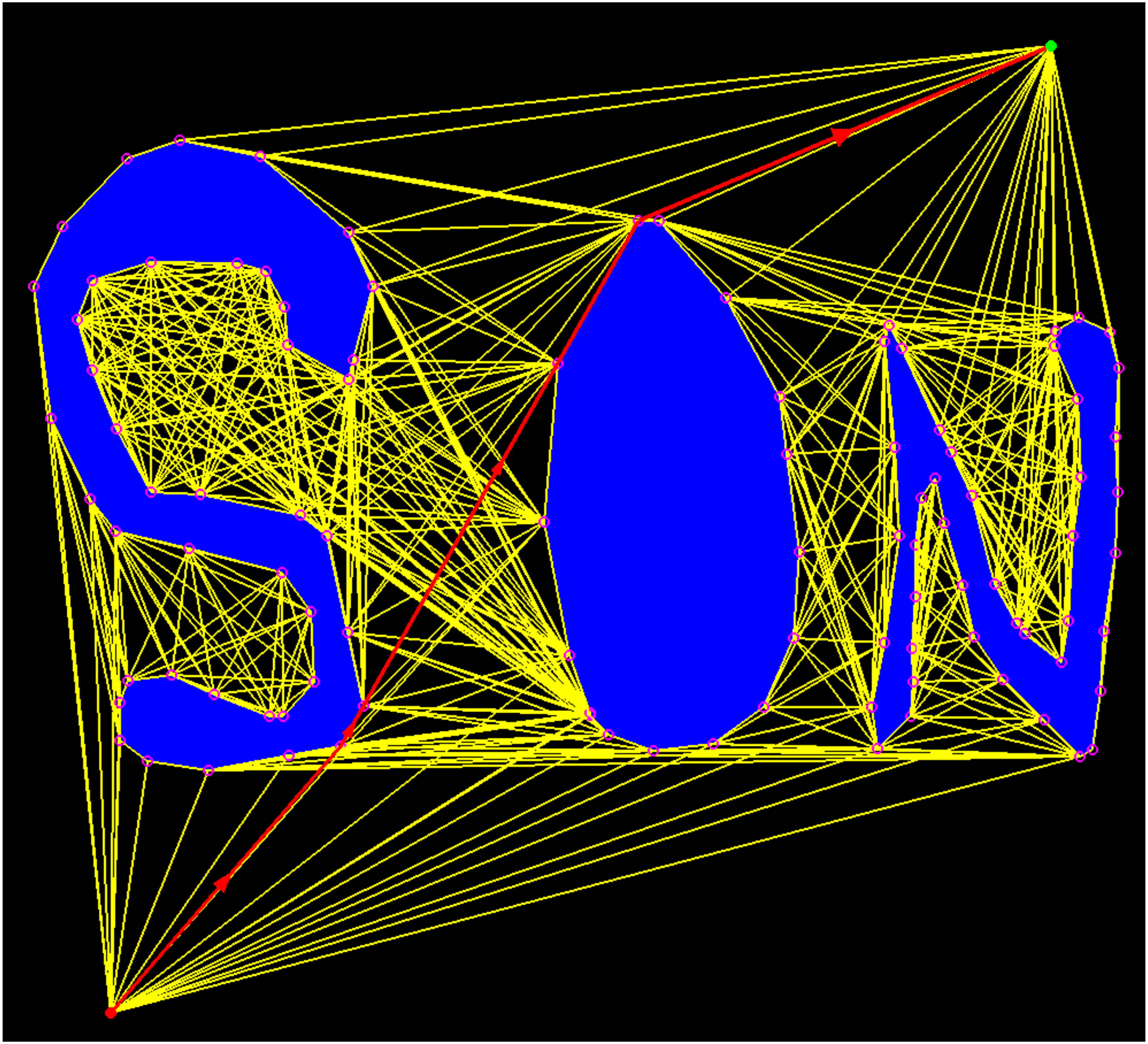
Path planning:

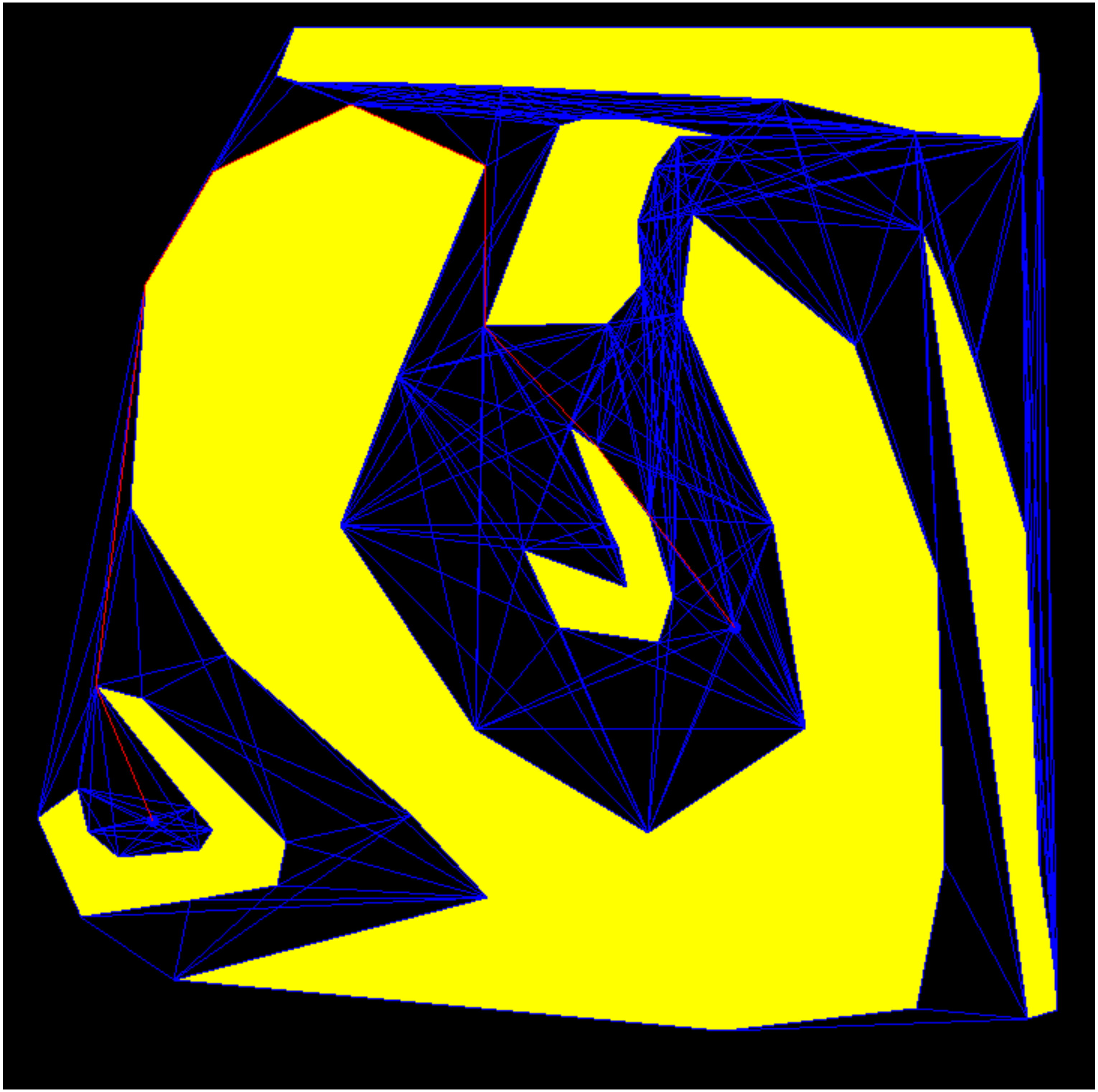
- Compute visibility graph
- SSSP (Dijkstra) in VG from p_{start} to p_{end}

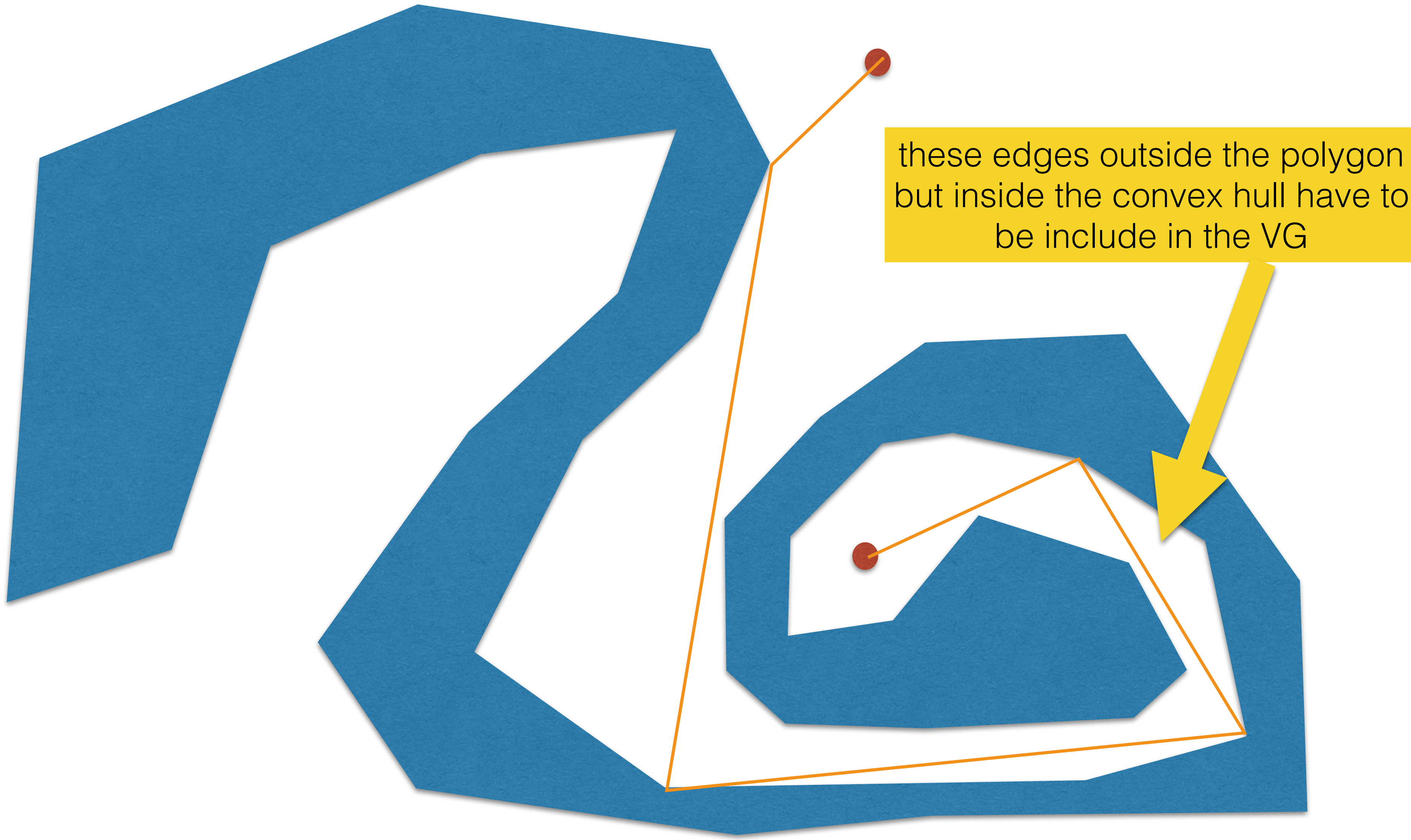
Visibility graph

- V = {set of vertices of obstacles + p_{start} + p_{end}}
- E = {all pairs of vertices (v_i, v_j) such that v_iv_j are visible to each other (and not inside a polygon)








these edges outside the polygon
but inside the convex hull have to
be include in the VG

n = complexity of obstacles (total number of edges)

Computing the Visibility Graph

- Straightforward:
  - V = {set of vertices of obstacles + p_{start} + p_{end}}
  - for each vertex u:
    - for each vertex v:
      - if segment uv does not intersect any edges of the polygon properly AND uv is NOT interior to a polygon: add uv as an edge

- Notes:
  - the edges of the polygons must be in the VG
  - interior edges: use $\texttt{inCone}(a, b)$ to determine if b is in the cone of $a^- a a^+$

- Running time: $O(n^3)$
- Size of visibility graph:
  - nb of vertices $V : n + 2 = \Theta(n)$
  - nb of edges: $\Omega(n), O(n^2)$

n = complexity of obstacles (total number of edges)

Optimal planning for point robot in 2D

Path planning:

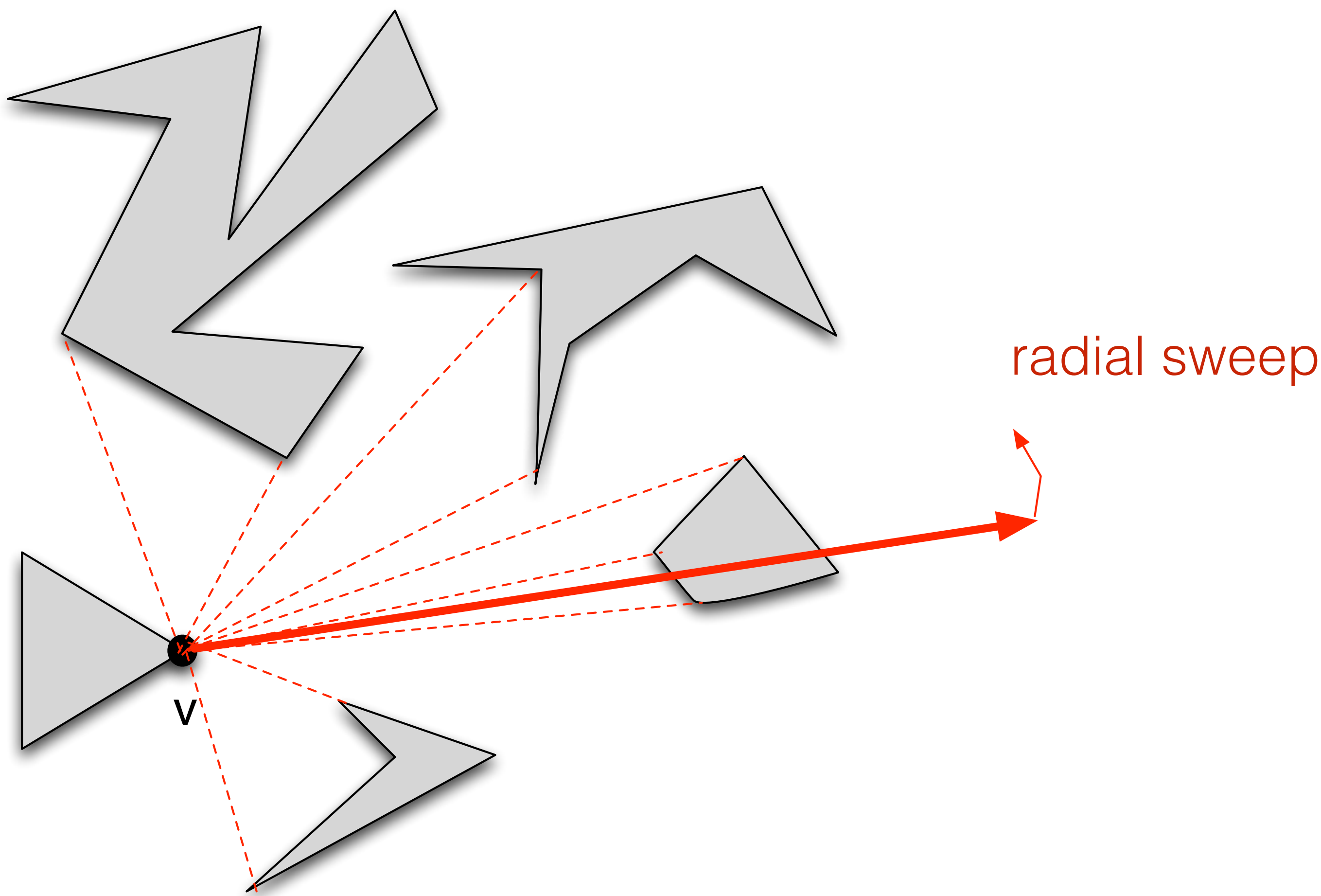
- Compute visibility graph ← can have quadratic size
- SSSP (Dijkstra) in VG from p_{start} to p_{end}

- Computing the visibility graph
  - $O(n^3)$ straightforward
  - $O(n^2 \lg n)$ improved
- Dijkstra in VG
  - $O(|E| \lg n)$

Computing the visibility graph in $O(n^2 \lg n)$

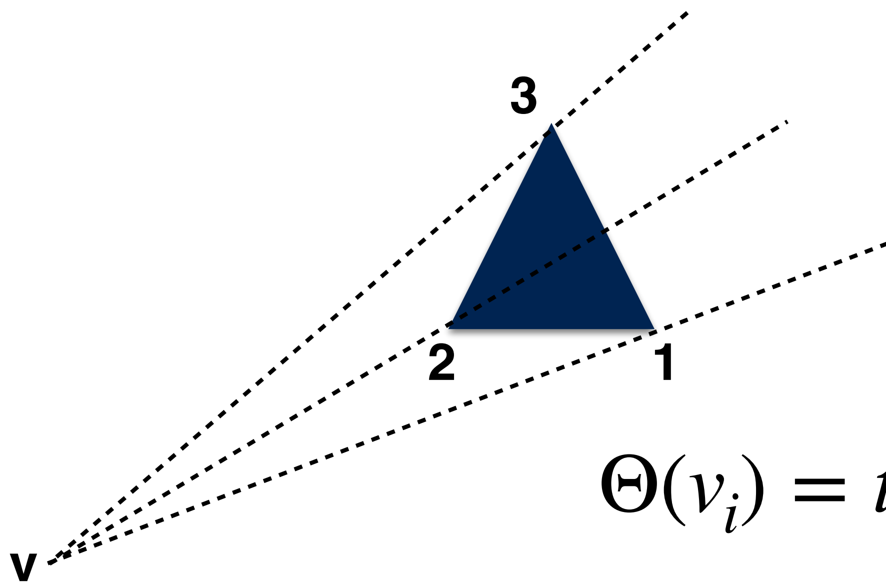
Improved computation of VG

- For every vertex v: compute all vertices visible from v in O(n lg n)



Improved computation of VG

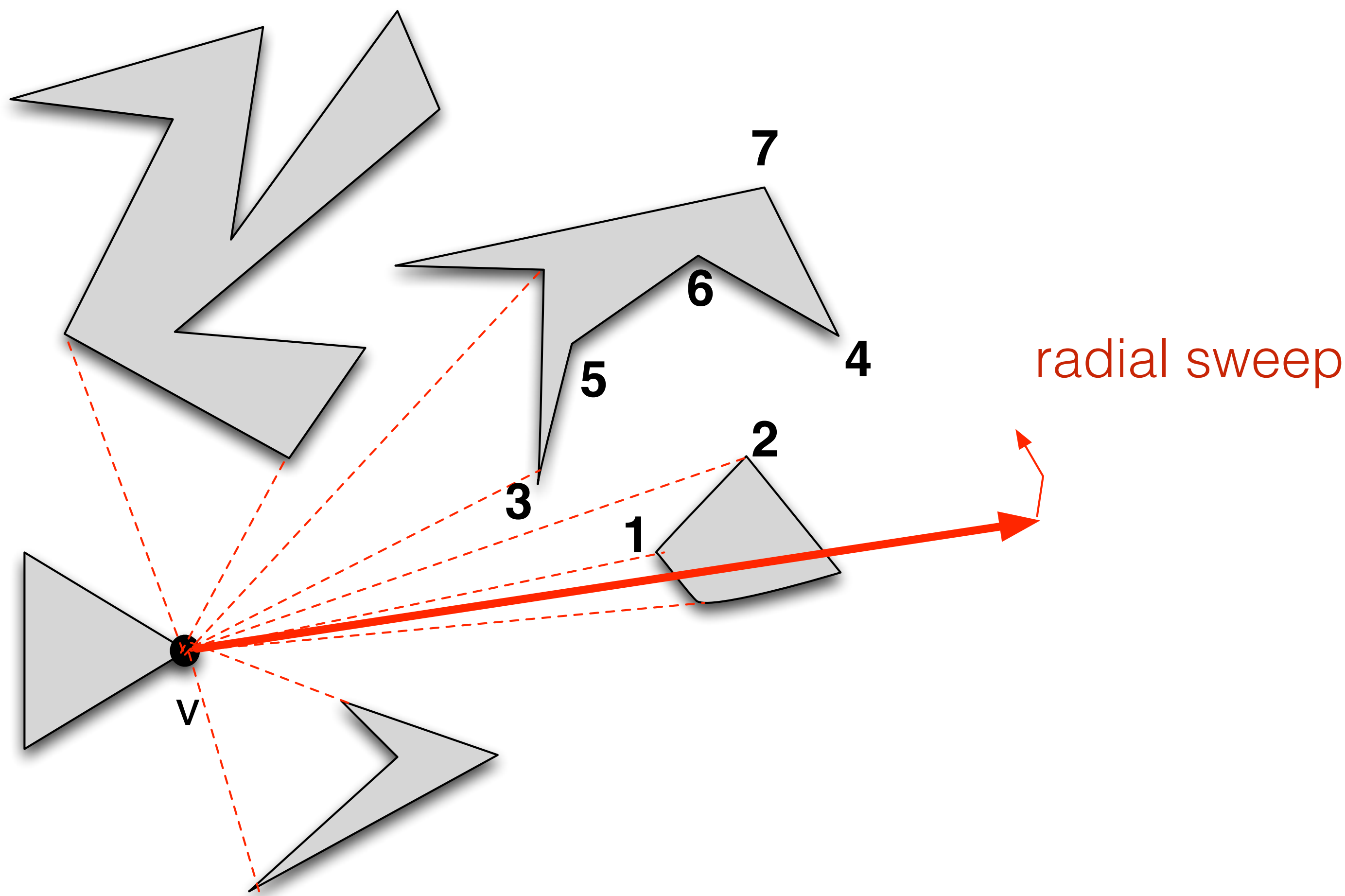
- Radial sweep: rotate a ray centered at v
- Events: vertices of polygons (obstacles) sorted in radial order
    - events of equal angle, sorted by distance from v



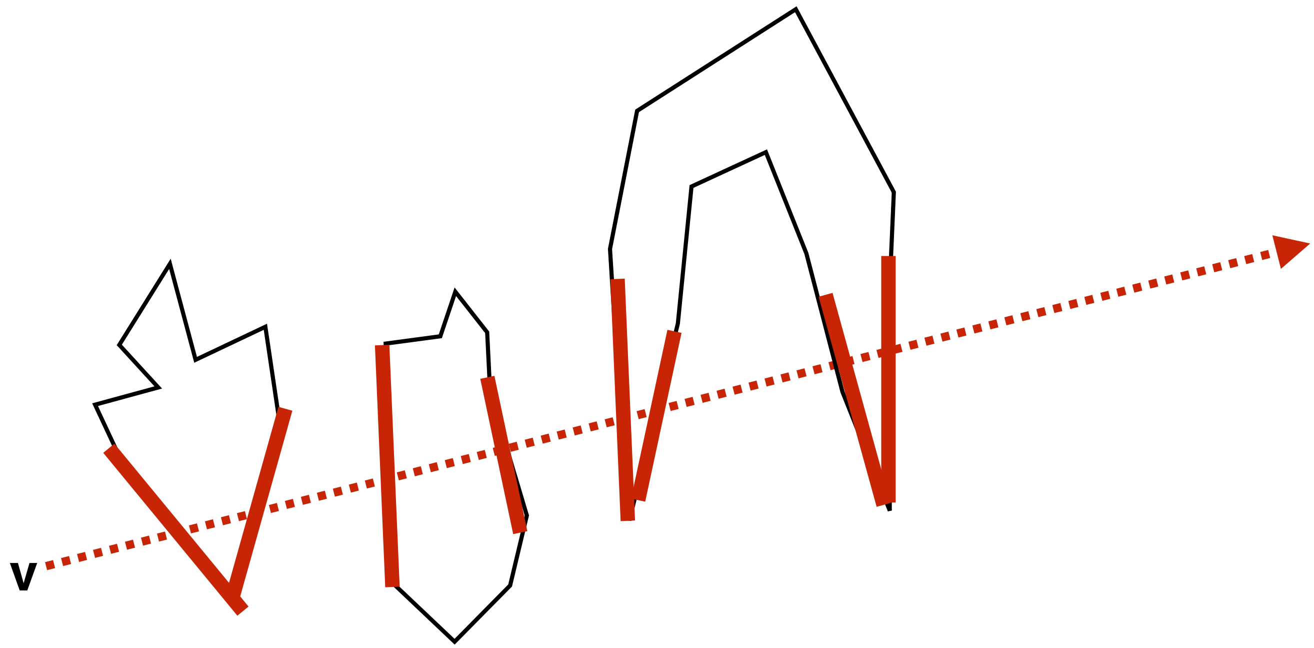
$$\Theta(v_i) = tan^{-1}\frac{y_i - y_v}{x_i - x_v}$$

VG via line sweep

- Radial sweep: rotate a ray centered at v
- Events: vertices of polygons (obstacles) sorted in radial order
    - events of equal angle, sorted by distance from v



VG via line sweep



Active structure (AS) stores all the edges that intersect the sweep line,

ordered by distance from v

VG via line sweep

//find all vertices visible from a vertex p

RadialSweep(polygon vertices V, vertex p)

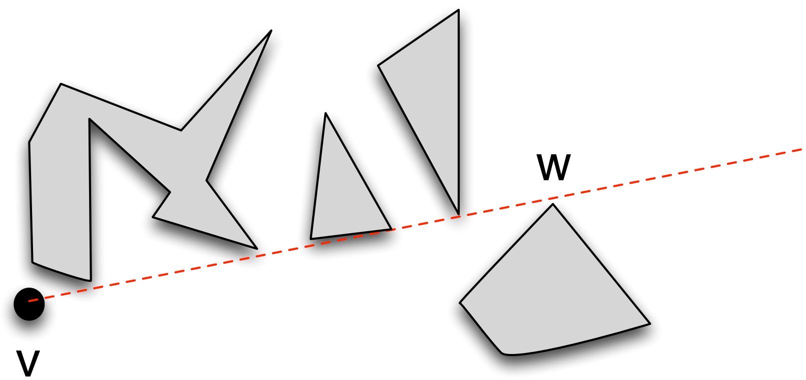
- sort V radially from p, and secondarily by distance from p
- initialize AS with all edges that intersect the horizontal ray from p
- For each vertex v in sorted order:
    - use AS to determine if v is visible from p
    - figure out if the edges incident to v are above/below the sweep line.

      If above -> insert edge in AS. If below => delete edge from AS

Runs in $O(n \lg n)$ time

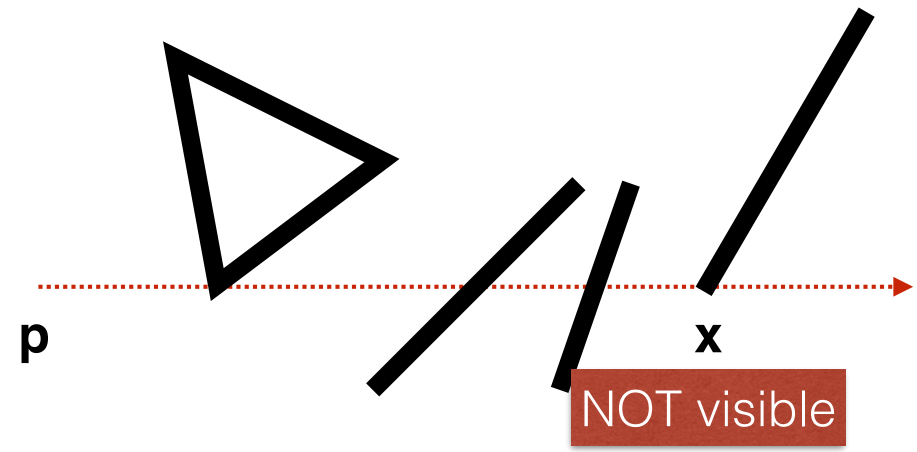
Repeat for all vertices p ==> $O(n^2 \lg n)$

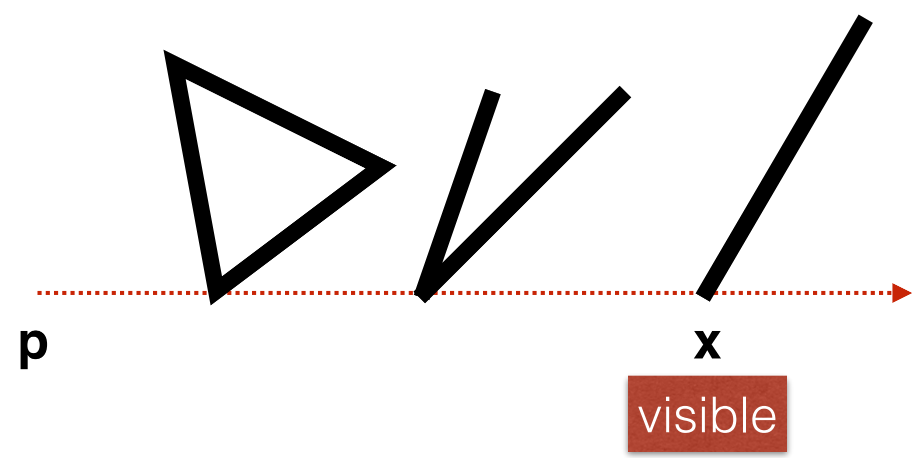
VG via line sweep



w visible if vw does not intersect the interior of any obstacle

Is vertex x visible from p? some cases





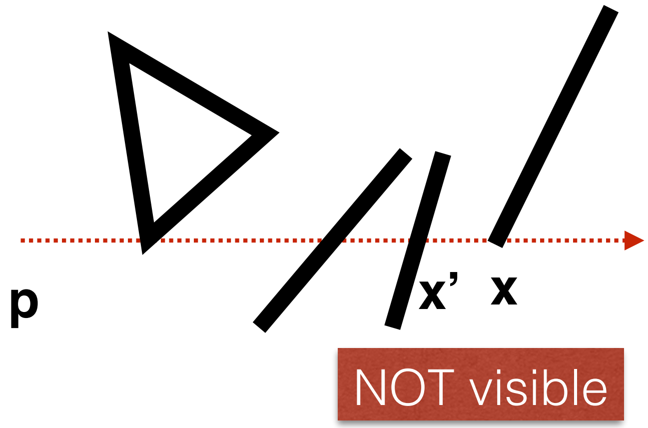
x is NOT visible:

If there is any edge in AS left of x, whose interior intersects the line

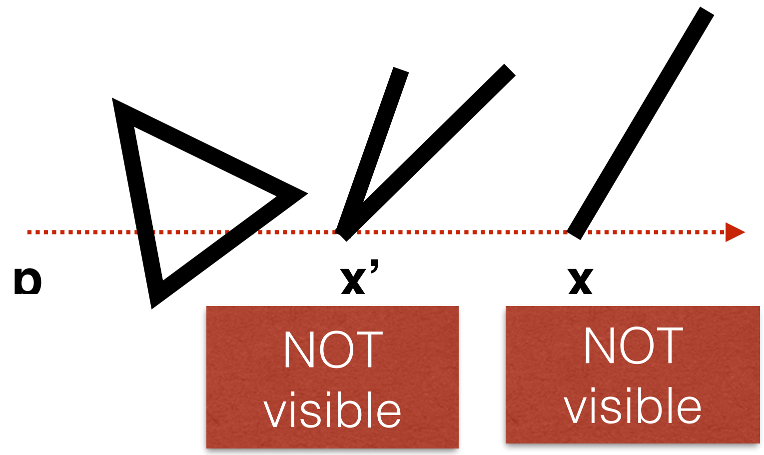
Is vertex x visible from p?

Let x’ be the edge just before x in the AS,   x’ = AS.predecessor(x)

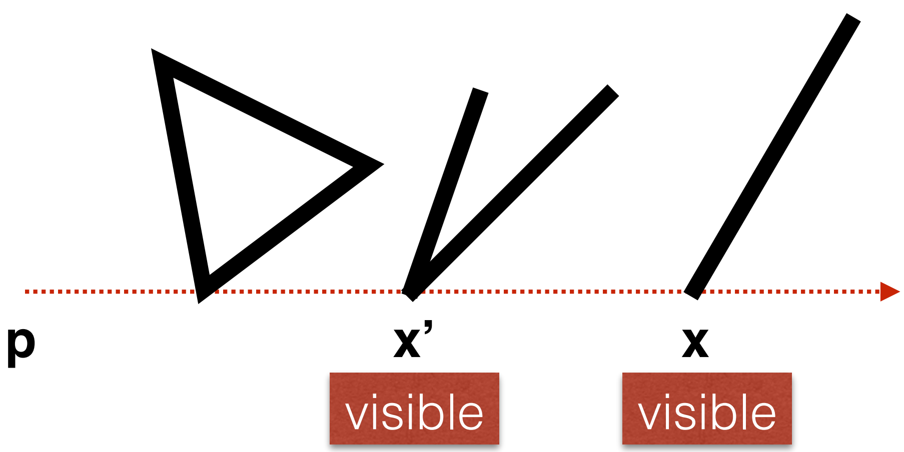
x’ interior intersects line



x’is a vertex and it’s not visible



x’is a vertex and it’s visible

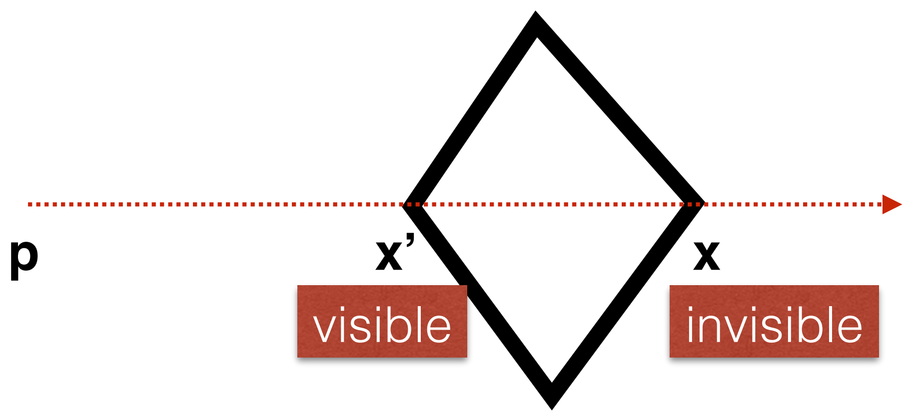


Is vertex x visible from p?

Let x’ be the edge just before x in the AS, x’ = AS.predecessor(x)

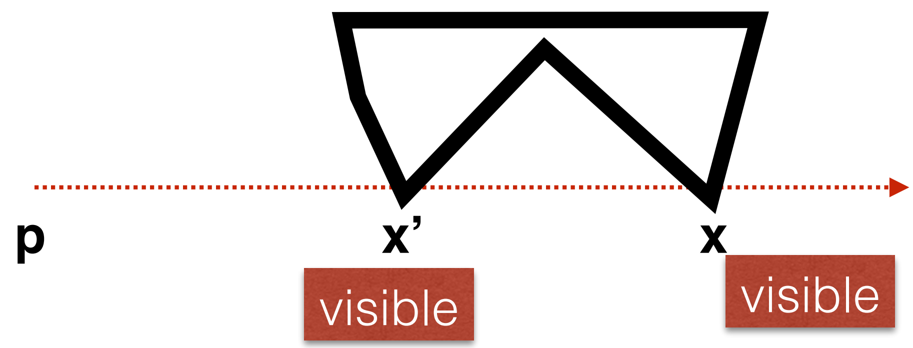
x’is a vertex and it’s visible

x, x’ part of same polygon



x’is a vertex and it’s visible

x, x’ part of same polygon



Is vertex x visible from p?

- check the event just before x in AS. Call this x’, x’=AS.predecessor(x)
- if x’ is an edge whose interior intersects sweep line => x is not visible
- if x’ has a vertex on the sweep line then:
    - if x’ is not visible => x not visible
    - if x’ is visible => x visible, unless they are both on the same polygon (a few cases to check)

Runs in $O(\lg n)$ time

Computing the visibility graph in $O(n^2 \lg n)$

END

Recap: Point robot in 2D

- Complete, not optimal
  - Compute the trapezoid decomposition of free space and a graph that represents it in $O(n \lg n)$ time
  - BFS in this graph in $O(n)$ time

- Complete and optimal
  - Compute visibility graph in $O(n^2 \lg n)$
  - Dijkstra in VG in $O(E_{VG} \lg n)$

+ Any shortest path must be a path in VG

+ VG needs to be computed only once, so we can think of it as pre-processing

- VG may be large, so this approach is doomed to $\Omega(n^2)$

Point robot in 2D

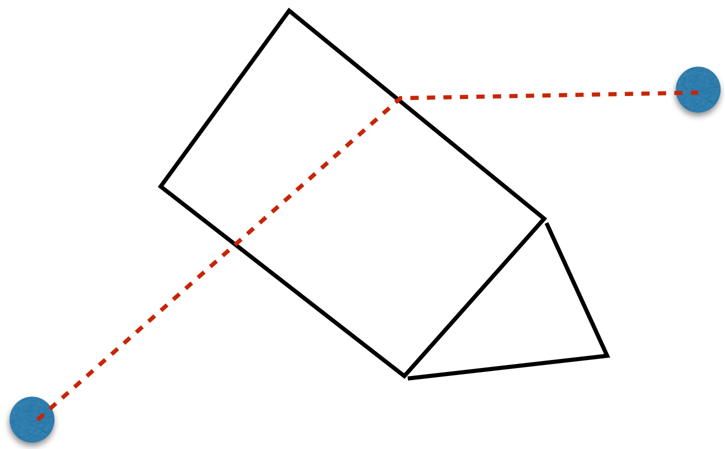
Long history of research and results

- $O(E_{VG} \lg n) = O(n^2 \lg n)$
- Improved to $O(n^2)$
- Quadratic barrier broken by Joe Mitchell: shortest path for a point robot moving in 2D can be computed in $O(n^{1.5+\epsilon})$
- Continuous Dijkstra approach: SP of a point robot moving in 2D can be computed in $O(n \lg n + k)$ [Hershberger and Suri 1993]
- Special cases can be solved faster:
    - e.g. SP inside a simple polygon w/o holes: $O(n)$ time

Point robot in 3D

Visibility graph does not generalize to 3D

- Inflection points of SP are not restricted to vertices of S, can be inside edges



- Shortest paths in 3D much harder
  - Computing 3D shortest paths among polyhedral obstacles is NP-complete
  - Complete and optimal planning in 3D is hopeless

Path planning in 2D

- point robot moving among arbitrary polygons in 2D

next

- polygonal robot moving among arbitrary polygons in 2D
    - translation only
    - translation+rotation