Week 3: Lab

1. Consider the linear-time merge algorithm discussed in the notes and a possible implementation below. How many element comparisons will the standard merge function take to merge the following left and right lists ?

$left = [1, 3, 4, 5, 6, 7, 8], right = [1, 5, 9, 11, 12, 16]$

Merge(left, right)

```
result = []
i=0
j=0
while i < len(left) and j < len(right):
    if left[i] < right[j]:
        result.append(left[i])
        i = i+1
  else:
        result.append(right[j])
        j = j+1

  # add any left overs
  while i < len(left):
      result.append(left[i])
      i = i+1
  while j < len(right):
      result.append(right[j])
      j = j+1

  return result
```

A 8

B 9

C 10

D 13

Find a $\Theta()$ bound for the following recurrences using iteration. Assume $T(1) = 1$. Show your work.

2. $T(n) = T(n/2) + 1$

3. $T(n) = T(n/3) + 1$ (assume $T(i) = 1$ for $i = 1, 2$).

4. $T(n) = T(n/10) + 1$ (assume $T(i) = 1$ for $i < 10$).

5. $T(n) = T(2n/3) + 1$ (assume $T(i) = 1$ for $i = 1, 2$).

6. $T(n) = T(n - 1) + 1$

7. $T(n) = T(n - 2) + 1$ (assume $T(i) = 1$ for $i = 1, 2$).

8. $T(n) = T(n - 3) + 1$ (assume $T(i) = 1$ for $i = 1, 2, 3$).

9. $T(n) = T(n/2) + n$

10. $T(n) = T(n/3) + n$ (assume $T(i) = 1$ for $i < 3$).

11. $T(n) = 3T(n/3) + \Theta(n)$ (assume $T(i) = 1$ for $i < 3$).

12. $T(n) = 5T(n/5) + \Theta(n)$ (assume $T(i) = 1$ for $i < 5$).

13. $T(n) = T(n-1) + n$

14. $T(n) = T(n-2) + \Theta(n)$ (assume $T(i) = 1$ for $i = 1, 2$).

15. $T(n) = 2T(n-1) + \Theta(1)$

challenge $T(n) = T(n/3) + T(2n/3) + \Theta(n)$ (only guess the solution)

challenge $T(n) = T(n/3) + T(n/4) + \Theta(n)$ (only guess the solution)

challenge $T(n) = T(n/2) + T(n/4) + T(n/10) + \Theta(n)$ (only guess the solution)

16. $T(n) = T(\sqrt{n}) + 1$

17. $T(n) = 7T(n/2) + n^3$

18. $T(n) = 7T(n/2) + n^2$

19. $T(n) = T(n-1) + 2n - 3$, with $(T(1) = 1$

20. $T(n) = 2T(n-1) + 1$, with $(T(1) = 1$

21. $T(n) = 6T(n/6) + 2n + 3$, with $(T(1) = T(2) = T(3) = T(4) = T(5) = 1$

22. $T(n) = 4T(n/3) + 2n - 1$, with $(T(1) = T(2) = 1$

23. $T(n) = 3T(n/2) + n^2$, with $(T(1) = 1$

24. $T(n) = 7T(n/2) + n^2$, with $(T(1) = 1$

Partial Answers

1. B (9 comparisons)

2. $\Theta(\lg n)$

3. $\Theta(\lg n)$

4. $\Theta(\lg n)$

5. $\Theta(\lg n)$

6. $\Theta(n)$

7. $\Theta(n)$

8. $\Theta(n)$

9. $\Theta(n)$

10. $\Theta(n)$

11. $\Theta(n \lg n)$

12. $\Theta(n \lg n)$

13. $\Theta(n^2)$

14. $\Theta(n^2)$

15. $\Theta(2^n)$

16.

17.

18.

19. $T(n) = T(n-1) + 2n - 3$, with $(T(1) = 1 : T(n) = \Theta(n^2)$

20. $T(n) = 2T(n-1) + 1$, with $(T(1) = 1: T(n) = \Theta(2^n)$ Note: For exponential recurrences we are usually happy with just a lower bound.

21. $T(n) = 6T(n/6) + 2n + 3$, with $(T(1) = T(2) = T(3) = T(4) = T(5) = 1: T(n) = \Theta(n \lg n)$

22. $T(n) = 4T(n/3) + 2n - 1$, with $(T(1) = T(2) = 1 \,$: $T(n) = \Theta(n^{log_3 4})$

23. $T(n) = 3T(n/2) + n^2$, with $(T(1) = 1: T(n) = \Theta(n^2)$

24. $T(n) = 7T(n/2) + n^2$, with $(T(1) = 1 : T(n) = \Theta(n^{\log_7 8})$