Assignment 6

csci2200, Algorithms

Instructions:

- Honor code: Work on this assignment alone, or with one partner. Between different teams, Collaboration is at level 1 [verbal collaboration only]
- Check out the Homework guidelines on class website.

---

1. **Finding k-quantiles:** Given an unsorted sequence S of n elements, and an integer k, we want to find $O(k)$ elements that have rank $\lceil n/k \rceil$, $2\lceil n/k \rceil$, $3\lceil n/k \rceil$, and so on. You may assume that k is a power of 2.

   (a) Describe the "naive" algorithm that works by repeated selection, and analyze its running time function of n and k (do not assume k to be a constant).

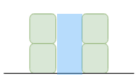
   (b) Describe an improved algorithm that runs in $O(n \lg k)$ time. After you describe it, argue why its running time is $O(n \lg k)$.

   We expect: pseudocode and an English description of your algorithm, and analysis of its running time.

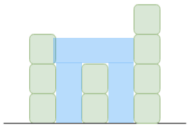
2. **Flooding 1D-terrain**: You are given an array whose values represent heights of a one-dimensional terrain, sampled at 1m resolution. Imagine an arbitrarily large amount of water falling from the sky and flooding the terrain, and also imagine that the terrain is surrounded by a giant sink/ocean. Water will accumulate on top of a cell in the terrain unless it can find a path to the ocean. At some point flooding will reach steady state when further rain will no longer increase the flooding.

   The goal is to compute the total amount of water that is accumulated in the terrain (aka volume of flood) when steady state is reached. You may consider that the elevations are all non-negative (i.e. ≥ 0), and the width of each "pixel" (ie the distance between two consecutive heights in the array) is 1 unit.

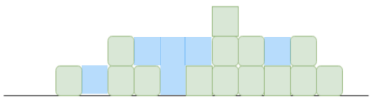
   Examples:



Input: A = [0, 2, 0, 2, 0]
Output: 2

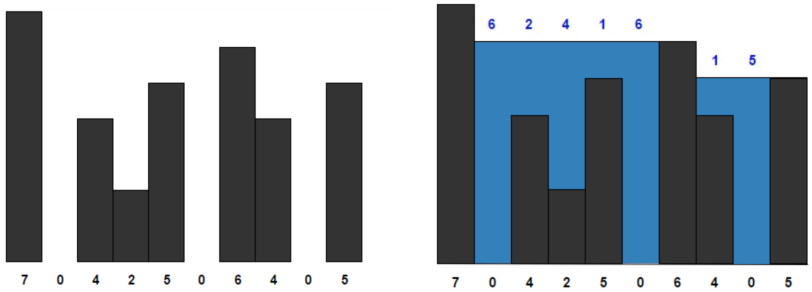


Input: A = [0, 3, 0, 2, 0, 4, 0]
Output: 7



Input: A = [0, 1, 0, 2, 1, 0, 1, 3, 2, 1, 2, 1, 0]
Output: 6

The maximum amount of water that can be trapped is 25, as shown below (blue).



Aim for a linear time algorithm.

We expect: (1) Formulate a claim on what is the height of water accumulated on $A[i]$, in terms of the elevation of the pixels to its left and right. Hint: consider min and max values. (2) high-level pseudocode ; (3) analysis of running time.