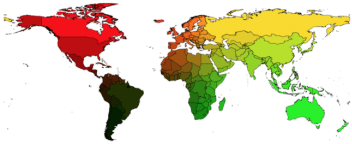


Advanced Network Analysis

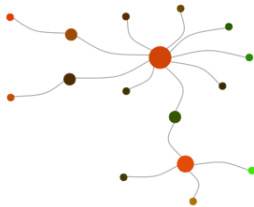
TERGM Introduction

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Longitudinal networks



-1965



Longitudinal networks

- Networks that change over time.
- Examples: networks of friends, conflict networks, trade networks.
- Want to model network dynamics within and across time periods.

Outline

- Setting up longitudinal network data
- Visualizing longitudinal data
- Descriptive statistics
- Inferential analysis

Getting the data ready

We are used to data in this format:

```
#install_github("ochyzh/networkdata")  
library(networkdata)  
data(allyData)  
head(dyadData)[,1:8]
```

| ## | cname1 | cname2 | year | ally | war | contiguity | id1_cinc | id2_cinc |
|------|--------|--------|------|------|-----|------------|-----------|-----------|
| ## 2 | CAN | USA | 1991 | 1 | 0 | 1 | 0.0119571 | 0.1364806 |
| ## 3 | MEX | USA | 1991 | 1 | 0 | 1 | 0.0125758 | 0.1364806 |
| ## 4 | COL | USA | 1991 | 1 | 0 | 0 | 0.0046681 | 0.1364806 |
| ## 5 | VEN | USA | 1991 | 1 | 0 | 0 | 0.0052502 | 0.1364806 |
| ## 6 | PER | USA | 1991 | 1 | 0 | 0 | 0.0033841 | 0.1364806 |
| ## 7 | BRA | USA | 1991 | 1 | 0 | 0 | 0.0240151 | 0.1364806 |

Getting the data ready

We need to convert this information such that:

- the dependent variable must be a list of network objects
- nodal covariates are vertex attributes in the list of network objects
- dyadic covariates are included separately in a list of matrices

Start with setting up war

Output should look like this:

```
class(war)
```

```
## [1] "list"
```

```
length(war)
```

```
## [1] 10
```

```
class(war[[1]])
```

```
## [1] "matrix" "array"
```

```
dim(war[[1]])
```

```
## [1] 50 50
```

```
war[[1]][1:3,1:3]
```

```
##      USA CHN IND
## USA    0   0   0
## CHN    0   0   1
## IND    0   0   0
```

contiguity should be easier

Output should look like this:

```
class(contiguity)
```

```
## [1] "matrix" "array"
```

```
dim(contiguity)
```

```
## [1] 50 50
```

```
contiguity[1:3,1:3]
```

```
##      USA CHN IND
## USA    0    0    0
## CHN    0    0    1
## IND    0    1    0
```


Now set up DV with vertex attributes

Output should look like this:

```
class(ally)
```

```
## [1] "list"
```

```
length(ally)
```

```
## [1] 10
```

```
class(ally[[1]])
```

```
## [1] "network"
```

```
list.vertex.attributes(ally[[1]])
```

```
## [1] "cinc"          "cname"         "polity"        "vertex.names" "year"
```

Exploring temporal network data

The `statnet` package includes a range of "sub-packages" that enable you to understand the characteristics of dynamic networks:

- `networkDynamic`: storage and management of temporal network data
- `tsna`: descriptive statistics and graphics for exploratory network analysis
- `ndtv`: utilities for plotting temporal networks (including network movies)

Prepping data

First step is going to be formatting our list of network objects into a format that these packages can recognize:

```
allyDyn = networkDynamic(network.list=ally)
```

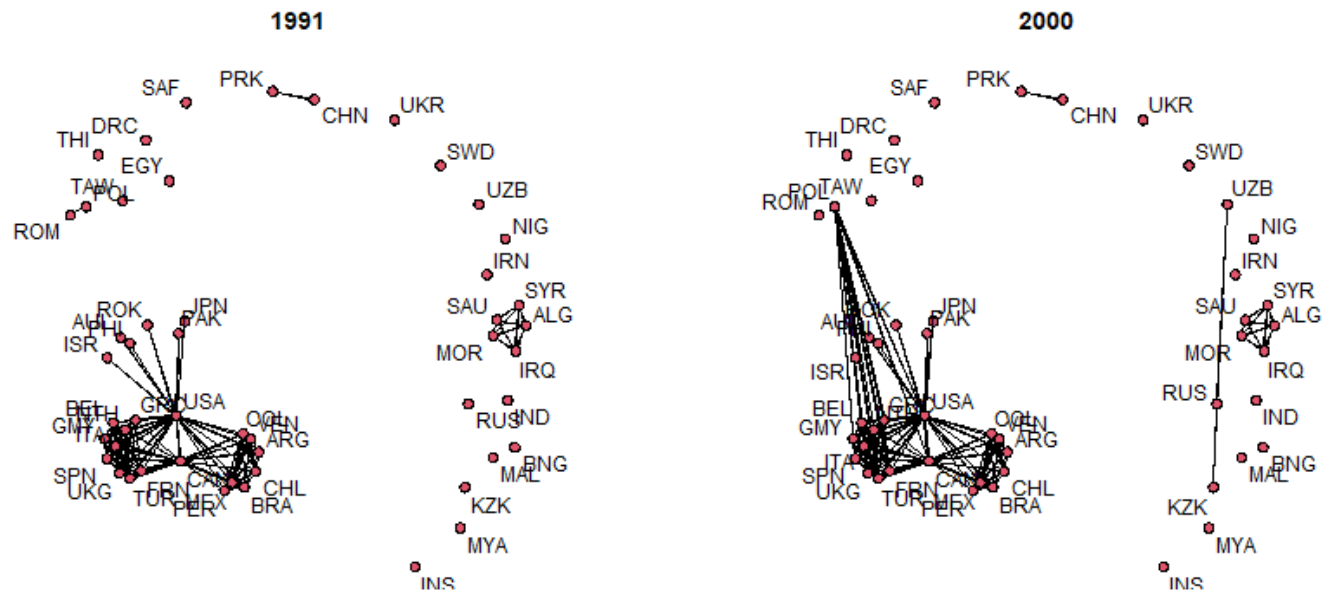
```
## Neither start or onsets specified, assuming start=0
## Onsets and termini not specified, assuming each network in network.list sh
## Argument base.net not specified, using first element of network.list inste
## Created net.obs.period to describe network
## Network observation period info:
##   Number of observation spells: 1
##   Maximal time range observed: 0 until 10
##   Temporal mode: discrete
##   Time unit: step
##   Suggested time increment: 1
```

Quick snapshots

networkDynamic makes it easy to generate some snapshots of a longitudinal network:

```
par(mfrow = c(1,2))
p<-plot(
  network.extract(allyDyn, at = 0),
  main = "1991", displaylabels = T)
plot(
  network.extract(allyDyn, at = 9),
  main = "2000", displaylabels = T, coord=p)
```

Quick snapshots



Quick movie

ndtv makes it pretty easy to render a simple D3 movie for a longitudinal network:

```
library(ndtv)
render.d3movie(allyDyn,
  plot.par=list(displaylabels=T),filename="AlliesNetwork.html", lau
```

Descriptive analyses

tsna enables us to quickly calculate some basic descriptive statistics such as the density of a graph:

```
library(tsna)
tSnaStats(allyDyn, "gden") # Changes in graph density
```

```
## Time Series:
## Start = 0
## End = 10
## Frequency = 1
##           Series 1
## [1,] 0.08816327
## [2,] 0.09061224
## [3,] 0.09061224
## [4,] 0.08979592
## [5,] 0.08979592
## [6,] 0.08979592
## [7,] 0.09795918
## [8,] 0.09795918
## [9,] 0.09795918
## [10,] 0.09795918
## [11,] NA
```

Descriptive analyses

Can also examine changes in transitivity over time:

```
tSnaStats(allyDyn, "gtrans") # Changes in graph transitivity
```

```
## Time Series:
## Start = 0
## End = 10
## Frequency = 1
##      Series 1
## [1,] 0.7617647
## [2,] 0.7768924
## [3,] 0.7768924
## [4,] 0.7768924
## [5,] 0.7768924
## [6,] 0.7768924
## [7,] 0.7974684
## [8,] 0.7974684
## [9,] 0.7974684
## [10,] 0.7974684
## [11,] NA
```


Descriptive analyses

The `tErgmStats` enables us to calculate changes in `ergm` terms over time:

```
tErgmStats(allyDyn, "~ edges+triangle")
```

```
## Time Series:
## Start = 0
## End = 10
## Frequency = 1
##      edges triangle
##  0      108      259
##  1      111      260
##  2      111      260
##  3      110      260
##  4      110      260
##  5      110      260
##  6      120      315
##  7      120      315
##  8      120      315
##  9      120      315
## 10         0         0
```

Your Turn

1. Format the war data from above as a `networkDynamic` object.
2. Summarize changes in the number of war initiation (edges) and triangles in the war network.
3. Plot the war network from 1991 and 2000 side-by-side using the `network.extract` function.
4. Make a quick movie of the war network over time using the `render.d3movie` function.

TERGM: Discrete time model

- Developed by [Robins & Pattison \(2001\)](#) and further developed by [Hanneke et al. \(2010\)](#)
- Scholars in political science most notably [Cranmer and Desmarais \(2011\)](#) have eased the use and highlighted the utility of these types of models for political science
- Extension of ERGM to the temporal setting is based on the idea of panel regression
- In a sequence of observations, lagged earlier observations or derived information thereof can be used as predictors for later observations.
 - In other words, some of the statistics are direct functions of an earlier realization of the network
 - In its most basic form, the TERGM is a conditional ERGM with an earlier observation of the network occurring among the predictors.

TERGM: Discrete time model

- To extend ERGM to a longitudinal context, [Hanneke et al. \(2010\)](#) make a Markov assumption on the network from one time step to the next
- Specifically, given an observed network Y^t , make the assumption that Y^t is independent of Y^1, \dots, Y^{t-2}
- Thus a sequence of network observations has the property that:

$$Pr(Y^2, Y^3, \dots, Y^t | Y^1) = Pr(Y^t | Y^{t-1}) Pr(Y^{t-1} | Y^{t-2}) \dots Pr(Y^2 | Y^1)$$

TERGM: Discrete time model

- With this assumption in mind we just need to choose a form for the conditional PDF of $P(Y^t|Y^{t-1})$
- $Y^t|Y^{t-1}$ can be expressed through an ERGM distribution, which then gives us what is referred to as a TERGM:

$$\Pr(Y^t|\theta, Y^{t-1}) = \frac{\exp(\theta^T g(Y^t, Y^{t-1},))}{k}$$

TERGM: Block-diag visualization

- TERGM is essentially estimated through an ERGM with the dependent variable modeled as a block-diagonal matrix (such as below)
- Constraints are put on the model such that cross-network edges in the off-diagonal blocks are prohibited

| | a ₁ | b ₁ | c ₁ | d ₁ | a ₂ | b ₂ | c ₂ | d ₂ | a ₃ | b ₃ | c ₃ | d ₃ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| a ₁ | 0 | 1 | 0 | 0 | × | × | × | × | × | × | × | × |
| b ₁ | 0 | 0 | 1 | 0 | × | × | × | × | × | × | × | × |
| c ₁ | 0 | 1 | 0 | 0 | × | × | × | × | × | × | × | × |
| d ₁ | 0 | 0 | 0 | 0 | × | × | × | × | × | × | × | × |
| a ₂ | × | × | × | × | 0 | 1 | 0 | 0 | × | × | × | × |
| b ₂ | × | × | × | × | 1 | 0 | 0 | 0 | × | × | × | × |
| c ₂ | × | × | × | × | 1 | 1 | 0 | 1 | × | × | × | × |
| d ₂ | × | × | × | × | 1 | 0 | 0 | 0 | × | × | × | × |
| a ₃ | × | × | × | × | × | × | × | × | 0 | 0 | 1 | 0 |
| b ₃ | × | × | × | × | × | × | × | × | 1 | 0 | 0 | 0 |
| c ₃ | × | × | × | × | × | × | × | × | 0 | 1 | 0 | 1 |
| d ₃ | × | × | × | × | × | × | × | × | 1 | 1 | 0 | 0 |

btergm Package

- The btergm package has been developed by **Leifeld, Cranmer, & Desmarais (2018)** to estimate longitudinal networks using TERGM



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Temporal Exponential Random Graph Models with btergm: Estimation and Bootstrap Confidence Intervals

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Bruce A. Desmarais
Pennsylvania State University

- Package provides two functions to estimate a TERGM, one using a pseudolikelihood (btergm) and the other using MCMC-MLE (mtergm)

Running a TERGM

- We are going to run a TERGM on the longitudinal alliance network, and will employ the following specification:
 - `edges`: density term
 - `edgecov(war)`: list of matrices where cross-sections denote war
 - `edgecov(contiguity)`: matrix of distances between countries
 - `absdiff(polity)`: Absolute difference between polity of i and j
 - `absdiff(cinc)`: Absolute difference between cinc of i and j
 - `gwesp(.5, fixed = TRUE)`: Geometric weighted triangle term

Running a TERGM

```
library(btergm)

tergmFit <- btergm(
  ally ~ edges +
  edgecov(war) + edgecov(contiguity) +
  nodecov('polity') + absdiff("polity") +
  nodecov('cinc') + absdiff("cinc") +
  gwesp(.5, fixed = TRUE)
)

#check model fit:
#GOF1<-gof(tergmFit)
```

Peaking at the results

```
summary(tergmFit)
```

| ## | Estimate | 2.5% | 97.5% |
|----------------------------|------------|---------|---------|
| ## edges | -5.3675652 | -5.5202 | -5.2208 |
| ## edgecov.war[[i]] | 1.0547194 | 0.7438 | 1.3382 |
| ## edgecov.contiguity[[i]] | -0.2176738 | -0.3353 | -0.1089 |
| ## nodecov.polity | 0.0774665 | 0.0655 | 0.0895 |
| ## absdiff.polity | -0.0099237 | -0.0161 | -0.0025 |
| ## nodecov.cinc | -5.5272184 | -7.9541 | -3.3051 |
| ## absdiff.cinc | 21.0799677 | 18.9316 | 23.3665 |
| ## gwesp.fixed.0.5 | 2.2031934 | 2.0987 | 2.2957 |

Duque (2018)

The DV is diplomatic ties, `dipl_ties`:

```
#Clear your memory and unload `btergm` as it clashes with `network`:
```

```
detach("package:btergm", unload=TRUE)  
data("duqueData")  
  
class(dipl_ties)
```

```
## [1] "list"
```

```
length(dipl_ties)
```

```
## [1] 8
```

```
class(dipl_ties[[1]])
```

```
## [1] "data.frame"
```

Get an error that need `vertex.pid` (persistent identifies), as our networks are not of equal size.

The Dependent Variable

we can see that `dipl_ties` is currently a list of `data.frames`. Let's convert it into a list of networks.

```
library(statnet)
for (i in 1:8) {
  dipl_ties[[i]]<-as.network(as.matrix(dipl_ties[[i]]))
}
class(dipl_ties[[1]])
```

```
## [1] "network"
```

Use networkDynamic for Visualizing the Network

```
diplDyn = networkDynamic(network.list=dipl_ties, vertex.pid='vertex.r
```

Get an error that need vertex.pid (persistent identifiers), as our networks are not of equal size.

Try Again

```
#Define network pids:
for (i in 1:8) {
  set.network.attribute(dipl_ties[[i]], 'vertex.pid', 'vertex.names')
}
#Takes 5 min to run:
diplDyn = networkDynamic(network.list=dipl_ties, vertex.pid='vertex.r
diplDyn
```

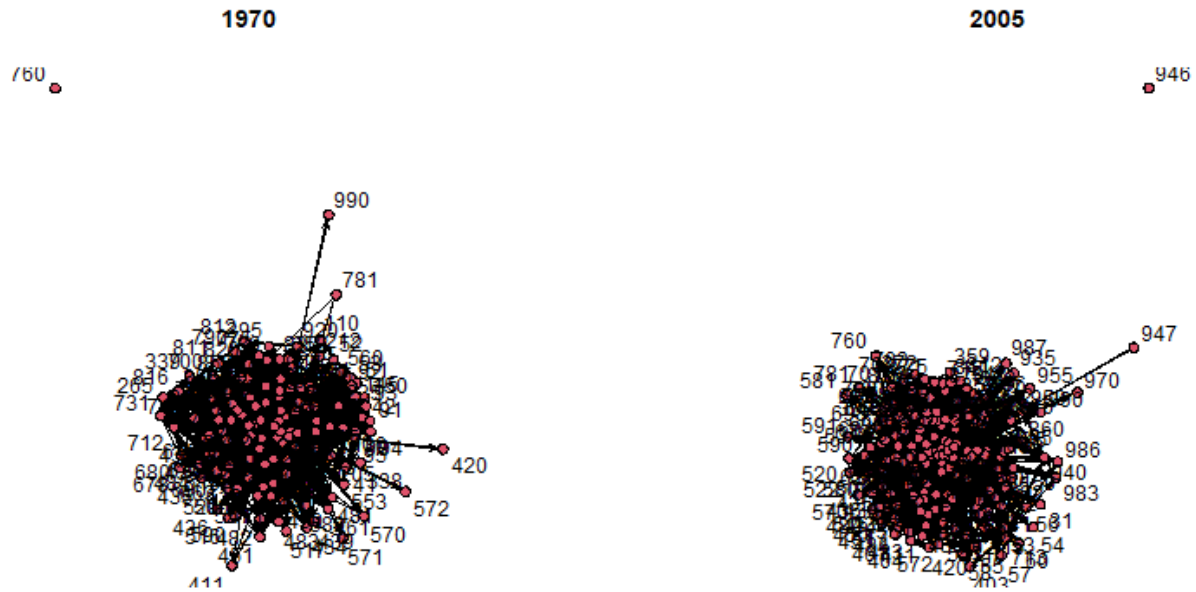
```
## NetworkDynamic properties:
##   distinct change times: 9
##   maximal time range: 0 until 8
##
## Includes optional net.obs.period attribute:
##   Network observation period info:
##     Number of observation spells: 1
##     Maximal time range observed: 0 until 8
##     Temporal mode: discrete
##     Time unit: step
##     Suggested time increment: 1
##
## Network attributes:
##   vertices = 194
```

Quick snapshots

```
par(mfrow = c(1,2))
plot(
  network.extract(diplDyn, at = 1),
  main = "1970", displaylabels = T)
plot(
  network.extract(diplDyn, at = 6),
  main = "2005", displaylabels = T)
```

Note: cannot use coordinates, because networks are not of equal size, include different actors.

Quick snapshots



What Have We Learned?

- Networks of embassies are dense.
- Some states host very few embassies.
- Note: network graphs of dense networks are not very esthetically pleasing or informative.

Descriptive analyses

tsna enables us to quickly calculate some basic descriptive statistics such as the density of a graph:

```
library(tsna)
tSnaStats(diplDyn, "gden") # Changes in graph density
```

```
## Time Series:
## Start = 0
## End = 8
## Frequency = 1
##      Series 1
## [1,] 0.2050836
## [2,] 0.2368542
## [3,] 0.2169688
## [4,] 0.2145924
## [5,] 0.2195870
## [6,] 0.1851018
## [7,] 0.1902708
## [8,] 0.2099215
## [9,]          NA
```

Descriptive analyses

Can also examine transitivity over time:

```
tSnaStats(diplDyn, "gtrans") # Changes in graph transitivity
```

```
## Time Series:  
## Start = 0  
## End = 8  
## Frequency = 1  
##      Series 1  
## [1,] 0.4830388  
## [2,] 0.4953440  
## [3,] 0.5264074  
## [4,] 0.5159558  
## [5,] 0.5269708  
## [6,] 0.4841114  
## [7,] 0.4875437  
## [8,] 0.5132955  
## [9,]          NA
```

Descriptive analyses

The `tErgmStats` enables us to calculate changes in `ergm` terms over time:

```
tErgmStats(diplDyn, "~ edges+triangle")
```

```
## Time Series:
## Start = 0
## End = 8
## Frequency = 1
##      edges triangle
## 0   3655    100235
## 1   5153    190717
## 2   5314    203523
## 3   5597    215072
## 4   5870    238736
## 5   6165    236593
## 6   6618    270378
## 7   7380    336926
## 8      0         0
```

Duque (2018)

Popularity hypothesis: High-status states should receive more recognition simply because of their position in the social structure, rather than because of the possession of status attributes (2-instars).

Reciprocity and transitivity: A state's existing relations should influence the state's ability to achieve status (mutual and triangle).

Homophily: States should recognize states that have similar values and resources as them (absdiff).

Dyadic Covariates

Contiguity (`contig`) and alliances (`allies`) are time-varying edge-level covariates. We must make sure that they are stored as lists of matrices.

```
#Contiguity:  
class(contig)
```

```
## [1] "list"
```

```
length(contig)
```

```
## [1] 8
```

```
class(contig[[1]])
```

```
## [1] "data.frame"
```

```
dim(contig[[1]])
```

```
## [1] 134 134
```

```
contig[[1]][1:3,1:3]
```

```
##      2 20 40  
## 2    0  1  0  
## 20   1  0  0  
## 40   0  0  0
```

Allies

```
#Allies:  
class(allies)
```

```
## [1] "list"
```

```
length(allies)
```

```
## [1] 8
```

```
class(allies[[1]])
```

```
## [1] "data.frame"
```

```
dim(allies[[1]])
```

```
## [1] 134 134
```

```
allies[[1]][1:3,1:3]
```

```
##      2 20 40  
## 2   0  1  0  
## 20  1  0  0  
## 40  0  0  0
```

Dyadic Covariates

It looks like `allies` and `contig` are currently stored as lists of `data.frames`. We must convert them to lists of matrices.

```
for (i in 1:8) {  
  contig[[i]]<-as.matrix(contig[[i]])  
  allies[[i]]<-as.matrix(allies[[i]])  
}
```


Now set up DV with vertex and dyadic attributes

Our node-level covariate, `polity$dem_dum` must be defined as a vertex attribute in each of the `dipl_ties` networks.

```
#Define Dem as a vertex attribute for each year of dipl_ties (didn't
set.vertex.attribute(dipl_ties[[1]], "dem", polity$dem_dum[polity$year])
set.vertex.attribute(dipl_ties[[2]], "dem", polity$dem_dum[polity$year])
set.vertex.attribute(dipl_ties[[3]], "dem", polity$dem_dum[polity$year])
set.vertex.attribute(dipl_ties[[4]], "dem", polity$dem_dum[polity$year])
set.vertex.attribute(dipl_ties[[5]], "dem", polity$dem_dum[polity$year])
set.vertex.attribute(dipl_ties[[6]], "dem", polity$dem_dum[polity$year])
set.vertex.attribute(dipl_ties[[7]], "dem", polity$dem_dum[polity$year])
set.vertex.attribute(dipl_ties[[8]], "dem", polity$dem_dum[polity$year])
```

```
dipl_ties[[1]] %v% "dem"
```

```
##      [1] 1 1 0 0 0 1 0 0 0 0 0 0 0 0 1 0 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0
##     [38] 0 1 0 0 1 0 0 0 0 1 0 0 0 1 1 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     [75] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0
##    [112] 0 0 0 1 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0
```

```
dipl_ties[[2]] %v% "dem"
```

Specify the Model:

```
library(btergm)
#This runs for 5 min:
tergm_Duque<-btergm(dipl_ties ~ edges + istar(2) + ostar(2) + mutual
                    absdiff("dem")+
                    +nodeicov("dem")+
                    +edgecov(allies)
                    +edgecov(contig),
                    R=1000)

summary(tergm_Duque)
```

Run the Model

| ## | Estimate | 2.5% | 97.5% |
|------------------------|-----------|---------|---------|
| ## edges | -5.377882 | -5.6133 | -5.1253 |
| ## istar2 | 0.028279 | 0.0267 | 0.0313 |
| ## ostar2 | 0.029487 | 0.0266 | 0.0340 |
| ## mutual | 2.485567 | 2.3500 | 2.5956 |
| ## triangle | 0.011750 | 0.0103 | 0.0127 |
| ## absdiff.dem | -0.287938 | -0.3656 | -0.2207 |
| ## nodeicov.dem | -0.204423 | -0.2515 | -0.1246 |
| ## edgecov.allies[[i]] | 1.205762 | 1.1238 | 1.3205 |
| ## edgecov.contig[[i]] | 1.765076 | 1.5732 | 2.0623 |

Results

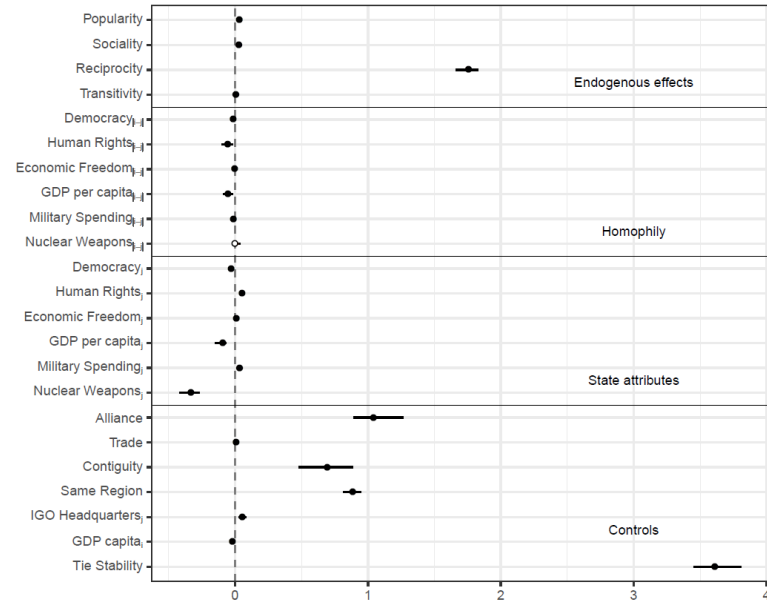


Figure 3. Temporal exponential random graph model of diplomatic ties for 1995-2005.

Temporal Terms

- `delrecip(mutuality = FALSE, lag = 1)`: checks for delayed reciprocity. For example, if node j is tied to node i at $t = 1$, does this lead to a reciprocation of that tie back from i to j at $t = 2$? If `mutuality = TRUE` is set, this extends not only to ties, but also non-ties. The `lag` argument controls the size of the temporal lag: with `lag = 1`, reciprocity over one consecutive time period is checked. Note that as lag increases, the number of time steps on the dependent variable decreases.
- `memory(type = "stability", lag = 1)`: controls for the impact of a previous network on the current network. Four different types of memory terms are available: positive autoregression (`type = "autoregression"`) checks whether previous ties are carried over to the current network; dyadic stability (`type = "stability"`) checks whether both edges and non-edges are stable between the previous and the current network; edge loss (`type = "loss"`) checks whether ties in the previous network have been dissolved and no longer exist in the current network; and edge innovation (`type = "innovation"`) checks whether previously unconnected nodes have the tendency to become tied in the current network.

Temporal Terms Cont'd

- `timecov(x = NULL, minimum = 1, maximum = NULL, transform = function(t) t)`: checks for linear or non-linear time trends with regard to edge formation. Optionally, this can be combined with a covariate to create an interaction effect between a dyadic covariate and time in order to test whether the importance of a covariate increases or decreases over time. In the default case, edges modeled as being linearly increasingly important over time. By tweaking the transform function, arbitrary functional forms of time can be tested. For example, `transform = sqrt` (for a geometrically decreasing time effect), `transform = function(x) x^2` (for a geometrically increasing time effect), `transform = function(t) t` (for a linear time trend) or polynomial functional forms (e.g., $0 + (1 \ t) + (1 \ t^2)$) can be used.

Example with Time Vars

#This runs for 5 min:

```
tergm_Duque1<-btergm(dipl_ties ~ edges + istar(2) + ostar(2) + mutual  
                    absdiff("dem")+  
                    +nodeicov("dem")+  
                    +edgecov(allies)+  
                    +edgecov(contig)+  
                    timecov(),  
                    R=1000)  
  
summary(tergm_Duque1)
```

Example with Time Vars

| ## | Estimate | 2.5% | 97.5% |
|--------------------------|-----------|---------|---------|
| ## edges | -4.915580 | -5.2673 | -4.5906 |
| ## istar2 | 0.029973 | 0.0283 | 0.0333 |
| ## ostar2 | 0.031485 | 0.0289 | 0.0353 |
| ## mutual | 2.428539 | 2.2767 | 2.5464 |
| ## triangle | 0.011518 | 0.0102 | 0.0126 |
| ## absdiff.dem | -0.298055 | -0.3669 | -0.2392 |
| ## nodeicov.dem | -0.122282 | -0.2013 | -0.0506 |
| ## edgecov.allies[[i]] | 1.199983 | 1.0998 | 1.3312 |
| ## edgecov.contig[[i]] | 1.837357 | 1.6100 | 2.1092 |
| ## edgecov.timecov1[[i]] | -0.126189 | -0.2050 | -0.0802 |

Your Turn

Re-specify the model to account for delayed reciprocity and stability. Estimate your new model.

Bayesing ERGM (BERGM)

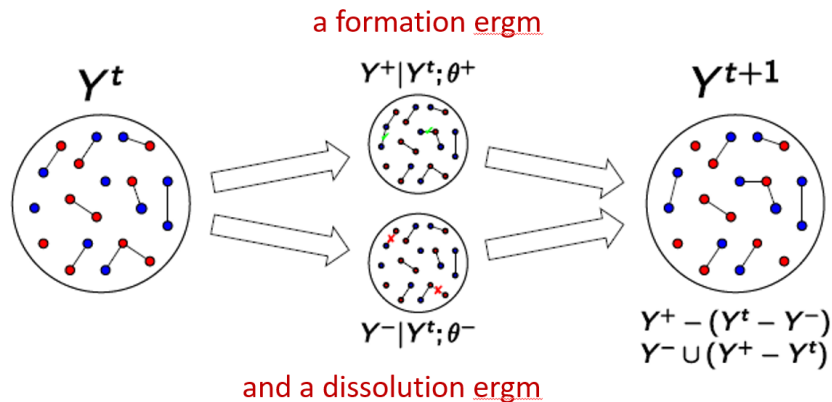
- Paper: Caimo & Friel (2011)
- Bergm on CRAN
- Vignette

ego-ERGM

- Paper: Salter-Townshend & Murphy (2016)

Separable temporal ERGM (STERGM)

- Paper: Krivitsky & Handcock (2012)
- `tergm` on CRAN
- Vignette



Hierarchical Exponential-Family Graph Model (HERGM)

- Paper: Schweinberger & Handcock (2015)
- hergm on CRAN
- Vignette

Multilevel ERGM

- Paper: Wang et al. (2016)

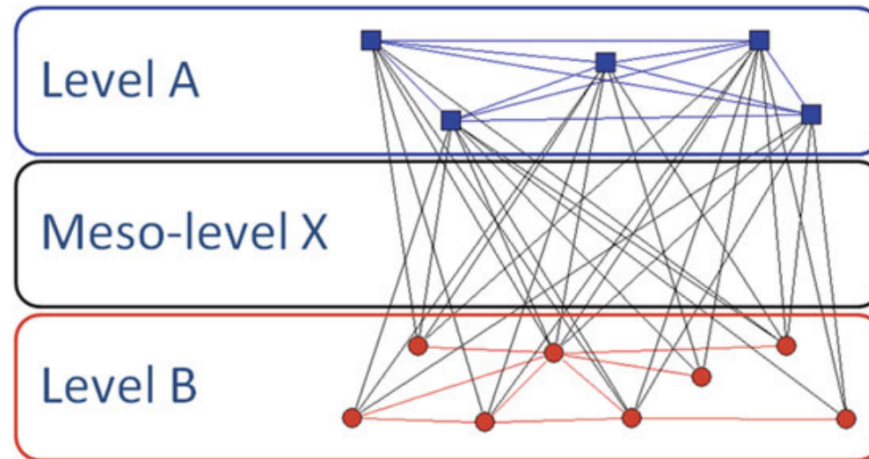


Fig. 6.1 A two-level network representation