Advanced Network Analysis

Inferential Modeling with Blocks

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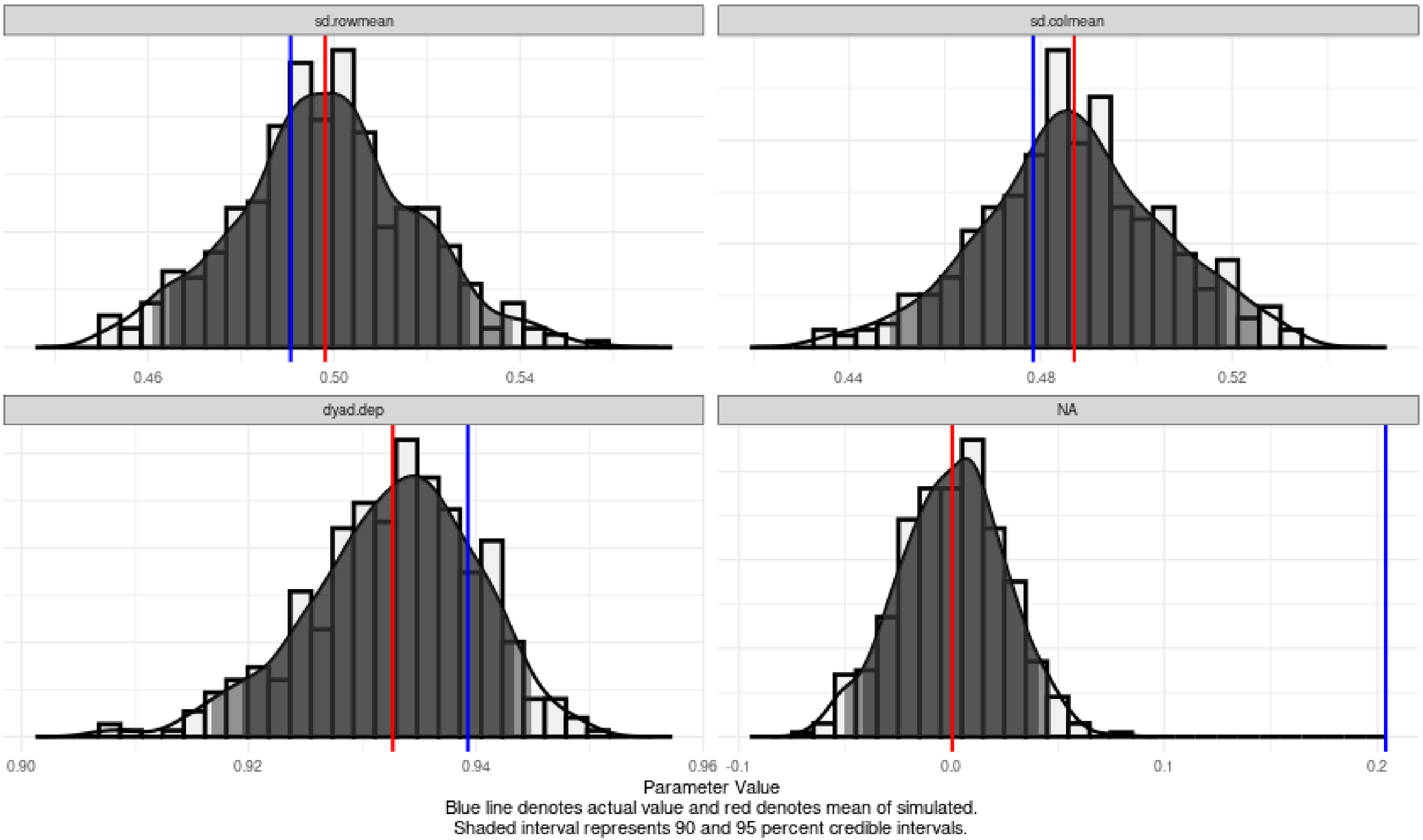
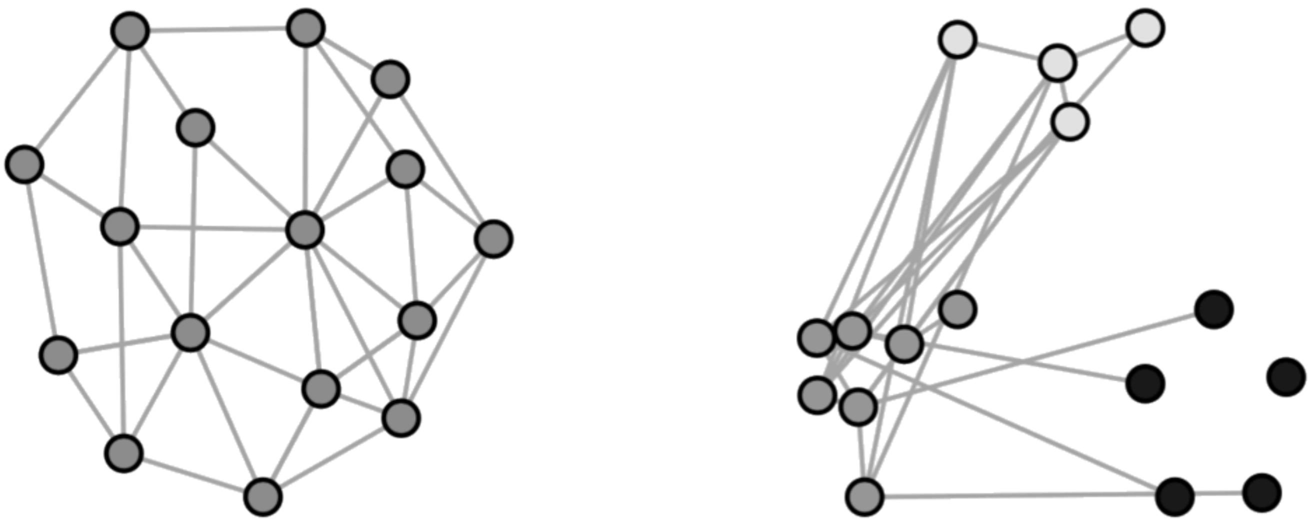
Capturing network features?

```
gofPlot(fitSRM$GOF, symmetric=FALSE)
```



What are we missing?



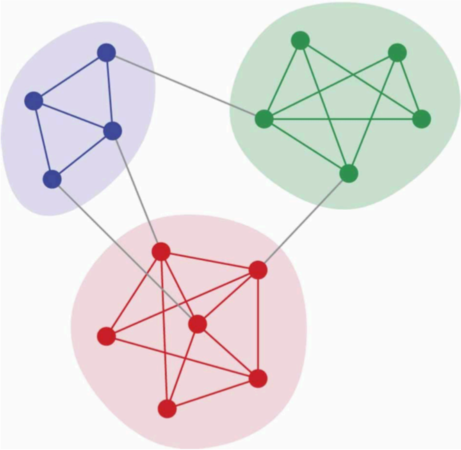
- **Homophily**: "birds of a feather flock together"
- **Stochastic equivalence**: nothing as pithy to say here, but this model focuses on identifying actors with similar roles

Basically, our inferential model is missing something, one way to think about it is that we need to find an expression for γ:

$$y_{ij} \approx \beta^T X_{ij} + a_i + b_j + \gamma(u_i, v_j)$$

One way of getting at this ... community detection

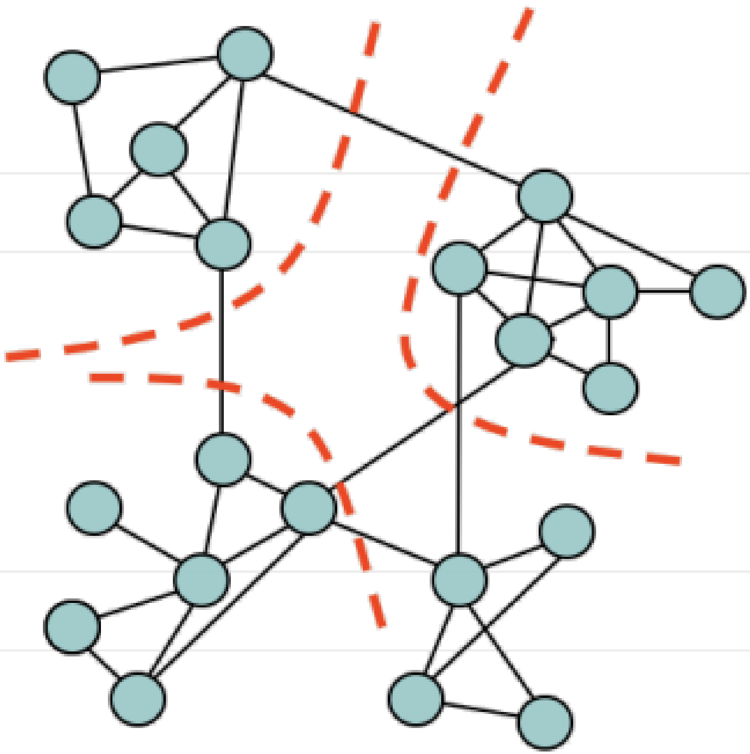
- What makes a community?
  - Cliques: Everyone in the group has connections to one another
  - Compact: The path between nodes in the community are small
  - Differentiation: High frequency of ties among people in the community versus with non-members
- Community structure
  - One way of thinking about this is that vertices often cluster into tight-knit groups with a high density of within-group edges and a lower density of between-group edges



Source: Newman (2012)

How does it work?

Most community detection techniques are graph partitioning problems:



Why would we detect communities?

- Exploratory: What structures my network
- Confirmatory: Do communities map onto some exogenous variable
- Predictive: Does community membership predict something else

Methods for community detection

There's a lot ... the list below is not comprehensive

- Edge-betweenness (Girvan and Newman 2001)
  - In `igraph` the relevant function is `cluster_edge_betweenness`
- Leading Eigenvector (Newman 2006)
  - `igraph::cluster_leading_eigen`
- Fast-Greedy (Clauset et al. 2004)
  - `igraph::cluster_fast_greedy`
- Multi-Level (Blondel et al. 2008)
  - `igraph::cluster_louvain`
- Walktrap (Pons and Latapy 2005)
  - `igraph::cluster_walktrap`
- Label propagation (Raghavan et al. 2007)
  - `igraph::cluster_label_prop`
- InfoMAP (Rosvall et al. 2009)
  - `igraph::cluster_infomap`

What "unites" these approaches?

- Modularity is a score for the fraction of the edges that fall within the given group minus the expected such fraction if edges were distributed at random (`igraph::modularity`)
- Formally, given a particular division of communities, the modularity of the division is:

$$Q = \frac{1}{2m} \sum_{ij} [Y_{ij} - \frac{k_i k_j}{2m}] I(c_i = c_j)$$

- Y_{ij} is the element of the A adjacency matrix in row i and column j
- k_i is the degree of i, k_j is the degree of j
- m is the number of edges in the network
- c is the group index; c_i is the type (or component) of i, c_j that of j

Basically ...

- Modularity provides us with an assessment of how good the communities we identified are at grouping together actors
- Ideally, we want communities where actors with dense connections are grouped together and actors with sparse are grouped separately

So those algorithms ...

Lets discuss them in the context of Gade et al. 2019:

```r
library(igraph)
load('gadeData.rda')
dim(Y)
```

```
## [1] 31 31
```

```r
Y[1:5,1:5]
```

```
##       101st 13th AARB AF ANF
## 101st     0    1    0  0   0
## 13th      1    0    0  0   0
## AARB      0    0    0  1   1
## AF        0    0    1  0   1
## ANF       0    0    1  1   0
```

Lets find communities, we'll start with edge-betweenness

- Edge-betweennness (aka Girvan-Newman method) starts by calculating betweenness at the edge level
- The idea:
  - Find edges that serve as bridges between communities
  - Remove those edges
  - Group remaining actors into communities
  - Repeat until some maximal level of modularity is reached

Edge-betweenness cont'd

- So how do we find those edges that serve as bridges?
- We can calculate the edge betweenness score for a given edge by: $\sum \frac{\sigma_{ij}(e)}{\sigma_{ij}}$, where
  - σ_{ij} is the total number of shortest paths from node i to j and
  - $\sigma_{ij}(e)$ is the number of those paths that pass through edge e

So how do we calculate?

`igraph::cluster_edge_betweenness`

```
# convert Y to graph object
g = graph_from_adjacency_matrix(Y,
    mode='undirected',
    weighted=NULL,
    diag=FALSE
    )
ebComm = cluster_edge_betweenness(g)
```

Who got assigned where

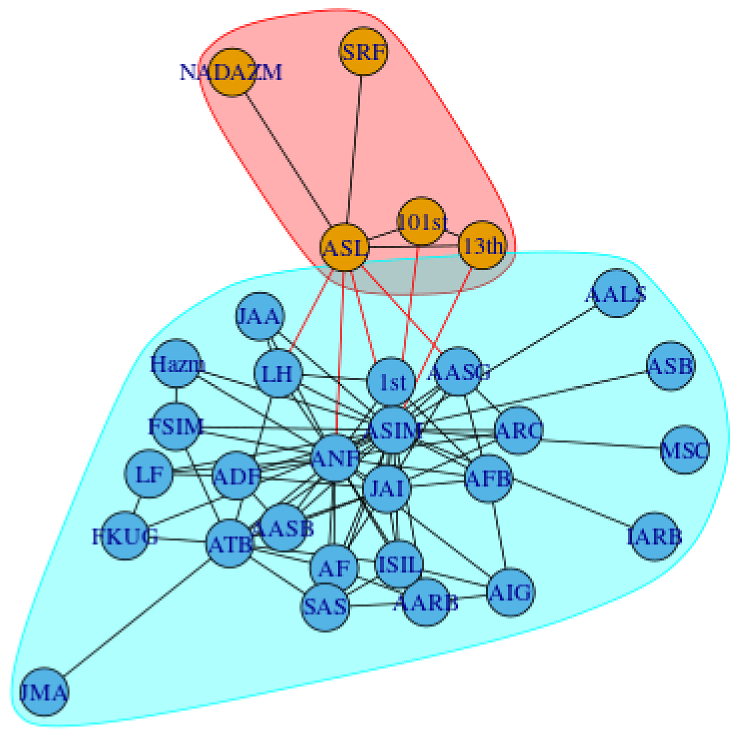
We can use the `membership` function to extract actor assignments

```
membership(ebComm)
```

```
##   101st   13th   AARB     AF    ANF   ASIM   ISIL   AASB    ADF   AASG    A
##       1      1      2      2      2      2      2      2      2      2
##      LF    ATB    JAI    AFB    1st    AIG   FSIM   Hazm    JAA     LH    S
##       2      2      2      2      2      2      2      2      2      2
##    AALS    ASB   FKUG    MSC    ASL NADAZM    SRF    JMA   IARB
##       2      2      2      2      1      1      1      2      2
```

We can also plot

```
plot(ebComm, g)
```



Now, how did this algorithm do?

Lets calculate the modularity score:

```
ebCommScore = modularity(ebComm, g)
ebCommScore
```

```
## [1] 0.09897242
```

How did we do?

- Modularity ranges from $< -1, 1 >$, where 1 indicates stronger community structure
- Values close to 1 indicate strong community structure
- Values close to 0 indicate the community division is not better than random

Lets try another: Leading Eigenvector

- Basically, a principal components analysis for networks
- In each step, the network is split into two parts such that the separation yields a significant increase in modularity
- At each stage, the split is determined by evaluating the results of the principal components analysis
- To run this we can use: `igraph::cluster_leading_eigen`

Newman's Leading Eigenvector

Does it do better?

```
leComm = cluster_leading_eigen(g)
leCommScore = modularity(leComm, g)
leCommScore
```

```
## [1] 0.1980124
```

Practical guidance

- How should we choose between community detection approaches (there are some problems with modularity)?
  - How do you choose topics in a topic model?

How is this used in modeling?

Latent class model/blockmodels (Holland et al. 1983; Nowicki & Snijders 2001; Rohe et al. 2011; Airoldi et al. 2013)

Each node i is a member of an (unknown) latent class:

$$\mathbf{u}_i \in \{1, \ldots, K\},\ i \in \{1, \ldots, n\}$$

The probability of a tie between i and j is:

$$Pr(Y_{ij} = 1|\mathbf{u}_i, \mathbf{u}_j) = \theta_{\mathbf{u}_i \mathbf{u}_j}$$

- Nodes in the network may have a small or high probability of ties: θ_{kk} may be small or large
- Nodes in the same class are stochastically equivalent

Software packages:

- CRAN: `blockmodels` (Leger 2015)
- CRAN: `sbm` (Chiquet et al 2021)
- CRAN: `mixedMem` (Wang & Erosheva 2015)
- CRAN: `dynsbm` (Matias & Miele 2018)
- CRAN: `NetMix` (Olivella et al 2021)

LCM for community detection

Newman (2006): Nouns

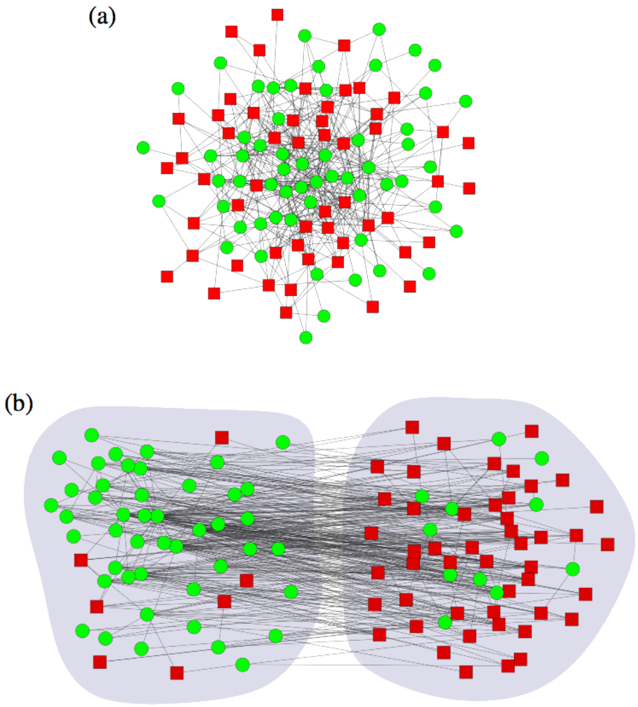


FIG. 7. (Color online) (a) The network of commonly occurring English adjectives (circles) and nouns (squares) described in the text. (b) The same network redrawn with the nodes grouped so as to minimize the modularity of the grouping. The network is now revealed to be approximately bipartite, with one group consisting almost entirely of adjectives and the other of nouns.

White & Murphy (2016): Mixed membership stochastic block model

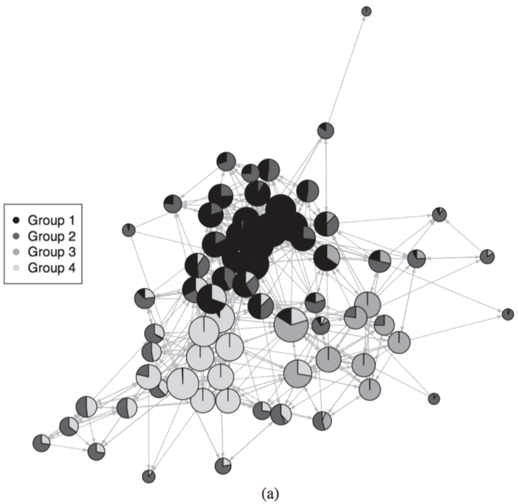


Fig. 7. Visualization of the 4 group MMESBM fitted to the Lazega Lawyers friendship

Apply LCM to trade

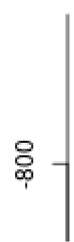
- Below we show how to implement a basic stochastic blockmodel using the `blockmodels` package (see vignette and CRAN function list for more details):

```
library(blockmodels)

# make sure there are no NAs
diag(Y) = 0

# gaussian stochastic blockmodel
set.seed(6886)
sbm = BM_gaussian('SBM', Y)

# to estimate model run
sbm$estimate()
```



Attributes of estimated object

The output from the `blockmodels` package is different than what most other packages do:

```
sbm$show()
```

```
## blockmodels object
##     model: gaussian
##     membership: SBM
##     network: 30 x 30 scalar network
##     maximum of ICL: 5 groups
##     Most usefull fields and methods:
##         The following fields are indexed by the number of groups:
##             $ICL : vector of ICL
##             $PL : vector of pseudo log liklihood
##             $memberships : list of memberships founds by estimation
##                            each membership is represented object
##             $model_parameters : models parameters founds by estimation
##         Estimation methods:
##             $estimate(reinitalization_effort=1) : to run again estimation
##                                                   higher reinitalization e
##         Plotting methods:
##             $plot_obs_pred(Q) : to plot the obeserved and predicted networ
##             $plot_parameters(Q) : to plot the model_parameters for Q group
```

Extracting membership vector

```
mem = sbm$memberships[[5]]$Z
memCat = apply(mem, 1, function(x){which(max(x)==x)})
memCat
```

```
##  [1] 3 5 4 3 5 2 2 3 3 4 5 5 3 4 1 5 4 3 5 3 2 3 5 4 5 2 5 3 4 1
```

Plotting results from stochastic blockmodel

```r
# create graph object
diag(Y) = NA
yGraph = igraph::graph.adjacency(Y,
  mode='directed',
  weighted=TRUE,
  diag=FALSE
  )

# add node attributes
V(yGraph)$size = rescale(
  apply(Y, 2, sum, na.rm=TRUE), c(10, 16) )

# Colors
library(RColorBrewer)
cols = brewer.pal(5, 'Set1')
nodeColors = cols[memCat]
```

How to set layout?

- We want to set the layout so it is at least somewhat reflective of the community assignments
- Here's a helper function that tries to do this

```
commPlotHelper = function(m, graph){
  el <- igraph::as_edgelist(graph, names = FALSE)
  m1 <- m[el[, 1]]
  m2 <- m[el[, 2]]
  res <- m1 != m2
  if (!is.null(names(m1))) {
      names(res) <- paste(names(m1), names(m2), sep = "|")
  }
  return(res) }

weights = commPlotHelper(memCat, yGraph)
set.seed(6886)
commLayout = layout_with_fr(yGraph, weights=weights)
```

Now put the pieces together

```
plot(yGraph,
  layout=commLayout,
  vertex.color=cols[memCat],
  vertex.label.color='white',
  vertex.size=V(yGraph)$size,
  vertex.label.cex =.75,
  edge.color='grey20',
  edge.width=E(yGraph)$weight,
  edge.arrow.size=.2,
  asp=FALSE
  )
```

Now put the pieces together