ECE 2504: Introduction to Computer Engineering
Homework Assignment 2 (40 points)

Make a reasonable effort to show your work. Clearly indicate your answers.

Problem 1 (12 points)
Even though we aren't trying to prove any of our axioms, a truth table is a useful means for justifying the truth of many axioms. If we can demonstrate that two seemingly distinct expressions give the same value for all combinations of a set of inputs, then the two expressions are actually the same.

For example, demonstrating that the AND function is associative amounts to showing that (AB)C = A(BC):

A	B	C	AB	C	(AB)C	A	BC	A(BC)
0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	1	0	1	0	0	1	0
1	0	0	0	0	0	1	0	0
1	0	1	0	1	0	1	0	0
1	1	0	1	0	0	1	0	0
1	1	1	1	1	1	1	1	1

(AB)C = A(BC) for all combinations of A, B, and C. Thus, the AND function exhibits associativity.
Using a truth table in the fashion shown above, demonstrate each of the following:

a. The distributive property A(B + C) = AB + AC

A	B	C	B+C	**A(B+C)**	AB	AC	**AB+AC**
0	0	0	0	**0**	0	0	**0**
0	0	1	1	**0**	0	0	**0**
0	1	0	1	**0**	0	0	**0**
0	1	1	1	**0**	0	0	**0**
1	0	0	0	**0**	0	0	**0**
1	0	1	1	**1**	0	1	**1**
1	1	0	1	**1**	1	0	**1**
1	1	1	1	**1**	1	1	**1**

Thus A(B+C) = AB+AC

b. The distributive property A + BC = (A + B)(A + C)

A	B	C	BC	**A+BC**	A+B	A+C	**(A+B)(A+C)**
0	0	0	0	**0**	0	0	**0**
0	0	1	0	**0**	0	1	**0**
0	1	0	0	**0**	1	0	**0**
0	1	1	1	**1**	1	1	**1**
1	0	0	0	**1**	1	1	**1**
1	0	1	0	**1**	1	1	**1**
1	1	0	0	**1**	1	1	**1**
1	1	1	1	**1**	1	1	**1**

Thus A+BC = (A+B)(A+C)

Problem 1 (continued)

c. The NAND form of DeMorgan's Theorem for three variables – that is, show that $(ABC)' = A' + B' + C'$.

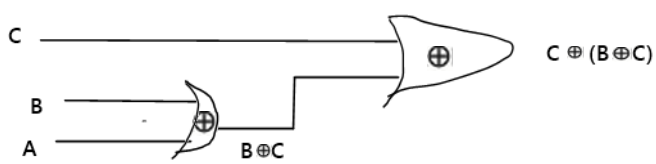
A	B	C	(ABC)	**(ABC)'**	A'	B'	C'	**(A'+B'+C')**
0	0	0	0	**1**	1	1	1	**1**
0	0	1	0	**1**	1	1	0	**1**
0	1	0	0	**1**	1	0	1	**1**
0	1	1	0	**1**	1	0	0	**1**
1	0	0	0	**1**	0	1	1	**1**
1	0	1	0	**1**	0	1	0	**1**
1	1	0	0	**1**	0	0	1	**1**
1	1	1	1	**0**	0	0	0	**0**

Thus $(ABC)' = A' + B' + C'$.

d. The NOR form of DeMorgan's Theorem for three variables – that is, show that $(A + B + C)' = A'B'C'$.

A	B	C	A'	B'	C'	(A+B+C)	**(A+B+C)'**	**(A'B'C')**
0	0	0	1	1	1	0	**1**	**1**
0	0	1	1	1	0	1	**0**	**0**
0	1	0	1	0	1	1	**0**	**0**
0	1	1	1	0	0	1	**0**	**0**
1	0	0	0	1	1	1	**0**	**0**
1	0	1	0	1	0	1	**0**	**0**
1	1	0	0	0	1	1	**0**	**0**
1	1	1	0	0	0	1	**0**	**0**

e. The three-input XOR function gives true output if and only if an odd number of its inputs are true. (Try "building" the three-input XOR from two-input XORs.)



f. The XOR function is associative – that is, show that $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

A	B	C	A+B	(A⊕B)	**(A⊕B) ⊕C**	(B ⊕ C)	**A ⊕ (B ⊕ C)**
0	0	0	0	0	**0**	0	**0**
0	0	1	0	0	**1**	1	**1**
0	1	0	1	1	**1**	1	**1**
0	1	1	1	1	**0**	0	**0**
1	0	0	1	1	**1**	0	**1**
1	0	1	1	1	**0**	1	**0**
1	1	0	1	0	**0**	1	**0**
1	1	1	1	0	**1**	0	**1**

<u>Problem 2</u> (12 points)
Using Boolean algebra manipulation, simplify the following expressions to their simplest terms:

a. A + AB (This is a Boolean algebra theorem called an absorption theorem.)

A+AB
= A(1+B) == A+AB
=A(1) == x+1 = 1
=A == X(1)= X

b. CD + CD′ (This is an important Boolean algebra theorem; it forms the basis of most logic minimization.)

CD+CD'

= C(D+D') ==CD+CD'
=C(1) == X+X' = 1
=C == X(1) = X

c. BC + ABC′ + A′BC′

BC+ABC'+A'BC'

=B(C+AC'+A'C') == BC+ABC'+A'BC'
=B((C+A)(C+C')+(A+C)') == x+yz = (x+y)(x+z)
(x+y)'=x'y'
=B((C+A)(1)+(C+A)'))
=B((C+A)+(C+A)')) == X*1=X

=B(1) == x+x' = 1

=B == x*1=x

Problem 2 (continued)

d. $BD + B'C'D$

$BD+B'C'D$

$=D(B+B'C')$ == $x(y+z)$ $xy+xz$
$=D(B+(B+C)')$ == $(x+y)' = x'y'$
$=D(B(1+(1+C)')$
$=D(B(1+(1)')$ == $x+1 =1$
$=D(B+B')$ == $x(y+z) = xy+xz$
$=D(1)$ == $x+x' = 1$
$=D$ == $x*1 = x$

e. $AB'D + AC'D + BD$
$=D(B'A+B+AC')$ == $x(y+z)$ $xy+xz$
$=D((B+B')(B+A)+AC')$ == $(x+yz) = (x+y)((x+z)$
$=D(1)(B+A)+(AC')$ == $(x+x')=1$
$=D(B+A +(AC'))$
$=D(B+A(1+C'))$ == $x(y+z)$ $xy+xz$
$=D(B+A(1))$ == $1+x' = 1$
$=D(B+A)$

f. $(BC + A'D)(AB' + C'D')$ (Hint: You can apply an approach similar to the multiplication of polynomials to expand this expression before you begin simplifying it.)

$(BC + A'D)(AB' + C'D')$
$= BCAB'+BCC'D'+A'DAB'+A'DC'D'$ ==$x(y+z)=xy+xz$
$= 0 + 0+0+0$ == $x*x'=0$
$=0$ == $x+0=x$

Problem 3 (8 points)
Consider the three-variable function $F(A,B,C) = AC' + A'BC$:

a. Using DeMorgan's Theorem, express F'. (I recommend that you express F' in SOP form.)
b. Using Boolean algebra and the function that you derived in part (a), show that $F \bullet F' = 0$ and that $F + F' = 1$.

(Hint: Remember the hint from Problem 2(f).)

a)
$(A+C)*(A+B'+C')$
$= (A'A + A'B' + A'C' + CA + CB' + CC')$

b)
$F*F'=0$

$(AC'+A'BC)* (A'A + A'B' + A'C' + CA + CB' + CC')$

$A'A= 0 == x*x'=0$
$CC' == x*x'=0$

$=(AC'+A'BC) * (A'B' + A'C' + CA + CB' + 0 + 0)$
$=0 \;\; == x*0 = 0$

Outputs 0 if any are 0 thus $F*F'=0$

b)
$F+F'=1$

$(AC'+A'BC)+ (A'A + A'B' + A'C' + CA + CB' + CC')$

$=AC'+A'BC+A'(B'+C')+C(A+B')$
$=AC'+A'(B'+C'+BC)+C(A+B')$
$=AC'+A'((B'+B) (B'+C) + C') + C(A+B')$
$=AC'+A'((1) (B'+C) + C')) + C(A+B')$
$=AC'+A' ((C'+C) +B') + C(A+B')$
$=AC'+ A'((1)+B') + C(A+B')$
$=AC' + A'+B'+C(A+B')$
$=(A'+A)(A'+C') + B' + C(A+B')$
$=(1)(A'+C')+B'+C(A+B')$
$=A'+C'+B'+CA+CB'$
$=(C'+C)(C'+C') +A'+B'+CA$
$=(1) (A'+B'+CA+C'+B')$
$=A'+B'+(C'+A)(C'+C)+B'$
$=A'+B'+(C'+A)(1)+B'$
$=A'+C'+A+B'$
$=1+C'+B'$
$1+x = 1$ thus $F+F' =1$ as there needs to result in just at least one 1 for an output of 1 to happen.

Problem 4 (8 points)
Consider the three-variable function F(A,B,C) = A'BC + ABC + ABC'.

a. Show the truth table for the function.
b. Using Boolean algebra, simplify the function to its simplest terms.
c. Show a truth table for the simplified function, and use it to demonstrate that the simplified function is equivalent to the original function.

(Hint: Sometimes, simplifying a function using Boolean algebra can involve making an expression less simple before making it simpler.)

a)

A	B	C	A'	B'	C'	A'BC	ABC	ABC'	A'BC + ABC + ABC'
0	0	0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0	0	0
0	1	0	1	0	1	0	0	0	0
0	1	1	1	0	0	1	0	0	1
1	0	0	0	1	1	0	0	0	0
1	0	1	0	1	0	0	0	0	0
1	1	0	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	1

b)
A'BC+ABC+ABC'

=B(A'B+AC+AC') == x(y+z) xy+xz
= B(A'C+ AC + AC') == x(y+z)=xy+xz
=B(C (A'+A) +AC') == x(y+z)=xy+xz
=B(C (1) +AC') == x+x'=1
=B (C +AC')
= B((C+A) (C+C')) == x+yz = (x+y)(x+z)
=B(C+A)

C)

A	B	C	A'	B'	C'	A'BC	ABC	ABC'	**A'BC + ABC + ABC'**	C+A	**B(C+A)**
0	0	0	1	1	1	0	0	0	**0**	0	**0**
0	0	1	1	1	0	0	0	0	**0**	1	**0**
0	1	0	1	0	1	0	0	0	**0**	0	**0**
0	1	1	1	0	0	1	0	0	**1**	1	**1**
1	0	0	0	1	1	0	0	0	**0**	1	**0**
1	0	1	0	1	0	0	0	0	**0**	1	**0**
1	1	0	0	0	1	0	0	1	**1**	1	**1**
1	1	1	0	0	0	0	1	0	**1**	1	**1**